Abstract Algebra Homework 11

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This problem set includes problems $3c, 4b, 24$, and an extra problem from section 17.4.

3) Use the division algorithm to find $q(x)$ and $r(x)$ such that $a(x) = q(x)b(x) + r(x)$ with $\deg r(x) < \deg b(x)$.

3c) $a(x) = 4x^5 - x^3 + x^2 + 4$ and $b(x) = x^3 - 2$ where $a(x), b(x) \in \mathbb{Z}_5[x]$.

Solution: Performing long division, we have

$$\begin{array}{r|l} & 4x^2 - 1 \\ \hline x^3 - 2 & 4x^5 - x^3 + x^2 + 4 \\ & \underline{-4x^5 \qquad + 8x^2} \\ & -x^3 + 9x^2 + 4 \\ & \underline{x^3 \qquad - 2} \\ & 9x^2 + 2 \end{array}$$

Thus,

$$\begin{aligned} a(x) &= (4x^2 - 1) \cdot (x^3 - 2) + (9x^2 + 2) \\ &\equiv (4x^2 - 1) \cdot (x^3 - 2) + (4x^2 + 2). \end{aligned}$$