Rotor Analysis

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Methods

Results and Discussion

The Blade Element Moment Theory

Axial induction factor a Tangential induction factor a'

1 Glossary

- Angle of Rotation, ϕ - The Angle of Rotation, sometimes denoted by the Greek letter ϕ, is the angle between the freest stream velocity and the velocity of the airfoil as it rotates. It is used in Blade Element Moment Theory calculations.
- Axial Induction Factor, a - The Axial Induction Factor is the ratio of the reduction in air velocity at an airfoil to its free stream velocity.
- Blade Element Moment Theory - The theory used to calculate local forces on a propellor or wind turbine blade. It employs both Blade Element Theory and Momentum Theory. These equations are used to recursively find the Axial Induction Factor, a, Tangential Induction Factor, and Angle of Rotation, ϕ

$$\begin{aligned} \frac{1}{2}W^2NcC_y &= 4\pi U_\infty(1-a)\times\Omega a' r^2 \\ \frac{1}{2}\rho W^2NcC_x &= 4\pi\rho[(a'\Omega r)^2+\Omega_\infty^2 a(1-a)]r \\ \sin\phi &= \frac{U_\infty}{W}(1-a) \end{aligned} \tag{1}$$

  In these equations, a, a', and ϕ are the previously mentioned axial and tangential induction factors and angle of rotation. The airfoil's apparent speed is represented by the letter W, N is the number of propellers, ρ is the fluid density, c is the chord length, C_x and C_y are obtained by the equation below, U_∞ is the fluid free velocity, Ω is the blade's angular speed, and r is the radius to the tip of the blade.

$$\begin{aligned} C_x &= c_l\cos\phi + c_d\sin\phi \\ C_y &= c_l\sin\phi + c_d\cos\phi \end{aligned} \tag{2}$$

- Blade Element Theory - Blade Element Theory calculates the forces on a turbine blade by dividing it into finite pieces and summing the forces on all of these pieces. This theory determines the induced velocity and efficiency of a point along a blade using these equations:

$$\begin{aligned} v_i &= \sqrt{\frac{T}{A}\frac{1}{2\rho}} \\ \eta &= \frac{\tan\phi}{\tan(\phi+\gamma)} \end{aligned} \tag{3}$$

  In these equations, v_i is the uniform induced velocity across the disk, T the thrust it experiences, A is its area, ρ is the air density, ϕ is the angle to the airfoil's plane of rotation as it moves forward, and γ is the difference between ϕ and β, what the airfoil's actual angle of rotation would be if it were stationary..

- Momentum Theory - Momentum Theory defines the power required to produce sufficient thrust to maintain momentum in a blade by the following equation, where T is thrust, ρ is density, A is disc area, and P is power:

$$P = \sqrt{\frac{T^3}{2\rho A}} \tag{4}$$

- Tangential Induction Factor, a' - The Tangential Induction Factor is the ratio of the increase in air velocity tangential to the airfoil to its free stream velocity.