Homework 2 - Quantum Algorithms

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I pledge my honor that I have abided by the Stevens Honor System.

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Problem 1

- $\overline{(AB)} = \overline{A}\,\overline{B}$

  Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$. Then:

$$A \cdot B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\overline{(AB)} = \begin{bmatrix} \overline{a_{11}b_{11} + a_{12}b_{21}} & \overline{a_{11}b_{12} + a_{12}b_{22}} \\ \overline{a_{21}b_{11} + a_{22}b_{21}} & \overline{a_{21}b_{12} + a_{22}b_{22}} \end{bmatrix}$$

  For the right hand side:

$$\begin{aligned} \overline{A}\,\overline{B} &= \begin{bmatrix} \overline{a_{11}} & \overline{a_{12}} \\ \overline{a_{21}} & \overline{a_{22}} \end{bmatrix} \begin{bmatrix} \overline{b_{11}} & \overline{b_{12}} \\ \overline{b_{21}} & \overline{b_{22}} \end{bmatrix} \\ &= \begin{bmatrix} \overline{a_{11}b_{11}} + \overline{a_{12}b_{21}} & \overline{a_{11}b_{12}} + \overline{a_{12}b_{22}} \\ \overline{a_{21}b_{11}} + \overline{a_{22}b_{21}} & \overline{a_{21}b_{12}} + \overline{a_{22}b_{22}} \end{bmatrix} \\ &= \begin{bmatrix} \overline{a_{11}b_{11} + a_{12}b_{21}} & \overline{a_{11}b_{12} + a_{12}b_{22}} \\ \overline{a_{21}b_{11} + a_{22}b_{21}} & \overline{a_{21}b_{12} + a_{22}b_{22}} \end{bmatrix} \end{aligned}$$

  Since complex conjugation is an isomorphism.

- $(AB)^T = B^T A^T$

$$\begin{aligned} (AB)^T &= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{21}b_{11} + a_{22}b_{21} \\ a_{11}b_{12} + a_{12}b_{22} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \\ &= \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \\ &= B^T A^T \end{aligned}$$

Problem 2

$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{1}{\sqrt{3}} & \frac{3+i}{2\sqrt{15}} \\ \frac{-1}{2} & \frac{1}{\sqrt{3}} & \frac{4+3i}{2\sqrt{15}} \\ \frac{1}{2} & \frac{-i}{\sqrt{3}} & \frac{5i}{2\sqrt{15}} \end{bmatrix}$$

A is unitary if $A^{\dagger}A = I$.

$$\begin{aligned} A^{\dagger}A &= \begin{bmatrix} \frac{1-i}{2} & \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{3}} \\ \frac{3-i}{2\sqrt{15}} & \frac{4-3i}{2\sqrt{15}} & \frac{-5i}{2\sqrt{15}} \end{bmatrix} \begin{bmatrix} \frac{1+i}{2} & \frac{1}{\sqrt{3}} & \frac{3+i}{2\sqrt{15}} \\ \frac{-1}{2} & \frac{1}{\sqrt{3}} & \frac{4+3i}{2\sqrt{15}} \\ \frac{1}{2} & \frac{-i}{\sqrt{3}} & \frac{5i}{2\sqrt{15}} \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{-i}{\sqrt{3}} & 0 \\ \frac{i}{\sqrt{3}} & 1 & \frac{(1+2i)\sqrt{5}}{15} \\ 0 & \frac{(1-2i)\sqrt{5}}{15} & 1 \end{bmatrix} \\ &\Rightarrow \text{A is not unitary} \end{aligned}$$