Optimization of Multiple Rotors

J. Spencerr *
Brigham Young University, Provo, Utah, 84601

This report describes a project completed as part of my research for the Brigham Young University FLOW Lab. this project drew on skills gained throughout the semester to design and execute a code in the julia programming language to find the optimal blade count, blade thickness magnification, and twist angle for a propellor to achieve its maximum efficiency, η, at a given advance ratio, J. This investigation found that thinner rotors are more efficient, especially at negative angles of attack. The implications of these results and possible areas of future research are then discussed.

Nomenclature

(Nomenclature entries are identified with their default units.)

J	=	Advance Ratio, *dimensionless*
α	=	Angle of Attack, rad
ϕ	=	Angle of Rotation, rad
Ω, n	=	Angular Speed, $rev./s$
W	=	Apparent Speed, m/s
a	=	Axial Induction Factor *dimensionless*
M_b	=	Bending Moment, $N \times m$
r	=	Blade Tip Radius, m
c	=	Chord length, m
D	=	Diameter, m
c_d	=	Drag Coefficient, *dimensionless*
D	=	Drag Force, N
ν	=	Dynamic Viscosity, $\frac{Ns}{m^2}$
η	=	Efficiency, *unitless*
ρ	=	Fluid Density, kg/m^3
v_a, U_∞	=	Freestream Velocity, m/s
μ	=	Kinematic Viscosity, m^2/s

*Undergraduate Researcher, Brigham Young University FLOW Lab.

c_l = Lift Coefficient, *dimensionless*

L = Lift Force, N

M = Mach Number, *dimensionless*

M_p = Pitching Moment, $N \times m$

c_m = Pitching Moment Coefficient, *dimensionless*

P = Power, W

C_P = Power Coefficient, *dimensionless*

N = Propellor Count, *integer*

ω = Radial Velocity, $\frac{rad.}{s}$

RPM = Revolutions Per Minute, $\frac{rev.}{min.}$

Re = Reynolds Number, *dimensionless*

n = Rotational Frequency, $\frac{rev.}{s}$

σ = Rotor Solidity *dimensionless*

n = Safety Factor, *dimensionless*

s = Spacing, m

A = Surface Area, $m2$

β = Stationary Angle of Rotation, $rad.$

a' = Tangential Induction Factor *dimensionless*

T = Thrust, N

C_T = Thrust Coefficient, *dimensionless*

λ = Tip Speed Ratio, *dimensionless*

Q = Torque, $N \times m$

C_Q = Torque Coefficient, *unitless*

v_i = Uniform Induced Velocity, m/s

u = Velocity, m/s

I. Introduction

PROPELLORS come in a variety of different shapes and sizes. This paper describes how one propellor, which started as an APC 10x7 rotor and a NACA 4412 airfoil, was optimized to perform better at a certain advance ratio. The code used for this report could also be applied to different rotors in the future.

This report shows how computer simulations can be used to simulate the performance of propellors without needing to actually create them. It finds that changing the twist angle and the thickness of a propellor doesn't just shift the curves for its efficiency, thrust coefficient, and torque coefficient; it entirely changes their shape. Propellors with different blade counts also have different curves for these three non-dimensional numbers. All code used for each section of this report can also be accessed through a GitHub repository *.

II. Procedure

This project was performed using julia programming language. † jula is available for free and is useful for a variety of reasons, including that It can store data as vectors and matrices and perform rapid calculations with these objects. It compiles functions in advance, so they can be run more quickly than in some other languages. Function files used for this project, including Xfoil ‡, CCBlade §, SNOW ¶, and FLOWMath ‖, are all designed for julia. These files provided useful functionality that simplified the design process.

In the rotor design process, an airfoil was first created using Xfoil.jl. This rotor was then attached to a rotor and evaluated using CCBlade.jl. Data about the rotor, including its moments in the normal and tangential directions and its torque, is recorded. This data is then multiplied by a factor, in this case $n = 1.1$, to determine the maximum allowable loads before the rotor would break or a different material would be required. After constraints are determined, rotor variable types called *rotortest* are created with variable thickness magnification and rotation angle and the program uses Optim.jl to find the rotor with the optimal twist angle and thickness magnification for the objective function.

The objective function used in this optimization is listed in equation (1).

$$f(x_1, x_2, x_3...) = \eta \tag{1}$$

.

Constraints were placed on the solution to ensure that the optimizer found a reasonable solution. In addition to restrictions on the moment and torque mentioned previously, the cord thickness was kept within a factor of two of the original and the twist angle was kept between $-90°$ and $90°$. These constraints are described by the following constraints table.

*This repository can be accessed at `https://github.com/JoeSpencer1/497R-Projects`

†julia can be found at `https://julialang.org`.

‡Xfoil.jl is available at `https://github.com/byuflowlab/Xfoil.jl`

§CCBlade.jl is available at `https://github.com/byuflowlab/CCBlade.jl`

¶SNOW.jl is available at `https://github.com/byuflowlab/SNOWl.jl`

‖FLOWMath.jl is available at `https://github.com/byuflowlab/FLOWMath.jl`

Table 1: Optimization objective, parameters, and constraints

Maximize:	η
By varying:	scale of the chord, c
	twist angle, ϕ
Subject to	total torque less than 110% of original
	normal moment less than 110% of original
	tangential moment less than 110% of original
	$-90° <$ twist angle $< 90°$
	50% < chord magnification < 200%

The optimized rotors each had some properties in common and some different between them. For reference, these properties are shown in table 2. Parameters kept constant in this table have the same maximum and minimum value. The advance ratio in table 2 can be found from other variables already listed in the table by equation 2, in which v_∞ is the free stream velocity, n is the rotational velocity in revolutions per second, and D is the outer diameter.

$$J = \frac{v_\infty}{nD} \tag{2}$$

Table 2: Input Value Limits in Rotor Design

These are the default values for each parameter, before the rotor is optimized.

<table>
<tr><th>Parameter</th><th>Default Value</th><th>Minimum Value</th><th>Maximum Value</th></tr>
<tr><td>Chord Length,</td><td>100% length</td><td>50%</td><td>200%</td></tr>
<tr><td>Twist Angle,</td><td>0°</td><td>−90°</td><td>90°</td></tr>
<tr><td>Rotational Velocity, RPM</td><td colspan="3">500 RPM</td></tr>
<tr><td>Blade Count</td><td>2 blades</td><td colspan="2">1 to 3)</td></tr>
<tr><td>Hub-to-tip ratio</td><td colspan="3">10%</td></tr>
<tr><td>Air Density</td><td colspan="3">1.225 kg/m^3</td></tr>
<tr><td>Diameter</td><td colspan="3">20 m</td></tr>
<tr><td>Velocity</td><td colspan="3">45 m/s</td></tr>
<tr><td>Advance Ratio</td><td colspan="3">0.27</td></tr>
</table>

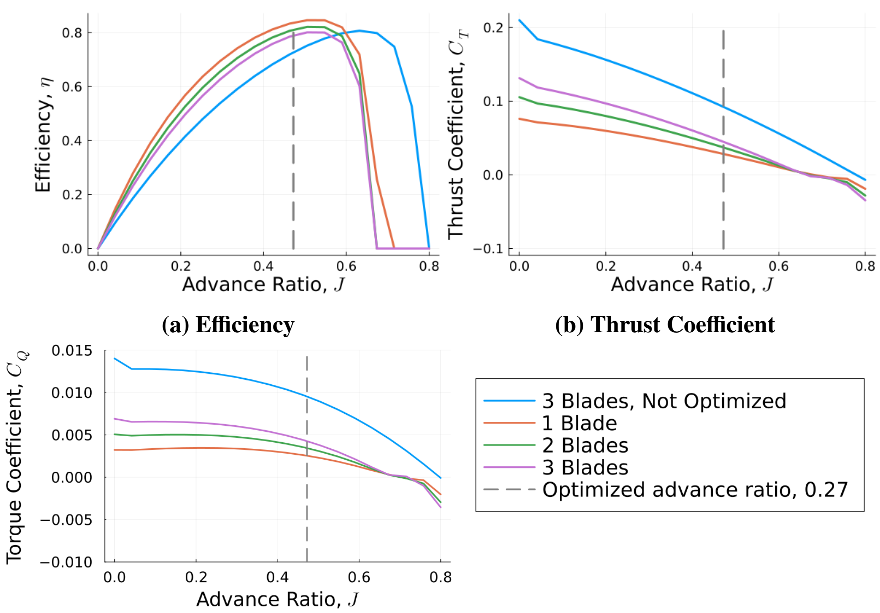
III. Results

As stated previously, an APC 10x7 rotor with what started as a NACA 4412 airfoil was optimized. While the APC rotor number stayed the same, the NACA airfoil number was changed. The maximum chord found in first digit and the maximum thickness from two digits at the end are multiplied by whatever the magnification of the chord is. So, a NACA 4412 airfoil becomes a NACA 2206 airfoil at 50% magnification and a NACA 8424 at 200%.

The propellor was checked at blade counts of 2, 3, and 4. These results are described and compared in plot 1. This report found that for all three blade counts, the optimal propellor was as thin as possible, with negative angles of attack. The finding about thinner rotor blades being more efficient was in line with **THE PAPER** **, but such a high negative rotation angle was surprising.

A. Results Plot

Four plots, displayed below, were output by the rotor analysis. Figure 1 shows that while the optimized design increased the efficiency of even the rotor with a higher blade count above the rotor pre-optimization, it dramatically decreases the thrust and torque coefficients. These decreases in other rotor properties may or may not be desired. This illustrates one problem with optimizers: they will do exactly what they are programmed to do and may sacrifice some desirable properties to further optimize the objective function.



(a) Efficiency **(b) Thrust Coefficient**

(c) Torque Coefficient

Fig. 1 Efficiency, Thrust Coefficients, and Torque Coefficients Compared at Different Advance Ratios
These plots visually represent how optimized rotors are different from the original rotor.

**Experimental study of blade thickness effects on the overall and local performances of a Controlled Vortex Designed axial-flow fan, `https://doi.org/10.1016/j.expthermflusci.2011.01.002`

If necessary, minimum thrust and torque coefficients could be provided as parameters along with those shown below the objective function in table 1. These could find the angle of rotation and thickness that would provide the maximum efficiency while still maintaining some required thrust. If a minimum thrust or torque coefficient is provided, the optimizer might not reduce the rotor to its minimum possible thickness in every case.

B. Results Table

Table 3 shows the optimal twist angles and chord magnifications for each different rotor blade count. The reader can observe that the optimal thickness for each propellor blade was the very thinnest possible. The thickness had to between 50% and 200% of the original. The optimal angle of rotation was slightly less than $-10^(\circ)$ for each rotor, gradually decreasing in magnitude as the blade count increased.

Table 3: Optimized Chord Magnification and Rotation Angles for Different Blade Counts

Blade Count	**Chord Thickness Multiplication**	**Twist Angle**
3 (Default)	1.0	0°
2	0.50	−9.54°
3	0.50	−9.47°
4	0.50	−9.39°

IV. Discussion

As stated previously, one lesson that could be learned from this optimization is that people should be careful what they optimize for, because the computer will optimize exactly what it is told, even if that is not what the user actually wants. An optimization that is written incorrectly or has unseen loopholes can not only give a misleading answer but also waste a lot of time. In the case of this optimization, I think other constraints could have been added to the optimization

A. Efficiency Equation

One issue with the optimization is that a rotor's efficiency can be found as a function of the advance ratio, the thrust coefficient, and the power coefficient. The equation is described by Andrew Ning in *Computational Aerodynamics* **CITE!** using the relation below.

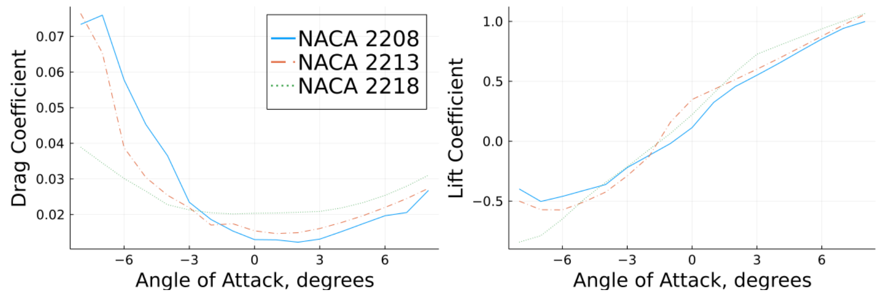
$$\eta = J\frac{C_T}{C_P} \tag{3}$$

With J kept constant, as it was in this research, this equation can be maximized in either by maximizing C_T or by minimizing C_P, which is equal to C_Q multiplied by 2π. Inspection of figure 1 reveals that the optimizer did the latter.

The torque coefficient C_Q is five times lower for each rotor. While the thrust coefficient C_T did not decrease by as much as C_Q, it is still much lower than before. Although more efficient, the newly optimized propellor is now designed for an entirely different function.

B. Angle of Attack

The optimal angle of attack was a surprising result. I had found, in my airfoil analysis, that the angle of attack that generally had the lowest drag coefficient was close to zero, like in figure 2.



(a) Drag Coefficient **(b) Lift Coefficient**

Fig. 2 Drag Coefficient and Lift Coefficient Compared to Angle of Attack
These plots show that an airfoil's drag and lift are both closest to zero when it has a low angle of attack.

The angle of rotation is different from the angle of attack, though. While the angle of attack is simply the angle the chord line forms with the normal, a propellor's rotation angle is the angle between its tangential velocity and the free stream velocity. (CITE) *Computational Aerodynamics, page 205* shows that the

https://www.sciencedirect.com/science/article/abs/pii/S0894177711000033?via%3Dihub https://www.sciencedirect.com/science/articl

V. Keeping this stuff so I can learn how to do references to my bibliography.

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Papers with many symbols may benefit from a nomenclature list that defines all symbols with units, inserted between the abstract and the introduction. If one is used, it must contain all the symbology used in the manuscript, and the definitions should not be repeated in the text. In all cases, identify the symbols used if they are not widely recognized in the profession. Define acronyms in the text, not in the nomenclature.

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Survey Papers and some Full-Length Papers include author biographies. These biographies are one paragraph each and should use the abstract formatting style.

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List and number all references at the end of the paper. Corresponding bracketed numbers are used to cite references in the text [1], including citations that are an integral part of the sentence (e.g., "It is shown in [2] that. . . ") or follow a mathematical expression: "$A^2 + B = C$ (Ref. [3])." For multiple citations, separate reference numbers with commas [4, 5], or use a dash to show a range [6–9]. Reference citations in the text should be in numerical order.

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Insert tables and figures within your document; they may be either scattered throughout the text or grouped all together at the end of the file. Do not insert your figures in text boxes. Figures should have no background, borders, or outlines. In the LaTeX template, use the "caption" command to type caption text. Captions are bold with a single tab (no hyphen or other character) between the figure number and figure description. See the Table 1 example for table style and column alignment. If you wish to center tables that do not fill the width of the page, simply highlight and "grab" the entire table to move it into proper position.

Table 1 Transitions selected for thermometry

	Transition					
Line	v''		J''	Frequency, cm^{-1}	FJ, cm^{-1}	Gv, cm^{-1}
a	0	P_{12}	2.5	44069.416	73.58	948.66
b	1	R_2	2.5	42229.348	73.41	2824.76
c	2	R_{21}	805	40562.179	71.37	4672.68
d	0	R_2	23.5	42516.527	1045.85	948.76

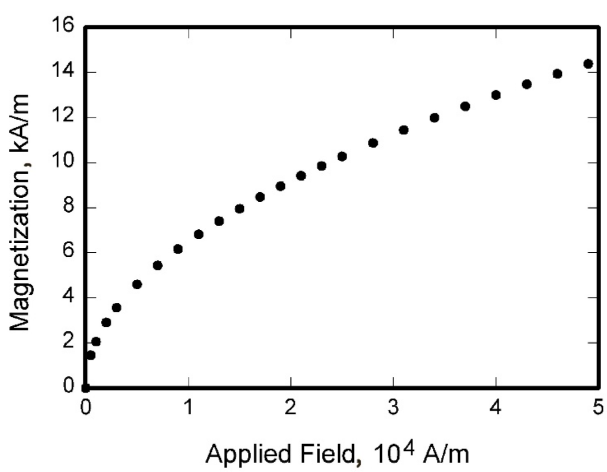

Fig. 3 Magnetization as a function of applied fields.

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A sample equation is included here, formatted using the preceding instructions:

$$\int_0^{r_2} F(r,\varphi)\mathrm{d}r\,\mathrm{d}\varphi = [\sigma r_2/(2\mu_0)]\int_0^{\infty} \exp(-\lambda|z_j - z_i|)\lambda^{-1}J_1(\lambda r_2)J_0(\lambda r_i\,\lambda\mathrm{d}\lambda) \tag{4}$$

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Use only one space after periods or colons. Hyphenate complex modifiers: "zero-field-cooled magnetization." Insert a zero before decimal points: "0.25," not ".25." Use "cm^2" not "cc."

A parenthetical statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within parenthesis.) Use American, not English, spellings (e.g., "color," not "colour"). The serial comma is preferred: "A, B, and C" instead of "A, B and C."

Be aware of the different meanings of the homophones “affect” (usually a verb) and “effect” (usually a noun), “complement” and “compliment,” “discreet” and “discrete,” “principal” (e.g., “principal investigator”) and “principle” (e.g., “principle of measurement”). Do not confuse “imply” and “infer.”

VI. Conclusion

Although a conclusion may review the main points of the paper, it must not replicate the abstract. A conclusion might elaborate on the importance of the work or suggest applications and extensions. Do not cite references in the conclusion. Note that the conclusion section is the last section of the paper to be numbered. The appendix (if present), funding information, other acknowledgments, and references are listed without numbers.

Appendix

An Appendix, if needed, appears **before** research funding information and other acknowledgments.

Funding Sources

Sponsorship information and acknowledgments of financial support should be included here. **Authors are responsible for accurately reporting funding data relevant to their research.** Please confirm that you have correctly entered **all sources** of funding and grant/award numbers **for all authors** in this section of your article. You will also be asked to select the appropriate funding organization from a drop-down menu in ScholarOne when you submit your manuscript. Be careful to choose the correct funder name, as organization names can be similar, and also be mindful to select sub-organizations within the registry hierarchy that are the actual funding sources, as appropriate, rather than choosing the name of the parent organization. Information provided in your manuscript must match the funding data entered in ScholarOne.

Acknowledgments

An Acknowledgments section, if used, **immediately precedes** the References. Individuals other than the authors who contributed to the underlying research may be acknowledged in this section. The use of special facilities and other resources also may be acknowledged.

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