Homework 3 - Quantum Algorithms

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I pledge my honor that I have abided by the Stevens Honor System.

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1 Problem 1

- It is deterministic turing machine because there is only one possible output for each input.
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  - For ϵ The machine starts with _, then it moves to q_1, write 0 and moving left. Since q_1 is the final state. The machine halts.
  - For '000'

$$\underbrace{\_}_{q_0}00\_$$
$$\underbrace{\_}_{q_0}000\_$$
$$\underbrace{\_}_{q_1}0000\_$$

    The machine halts.
  - For '001'

$$\underbrace{\_}_{q_0}01\_$$
$$\underbrace{\_}_{q_0}001\_$$
$$\underbrace{\_}_{q_1}0001\_$$

    The machine halts.

– For '111'

$$\underbrace{\_1}_{q_0}11\_$$
$$\_1\underbrace{1}_{q_0}1\_$$
$$\_11\underbrace{1}_{q_0}\_$$
$$\_111\underbrace{\_}_{q_0}$$
$$\_11\underbrace{1}_{q_1}0\_$$

The machine halts.

– For '101'

$$\underbrace{\_1}_{q_0}01\_$$
$$\_1\underbrace{0}_{q_0}1\_$$
$$\underbrace{\_1}_{q_0}01\_$$
$$\_1\underbrace{0}_{q_0}1\_$$
$$\ldots$$

The machines does not halt.

- The machine halts on inputs that does not starts with 1s and have 0s. Example:
  Set that accepted by the machine: {010, 01010, ...}
  Set that rejected by the machine: {1110, 1010, 1011, ...}

Problem 2

- The machine halts on all inputs. $\Sigma = \{0, 1\}$
- If the length of the input is not modulo 3, the machine will adds 0 to the rightmost of the input else it will add 1 to the rightmost of the input and halts. From the transition functions, we can see that the machine will move left to right from state $q_0 \to q_1 \to q_2 \to q_0 \to \ldots$ without changing the input string. If it reaches the end (blank) at state q_0, it will add 1 to the rightmost of the input and halts, else if it's state q_1 or q_2, it will add 0 to the rightmost of the input and halts.

  For the string w. If $|w| \mod 3 = 0$ then the machine will add 1 to the rightmost of the input and halts. If $|w| \mod 3 \neq 0$ then the machine will add 0 to the rightmost of the input and halts.