MA565: Homework 1

Exercise 1.1. [10pts] Prove polarization identity, i.e., for $\overline{u}, \overline{v} \in \mathbb{C}^n$ show that the following holds:

$$\langle u, v\rangle = \frac{1}{4}\big(\|u+v\|^2 - \|u-v\|^2 - i\|u+iv\|^2 + i\|u-iv\|^2\big).$$

Exercise 1.2. [6pts] Let $\overline{u} = (i, 1+i)^T$ and $\overline{v} = (2, 1-i)^T$.

(1) Compute $\langle \overline{u}, \overline{v}\rangle$.
(2) Compute $\|\overline{u}\|$.
(3) Express $\frac{1}{2-i}$ in the form $a + bi$ with $a, b \in \mathbb{R}$.

Exercise 1.3. [4pts] Prove that a composition of linear transformations $\varphi, \psi : V \to V$ is linear.

Exercise 1.4. [4pts] Prove that $\|1 - e^{2ix}\|^2 = 4\sin^2(x)$.