Abstract Algebra Homework 8

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This problem set includes problems $2, 24, 28, 34$, and 38 from section 16.6.

2) Let R be the ring of 2×2 matrices of the form

$$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix},$$

where $a, b \in \mathbb{R}$. Show that although R is a ring that has no identity, we can find a subring S of R with an identity.

Proof: $\square$

24) Let R be a ring with a collection of subrings $\{R_\alpha\}$. Prove that $\bigcap R_\alpha$ is a subring of R. Give an example to show that the union of two subrings cannot be a subring.

Proof: $\square$

28) A ring R is a boolean ring if for every $a \in \mathbb{R}, a^2 = a$. Show that every boolean ring is a commutative ring.

Proof: $\square$

34) Let p be prime. Prove that

$$Z_{(p)} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z} \text{and } \gcd(b, p) = 1 \right\}$$

is a ring.

Proof: $\square$

38) An element x in a ring is called idempotent if $x^2 = x$. Prove that the only idempotent in an integral domain are 0 and 1. Find a ring with a idempotent x not equal to 0 or 1.

Proof: $\square$