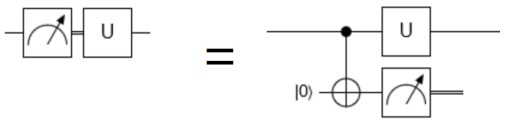
MA565: Homework 8

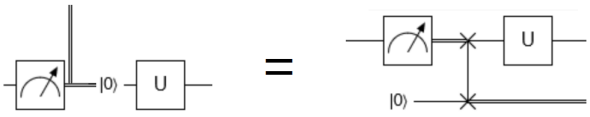
(**Deferred measurement principle**) It is possible to avoid doing any intermediate measurements in a quantum circuit, using one auxiliary qubit for each measurement that needs to be delayed until the end of the computation.

Exercise 8.1. [10pts] (Moving measurements onto a fresh ancilla qubit.)
Show that for any unitary U the following two circuits produce the same result (in the first qubit).

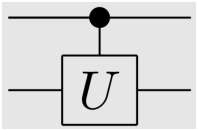


Swap gate swaps the classical values $|xy\rangle \mapsto |yx\rangle$

Exercise 8.2. [10pts] (Using fresh qubits instead of resets.)
Show that for any unitary U the following two circuits produce the same result.



(Controlled-U gate). If U is a unitary n-qubit operator, then Controlled-U is an $n+1$-qubit operator that applies U to the last n qubits if the first qubit is 1.



Exercise 8.3. [10pts] (Deferring measurements until later.)
Show that for any unitary U the following two circuits produce the same result.