MA228 NUMERICAL ANALYSIS - EXERCISE SHEET 3

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These exercises need to be handed in on Friday, 12 February 2016, 12:00 noon. Note you have 2 weeks in contrast to the previous exercises.

1. Task

Implement the pseudo-code of Euler's method as given in the lecture inside MATLAB (or any other programming language). Choose a function $f : [a, b] \times \mathbb{R} \to \mathbb{R}$ and the time interval with $a < b$, and solve the differential equation for the with a fixed step width $h = h_0 = \frac{b-a}{N} > 0$, for some $N \in \mathbb{N}$. The ODE is given by

$$\frac{d}{dt}x = f(t, x), \quad x(a) = x_0,$$

with x_0 the given initial state. Repeat the computation using the Euler method, but now with step width $h = \frac{h_0}{2}$, and increase the number of computational steps N until you reach again 'time' b as before. Print the results of both computations in a table, where you compare the distance between the values of the states x in absolute value at times that coincide in both computations (i.e. leave out every half step from the second computation, for which you have no computed state when compared with the first computation).

2. Task

Formulate pseudo-code for the Runge-Kutta method. Implement the pseudo-code of the Runge-Kutta method inside MATLAB (or any other programming language). Choose the same function $f : [a, b] \times \mathbb{R} \to \mathbb{R}$ and the same time interval with $a < b$, and solve the differential equation for the same fixed step width $h = h_0 = \frac{b-a}{N} > 0$, and for the same $N \in \mathbb{N}$ as in task 1. The ODE is again given by

$$\frac{d}{dt}x = f(t, x), \quad x(a) = x_0,$$

with the same x_0, the given initial state. Repeat the computation using the Runge-Kutta method, but now with step width $h = \frac{h_0}{2}$, and increase the number of computational steps N until you reach again 'time' b as before. Print the results of both computations in a table, where you compare the distance between the values of the states x in absolute value at times that coincide in both computations (i.e. leave out every half step from the second computation, for which you have no computed state when compared with the first computation).

Date: February 1, 2016.

3. Task

Choose a function f for which you can find an explicitly known solution of the first order differential equation used in tasks 1 and 2. Plot the distances of the true solution with solutions of the Euler method and the Runge-Kutta method at given time points using a fixed step width h_0 as given in tasks 2 and 3. What trend can you observe?

4. Task

Let $\Omega \subset \mathbb{R}^2$ be the square $(0,1) \times (0,1)$. Consider the problem

$$-\triangle u = f, x \in \Omega, \tag{1}$$

$$u = g, x \in \partial\Omega. \tag{2}$$

for two given functions $f : \Omega \to \mathbb{R}$, and $g : \partial\Omega \to \mathbb{R}$ of your own choice. Perform a finite difference discretisation with fixed step width h, and solve the problem using a Gauss-Seidel iteration numerically.