Convex Prog 5232 Part3 Lec3

Algorithms & Models

1) Sub-gradient calculus (convex f

(x) te of (x)

set-value mapping

 $\Leftrightarrow f(x) \geqslant f(\bar{x}) + \bar{z}^{\tau}(x - \bar{x}) \quad \forall x$

→ 28(X):= {g: f(Z)-f(X) 7 g7 (Z-X), for all Z }

 \bigcirc [prop] X is local minima \Rightarrow $0 \in \partial f(x) \rightarrow optimality condition$ f is convex \ o ∈ ∂ f(x)

2 [prop] df(x) singleton

 \Leftrightarrow f is differentiable at x and $\{\nabla f(x)\} = \partial f(x)$

Norm-2 Subgradient

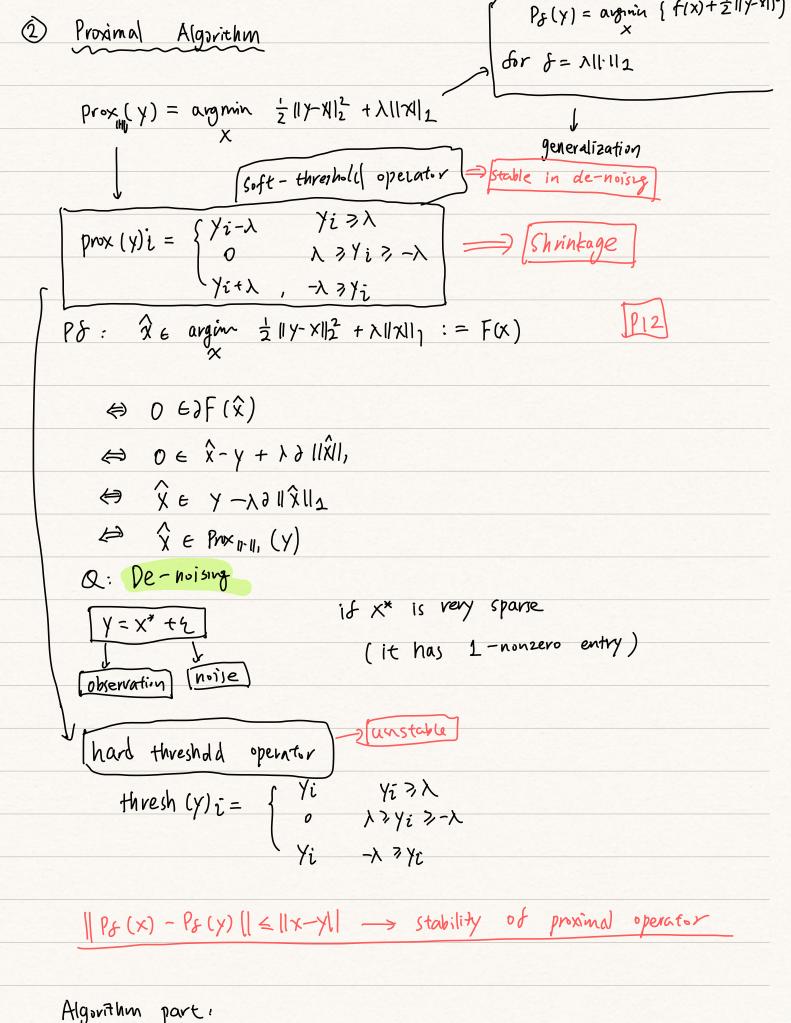
f(x) = |121/2

$$\frac{9 \, \ell(\bar{x})}{4} = \begin{pmatrix} x \\ y \\ x \end{pmatrix}$$

Norm-2 Subgradient

 $f(X) = ||X||_2^2 \Rightarrow \partial f(X) = 2X$

 $\mathcal{S}(X) = \|X\|_2 \Rightarrow 2\mathcal{S}(X) = \begin{cases} \frac{X}{\|A\|_2} & \text{if } X \neq 0 \\ 0 & \text{if } X = 0 \end{cases}$



min f(x) + glx)

Application [LASSO]

argmin
$$\frac{1}{2} \|y - Ax\|_{2}^{2} + \lambda \|x\|_{1}$$

$$\frac{f(x) := \frac{1}{2} \|y - Ax\|_{2}^{2}}{g(x)} = \lambda \|x\|_{1}$$

Step 2:
$$prox_{19C}$$
) (x) $\rightarrow \times$

prosonal operator with oper to nudear norm

$$\frac{|P(Y)|}{|P(Y)|} = \frac{1}{2} ||Y - X||_F^2 + \lambda ||X|| ||A|| ||A||$$

easy shrink (min (6=->10)

ADMM \Rightarrow $L(x;y) = f(x) + y^{T}(Ax-b)$ Graliet Ascent -> Saddle L(x;y) $\begin{cases} x \leftarrow \operatorname{argunn} L(x,y) & \text{fix } y \rightarrow \text{convex} \\ y \leftarrow y + \eta (Ax - b) \end{cases}$ Augmented Lagrangian $L_{A}(x;y) = L(x;y) + \frac{e}{2} ||Ax-b||_{2}^{2}$ First -Order: Cheap each + More iteration very accurate solution Large Scale Second-Order: Expensive t fewer iteration

2 hour

Optimal Power Flow