Optical Experiment Manuscript

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1 Introduction

In this manuscript, we will go through the detail of the optical experiment setup of the quantum circuit for Variational Quantum Eigensolver (VQE).

2 Components

Reference

Quarter-wave plate with fast axis at angle θ with respect to the horizontal axis (QWP):

$$e^{\frac{-i\pi}{4}} \begin{bmatrix} \cos^2(\theta) + i\sin^2(\theta) & (1-i)\sin(\theta)\cos(\theta) \\ (1-i)\sin(\theta)\cos(\theta) & \sin^2(\theta) + i\cos^2(\theta) \end{bmatrix} \tag{1}$$

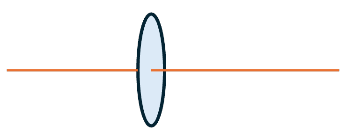

Figure 1: Quarter-wave plate

Quarter-wave plate with fast axis vertical (QWPv):

$$e^{\frac{i\pi}{4}} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \tag{2}$$

Quarter-wave plate with fast axis horizontal (QWPh):

$$e^{\frac{-i\pi}{4}} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \tag{3}$$

Half-wave plate (HWP):

$$\begin{bmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{bmatrix} \tag{4}$$

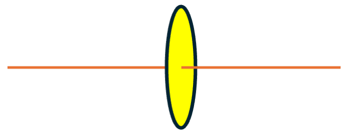

Figure 2: Half-wave plate

Dove Prism (DP) using Rotation matrix acting on the spatial mode:

$$\begin{bmatrix} \cos(\omega) & -\sin(\omega) \\ \sin(\omega) & \cos(\omega) \end{bmatrix} \tag{5}$$

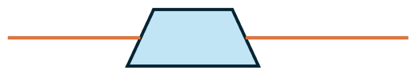

Figure 3: Dove Prism

Beamsplitter (BS):

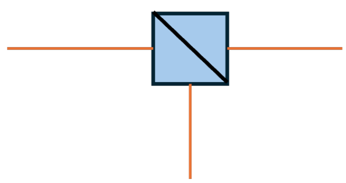

Figure 4: Beamsplitter

Polarizing beamsplitter (PBS):

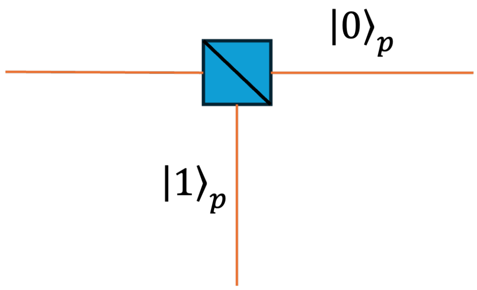

Figure 5: Polarizing Beamsplitter

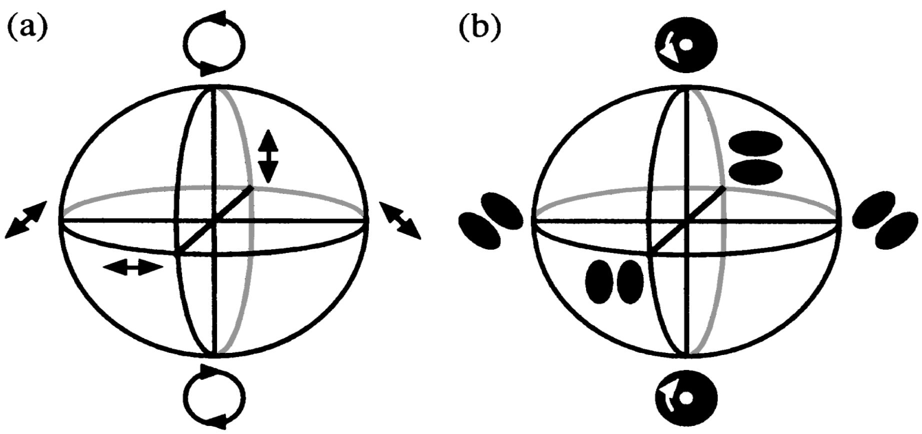

Figure 6: Poincare

For this experiment, we will use "01"/"10" Spatial Mode as our second qubit and Polarization as our first qubit.

$$|0\rangle = \text{Spatial Mode 10 and Horizontal Polarization or } |0\rangle_s, |0\rangle_p$$
$$|1\rangle = \text{Spatial Mode 01 and Vertical Polarization or } |1\rangle_s, |1\rangle_p$$
$$\frac{|0\rangle - i|1\rangle}{\sqrt{2}} = \text{South Pole of the Poincare Sphere or Left-circular Polarization}\frac{|L\rangle_s}{\sqrt{2}}, \frac{|L\rangle_p}{\sqrt{2}}$$
$$\frac{|0\rangle + i|1\rangle}{\sqrt{2}} = \text{North Pole of the Poincare Sphere or Right-circular Polarization}\frac{|R\rangle_s}{\sqrt{2}}, \frac{|R\rangle_p}{\sqrt{2}}$$
$$|+\rangle = \text{Diagonal Polarization}\frac{|D\rangle_s}{\sqrt{2}}, \frac{|D\rangle_p}{\sqrt{2}}$$
$$|-\rangle = \text{Anti-diagonal Polarization}\frac{|A\rangle_s}{\sqrt{2}}, \frac{|A\rangle_p}{\sqrt{2}}$$

Therefore our initial state is $|0\rangle_p \otimes |0\rangle_s \equiv |0_p 0_s\rangle$

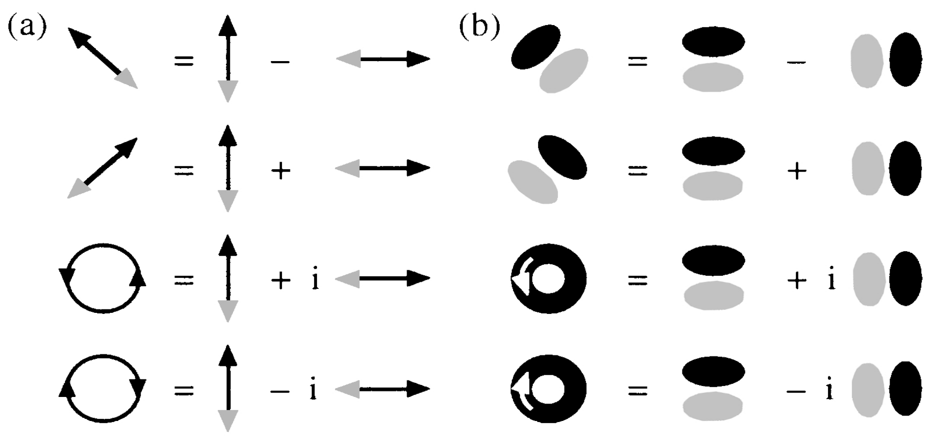

Figure 7: Geometric Phase Shift

3 Gates Realization

3.1 R_x Gate

$$R_x(\theta) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -i\sin(\frac{\theta}{2}) \\ -i\sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{bmatrix} \tag{6}$$

$$R_x(\frac{\pi}{2}) = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \tag{7}$$

R_x acting on $|0\rangle$ gives:

$$R_x(\frac{\pi}{2})\,|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} = \frac{|0\rangle - i\,|1\rangle}{\sqrt{2}} \tag{8}$$

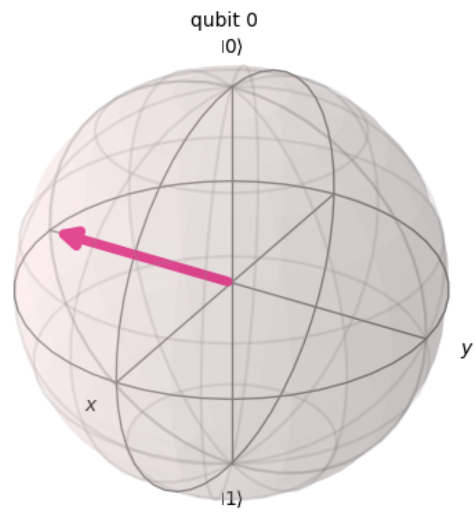

Figure 8: $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$

Quantum R_x gate can be realized for polarization by using quarter-wave plate with fast axis at angle θ with respect to the horizontal axis, where $\theta = \frac{\pi}{4}$:

$$\mathrm{QWP}(\frac{\pi}{4}) = \begin{bmatrix} \cos^2(\frac{\pi}{4}) + i\sin^2(\frac{\pi}{4}) & (1-i)\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) \\ (1-i)\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) & \sin^2(\frac{\pi}{4}) + i\cos^2(\frac{\pi}{4}) \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} \frac{1}{2} + i\frac{1}{2} & (1-i)\frac{1}{2} \\ (1-i)\frac{1}{2} & \frac{1}{2} + i\frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1+i & (1-i) \\ (1-i) & 1+i \end{bmatrix} = \frac{1+i}{2} \begin{bmatrix} 1 & \frac{1-i}{1+i} \\ \frac{1-i}{1+i} & 1 \end{bmatrix} \tag{10}$$

$$= \frac{1+i}{2} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \tag{11}$$

We got the horizontal polarization $|0\rangle_p$ going through QWP at $\theta = \frac{\pi}{4}$ gives:

$$\mathrm{QWP}(\frac{\pi}{4})\,|0\rangle_p = \frac{1+i}{2}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \underbrace{\frac{1+i}{2}\begin{bmatrix}1\\-i\end{bmatrix}}_{\text{Jone's vector}} = \frac{1+i}{2}\,|L\rangle_p \tag{12}$$

We can extract the time-dependent electric field from the Jone's vector:

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$$\vec{E}(t) = \mathrm{Re}\left\{|E\rangle\, e^{-iwt}\right\} \tag{13}$$

$$\Rightarrow \vec{E}_x(t) = \mathrm{Re}(E_x e^{-iwt}) \tag{14}$$

$$\Rightarrow \vec{E}_y(t) = \mathrm{Re}(E_y e^{-iwt}) \tag{15}$$

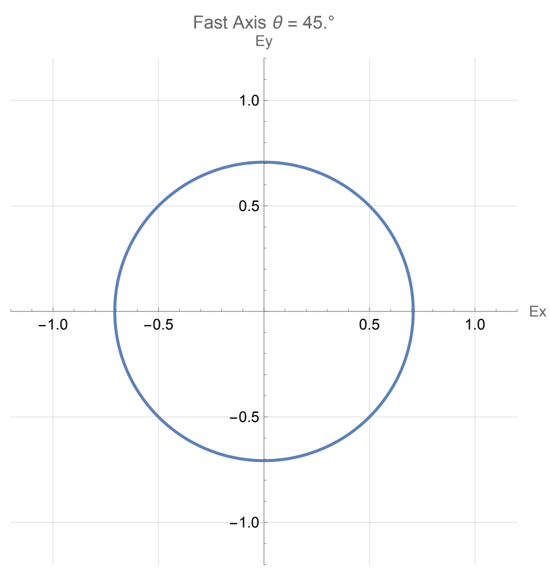

Figure 9: $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$ Time-dependent electric field

3.2 Stokes vector measurement

The Stokes vector component can be calulated from E_x and E_y of the Jones vector:

$$\text{Stokes} = \begin{bmatrix}I\\Q\\U\\V\end{bmatrix} = \begin{bmatrix}|E_x|^2 + |E_y|^2\\ |E_x|^2 - |E_y|^2\\ 2\mathrm{Re}(E_x E_y^*)\\ 2\mathrm{Im}(E_x E_y^*)\end{bmatrix} \tag{16}$$

Measuring the Stokes polarization parameters (Beth Schaefer)