QUBO to Ising Model Transformation

June 13, 2025

Starting from the equation (13) from the paper, we have the following QUBO problem:

$$\max_b L(b) = \max_b \left(\mu''^T b - q b^T \Sigma'' b \underbrace{-\lambda (P''^T b - 1)^2}_{penalty} \right) \tag{1}$$

Now we expand its component:

$$\mu''^T b = \sum_i u''_i b_i, \tag{2}$$

$$-q b^T \Sigma'' b = -q \sum_{i,j} \Sigma''_{ij} b_i b_j, \tag{3}$$

$$-\lambda (P''^T b - 1)^2 = -\lambda \left(\sum_i P''_i b_i - 1 \right)^2. \tag{4}$$

We have the transformation of $b_i = \frac{1+s_i}{2}; b_j = \frac{1+s_j}{2}$

$$\sum_i u''_i b_i = \sum_i u''_i \frac{1+s_i}{2} \tag{5}$$

$$= \frac{1}{2} \sum_i u''_i + \frac{1}{2} \sum_i u''_i s_i, \tag{6}$$

$$\tag{7}$$