Scale Recurrence Across Cosmic Structures: A Statistical Analysis of the 10^{24}-Meter Pattern

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Abstract

We document a pattern of scale recurrence across the universe in which selected canonical, directly-observable organizational structures appear separated by characteristic length ratios near 10^{24}. We term these separations "cosmic octaves" by analogy to musical octave spacing (logarithmic recurrence, not a proposed causal mechanism). Using a predefined structure ladder and a consistent characteristic-length definition across domains, we compile 15 structures spanning quantum to cosmological scales (≈42 orders of magnitude). We apply a fixed pairing rule that generates 7 octave pairs (one per rung, from microscopic to macroscopic).

Three of these seven pairs exhibit deviations ≤0.2 from the ideal $\log_{10}$ ratio of 24.0. Using a permutation test (n=200,000, fixed seed) that shuffles the 15 measured log-lengths among labels while keeping the 7 pairs fixed, we obtain **p=0.000055** for observing ≥3 strong matches (deviation ≤0.2) by chance. A conservative "look-elsewhere" scan over $\Delta \in [22, 26]$ in steps of 0.05 (maximizing strong matches per permutation) still yields a highly small probability (upper bound $\approx 1\times10^{-5}$ under this scan protocol with 200,000 permutations).

We present this as an empirical scaling observation that warrants further investigation, not as proof of a universal law. All assumptions, data, calculations, and code are provided for independent verification. We discuss measurement ambiguities, alternative explanations, and falsifiable predictions.

Keywords: fractal universe, scale invariance, cosmic scaling, logarithmic periodicity, permutation test, organizational structures, Our Fractal Universe

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1. Introduction

1.1 Motivation

Nature shows recurring structure across scales—branching networks in organisms, cities, and river basins; hierarchical clustering in cosmic matter distribution; and repeated statistical regularities in complex systems. Fractal geometry and allometric scaling have proven useful in biology and urban systems (Mandelbrot, 1982; West et al., 1997; West, 2017). Whether specific scale recurrences extend from biology to cosmology remains open and easy to mis-handle statistically due to selection effects.

This paper tests a specific quantitative claim: some canonical organizational structures recur at characteristic logarithmic intervals near 24 in base-10 length space, i.e., ratios near 10^{24}. We approach with skepticism and explicitly design methodology to reduce (not eliminate) selection bias, definition drift, and post-hoc tuning.

1.2 The "Cosmic Octave" Hypothesis

We test whether a set of fundamental structures, when represented by a consistent characteristic length L, form repeated separations of approximately:

$$\log_{10}(L\_large / L\_small) \approx 24$$

This is a **pattern claim, not a mechanism claim**. The paper does not propose a physical cause for octave spacing. It asks whether the observed recurrence is unlikely under a transparent null model.

1.3 Potential Explanations (Speculative)

If the pattern is real and replicable, candidate explanations include:

1. **Artifact** (selection/definition freedom)
2. **Privileged physical scales** (constraints from force regimes)

3. **Organizational convergence** (energy/information optimization across substrates)
4. **Anthropic observation bias** (structures we name and measure best are not random)

We do not endorse any explanation here; we only quantify the observation and its statistical rarity under a controlled null.

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2. Methods

2.1 Structure Selection Criteria

To reduce bias, we use a predefined ladder of canonical organizational units and apply explicit rules.

Inclusion requirements:

1. Structures represent widely recognized organizational units at their scale (canonical "rungs")
2. Length values are taken from peer-reviewed sources or international standards where available (CODATA, IAU, Planck), with a stated sensitivity when definitions vary
3. Structures are directly observable / empirically supported as organizational units at their scale
4. No value is adjusted to improve fit after ratio calculation

Exclusion rule (important change):

We exclude structures that are not directly observed as a confirmed organizational unit at the relevant scale within the scope of this paper's "canonical rung" definition. In particular, we **exclude the Oort Cloud** (hypothesized reservoir with large uncertainty and indirect inference), to avoid double-counting a partially speculative rung that overlaps the open-cluster scale in our ladder.

This yields **15 structures** spanning approximately 10^{-15} to 10^{27} meters.

2.2 Measurement Methodology: Characteristic Length L

Many structures do not have a well-defined "radius." We therefore use a characteristic length scale L, chosen consistently by structure class:

Structure Type	L definition	Rationale
Subatomic	RMS charge radius	Standard particle physics measurement
Atomic	Bohr radius a_0	Standard textbook bound-length scale
Molecular/Cellular	Half maximum dimension	Consistent proxy across irregular shapes
Multicellular	Half body length/height	Matches "half-length" cellular convention
Planetary/Stellar	Mean radius / photosphere	IAU standard quantities
Orbital systems	Bound extent (outer stable orbit proxy)	Consistent "extent of bound influence"
Galactic/ supercluster	Half-extent proxy	Common astronomical convention for scale

Crucially, we apply matching measurement logic across octave partners (e.g., "extent of bound influence" ↔ "extent of bound influence").

2.3 Verified Scale Table

Table 1. Characteristic length scales L (15 structures)

Structure	L (m)	$\log_{10}(L)$	Measurement Type	Source	Sensitivity / Notes
Proton	8.4×10^{-16}	-15.08	RMS charge radius	CODATA 2018	±2%
Atomic Orbital (H)	5.29×10^{-11}	-10.28	Bohr radius	CODATA 2018	"90% boundary" ~3× larger
Ribosome (70S)	1.1×10^{-8}	-7.96	Half diameter	BioNumbers	±20%
Bacterium (E. coli)	1.0×10^{-6}	-6.00	Half cell length	BioNumbers	±30%
C. elegans	5.0×10^{-4}	-3.30	Half body length	NCBI (≈1 mm adult)	±15%
Human	9.0×10^{-1}	-0.046	Half body height	Representative 1.8 m	±10%
City	1.0×10^{3}	3.00	Coordination radius	West (2017)	Factor ~5 (1–5 km)
Earth	6.37×10^{6}	6.80	Mean radius	IAU 2015	±0.01%
Sun	6.96×10^{8}	8.84	Photosphere radius	IAU 2015	±0.01%
Solar System	4.5×10^{12}	12.65	Neptune orbit (bound extent)	IAU 2015	Heliopause ~2×
Open Cluster	4.7×10^{16}	16.67	Half-mass/ characteristic cluster radius	Literature typical	Factor ~4 across clusters
Local Bubble	4.629×10^{18}	18.665	Median boundary distance	Pelgrims+ (2020)	80–360 pc (irregular)
Milky Way	5.0×10^{20}	20.70	Half stellar disk scale	Bland-Hawthorn & Gerhard (2016)	±10%
Virgo Supercluster	6.9×10^{23}	23.84	Half density-extent proxy	Standard value	Laniakea alternative noted
Observable Universe	4.4×10^{26}	26.64	Particle horizon	Planck 2018	±2%

Note (Virgo vs. Laniakea): Virgo is used as the representative "supercluster" rung by a traditional density-extent convention. Laniakea (Tully et al., 2014) is a larger flow-defined superstructure and is treated as a sensitivity alternative rather than the canonical rung value.

2.4 Pairing Rule and Deviation Metric

In this paper's canonical ladder, we define one octave partner per rung, producing **7 pairs**:

1. Proton → Sun
2. Atomic orbital → Solar System
3. Ribosome → Open Cluster
4. Bacterium → Local Bubble
5. C. elegans → Milky Way
6. Human → Virgo Supercluster
7. City → Observable Universe

For each pair:

$R = \log_{10}(L\_large) - \log_{10}(L\_small)$
$\Delta = |R - 24.0|$

Quality thresholds (predefined):

- **Perfect**: $\Delta \leq 0.05$
- **Excellent**: $\Delta \leq 0.2$
- **Very Good**: $\Delta \leq 0.7$
- **Good**: $\Delta \leq 1.0$
- **Fair**: $\Delta > 1.0$

2.5 Octave Pairs

Table 2. Seven octave pairs (canonical ladder pairing)

Pair	Small	Large	Ratio R	Deviation Δ	Quality
1	Proton (-15.08)	Sun (8.84)	23.92	0.08	Excellent
2	Atomic Orbital (-10.28)	Solar System (12.65)	22.93	1.07	Fair
3	Ribosome (-7.96)	Open Cluster (16.67)	24.63	0.63	Very Good
4	Bacterium (-6.00)	Local Bubble (18.665)	24.665	0.665	Very Good
5	C. elegans (-3.30)	Milky Way (20.70)	24.00	0.00	Perfect
6	Human (-0.046)	Virgo SC (23.84)	23.886	0.114	Excellent
7	City (3.00)	Observable Universe (26.64)	23.64	0.36	Very Good

Summary:

- **Strong matches ($\Delta \leq 0.2$):** 3/7 (one perfect + two excellent)
- **Within $\Delta \leq 0.7$:** 6/7
- One pair is "fair" (atomic orbital → solar system) under the present characteristic-length definitions

2.6 Statistical Significance (Permutation Test)

Null hypothesis:

If the 15 measured log-lengths are randomly assigned to the 15 labels, while the 7 canonical pairs remain fixed, how often do we obtain ≥3 strong matches ($\Delta \leq 0.2$)?

Why permutation testing:

A permutation test directly matches the claim: these fixed pairs (from a predefined ladder) are near $\Delta = 24.0$ more often than expected under random assignment. This avoids testing a mismatched null (e.g., "any adjacent logs").

Reproducible code:

```python
import numpy as np

# 15 log10(L) values from Table 1, in the same order as the label list below
# [Proton, AtomicOrbital, Ribosome, Bacterium, C_elegans, Human, City,
#  Earth, Sun, SolarSystem, OpenCluster, LocalBubble, MilkyWay, VirgoSC, ObsUniverse]
logs = np.array([
    -15.08, -10.28, -7.96, -6.00, -3.30, -0.046, 3.00,
     6.80,   8.84,  12.65, 16.67, 18.665, 20.70, 23.84, 26.64
])

# 7 fixed pairs (small_index, large_index) using the label order above:
pairs = [
    (0, 8),   # Proton -> Sun
    (1, 9),   # Atomic orbital -> Solar System
    (2, 10),  # Ribosome -> Open Cluster
    (3, 11),  # Bacterium -> Local Bubble
    (4, 12),  # C. elegans -> Milky Way
    (5, 13),  # Human -> Virgo Supercluster
    (6, 14)   # City -> Observable Universe
]

def deviations(arr, delta=24.0):
    return np.array([abs((arr[j] - arr[i]) - delta) for i, j in pairs])

obs = deviations(logs)
obs_strong = np.sum(obs <= 0.2)

print("Observed deviations:", np.round(obs, 3))
print("Observed strong matches (<=0.2):", int(obs_strong))

# permutation test
n_trials = 200000
rng = np.random.default_rng(42)
count = 0

for _ in range(n_trials):
    perm = rng.permutation(logs)
    if np.sum(deviations(perm) <= 0.2) >= obs_strong:
        count += 1

p_value = count / n_trials
print("Permutation p-value:", p_value, f"({p_value*100:.4f}%)")
print("Successes:", count, "out of", n_trials)
```

Verified output (this manuscript configuration):

```
 Observed deviations: [0.08  1.07  0.63  0.665 0.    0.114 0.36 ]
Observed strong matches (<=0.2): 3
Permutation p-value: 0.000055 (0.0055%)
Successes: 11 out of 200000
```

This corresponds to approximately **3.9σ** (one-sided normal equivalent) for the fixed-Δ test.

2.7 Look-Elsewhere / Δ-Scan (Conservative Check)

Because the interval "24" was initially noticed empirically, we also report a conservative correction: allow Δ to vary from 22 to 26 (step 0.05), and for each permutation record the maximum number of strong matches achieved at any scanned Δ. The observed data achieves a maximum of 4 strong matches for some Δ within the scan window. Under 200,000 permutations, this event occurred once.

Because one success in 200,000 implies limited resolution, we report:

- **Empirical estimate:** p_scan $\approx 5\times10^{-6}$
- **Conservative upper bound** (add-one): $(1+1)/(200000+1) \approx 1.0\times10^{-5}$

Either way, the scan correction remains very small under this defined protocol.

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3. Results

- Using a canonical ladder and consistent length definitions, **6/7 pairs** fall within $\Delta \le 0.7$ of 24.0, and **3/7 are strong** ($\Delta \le 0.2$)
- A permutation test on the fixed 7 pairs yields **p = 0.000055** for ≥3 strong matches (n = 200,000)
- A conservative Δ-scan protocol (22–26) still yields a very small probability (upper bound $\approx 10^{-5}$)

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4. Discussion

4.1 What This Result Does and Does Not Imply

It does: Show that, under a clearly defined null model (random reassignment of the measured log-lengths to labels), the observed number of strong octave separations in the fixed canonical pairs is rare.

It does not: Prove a mechanism, a universal law, or that "everything is fractal." Statistical rarity is not causation.

4.2 Remaining Vulnerabilities / Honest Limitations

Post-hoc origin of $\Delta \approx 24$: Even with a scan correction, the hypothesis originated from noticing alignments; independent replication is essential.

Definition freedom remains: Although constrained, L is not uniquely defined for cities, the Local Bubble, and clusters. Sensitivity ranges reduce but do not eliminate degrees of freedom.

Small N: With only 7 pairs, the result is sensitive to classification thresholds and the ladder definition.

Canonical ladder assumption: The ladder itself is a modeling decision; alternative ladders could change the pair set.

4.3 Falsifiable Predictions

If the octave recurrence reflects something real (not artifact), then:

1. Intermediate rungs should exist and/or tighten: Between the Solar System ($\sim 10^{12.6}$) and open clusters ($\sim 10^{16.7}$), the ladder predicts additional empirically robust organizational structures with characteristic scales that preserve approximate octave relationships.

2. Independent re-measurement should preserve the deviations: Using different datasets/definitions for Local Bubble boundary, open cluster characteristic radius, and city coordination radius should not destroy the overall concentration near $\Delta \approx 24$ under equivalent pairing rules.

3. Cross-domain constraints should appear: If recurrence is physical rather than classificatory, constraints from formation physics should correlate with the octave separations (e.g., stability/formation efficiency changes near octave boundaries).

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5. Conclusion

We report an empirical scale-recurrence observation: a canonical set of seven micro-to-cosmic pairs shows clustering near a log-length separation of 24, with 3 strong matches ($\Delta \leq 0.2$) and 6/7 within $\Delta \leq 0.7$. Under a permutation null model that directly matches the fixed-pair hypothesis, the probability of ≥3 strong matches is **p = 0.000055** (n = 200,000; fixed seed). A conservative Δ-scan protocol remains highly unlikely under the same permutation framework.

This does not establish mechanism, but it provides a transparent, testable statistical claim. We invite independent replication, alternate ladder definitions, and re-measurement with conservative definitions to test whether this "10^{24}" spacing reflects organizing principles or an artifact of classification and measurement choice.

Data and Code Availability

All data tables and the exact Python scripts used for permutation and Δ-scan testing are provided in the **cosmic-octaves-analysis** repository. For replication, the paper includes complete arrays and pair indices.

Repository: https://github.com/Chris-L78/cosmic-octaves-analysis

Video explanations: Our Fractal Universe YouTube channel (@OurFractalUniverse)

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