

Optimization of Multiple Rotors

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This report describes a project completed as part of my research with the Brigham Young University FLOW Lab. this project drew on skills gained throughout the semester to design and execute a code in the Julia programming language to find the optimal blade count, blade thickness magnification, and angle of rotation for a propellor to achieve its maximum efficiency, η , at a known advance ratio, J . This investigation found that thinner rotors are more efficient, especially at negative angles of attack. The implications of these results and possible areas of future research are then discussed.

Nomenclature

(Nomenclature entries are identified with their default units.)

J	=	Advance Ratio, <i>dimensionless</i>
α	=	Angle of Attack, <i>rad</i>
ϕ	=	Angle of Rotation, <i>rad</i>
Ω	=	Angular Speed, <i>rev./s</i>
W	=	Apparent Speed, <i>m/s</i>
a	=	Axial Induction Factor <i>dimensionless</i>
M_b	=	Bending Moment, $N \times m$
r	=	Blade Tip Radius, <i>m</i>
c	=	Chord length, <i>m</i>
D	=	Diameter, <i>m</i>
c_d	=	Drag Coefficient, <i>dimensionless</i>
D	=	Drag Force, <i>N</i>
ν	=	Dynamic Viscosity, $\frac{Ns}{m^2}$
η	=	Efficiency, <i>unitless</i>
ρ	=	Fluid Density, kg/m^3
v_a, U_∞	=	Freestream Velocity, <i>m/s</i>
μ	=	Kinematic Viscosity, m^2/s

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c_l	=	Lift Coefficient, <i>dimensionless</i>
L	=	Lift Force, N
M	=	Mach Number, <i>dimensionless</i>
M_p	=	Pitching Moment, $N \times m$
c_m	=	Pitching Moment Coefficient, <i>dimensionless</i>
P	=	Power, W
C_P	=	Power Coefficient, <i>dimensionless</i>
N	=	Propellor Count, <i>integer</i>
ω	=	Radial Velocity, $\frac{rad.}{s}$
RPM	=	Revolutions Per Minute, $\frac{rev.}{min.}$
Re	=	Reynolds Number, <i>dimensionless</i>
n	=	Rotational Frequency, $\frac{rev.}{s}$
σ	=	Rotor Solidity <i>dimensionless</i>
n	=	Safety Factor, <i>dimensionless</i>
s	=	Spacing, m
A	=	Surface Area, m^2
β	=	Stationary Angle of Rotation, $rad.$
a'	=	Tangential Induction Factor <i>dimensionless</i>
T	=	Thrust, N
C_T	=	Thrust Coefficient, <i>dimensionless</i>
λ	=	Tip Speed Ratio, <i>dimensionless</i>
Q	=	Torque, $N \times m$
C_Q	=	Torque Coefficient, <i>unitless</i>
v_i	=	Uniform Induced Velocity, m/s
u	=	Velocity, m/s

I. Introduction

PROPELLORS come in a variety of different shapes and sizes. This paper describes how one propellor, which started as an APC 10x7 rotor and a NACA 4412 airfoil, was optimized to perform better at a certain advance ratio. The code used for this report could also be applied to different rotors in the future.

This report shows how computer simulations can be used to simulate the performance of propellers without needing to actually create them. It finds that changing the angle of rotation and the thickness of a propellor shifts the curves for

its efficiency, thrust coefficient, and torque coefficient, and that propellers with different blade counts also have different curves for these three non-dimensional numbers. All code used for each section of this report can also be accessed through a GitHub repository ^{*}.

II. Procedure

This project was performed using julia programming language. [†] julia is available for free and is useful for a variety of reasons. It can store data as vectors and matrices, and perform rapid calculations with these objects. It compiles functions in advance, so they can be run more quickly than other languages like python. Header files needed for this project, including Xfoil [‡], CCBlade [§], SNOW [¶], and FLOWMath ^{||}, are all designed for julia. These files provided useful functionality that simplified the design process.

In the rotor design process, an airfoil was first created using Xfoil.jl. This rotor was then attached to a rotor and evaluated using CCBlade.jl. Data about the rotor, including its moments in the normal and tangential directions and its torque, is recorded. This data is then multiplied by a factor, in this case $n = 1.1$, to determine the maximum allowable loads before the rotor would break or a different material would be required. After constraints were determined, rotor variable types called *rotortest* are created with variable thickness magnification and rotation angle and the program uses Optim.jl to find the rotor with the optimal rotation angle and thickness magnification for the objective function.

The objective function used in this case is shown in equation 1.

$$f(x_1, x_2, x_3 \dots) = \eta \quad (1)$$

Constraints were placed on the solution to ensure that the optimizer found a reasonable solution. In addition to restrictions on the moment and torque, the cord thickness was kept within a factor of two of the original and the twist angle magnitde was kept below 90°.

^{*}This repository can be accessed at <https://github.com/JoeSpencer1/497R-Projects>

[†]julia can be found at <https://julialang.org>.

[‡]Xfoil.jl is available at <https://github.com/byuflowlab/Xfoil.jl>

[§]CCBlade.jl is available at <https://github.com/byuflowlab/CCBlade.jl>

[¶]SNOW.jl is available at <https://github.com/byuflowlab/SNOW1.jl>

^{||}FLOWMath.jl is available at <https://github.com/byuflowlab/FLOWMath.jl>

Table 1: Optimization objective, parameters, and constraints

Maximize:	η
By varying:	scale of the chord, c twist angle,
Subject to	total torque less than 110% of original normal moment less than 110% of original tangential moment less than 110% of original $-90^\circ < \text{twist angle} < 90^\circ$ $50\% < \text{chord magnification} < 200\%$

III. Results

As stated previously, an APC 10x7 rotor with what started as a NACA 4412 airfoil was optimized. While the APC rotor number stayed the same, the NACA airfoil number was changed. The maximum chord found in first digit and the maximum thickness from two digits at the end are multiplied by whatever the magnification of the chord is. So, a NACA 4412 airfoil becomes a NACA 2206 airfoil at 50% magnification and a NACA 8424 at 200%.

The propellor was checked at blade counts of 2, 3, and 4. These results are described and compared in plot 1. This report found that for all three blade counts, the optimal propellor was as thin as possible, with negative angles of attack. The finding about thinner rotor blades being more efficient was in line with **THE PAPER** **, but such a high negative rotation angle was surprising.

A. Results Table

Table 1 shows the optimal twist angles and chord magnifications for each different rotor blade count compared against

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Table 1 Transitions selected for thermometry

Line	Transition		J''	Frequency, cm^{-1}	FJ , cm^{-1}	$G\nu$, cm^{-1}
	ν''					
a	0	P ₁₂	2.5	44069.416	73.58	948.66
b	1	R ₂	2.5	42229.348	73.41	2824.76
c	2	R ₂₁	805	40562.179	71.37	4672.68
d	0	R ₂	23.5	42516.527	1045.85	948.76

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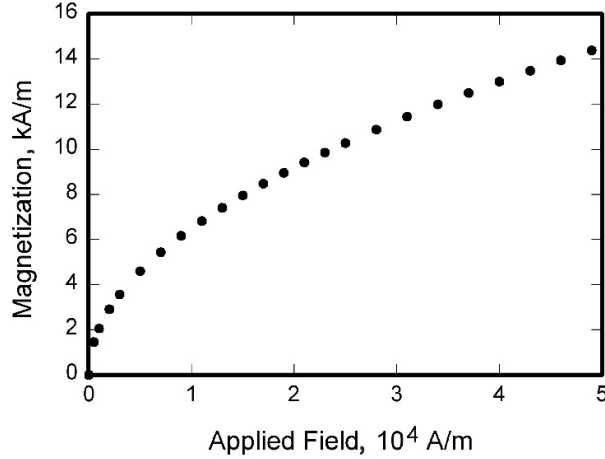


Fig. 1 Magnetization as a function of applied fields.

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