# The Minority Game: the emergence of cooperation from selfishness

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# Introduction

# Introduction

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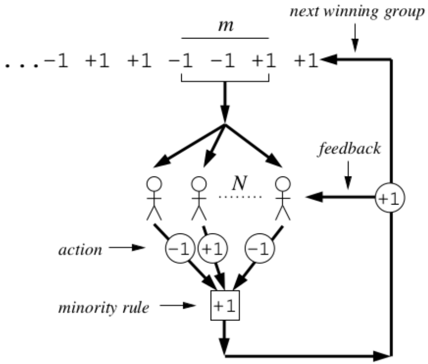
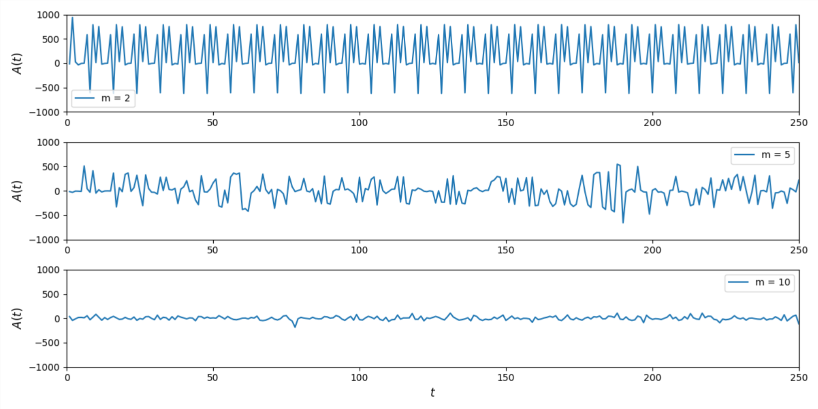
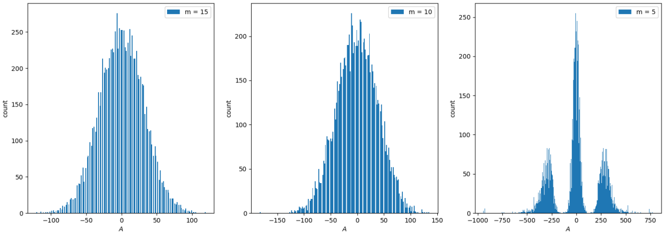



The main core of the code is an object-oriented implementation in *FORTRAN90*, with *openMP*

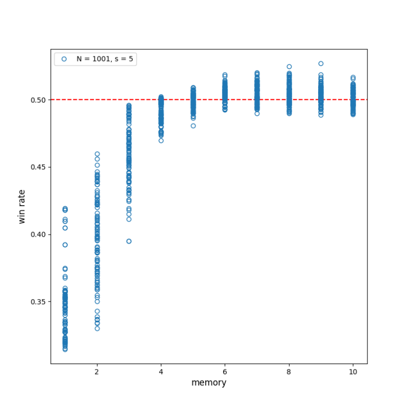
# Statistical analysis of the classical Minority Game




1. the temporal signal of $A(t)$ fluctuates around zero $\implies \langle A(t) \rangle = 0$.
2. The fluctuations are in decreasing order for ever increasingly intelligent populations (i.e. with $m = 2, 5, 10$)
3. for small values of $m$ (such as $m = 2$), a periodic behavior is observed: $\mu(t+1) = [2\mu(t) + (W(t) - 1)/2] \mod P$

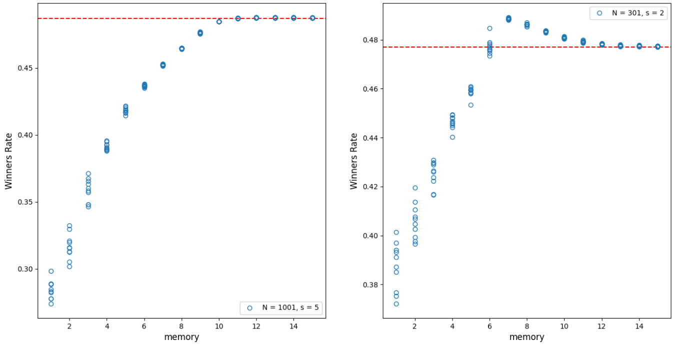


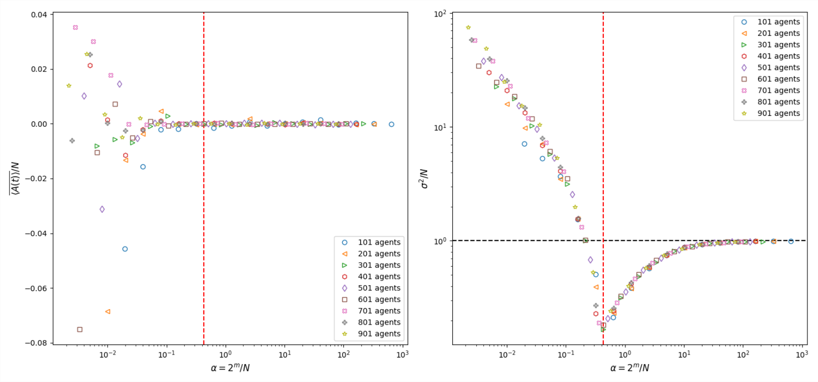

1. All the plots are symmetrical, and this confirms that $\langle A(t) \rangle = 0$
2. for small values of $m$, for example $m = 5$, the histogram shows two smaller side peaks. For $m \geq 10$ the histogram has only one central peak, for $1 \leq m < 10$ the distribution has also side peaks.
3. existence of a transition-phase point



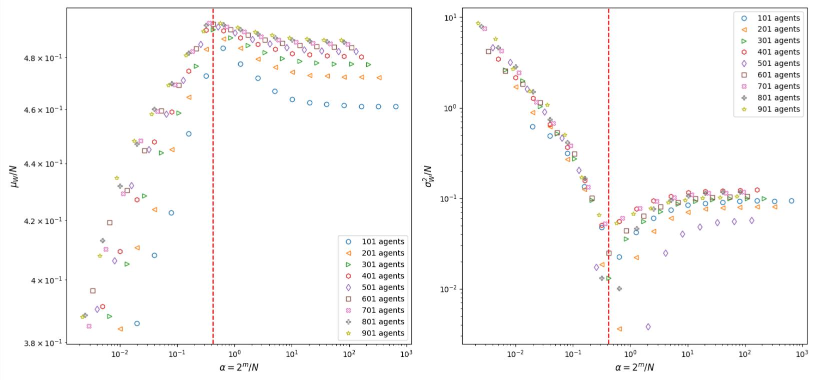

1. simulations of a mixed population of $N$ agents, with different memory values ($m = 1, ..., 10$), forced to play together
2. larger memory of a group of agents implies higer average win rate, and smaller spread
3. above a certain size ($m = 6$), average performance of a population appears to saturate

Winners rate for different populations with the **same** parameters:

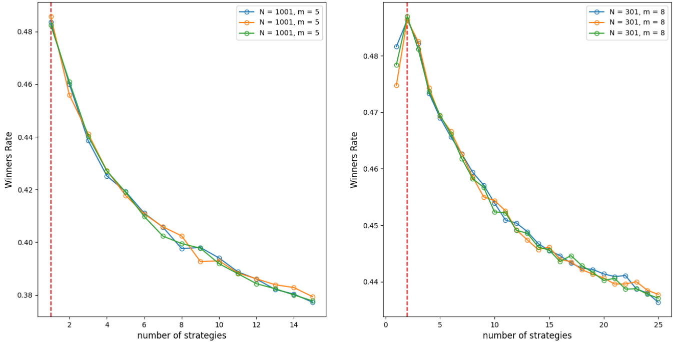




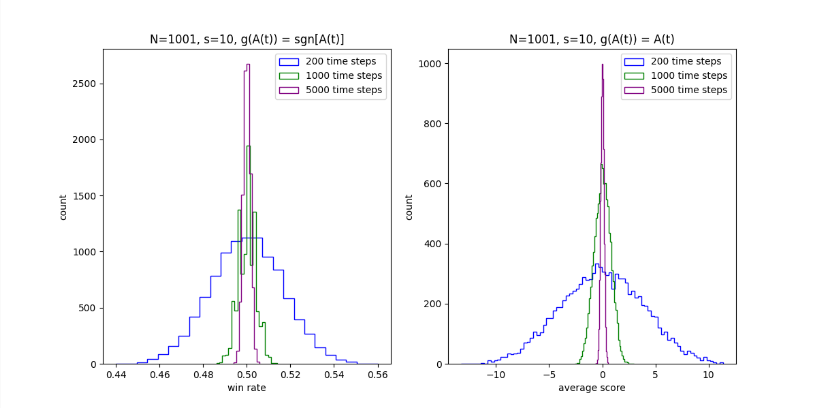

1. for large values of $\alpha$ (in the termodynamic limit $\alpha \to \infty$), $\sigma^2/N$ approaches the value of the random choice game
2. At low values of $\alpha$, there are large fluctuations and a waste of global gain: global inefficiency
3. At intermediate values of $\alpha$, agents cooperate better in order to reach a state in which less resources are globally wasted. The transition-phase point is $\alpha = \alpha_c \approx 0.425$.




1. the variance has a similar behavior as before
2. the mean value of the number of winners reaches a maximum for $\alpha = \alpha_c \approx 0.425$

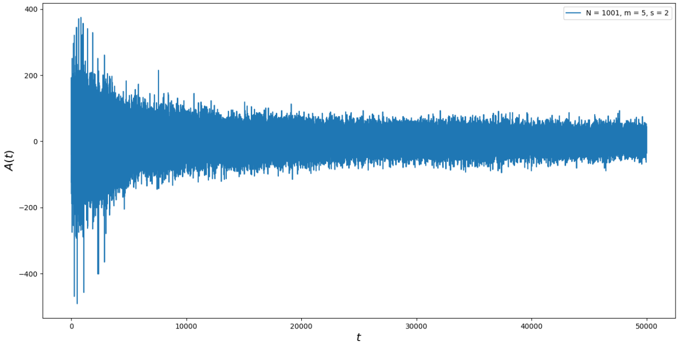



1. the winners rate for various populations with the same parameters, varying the number of strategies
2. with increasing number of strategies, the agents tend to perform worse
3. players tend to switch strategies often and are more likely choose some outperforming strategy

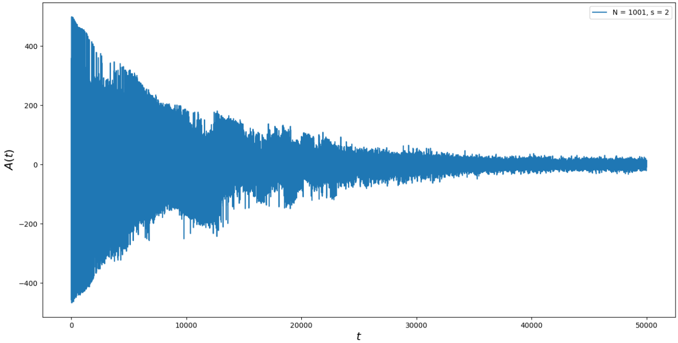



1. The longer the simulation time, the more concentrate is the distribution
2. all the strategies are equivalent to each other, in the limit $t \to \infty$, since the distribution tends to be increasingly peaked at zero
3. specific strategy compositions are to blame

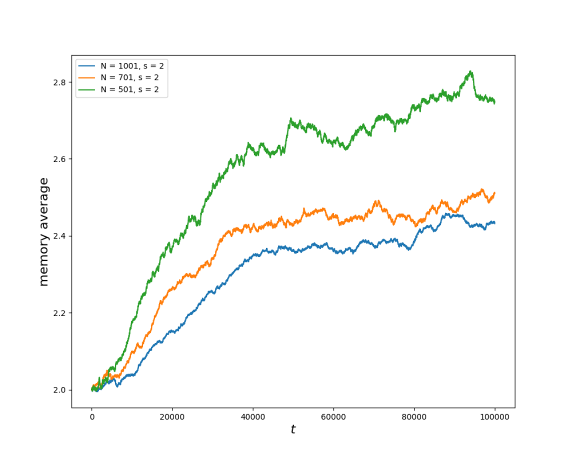
# evolutionary dynamics




1. Darwin-ist selection: the worst agent is replaced by a new one after a finite time steps, and the new agent is a clone of the best agent
2. fluctuations reduce with time steps. This implies a more efficient way to use the available resources.
3. this population is capable of learning



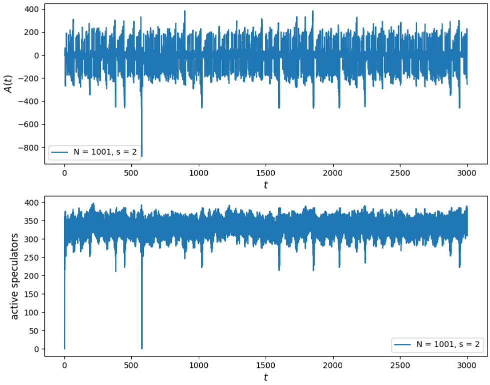

1. let the memory of the new born agents to be one bit grater or smaller (with equal probability)
2. sharp learning behaviour, across time steps



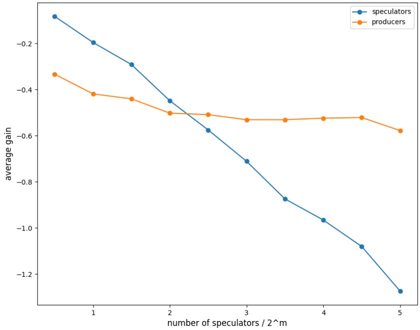

1. evolution selects the agents with higher values of $m$, and discard agents with little memory
2. such an evolution appears to saturate at a given level, that is not the same for all the populations
3. larger populations ($N = 1001$) have smaller learning rate with respect to smaller populations ($N = 501$)

# MG and financial markets

1. The money does not come from the speculators themselves, but there are other types of agents, not interested in making money inside markets, but who use markets for exchanging goods
2. we consider $N_p$ producers (non-adaptive agents, with signicantly less strategies) and $N_s$ speculators (standard inductive agents).
3. in the basic MG agents are forced to play at each time step: now an agent only plays if he has at least one strategy with a score higher than $\epsilon t/2^m$. So agents stay in the market if they perform well in the market and not only outside it.




1. when the number of speculators increases, the market becomes more efficient
2. presence of clustered volatility




1. number of producers fixed at $N_p = 256$. The number of speculators is variable.
2. with fixed number of producers, more speculators in the market means less average gain for them, whereas the gain of producers remains approximately constant
3. again: when the number of speculators increases, the market becomes more efficient