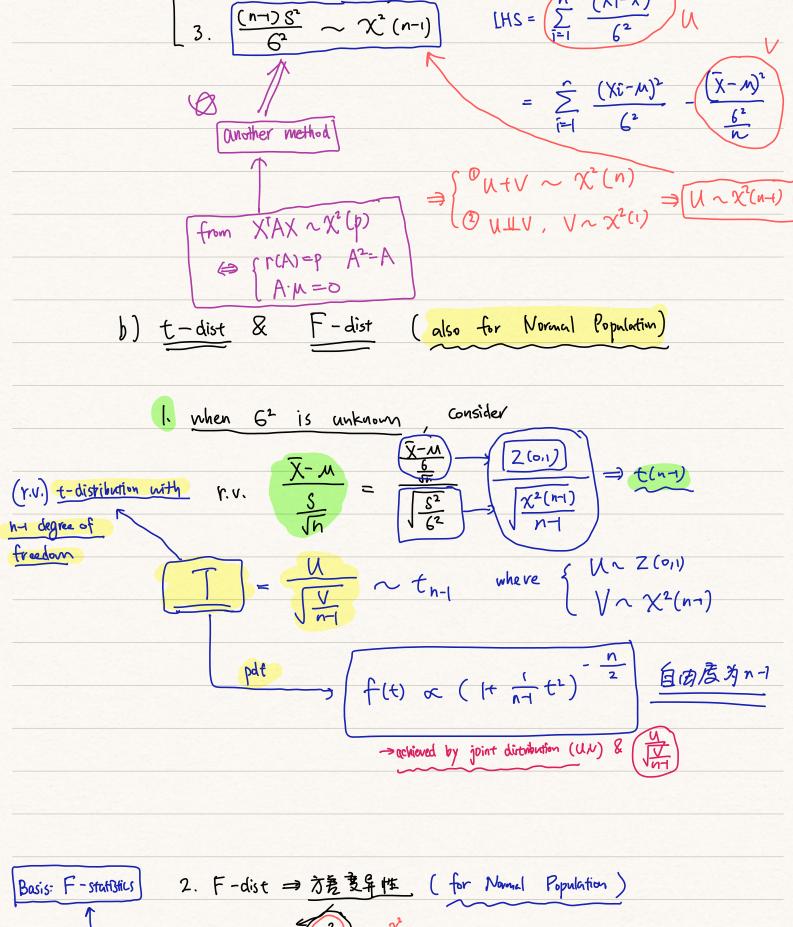
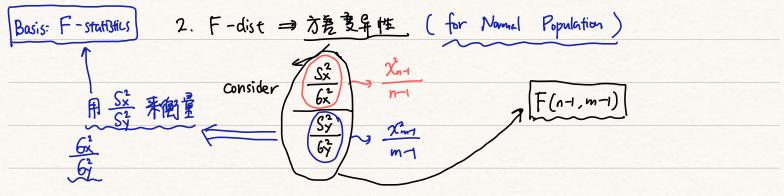


2. $\overline{\chi} \sim N(\mu, \frac{6^2}{n})$





Property
$$\bigcirc$$
 $X \sim F_{p,q} \Rightarrow X^{-1} \sim F_{q,p}$

$$\bigcirc$$
 $X \sim t_q = \frac{u}{\sqrt{2}} \Rightarrow X^2 = \frac{u^2}{\sqrt{2}} \sim F_{1,q}$

$$\bigcirc$$
 \bigcirc $X \sim F_{p,q} \Rightarrow \frac{\frac{p}{q} \times p}{1+\frac{p}{q} \times q} \sim \frac{\text{Beta}(\frac{p}{2}, \frac{q}{2})}{1+\frac{p}{q} \times q}$

$$X_{(i)} \longrightarrow [i-th \text{ order}] \text{ in } \{X_1, \dots, X_n\}$$

$$f_{X_{(i)}}(x) = {n \choose i+1 \quad n-i} \cdot f_{X}(x) \cdot F_{X}(x)^{i-1} \cdot (1-F_{x}(x))^{n-i}$$

$$S^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (\chi_{i} - \overline{\chi})^{2}$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} \chi_{i}^{2} - n \overline{\chi}^{2} \right]$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} (X_i - \mu)^2 - n (\tilde{\chi} - \mu)^2 \right)$$

$$\mathbb{E}[S^{2}] = \frac{1}{n-1} \left[n \mathbb{E}[X^{2}] - n \mathbb{E}[\bar{X}^{2}] \right]$$

$$= \frac{1}{n-1} \left[n \text{ var}[X] - n \text{ var}[\bar{X}] \right]$$

$$= \frac{n}{n-1} \left(6^{2} - \text{ var}[\bar{X}] \right) \in \left[0, \frac{n}{n-1} 6^{2} \right]$$