Optimization of Multiple Rotors

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This report describes a project completed as part of my research with the Brigham Young University FLOW Lab. this project drew on skills gained throughout the semester to design and execute a code in the Julia programming language to find the optimal blade count, blade thickness magnification, and angle of rotation for a propellor to achieve its maximum efficiency, η, at a known advance ratio, J. This investigation found that thinner rotors are more efficient, especially at negative angles of attack. The implications of these results and possible areas of future research are then discussed.

Nomenclature

(Nomenclature entries are identified with their default units.)

J	=	Advance Ratio, *dimensionless*
α	=	Angle of Attack, rad
ϕ	=	Angle of Rotation, rad
Ω	=	Angular Speed, $rev./s$
W	=	Apparent Speed, m/s
a	=	Axial Induction Factor *dimensionless*
M_b	=	Bending Moment, $N \times m$
r	=	Blade Tip Radius, m
c	=	Chord length, m
D	=	Diameter, m
c_d	=	Drag Coefficient, *dimensionless*
D	=	Drag Force, N
ν	=	Dynamic Viscosity, $\frac{Ns}{m^2}$
η	=	Efficiency, *unitless*
ρ	=	Fluid Density, kg/m^3
v_a, U_∞	=	Freestream Velocity, m/s
μ	=	Kinematic Viscosity, m^2/s

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c_l	=	Lift Coefficient, *dimensionless*
L	=	Lift Force, N
M	=	Mach Number, *dimensionless*
M_p	=	Pitching Moment, $N \times m$
c_m	=	Pitching Moment Coefficient, *dimensionless*
P	=	Power, W
C_P	=	Power Coefficient *dimensionless*
N	=	Propellor Count, *integer*
ω	=	Radial Velocity, $\frac{rad.}{s}$
RPM	=	Revolutions Per Minute, $\frac{rev.}{min.}$
Re	=	Reynolds Number, *dimensionless*
n	=	Rotational Frequency, $\frac{rev.}{s}$
σ	=	Rotor Solidity *dimensionless*
s	=	Spacing, m
A	=	Surface Area, $m2$
β	=	Stationary Angle of Rotation, $rad.$
a'	=	Tangential Induction Factor *dimensionless*
T	=	Thrust, N
C_T	=	Thrust Coefficient, *dimensionless*
λ	=	Tip Speed Ratio, *dimensionless*
Q	=	Torque, $N \times m$
C_Q	=	Torque Coefficient, *unitless*
v_i	=	Uniform Induced Velocity, m/s
u	=	Velocity, m/s

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Table 1 Transitions selected for thermometry

	Transition					
Line	v''		J''	Frequency, cm^{-1}	FJ, cm^{-1}	Gv, cm^{-1}
a	0	P_{12}	2.5	44069.416	73.58	948.66
b	1	R_2	2.5	42229.348	73.41	2824.76
c	2	R_{21}	805	40562.179	71.37	4672.68
d	0	R_2	23.5	42516.527	1045.85	948.76

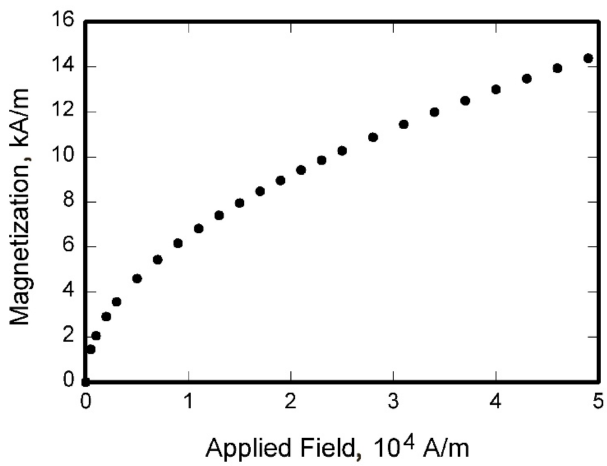

Fig. 1 Magnetization as a function of applied fields.

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$$\int_0^{r_2} F(r,\varphi)\mathrm{d}r\,\mathrm{d}\varphi = [\sigma r_2/(2\mu_0)]\int_0^{\infty} \exp(-\lambda|z_j - z_i|)\lambda^{-1}J_1(\lambda r_2)J_0(\lambda r_i\,\lambda \mathrm{d}\lambda) \tag{1}$$

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References

[1] Vatistas, G. H., Lin, S., and Kwok, C. K., "Reverse Flow Radius in Vortex Chambers," *AIAA Journal*, Vol. 24, No. 11, 1986, pp. 1872, 1873. https://doi.org/10.2514/3.13046.

[2] Dornheim, M. A., "Planetary Flight Surge Faces Budget Realities," *Aviation Week & Space Technology*, Vol. 145, No. 24, 1996, pp. 44–46.

[3] Terster, W., "NASA Considers Switch to Delta 2," *Space News*, Vol. 8, No. 2, 1997, pp. 13–19.

[4] Peyret, R., and Taylor, T. D., *Computational Methods for Fluid Flow*, 2nd ed., Springer-Verlag, New York, 1983, Chaps. 7, 14.

[5] Oates, G. C. (ed.), *Aerothermodynamics of Gas Turbine and Rocket Propulsion*, AIAA Education Series, AIAA, New York, 1984, pp. 19, 136.

[6] Volpe, R., "Techniques for Collision Prevention, Impact Stability, and Force Control by Space Manipulators," *Teleoperation and Robotics in Space*, edited by S. B. Skaar and C. F. Ruoff, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1994, pp. 175–212.

[7] Thompson, C. M., "Spacecraft Thermal Control, Design, and Operation," *AIAA Guidance, Navigation, and Control Conference*, CP849, Vol. 1, AIAA, Washington, DC, 1989, pp. 103–115.

[8] Chi, Y. (ed.), *Fluid Mechanics Proceedings*, NASA SP-255, 1993.

[9] Brandis, A. M., Johnston, C. O., and Cruden, B. A., “Nonequilibrium Radiation for Earth Entry,” *AIAA Paper 2016–3690*, June 2016. https://doi.org/10.2514/6.2016-3690.