Homework 2 - Quantum Algorithms

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I pledge my honor that I have abided by the Stevens Honor System.

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Problem 1

- $\overline{(AB)} = \overline{A}\,\overline{B}$

  Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$. Then:

$$A \cdot B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\overline{(AB)} = \begin{bmatrix} \overline{a_{11}b_{11} + a_{12}b_{21}} & \overline{a_{11}b_{12} + a_{12}b_{22}} \\ \overline{a_{21}b_{11} + a_{22}b_{21}} & \overline{a_{21}b_{12} + a_{22}b_{22}} \end{bmatrix}$$

  For the right hand side:

$$\begin{aligned}
\overline{A}\,\overline{B} &= \begin{bmatrix} \overline{a_{11}} & \overline{a_{12}} \\ \overline{a_{21}} & \overline{a_{22}} \end{bmatrix} \begin{bmatrix} \overline{b_{11}} & \overline{b_{12}} \\ \overline{b_{21}} & \overline{b_{22}} \end{bmatrix} \\
&= \begin{bmatrix} \overline{a_{11}b_{11}} + \overline{a_{12}b_{21}} & \overline{a_{11}b_{12}} + \overline{a_{12}b_{22}} \\ \overline{a_{21}b_{11}} + \overline{a_{22}b_{21}} & \overline{a_{21}b_{12}} + \overline{a_{22}b_{22}} \end{bmatrix} \\
&= \begin{bmatrix} \overline{a_{11}b_{11} + a_{12}b_{21}} & \overline{a_{11}b_{12} + a_{12}b_{22}} \\ \overline{a_{21}b_{11} + a_{22}b_{21}} & \overline{a_{21}b_{12} + a_{22}b_{22}} \end{bmatrix}
\end{aligned}$$

  Since complex conjugation is an isomorphism

- $(AB)^T = B^T A^T$