Visualizing uncertainty

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Week 7, Class 2

Reviewing

Lab 3

Data viz in the wild

Ann-Marie

Murat

Sarah Donaldson & Hyeonjin on deck

Agenda

- Framing uncertainty as relative frequencies
    - Discrete probabilities
    - Non-discrete probabilities
    - Understanding AUC calculations
- Understanding standard errors
    - Non-standard ways of visualizing SEs
- HOPs (briefly)
    - Also bootstrapping

Learning objectives

1. Understand there are lots of different ways to visualize uncertainty, and the best method may often be non–standard.

2. Understand how to implement basic methods, and the resources available to you to implement more advanced methods

Thinking about uncertainty

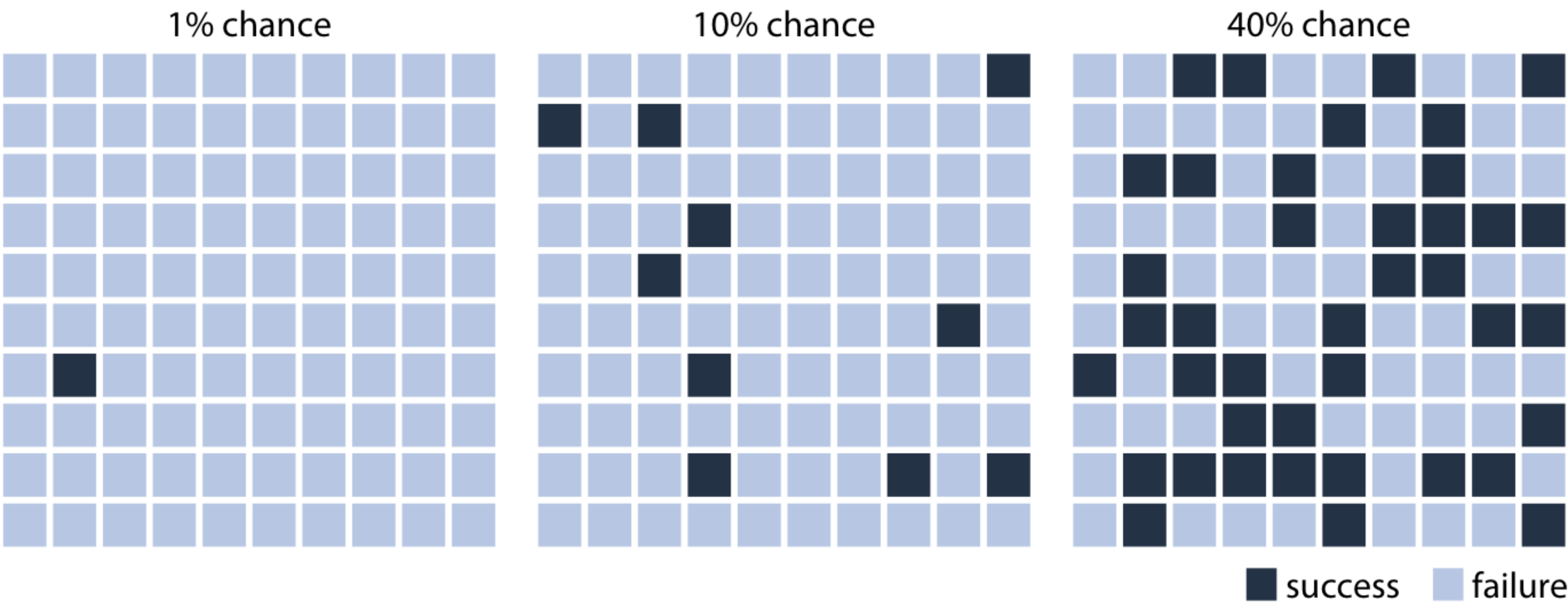
Uncertainty means exactly what it sounds like – we are not 100% sure.

- We are nearly always uncertain of future events (forecasting)
- We can also be uncertain about past events
  - I saw a parked car at 8 AM, but the next time I looked at 2PM it was gone. What time did it leave?

Quantifying uncertainty

- We quantify our uncertainty mathematically using probability
- Framing probabilities as frequencies is generally more intuitive

Framing a single uncertainty


1% chance
10% chance
40% chance
success
failure

How do we make these?

- Start out by making a grid

```
grid <- expand.grid(x = 1:20, y = 1:20)
head(grid)
```

```
##   x y
## 1 1 1
## 2 2 1
## 3 3 1
## 4 4 1
## 5 5 1
## 6 6 1
```

```
tail(grid)
```

```
##      x  y
## 395 15 20
## 396 16 20
## 397 17 20
## 398 18 20
## 399 19 20
## 400 20 20
```

Look at the grid

```
ggplot(grid, aes(x, y)) +
  geom_tile(color = "gray40",
            fill = "white") +
  theme_void()
```

Create occurrence rate

- For each sequence of `x`, create a variable that has the given occurence rate

How?

- Plenty of options, here's one

Consider 10%

```
nrow(grid)*.10 # n to sample
```

```
## [1] 40
```

```
set.seed(86753098)
samp <- sample(seq_len(nrow(grid)), nrow(grid)*.10)
head(samp)
```

```
## [1] 318 134 180 283 177 248
```

```
length(samp)
```

```
## [1] 40
```

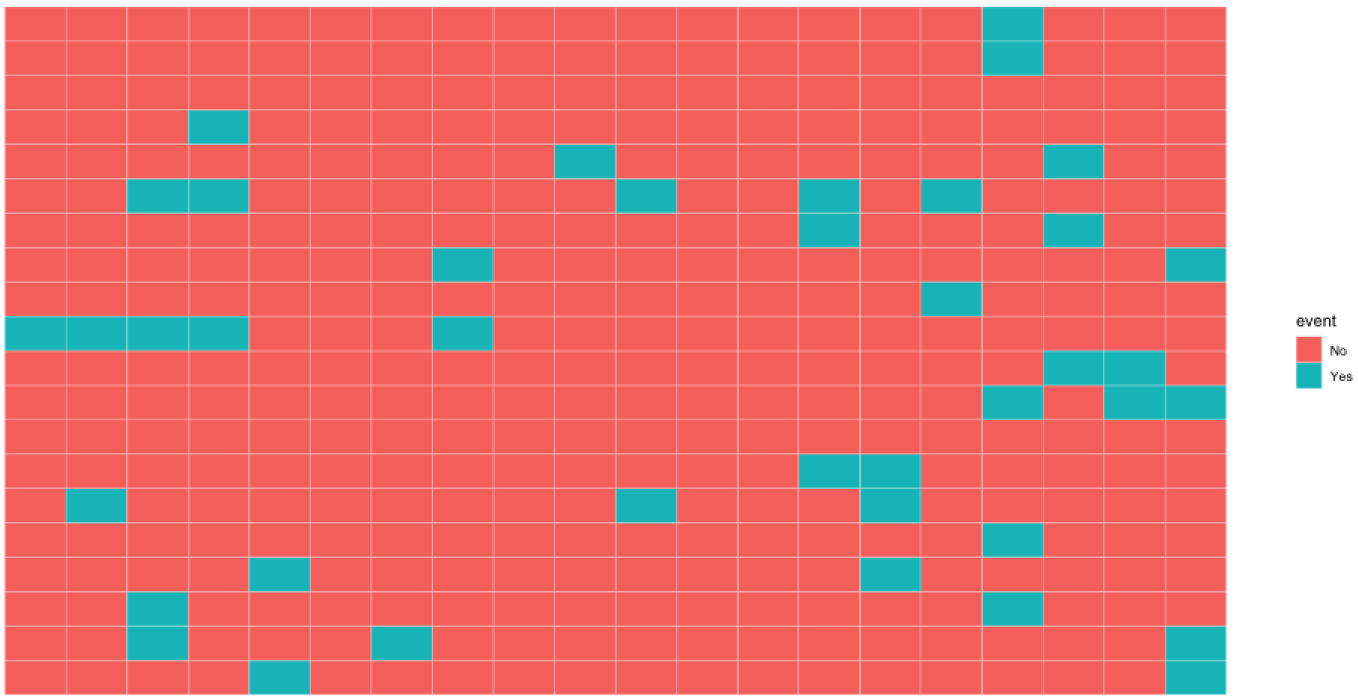
Create the variable

```r
grid <- grid %>%
  rownames_to_column("row_id") %>%
  mutate(event = ifelse(row_id %in% samp, "Yes", "No"))
head(grid)
```

```
##   row_id x y event
## 1      1 1 1    No
## 2      2 2 1    No
## 3      3 3 1    No
## 4      4 4 1    No
## 5      5 5 1   Yes
## 6      6 6 1    No
```

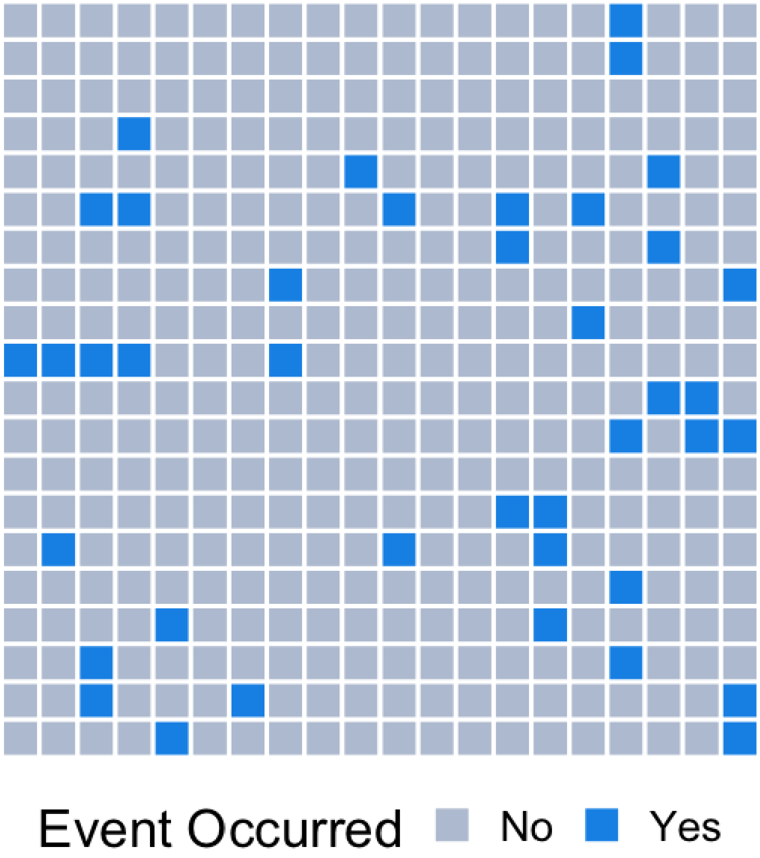
Fill in

```
ggplot(grid, aes(x, y)) +
  geom_tile(aes(fill = event), color = "white") +
  theme_void()
```



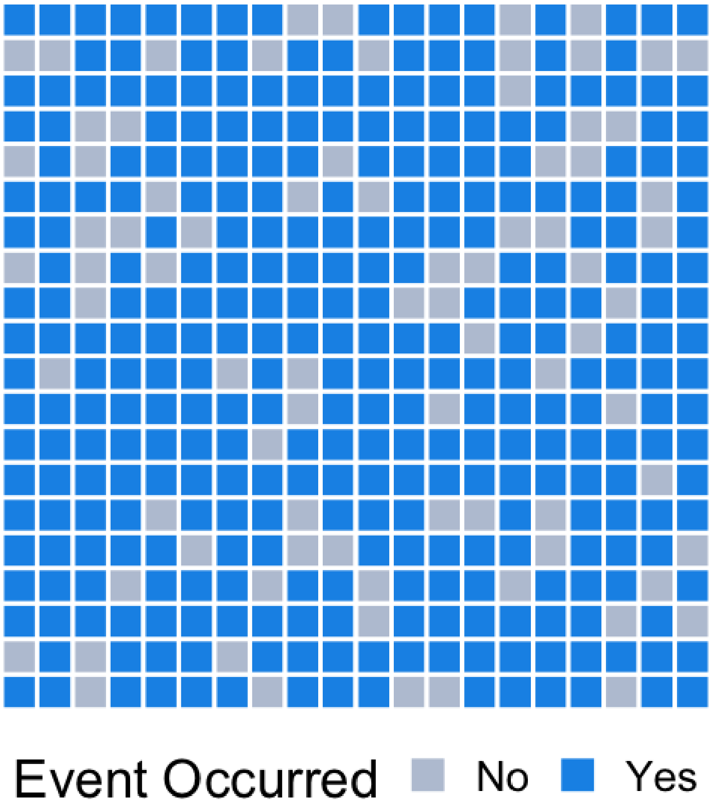
Customize

```r
library(colorspace)
ggplot(grid, aes(x, y)) +
  geom_tile(aes(fill = event), color = "white", size = 1.4) +
  scale_fill_manual(
    name = "Event Occurred",
    values = c(
      desaturate(
        lighten("#1694E8", 0.5),
        0.7),
      "#1694E8"
      )
    ) +
  coord_fixed() +
  theme_void() +
  theme(legend.position = c(0.75, 0),
        legend.direction = "horizontal",
        plot.margin = margin(b = 1, unit = "cm"))
```


Event Occurred
No
Yes

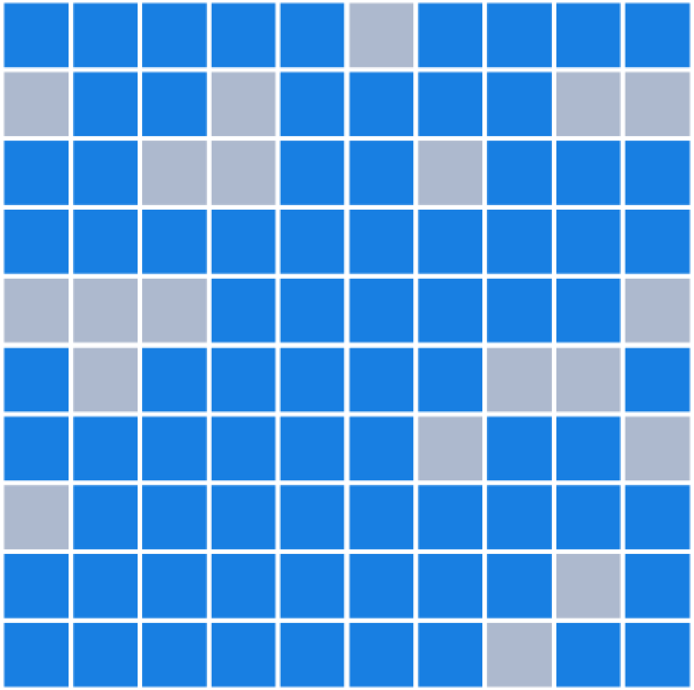
Chance of rain

80%



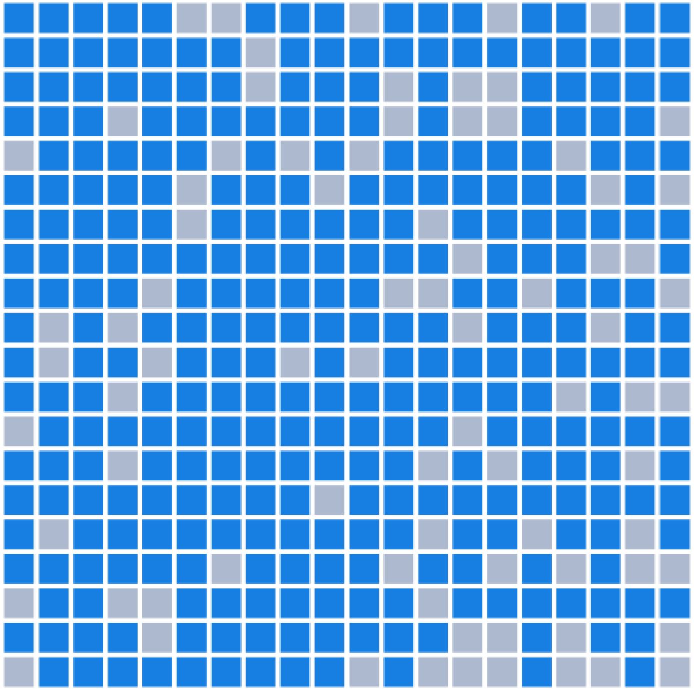
Vary grid size

10 x 10



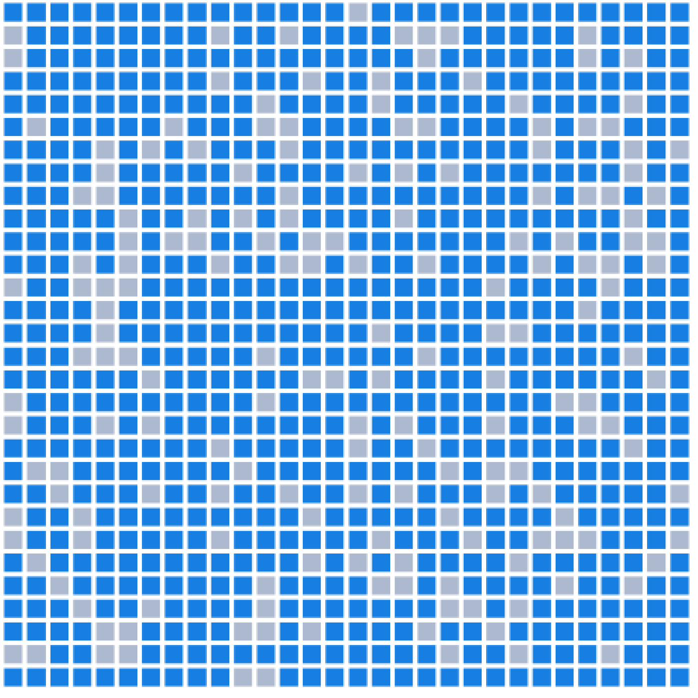
Vary grid size

20 x 20



Vary grid size

30 x 30



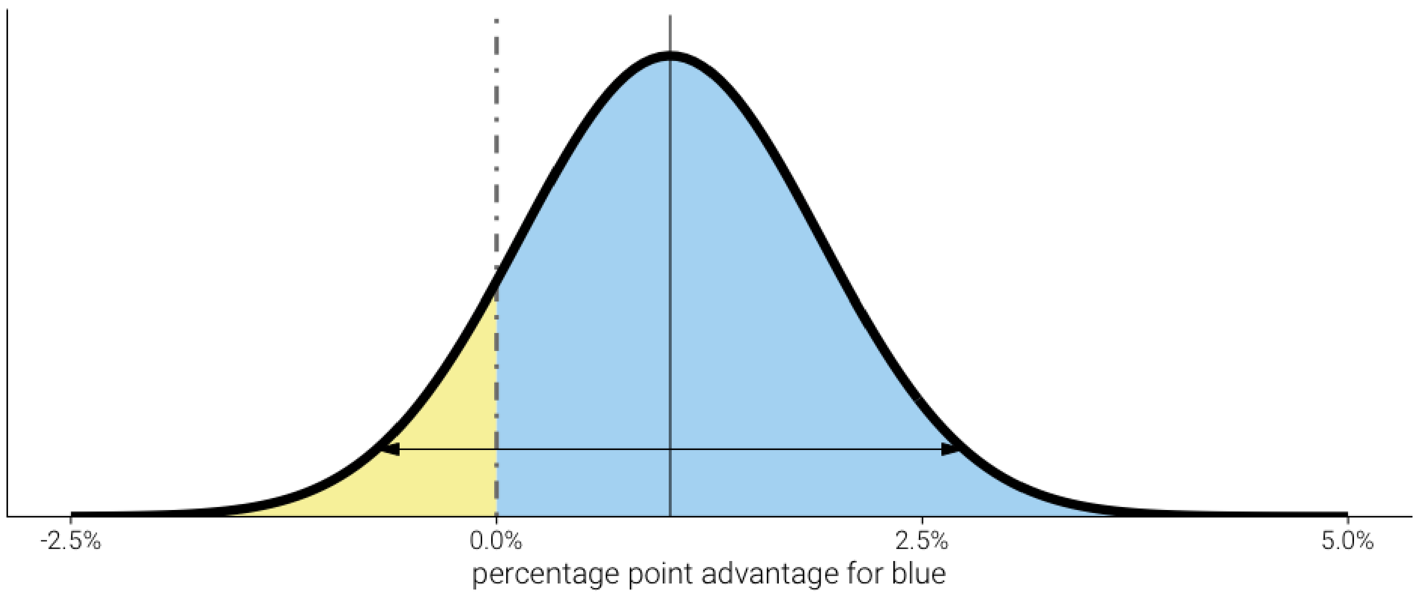
(probs too many)

Non–discrete probabilities

Hypothetical

Blue party has 1% advantage (technically 1.02%) w/ margin of error of 1.76 points

Who will win?



A bit of math

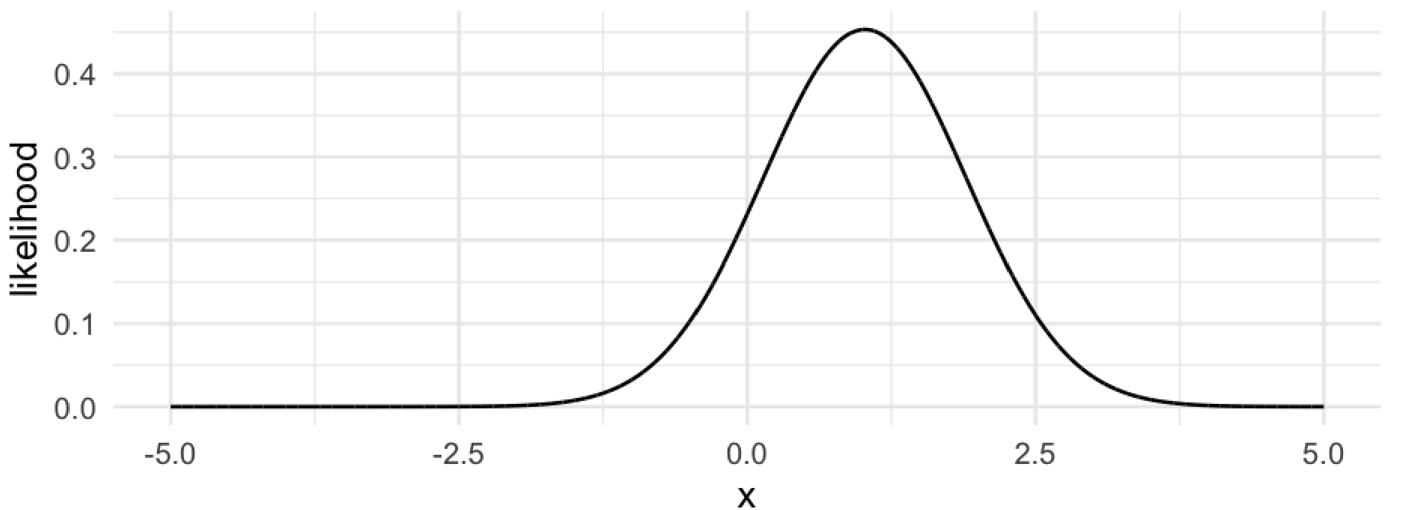
Our prior distribution was defined by $\boldsymbol{\mu = 1.02}$ and $\boldsymbol{sd = 0.88}$ (margin of error divided by two).

- What's the chance the end result is below zero?

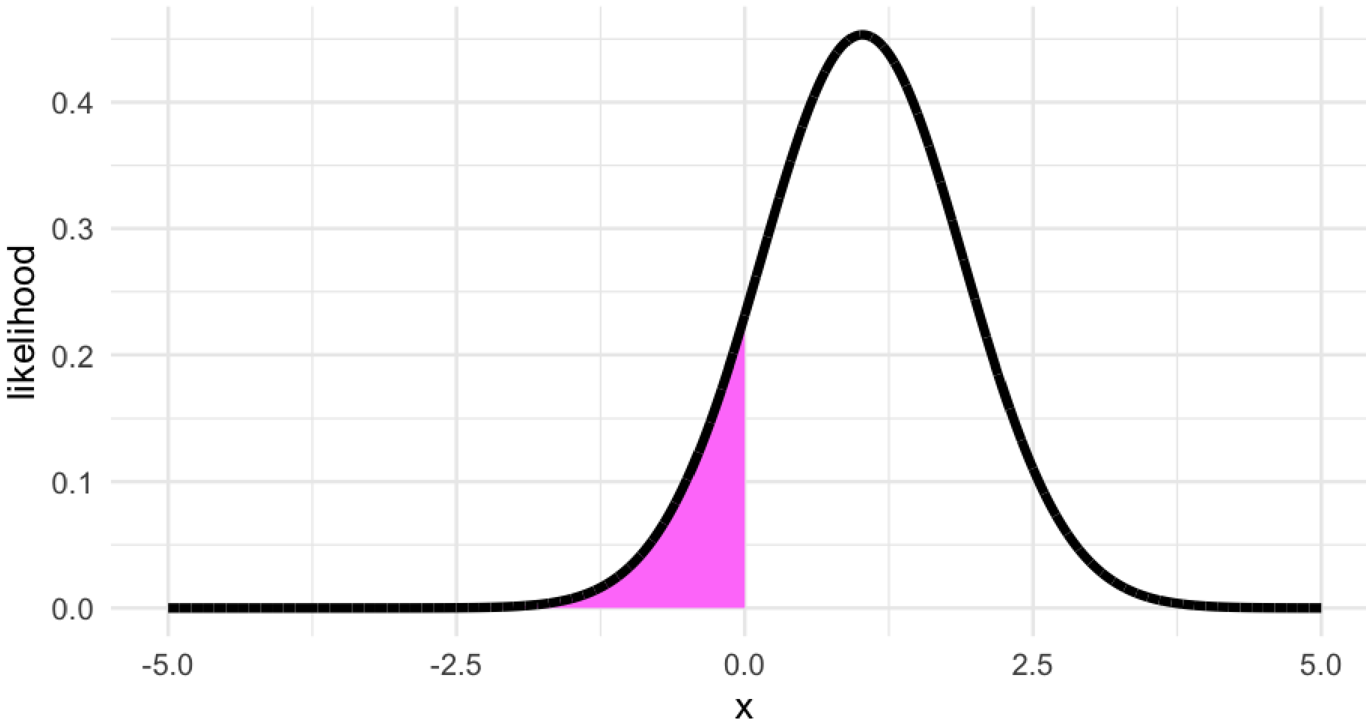
The hard way

Calculate the exact probability of data below zero under this distribution

```
x <- seq(-5, 5, 0.001)
likelihood <- dnorm(x, 1.02, 0.88)
sim <- data.frame(x, likelihood)
ggplot(sim, aes(x, likelihood)) +
  geom_line(size = 1.2)
```



How do we calculate this portion?



Integrate

```
zab <- filter(sim, x <= 0)
pracma::trapz(zab$x, zab$likelihood)
```

```
## [1] 0.1232096
```

Easier: Simulate

```r
random_draws <- rnorm(1e5, 1.02, 0.9)
table(random_draws > 0) / 1e5
```

```
## 
##   FALSE    TRUE 
## 0.12968 0.87032
```

Discretized plot

```r
ppoints(50)
```

```
##  [1] 0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.21 0.23 0.25 0.
## [18] 0.35 0.37 0.39 0.41 0.43 0.45 0.47 0.49 0.51 0.53 0.55 0.57 0.59 0.
## [35] 0.69 0.71 0.73 0.75 0.77 0.79 0.81 0.83 0.85 0.87 0.89 0.91 0.93 0.
```

```r
qnorm(ppoints(50), 1.02, 0.9)
```

```
##  [1] -1.073713087 -0.672714247 -0.460368264 -0.308211925 -0.186679530 -0
##  [7]  0.006247984  0.087209949  0.161251272  0.229893334  0.294220878  0
## [13]  0.412959225  0.468468308  0.521953752  0.573734687  0.624078151  0
## [19]  0.721331988  0.768612869  0.815209521  0.861263252  0.906904788  0
## [25]  0.997437983  1.042562017  1.087742876  1.133095212  1.178736748  1
## [31]  1.271387131  1.318668012  1.366788420  1.415921849  1.466265313  1
## [37]  1.571531692  1.627040775  1.684962164  1.745779122  1.810106666  1
## [43]  1.952790051  2.033752016  2.123875308  2.226679530  2.348211925  2
## [49]  2.712714247  3.113713087
```

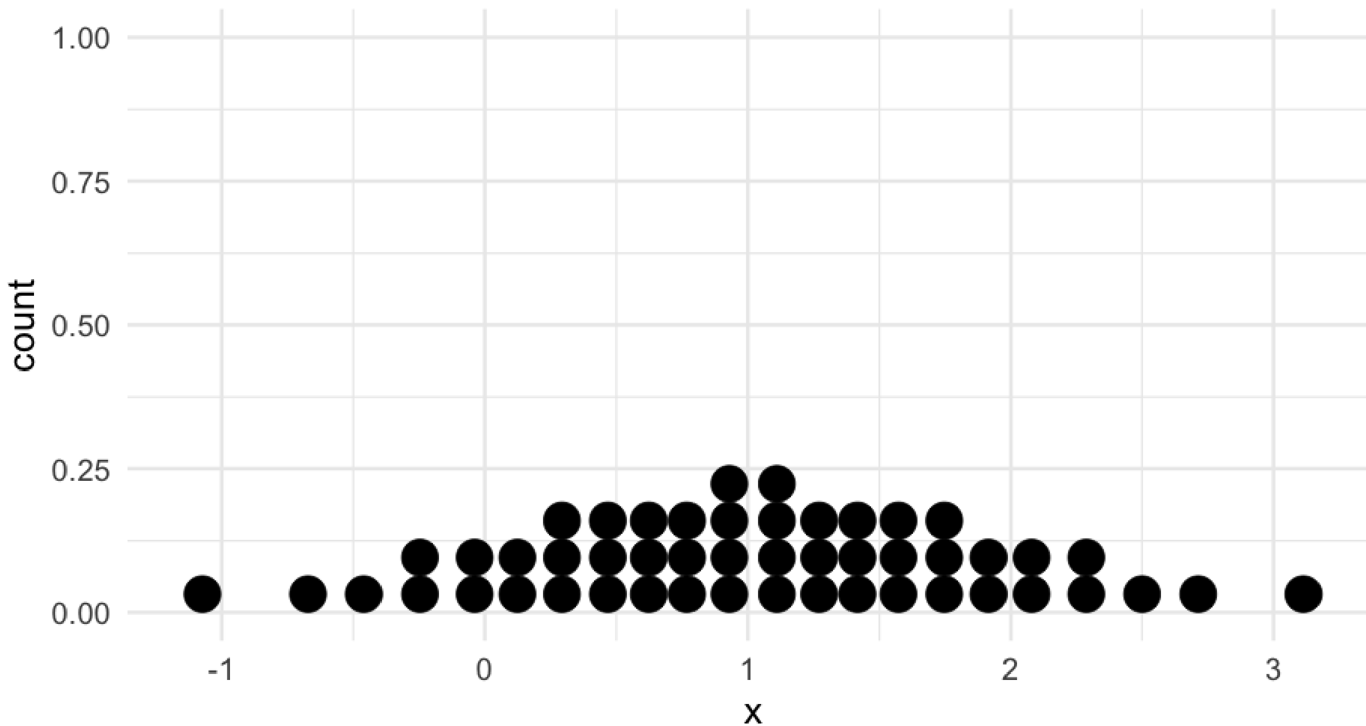
```r
discretized <- data.frame(
  x = qnorm(ppoints(50), 1.02, 0.9)
  ) %>%
  mutate(winner = ifelse(x <= 0, "#b1daf4", "#f8f1a9"))

head(discretized)
```

```
##             x  winner
## 1 -1.07371309 #b1daf4
## 2 -0.67271425 #b1daf4
## 3 -0.46036826 #b1daf4
## 4 -0.30821193 #b1daf4
## 5 -0.18667953 #b1daf4
## 6 -0.08387531 #b1daf4
```

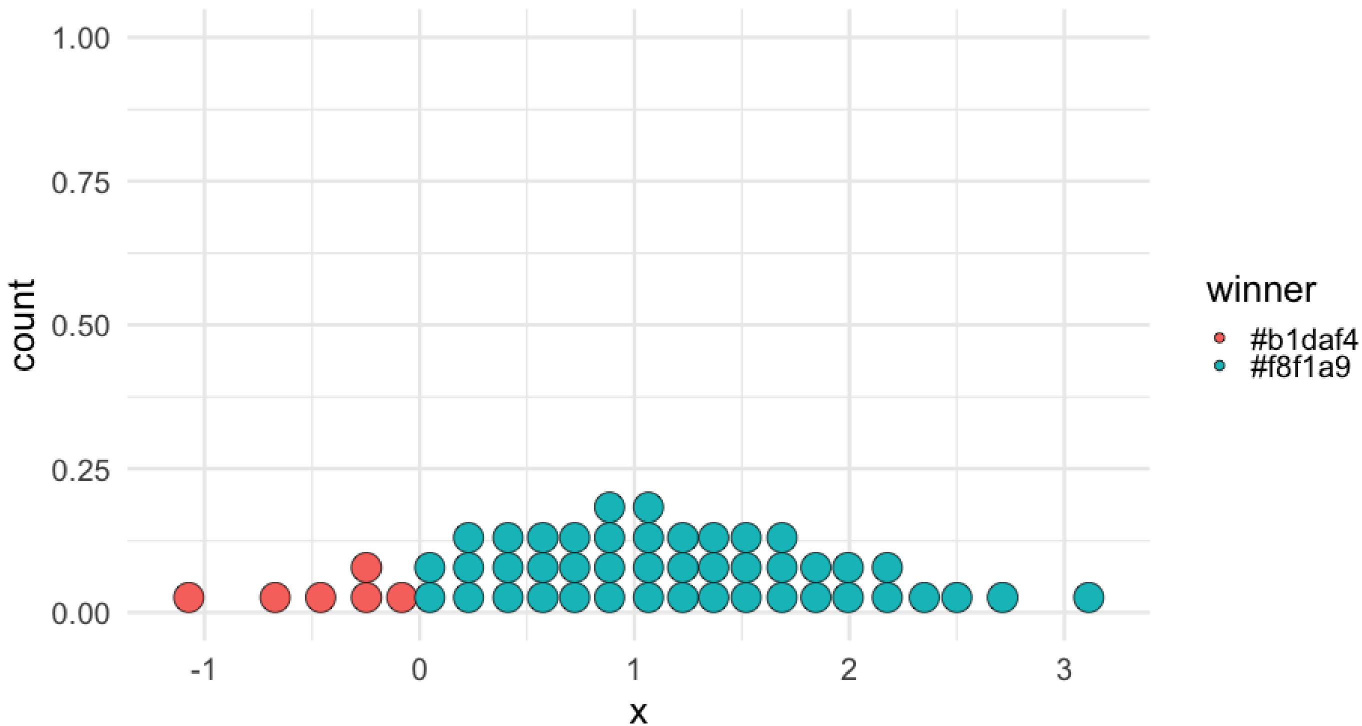
```r
tail(discretized)
```

```
##           x  winner
## 45 2.123875 #f8f1a9
## 46 2.226680 #f8f1a9
## 47 2.348212 #f8f1a9
## 48 2.500368 #f8f1a9
## 49 2.712714 #f8f1a9
## 50 3.113713 #f8f1a9
```

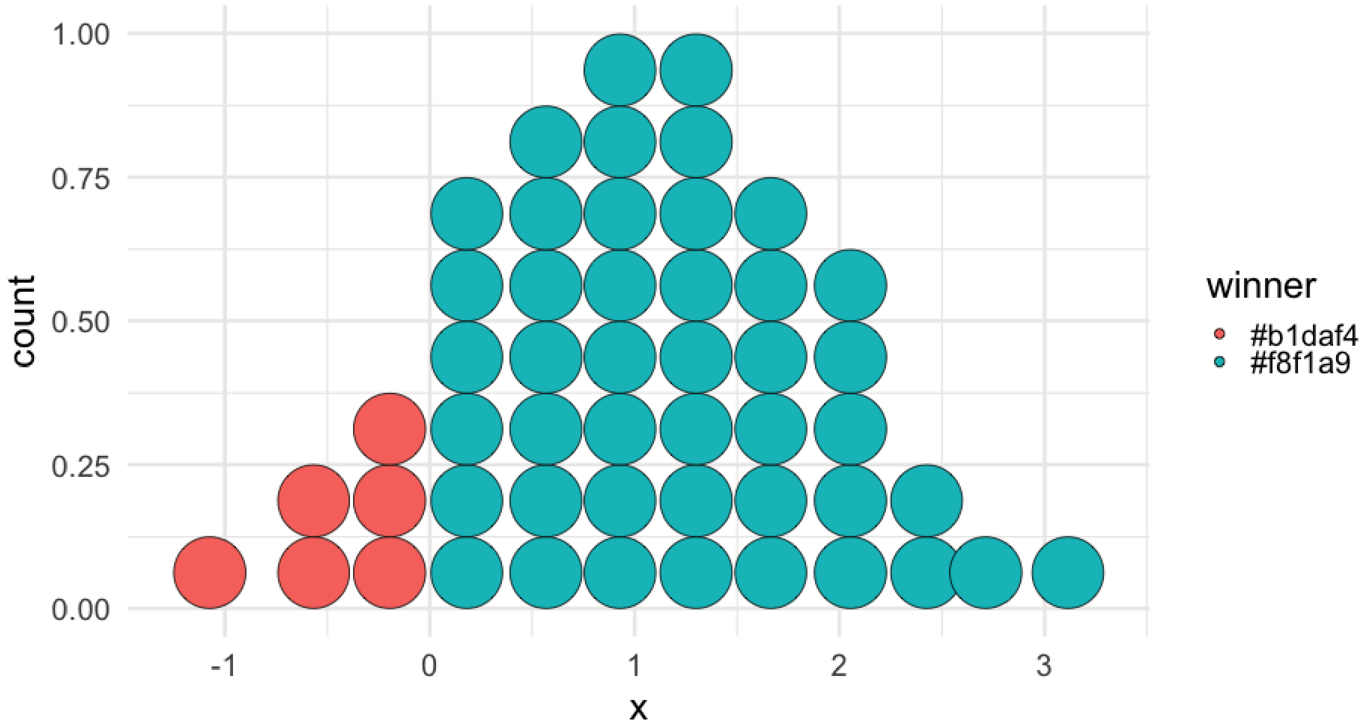
```
ggplot(discretized, aes(x)) +
  geom_dotplot()
```



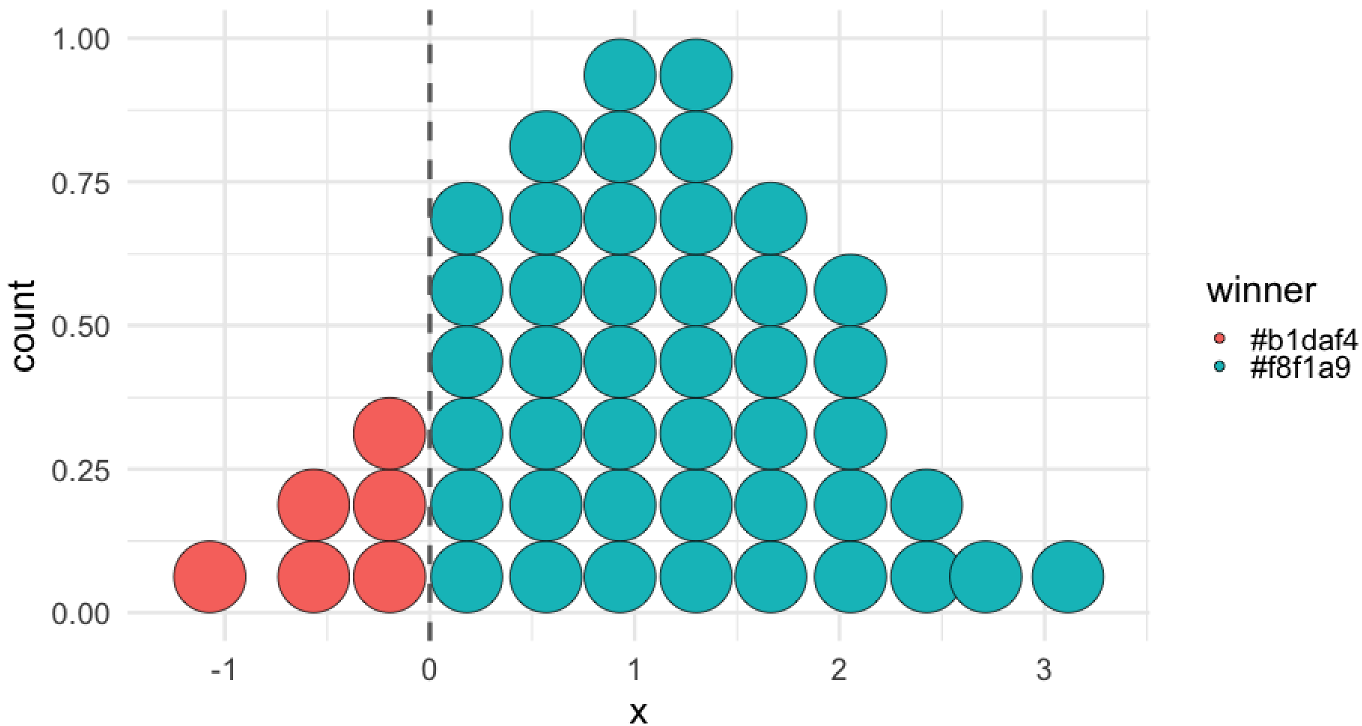
```r
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner))
```



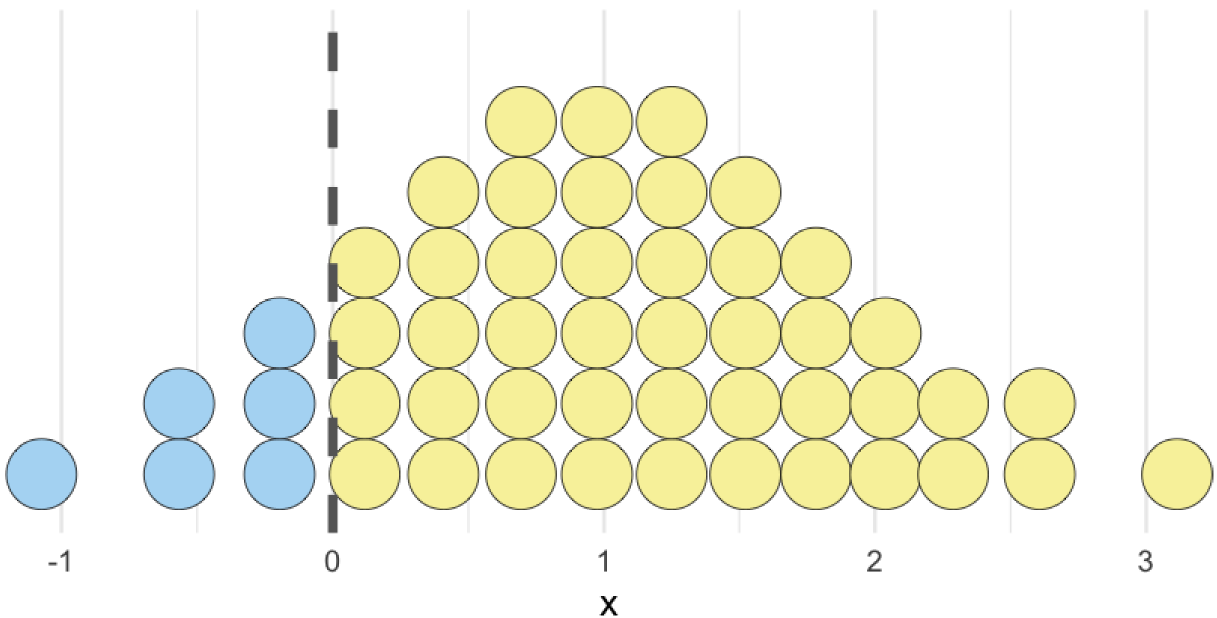
```r
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.35)
```



```r
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.35) +
  geom_vline(xintercept = 0,
             color = "gray40",
             linetype = "dashed",
             size = 1.5)
```

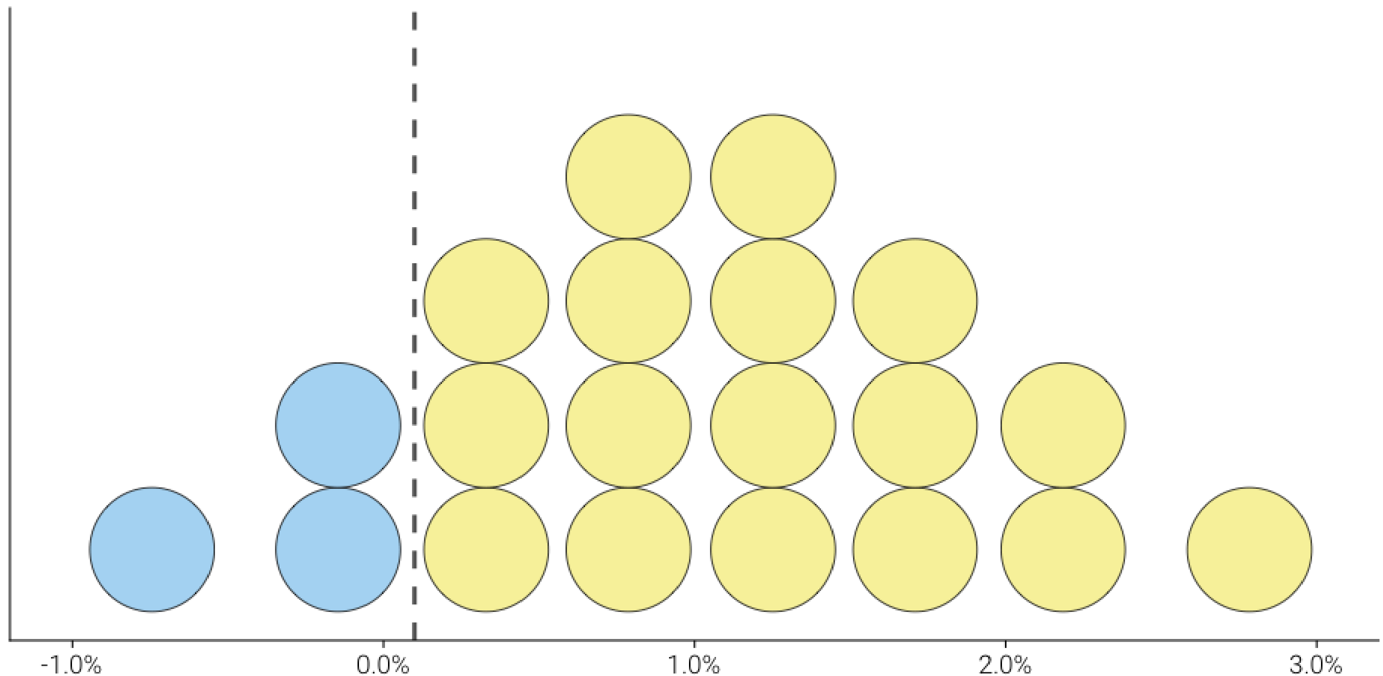


```r
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.26) +
  geom_vline(xintercept = 0,
             color = "gray40",
             linetype = 2,
             size = 3) +
  scale_fill_identity(guide = "none") +
  scale_y_continuous(name = "",
                     breaks = NULL)
```



Probs too many though

```
discretized2 <- data.frame(
  x = qnorm(ppoints(20), 1.02, 0.9)
  ) %>%
  mutate(winner = ifelse(x <= 0, "#b1daf4", "#f8f1a9"))

ggplot(discretized2, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.4) +
  geom_vline(
    xintercept = 0.1,
    color = "gray40",
    linetype = 2,
    size = 1.4) +
  scale_fill_identity(guide = "none") +
  scale_x_continuous(
    name = "",
    limits = c(-1, 3),
    labels = scales::percent_format(scale = 1)
  ) +
  theme_dviz_open(20, font_family = "Roboto Light") +
  scale_y_continuous(breaks = NULL,
                     name = "") +
  labs(caption = "Each ball represents 5% probability")
```



Each ball represents 5% probability

Uncertainty of point estimates

Quick review (hopefully a review)

- What is a standard error?
- Standard deviation of the sampling distribution
- What is the sampling distribution?
- Samples from the underlying, population–based, generative distribution
- What does this mean, exactly?
- Let's simulate to explore

Simulation

- Imagine the "real" distribution has $\mu = 100$ and $\sigma = 10$.
- Let's draw a sample of 10 from this distribution

```
set.seed(123)
samp10a <- rnorm(n = 10, mean = 100, sd = 10)
samp10a
```

```
##  [1]  94.39524  97.69823 115.58708 100.70508 101.29288 117.15065 104.609
##  [9]  93.13147  95.54338
```

- Calculate the mean

```
mean(samp10a)
```

```
## [1] 100.7463
```

Do it a second time

```r
samp10b <- rnorm(n = 10, mean = 100, sd = 10)
samp10b
```

```
##  [1] 112.24082 103.59814 104.00771 101.10683  94.44159 117.86913 104.978
##  [9] 107.01356  95.27209
```

```r
mean(samp10b)
```

```
## [1] 102.0862
```

Do it a bunch of times

```r
samples <- replicate(1000, rnorm(10, mean = 100, sd = 10),
                     simplify = FALSE)
samples
```

```
## [[1]]
##  [1]  89.32176  97.82025  89.73996  92.71109  93.74961  83.13307 108.377
##  [9]  88.61863 112.53815
##
## [[2]]
##  [1] 104.26464  97.04929 108.95126 108.78133 108.21581 106.88640 105.539
##  [9]  96.94037  96.19529
##
## [[3]]
##  [1]  93.05293  97.92083  87.34604 121.68956 112.07962  88.76891  95.971
##  [9] 107.79965  99.16631
##
## [[4]]
##  [1] 102.53319  99.71453  99.57130 113.68602  97.74229 115.16471  84.512
##  [9] 101.23854 102.15942
##
## [[5]]
##  [1] 103.79639  94.97677  96.66793  89.81425  89.28209 103.03529 104.482
##  [9] 109.22267 120.50085
##
## [[6]]
##  [1]  95.08969  76.90831 110.05739  92.90799  93.11991 110.25571  97.152
```

Calculate all means

```
map_dbl(samples, mean) %>%
  head()
```

```
## [1]  95.75441 103.22045  99.91284 102.21686 101.23084  96.37082
```

- What's the sd of these means? That's the standard error.

```
map_dbl(samples, mean) %>%
  sd()
```

```
## [1] 3.144175
```

Sample size

Let's re–do this, pulling a sample of 100 each time.

```
samples2 <- replicate(1000, rnorm(100, mean = 100, sd = 10),
                      simplify = FALSE)
map_dbl(samples2, mean) %>%
  sd()
```

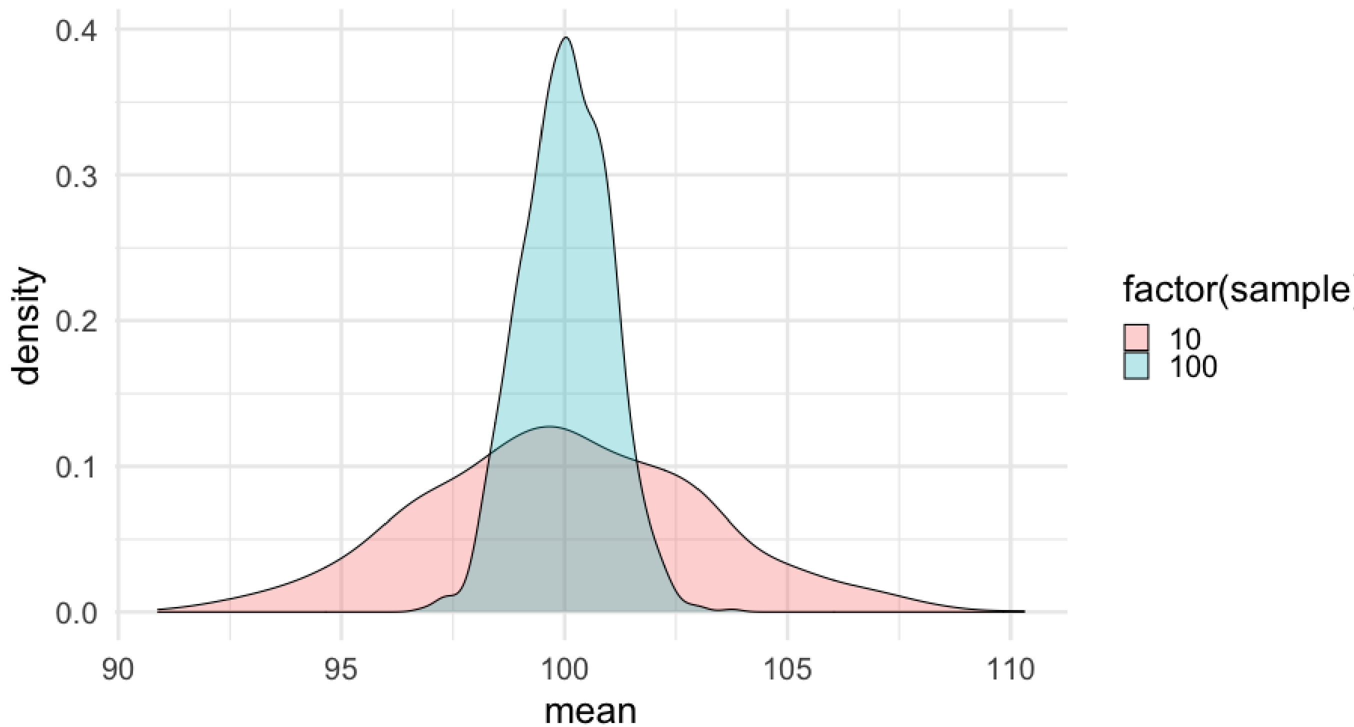
```
## [1] 0.9728883
```

Visualize the sampling distributions

```
sample_means <- tibble(
  iter = rep(1:1000, 2),
  sample = rep(c(10, 100), each = 1000),
  mean = c(map_dbl(samples, mean),
           map_dbl(samples2, mean))
)
sample_means
```

```
## # A tibble: 2,000 x 3
##     iter sample      mean
##    <int>  <dbl>     <dbl>
##  1     1     10  95.75441
##  2     2     10 103.2204
##  3     3     10  99.91284
##  4     4     10 102.2169
##  5     5     10 101.2308
##  6     6     10  96.37082
##  7     7     10 103.1310
##  8     8     10 104.3709
##  9     9     10  96.04152
## 10    10     10  96.77087
```

```r
ggplot(sample_means, aes(mean)) +
  geom_density(aes(fill = factor(sample)), alpha = 0.3)
```



Fit a model

```
m <- lm(cty ~ displ + class, mpg)
summary(m)
```

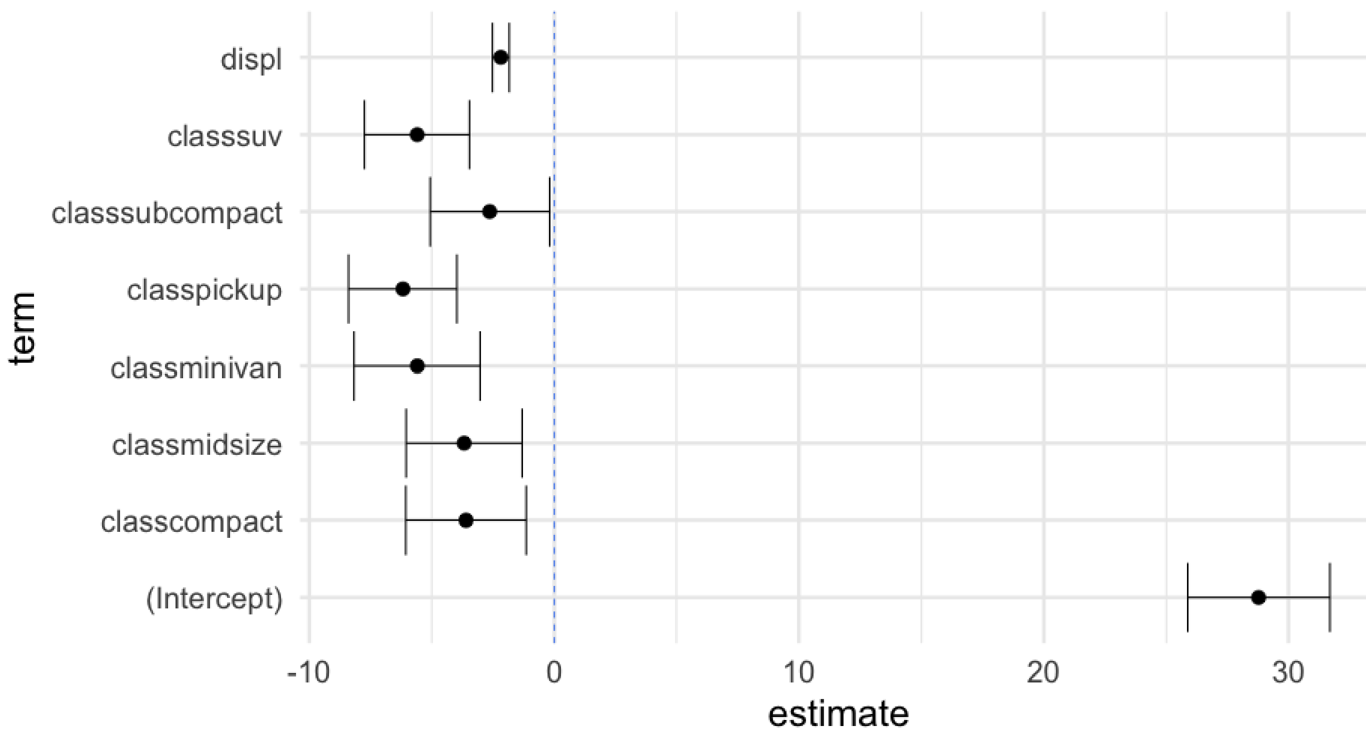
```
##
## Call:
## lm(formula = cty ~ displ + class, data = mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.2689 -1.1503 -0.0156  1.0341 12.9782
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      28.7768     1.4729  19.538  < 2e-16 ***
## displ            -2.1716     0.1747 -12.433  < 2e-16 ***
## classcompact     -3.5991     1.2522  -2.874  0.00444 **
## classmidsize     -3.6755     1.2063  -3.047  0.00259 **
## classminivan     -5.5951     1.3060  -4.284 2.71e-05 ***
## classpickup      -6.1825     1.1214  -5.513 9.60e-08 ***
## classsubcompact  -2.6290     1.2369  -2.125  0.03464 *
## classsuv         -5.5994     1.0872  -5.150 5.65e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.249 on 226 degrees of freedom
## Multiple R-squared:  0.7291,    Adjusted R-squared:  0.7207
```

Visualize with standard errors

```r
tidied_m <- broom::tidy(m, conf.int = TRUE)

tidied_m
```

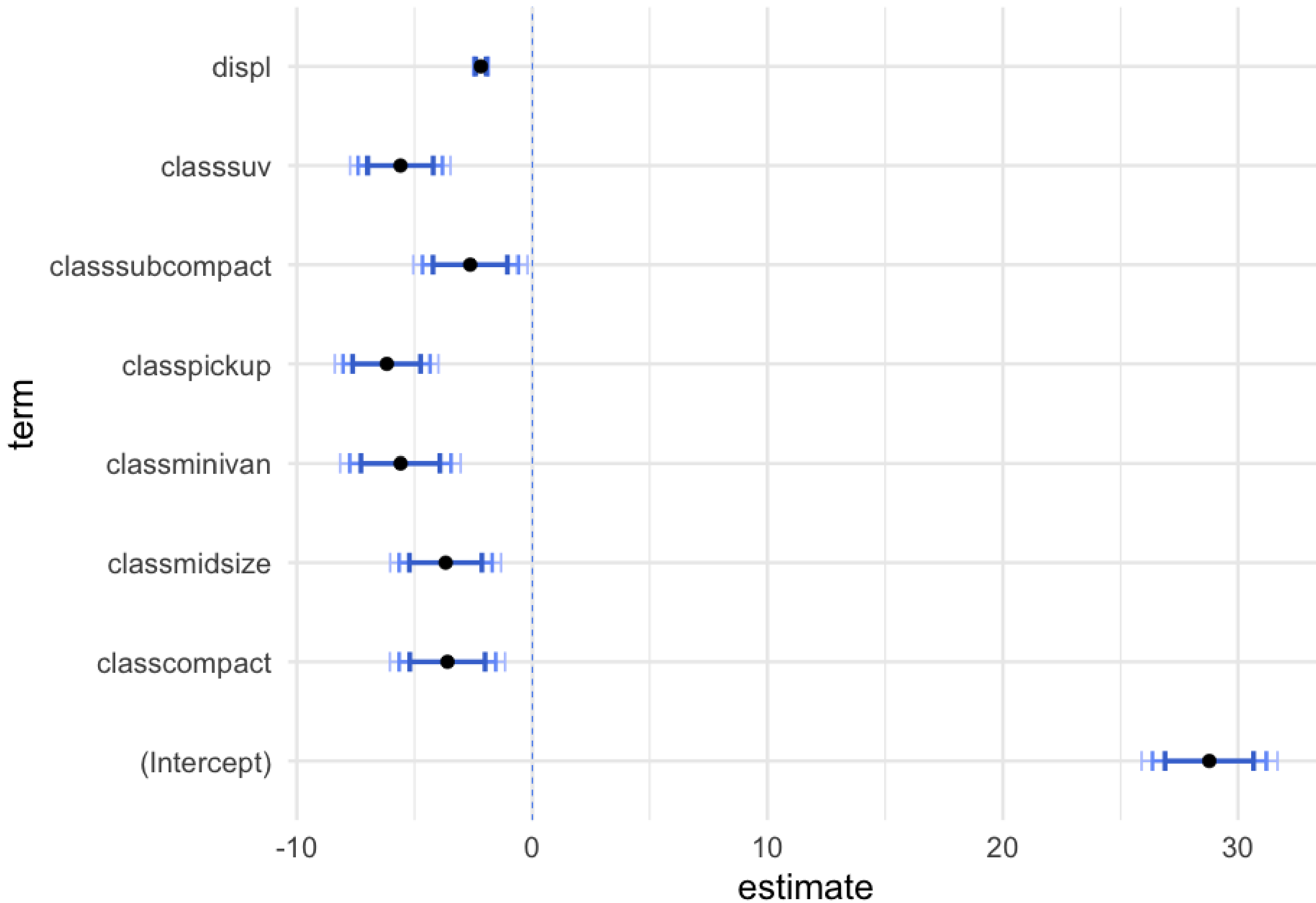
```
## # A tibble: 8 x 7
##   term            estimate std.error  statistic      p.value  conf.low
##   <chr>              <dbl>     <dbl>      <dbl>        <dbl>     <dbl>
## 1 (Intercept)     28.77682  1.472892   19.53763  1.905873e-50 25.87446
## 2 displ          -2.171562 0.1746638  -12.43281  2.197130e-27 -2.515740
## 3 classcompact   -3.599125  1.252190  -2.874265 4.436052e- 3 -6.066585
## 4 classmidsize   -3.675526  1.206253  -3.047061 2.585762e- 3 -6.052466
## 5 classminivan   -5.595070  1.305993  -4.284151 2.714490e- 5 -8.168550
## 6 classpickup    -6.182466  1.121448  -5.512931 9.600087e- 8 -8.392297
## 7 classsubcompact -2.629038 1.236950  -2.125420 3.463687e- 2 -5.066467
## 8 classsuv       -5.599361  1.087160  -5.150446 5.652249e- 7 -7.741628
```

```r
ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = conf.low, ymax = conf.high)) +
  geom_point() +
  coord_flip()
```



Multiple error bars

```
library(colorspace)
ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.025)*std.error,
                    ymax = estimate + qnorm(.975)*std.error),
                color = lighten("#4375D3", .6),
                width = 0.2,
                size = 0.8) + # 95% CI
  geom_errorbar(aes(ymin = estimate + qnorm(.05)*std.error,
                    ymax = estimate + qnorm(.95)*std.error),
                color = lighten("#4375D3", .3),
                width = 0.2,
                size = 1.2) + # 90% CI
  geom_errorbar(aes(ymin = estimate + qnorm(.1)*std.error,
                    ymax = estimate + qnorm(.9)*std.error),
                color = "#4375D3",
                width = 0.2,
                size = 1.6) + # 80% CI
  geom_point() +
  coord_flip()
```

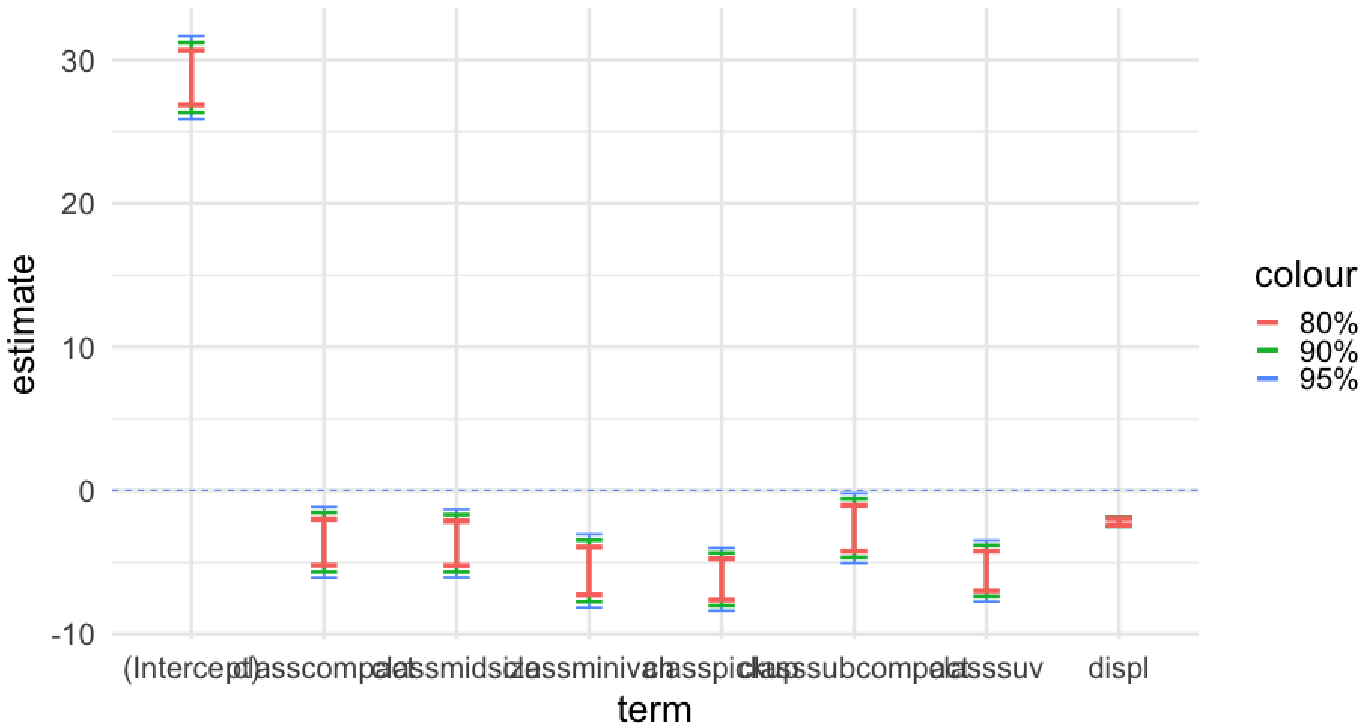

displ
classsuv
classsubcompact
classpickup
classminivan
classmidsize
classcompact
(Intercept)
term
-10
0
10
20
30
estimate

Add levels to legend

Include the color spec in `aes()`

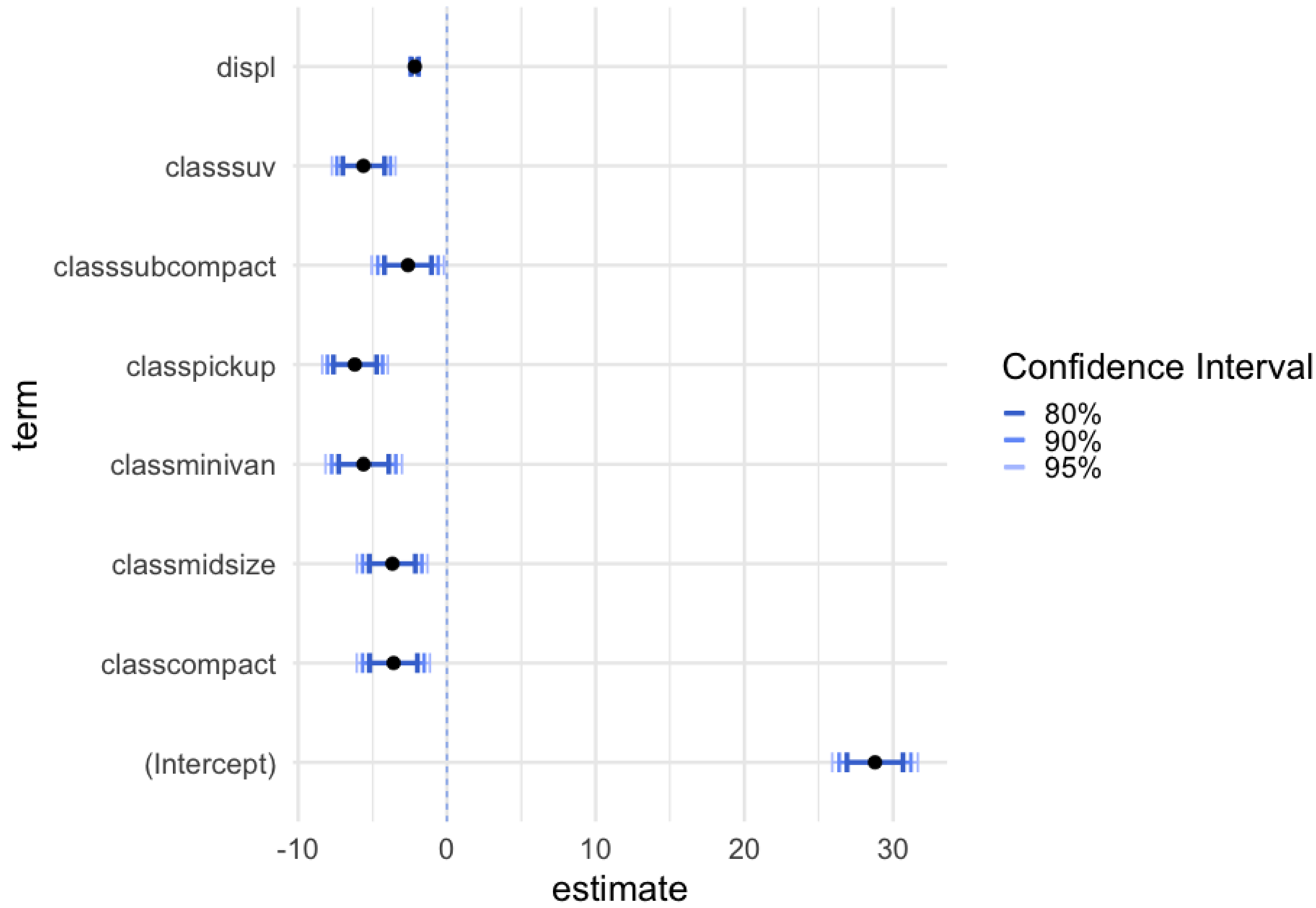
```
p <- ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.025)*std.error,
                    ymax = estimate + qnorm(.975)*std.error,
                    color = "95%"),
                width = 0.2,
                size = 0.8) +
  geom_errorbar(aes(ymin = estimate + qnorm(.05)*std.error,
                    ymax = estimate + qnorm(.95)*std.error,
                    color = "90%"),
                width = 0.2,
                size = 1.2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.1)*std.error,
                    ymax = estimate + qnorm(.9)*std.error,
                    color = "80%"),
                width = 0.2,
                size = 1.6)
```

```
p
```



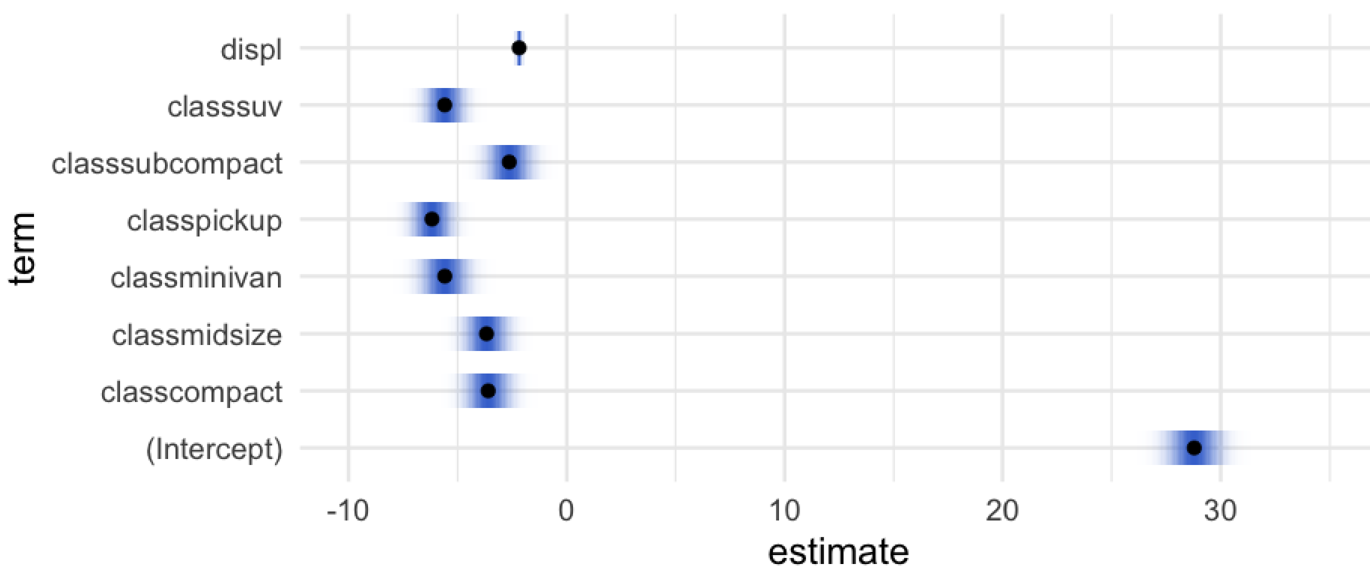
30
20
10
0
-10
estimate
(Intercept)
classcompact
classmidsize
classminivan
classpickup
classsubcompact
classsuv
displ
term
colour
80%
90%
95%

```r
p +
  scale_color_manual("Confidence Interval",
                     values = c("#4375D3",
                                lighten("#4375D3", .3),
                                lighten("#4375D3", .6))) +
  geom_point() +
  coord_flip()
```


displ
classsuv
classsubcompact
classpickup
classminivan
classmidsize
classcompact
(Intercept)
term
-10
0
10
20
30
estimate
Confidence Interval
80%
90%
95%

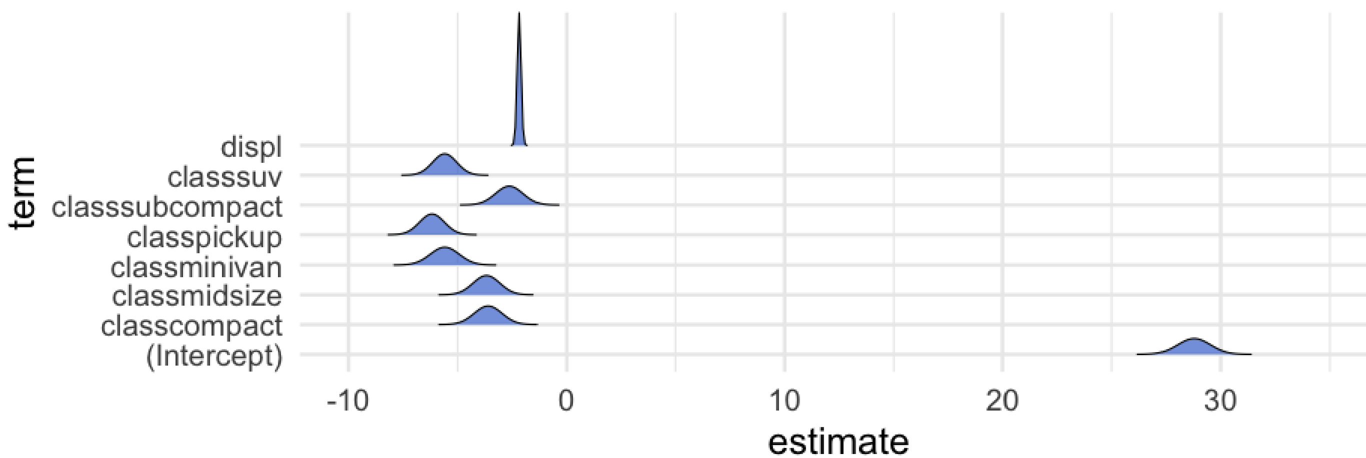
Density stripes

```
#devtools::install_github("wilkelab/ungeviz")
library(ungeviz)
ggplot(tidied_m, aes(estimate, term)) +
  stat_confidence_density(aes(moe = std.error),
                          fill = "#4375D3",
                          height = 0.6) +
  xlim(-10, 35) +
  geom_point()
```



Actual densities

```r
library(ggridges)
ggplot(tidied_m, aes(estimate, term)) +
  stat_confidence_density(aes(moe = std.error,
                              height = stat(density)),
                          geom = "ridgeline",
                          confidence = 0.95,
                          min_height = 0.001,
                          alpha = 0.7,
                          fill = "#4375D3") +
  xlim(-10, 35)
```

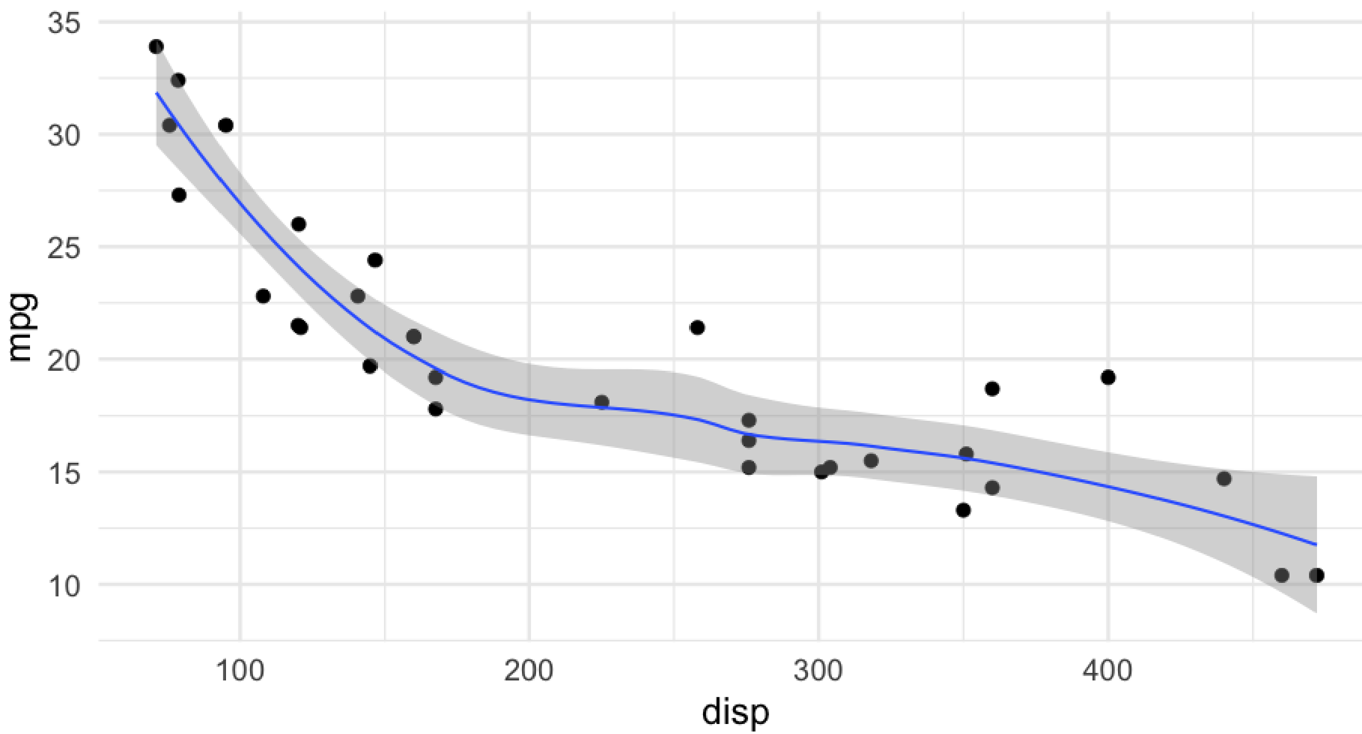


HOPs

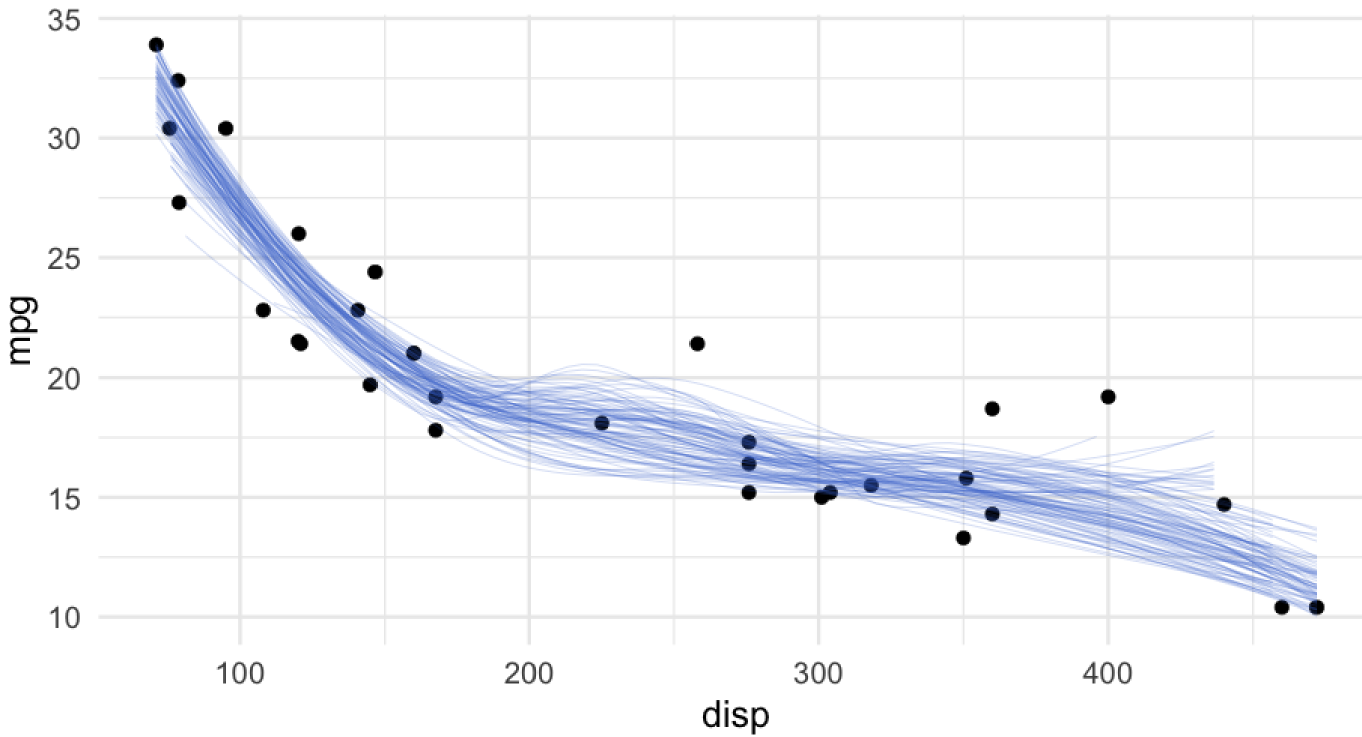
Hypothetical Outcome Plots (and related plots)

Standard regression plot

```
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  geom_smooth()
```



Alternative



How?

Bootstrapping

```r
row_samps <- replicate(100,
                       sample(seq_len(nrow(mtcars)),
                              nrow(mtcars),
                              replace = TRUE),
                       simplify = FALSE)
row_samps
```

```
## [[1]]
##  [1] 31  6 32 12  1 14  2 11 20 10 26 10 22 30 25 25  5 31 19 13 19 20 1
## [29] 20 21 16 23
##
## [[2]]
##  [1] 11 23 14  1 24 20 10 30 27 24 22 23 25  1 18 18 25  8  8 16 25 19 3
## [29] 14 14 12 24
##
## [[3]]
##  [1] 27 29 22  5  6  8 14 16  7 13 17 13 21 10  7 21  7 20 30 30  5 10
## [29] 27 23 19  7
##
## [[4]]
##  [1] 16  7  8 28  3 17 13 26  8 30  3 32 20 10  2  6 19 21 11  6 16  9 1
## [29] 19 21  1 16
```

Extract samples

```r
d_samps <- map_df(row_samps, ~mtcars[.x, ], .id = "sample")
head(d_samps)
```

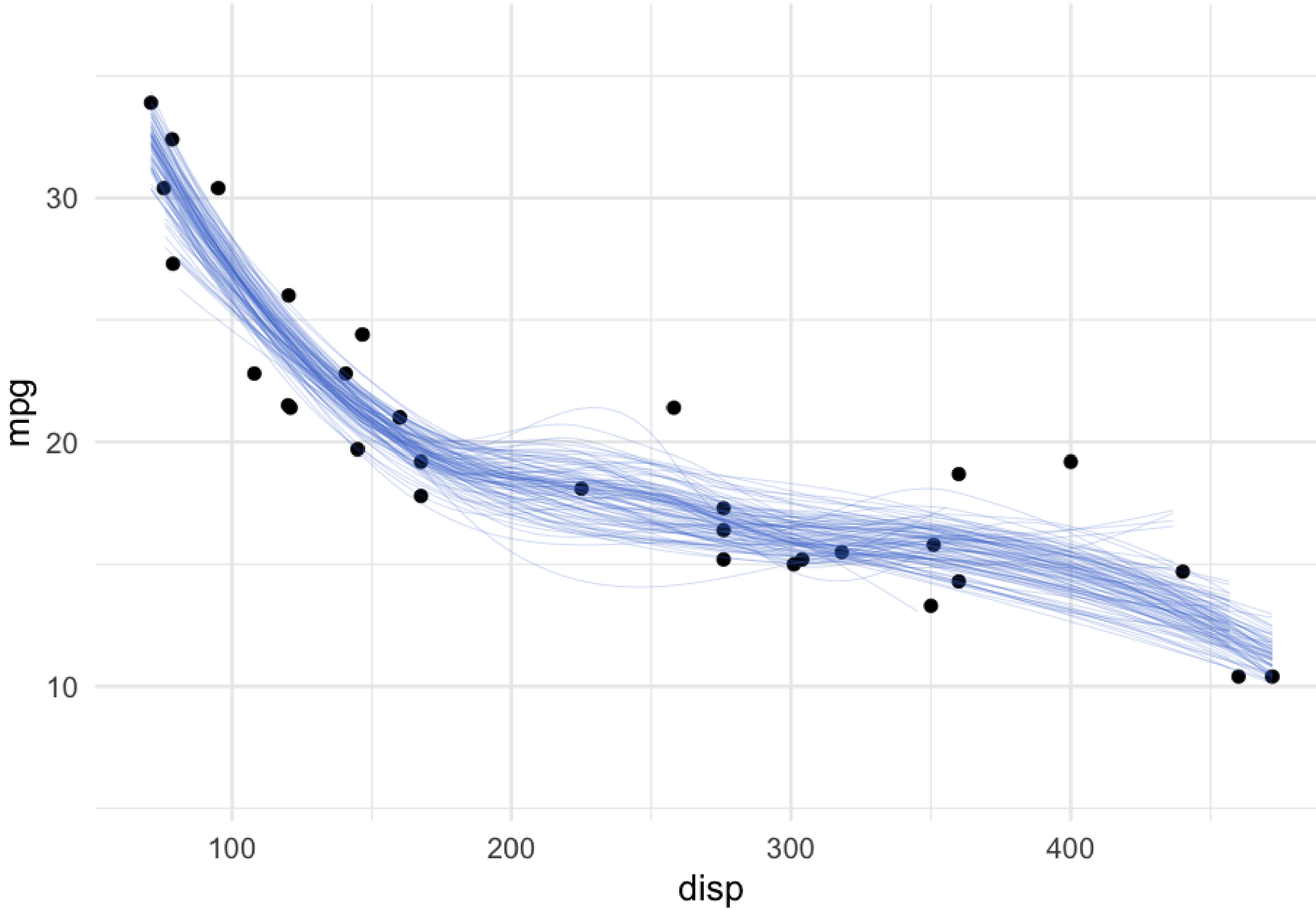
```
##                        sample  mpg cyl  disp  hp drat   wt  qsec vs am gear c
## Maserati Bora...1           1 15.0   8 301.0 335 3.54 3.57 14.60  0  1    5
## Valiant...2                 1 18.1   6 225.0 105 2.76 3.46 20.22  1  0    3
## Volvo 142E...3              1 21.4   4 121.0 109 4.11 2.78 18.60  1  1    4
## Merc 450SE...4              1 16.4   8 275.8 180 3.07 4.07 17.40  0  0    3
## Mazda RX4...5               1 21.0   6 160.0 110 3.90 2.62 16.46  0  1    4
## Merc 450SLC...6             1 15.2   8 275.8 180 3.07 3.78 18.00  0  0    3
```

```r
tail(d_samps)
```

```
##                                   sample  mpg cyl  disp  hp drat    wt  qsec
## Lincoln Continental.1...3195         100 10.4   8 460.0 215 3.00 5.424 17.82
## Pontiac Firebird...3196              100 19.2   8 400.0 175 3.08 3.845 17.05
## Fiat X1-9.1...3197                   100 27.3   4  79.0  66 4.08 1.935 18.90
## Mazda RX4 Wag.3...3198               100 21.0   6 160.0 110 3.90 2.875 17.02
## Hornet Sportabout...3199             100 18.7   8 360.0 175 3.15 3.440 17.02
## Lotus Europa...3200                  100 30.4   4  95.1 113 3.77 1.513 16.90
```

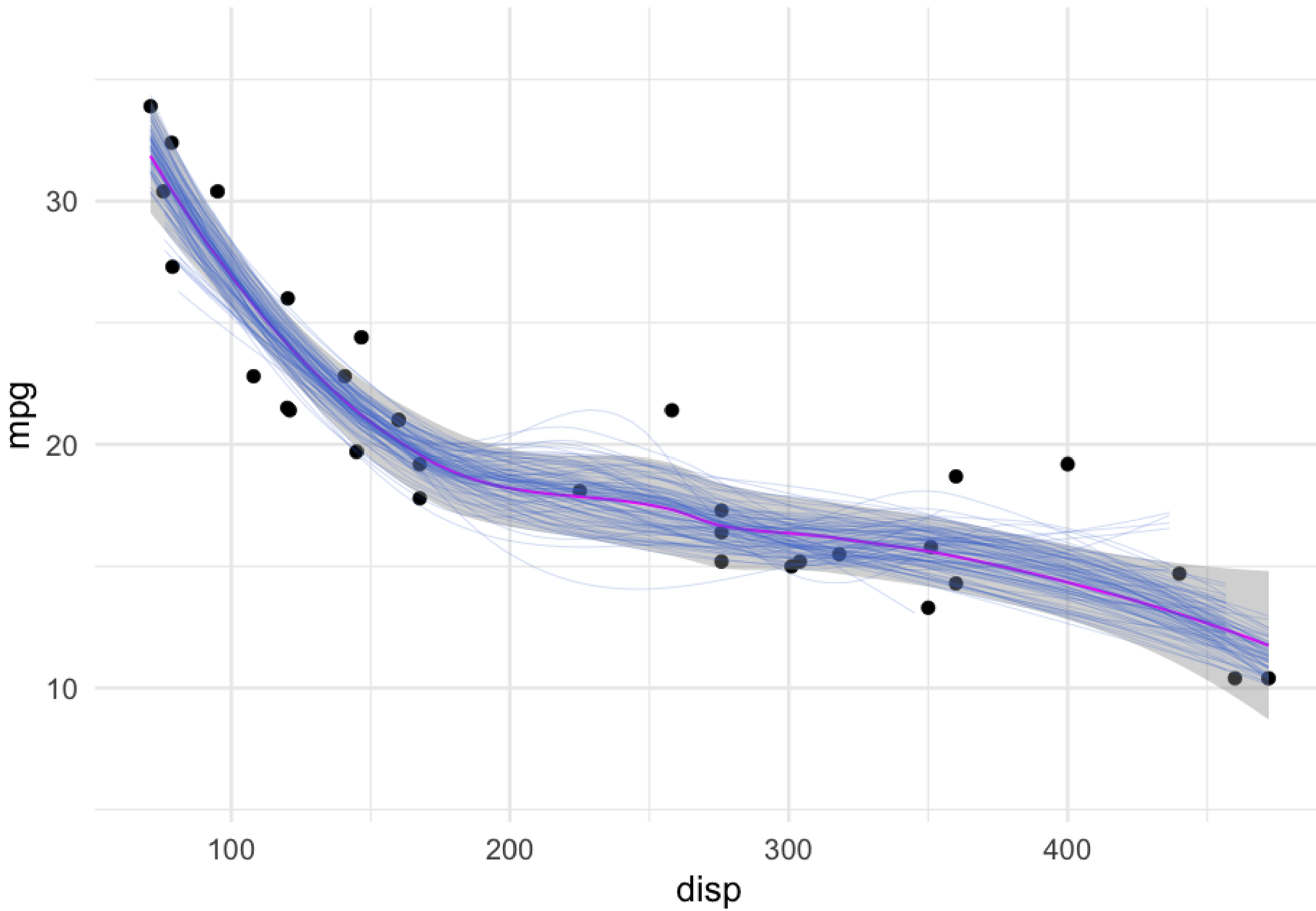
Plot both data sources

```r
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  stat_smooth(aes(group = sample),
              data = d_samps,
              geom = "line",
              color = "#4375D3",
              fullrange = TRUE,
              size = 0.1)
```


30
20
10
mpg
100
200
300
400
disp

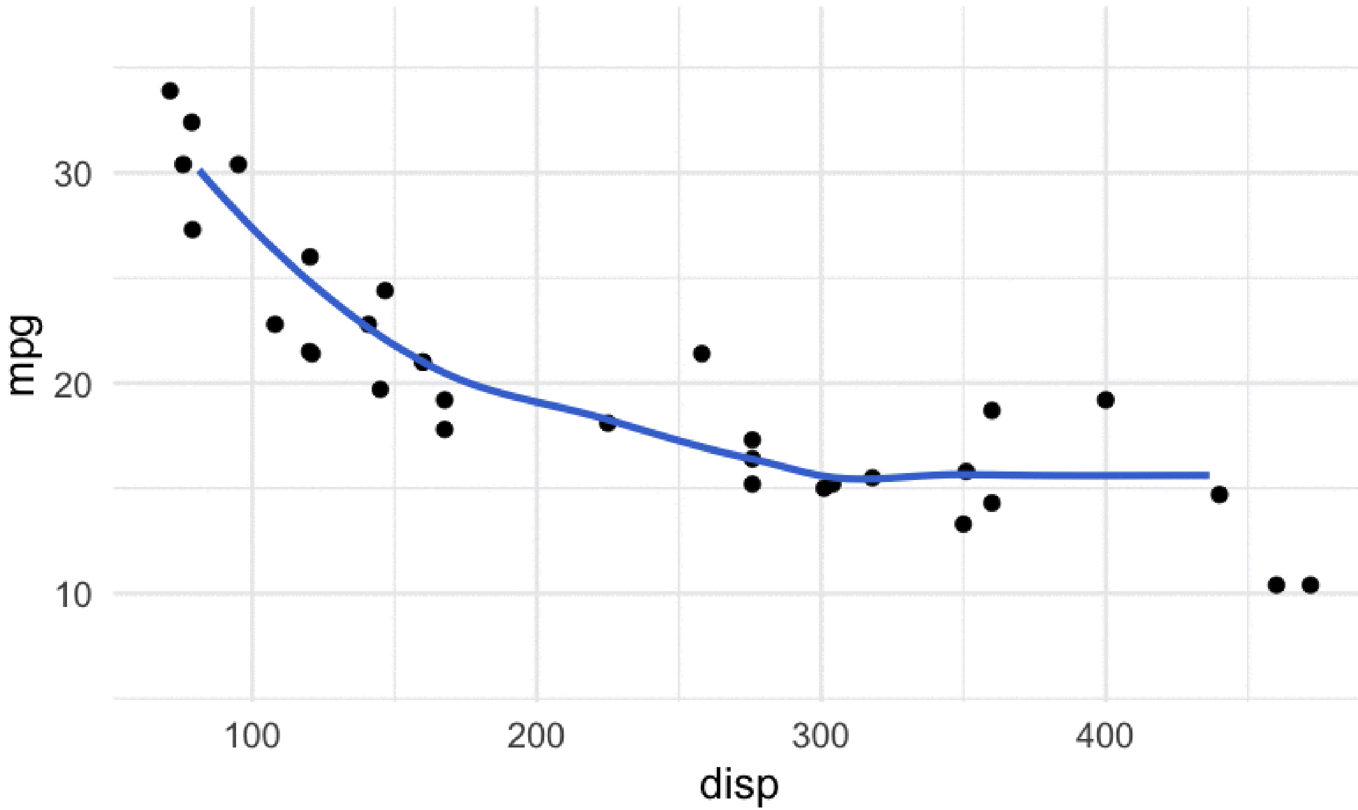
Note, they match up

```
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  geom_smooth(color = "magenta") +
  stat_smooth(aes(group = sample),
              data = d_samps,
              geom = "line",
              color = "#4375D3",
              fullrange = TRUE,
              size = 0.1)
```


mpg
30
20
10
100
200
300
400
disp

HOPs

Hops animate the process, so you can't settle on one "truth"



How?

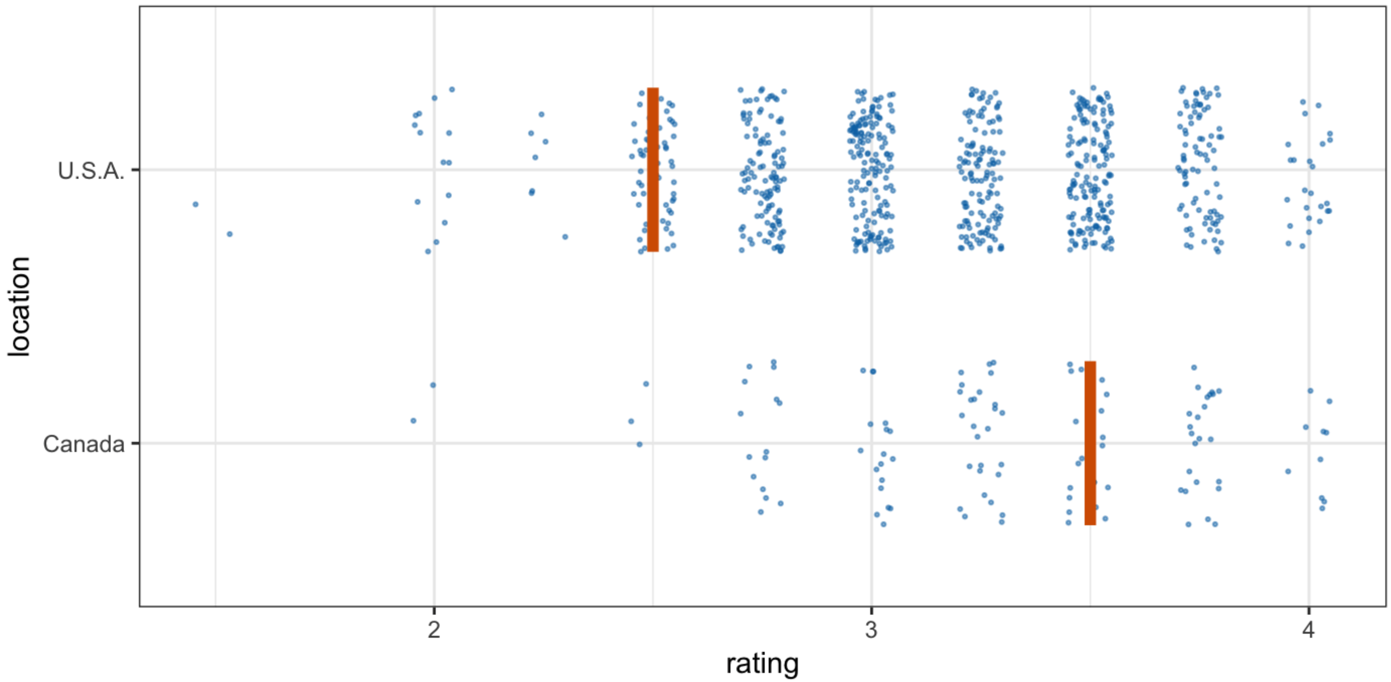
gganimate::transition_states

```
library(gganimate)
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  stat_smooth(data = d_samps,
              geom = "line",
              size = 2,
              color = "#4375D3",
              fullrange = TRUE) +
  transition_states(sample,
                    transition_length = 0.5,
                    state_length = 0.5) +
   ease_aes('linear') # Smoother transitions
```

Another example

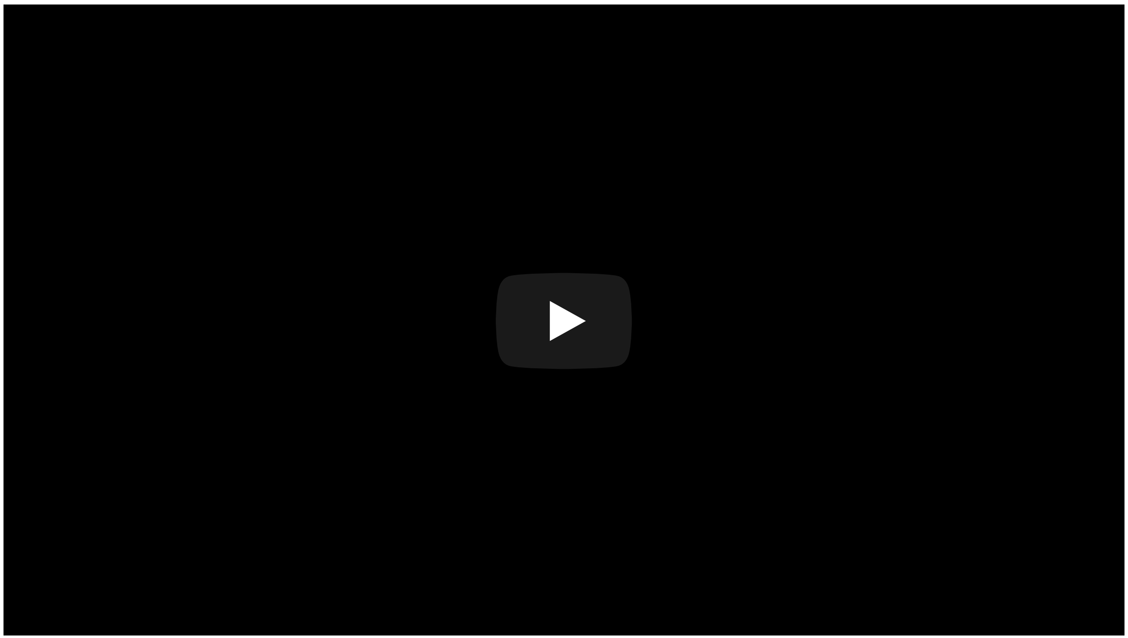
Canada has the highest average rating for their chocolate bars in the world, according to a recent study. But the sample size is also smaller than other countries, like the US.

How certain are we that Canada's chocolate is better? Compare the ratings of a randomly chosen Canadian chocolate bar to a randomly chosen US Chocolate bar.


U.S.A.
Canada
location
2
3
4
rating

Another example

From Dr. Kay again



Conclusions

- Lots of tools at your disposal (perhaps so many it can be difficult to choose)
- Do try to communicate uncertainty whenever possible
- I'd recommend checking out Clause Wilke's talk from `rstudio::conf(2019L)`, where he talks about the ungeviz package.

Next time

Tables and Fonts