```
Duality
          conjugate fun
            f^*(y) = \sup_{x} \{ \langle x, y \rangle - \delta(x) : x \in \mathbb{R}^n \}
                     = sup { (x,y>- M : (x,M) & epilf) }
             f* is closed
                                        Convex
                                and
                          f(y) = sup { hely): t } => f(y) is < closed => epi(f) is closed convex
       a special conjugate f^2 \rightarrow \delta(x|L) \iff \delta^*(x|L)
               S(x(c) = { 0, xec | is conjugate if c is closed and convex
               >> 5x(x(L) = sup { <xiy> ; y ∈ C}
                                                       f(x)= sup { h(x): h < &
     for closed, proper, convex f?
                      1. (f*)*= 8 -> + 是比它+的 Attine + 的上界.
                                           f(x) = sup { (x, y>-11 : h(x) & food for box }
                                                       (xy)-11 = 8100 /x
                                                       M > (xy>-810) Vx
                                                       => M > sup { (x,y) - /1x): 1/x]
                                                       => M = 8*(9)
                                                       ⇒ (y, M) ∈ epi(8*)
                                             = sup { (y,x>-u: (y,n)=epi(x))}
                                             = (f*)* (x)
                                                                     (= ) S(0)
                      2. f → positive homo geneous

⇔ f* → non-empty closed & onvex set C (indicator function)

                                             5x proper & chied & comes
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$$\bigoplus_{f(x)=sup\{(\kappa x^*)-sty:\kappa(K)\}} + f(x) = \sup_{f(x)=sup\{(\kappa x^*)-sty:\kappa(K)\}} + f(x) = \exp_{f(x)}(\kappa x^*)-sty:\kappa(K)\}$$

$$\Leftrightarrow x^* \in \partial f(x)$$

$$\Leftrightarrow x \in \partial f(x)$$

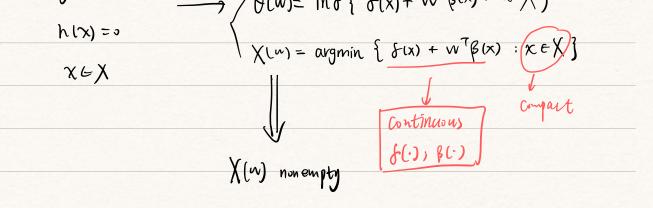
$$\Leftrightarrow x \in \partial f(x)$$

$$\Leftrightarrow (x, x^*) - f'(x) = \sup_{f(x)=sup} \{(z, x) - f'(z^*)^*: z^*\}$$

$$\Leftrightarrow (x, x^*) - f(x) = \sup_{f(x)=sup} \{(z, x^*) - f(z): z\}$$

$$\oint_{f(x)=sup} f(x)$$

$$f(x) = f$$



- () O(-) → concave
- 2) subgradient of 9(·)
  - 1. \$\frac{1}{\infty} \( \sigma \) \( \sigma
  - 2. X(W) → singleton

J

O(1) is differentiable at  $\overline{w} \Rightarrow PO(\overline{w}) = \beta(\overline{x})$ 

- ③ 方向子数  $\Rightarrow$   $\Theta(\bar{w}/4) = in \delta \{d^{T}\}, \{\epsilon \} \Theta(\bar{w})\}$
- (4) subgradient  $\Rightarrow \partial \theta(\overline{w}) = \text{conv} \{ \beta(y) : y \in \chi(\overline{w}) \}$

(E4)

⑤ 最上下降方向 らE → O(w) → has the smallest Euclidean norm

ANALY .