

Homework 3 - Quantum Algorithms

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I pledge my honor that I have abided by the Stevens Honor System.

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1 Problem 1

- It is deterministic turing machine because there is only one possible output for each input.
- – For ϵ The machine starts with $_$, then it moves to q_1 , write 0 and moving left. Since q_1 is the final state. The machine halts.
- For '000'

$$\begin{array}{c} \underbrace{_ 0 _}_{q_0} 00_ \\ \underbrace{_ _}_{q_0} 000_ \\ \underbrace{_ _ _}_{q_1} 0000_ \end{array}$$

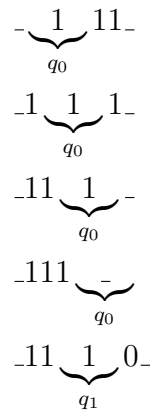
The machine halts.

- For '001'

$$\begin{array}{c} \underbrace{_ 0 _}_{q_0} 01_ \\ \underbrace{_ _}_{q_0} 001_ \\ \underbrace{_ _ _}_{q_1} 0001_ \end{array}$$

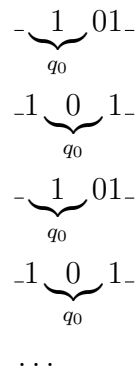
The machine halts.

– For '111'



The machine halts.

– For '101'



The machine does not halt.

- The machine halts on inputs that do not start with 1s and have 0s. Example:
Set that accepted by the machine: $\{010, 01010, \dots\}$
Set that rejected by the machine: $\{1110, 1010, 1011, \dots\}$

Problem 2

- The machine halts on all inputs. $\Sigma = \{0, 1\}$
- If the length of the input is not modulo 3, the machine will add 0 to the rightmost of the input else it will add 1 to the rightmost of the input and halts. From the transition functions, we can see that the machine will move left to right from state $q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow q_0 \rightarrow \dots$ without changing the input string. If it reaches the end (blank) at state q_0 , it will add 1 to the rightmost of the input and halts, else if it's state q_1 or q_2 , it will add 0 to the rightmost of the input and halts.

For the string w . If $|w| \bmod 3 = 0$ then the machine will add 1 to the rightmost of the input and halts. If $|w| \bmod 3 \neq 0$ then the machine will add 0 to the rightmost of the input and halts.