Generative Model

① Idea

a) $z_n \sim p(z|\theta)$ → hidden state (latent)

b) $x_n | z_n \sim p(x | z = z_n, \theta)$

c) $(x_n, z_n) \sim p(x, z|\theta) = p(z|\theta)\, p(x|z, \theta)$

Rmk: if our model does not include 'latent variable' z_n, then this goes back to statistical model:

$x_n \sim p(x|\theta)$

↓

goal: estimate parameter θ

② Some common example [Generative Story]

a) P-PCA (Probabilistic PCA)

$$\begin{cases} z_n \sim N(0, I_k) \\ x_n | z_n \sim N(Wz_n, \sigma^2 I_D) \end{cases}$$

Rmk: 1. $k \ll D$

2. z_n is the low-dimension representation for x_n

3. idea is: we can reconstract x_n through z_n under linear transformation with low cost

b) GMM

$$\begin{cases} z_n \sim \text{multi-nomial}(\pi_1, \ldots, \pi_K) \\ x_n | z_n \sim N(\mu_{z_n}, \Sigma_{z_n}) \end{cases}$$

c) Generative Classification

$$\begin{cases} Y_n \sim \text{multi-nomial}(\pi_1, \dots, \pi_K) \\ X_n | Y_n \sim N(\mu_{Y_n}, \Sigma_{Y_n}) \end{cases}$$

$\pi_1, \dots, \pi_K$ → Prior Knowledge

d) Discriminative Model for { Regression / Classification

→ $Y_n | X_n \sim N(w^T x_n, \sigma^2)$

→ $Y_n | X_n \sim \text{Bernoulli}(\sigma(w^T x))$

Rmk: This part can also be viewed as GLM framework!