$\boxed{\text{SLLN}}$ & $\boxed{\text{WLLN}}$

$\Rightarrow X_1, \dots, X_n$ independent & $\begin{cases} \mathbb{E}[X_i] = \mu \\ Var[X_i] = \sigma^2 \end{cases}$

if $X_1, \dots, X_n$ i.i.d, then

$\boxed{\text{CLT}} \Rightarrow \dfrac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{d} N(0,1)$

$\Rightarrow$ ① WLLN $\quad \bar{X}_n \xrightarrow{P} \mu$ as $n \to \infty$

$\Rightarrow$ ② SLLN $\quad \bar{X}_n \xrightarrow{a.s} \mu$ as $n \to \infty$

Difference between $X_n(\omega) \xrightarrow{P} X(\omega)$ & $X_n(\omega) \xrightarrow{a.s.} X(\omega)$

① Definition

P converge $\Rightarrow X_n(\omega) \xrightarrow{P} X(\omega)$

$\Leftrightarrow \boxed{\lim_{n\to\infty} \mathbb{P}(\omega : |X_n(\omega) - X(\omega)| \geq \varepsilon) = 0}$

$\boxed{\text{interpretation}}$: for arbitrary tolerated error $\varepsilon > 0$,

the event $\{|X_n - X| \geq \varepsilon\}$ have the probability $< \forall \delta$

when n is sufficient large

quality of estimator

Confidence $> 1-\delta$

$\Downarrow$

much confidence

the quality of our estimator is very good

enough trials (samples)

a.s. converge $\Rightarrow X_n(\omega) \xrightarrow{a.s.} X(\omega)$

$\Leftrightarrow \mathbb{P}(A_X) = 1$

$\boxed{\bigcap_{k=1}^{\infty} \bigcup_{i=1}^{\infty} \bigcap_{n=i}^{\infty} \{\omega : |X_n(\omega) - X(\omega)| < \frac{1}{k}\}}$

$A_X := \{\omega : \lim_{n\to\infty} X_n(\omega) = X(\omega)\}$

$\Leftrightarrow \mathbb{P}(\bigcap_{i=1}^{\infty} \bigcup_{n=i}^{\infty} \{\omega : |X_n(\omega) - X(\omega)| \geq \varepsilon\}) = 0 \quad \forall \varepsilon > 0$

$\boxed{\text{发生无限次}}$

$\Leftrightarrow \lim_{n\to\infty} \mathbb{P}(\bigcup_{i=n}^{\infty} \{\omega : |X_i(\omega) - X(\omega)| \geq \varepsilon\}) = 0 \quad \forall \varepsilon > 0$

$\Leftrightarrow \mathbb{P}(\overline{\lim_{n\to\infty}} \{\omega : |X_i(\omega) - X(\omega)| \geq \varepsilon\}) = 0$

P converge $\Leftrightarrow \lim_{n\to\infty} \mathbb{P}(\{\omega : |X_n(\omega) - X(\omega)| \geq \varepsilon\}) = 0 \quad \forall \varepsilon > 0$

① $\limsup_{n\to\infty} A_n := \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

$x \in \limsup_{n\to\infty} A_n \Leftrightarrow \forall n \in \mathbb{N}^+, \exists k \geq n$ s.t. $x \in A_k$

$\Leftrightarrow$ infinitely often happen

② $\liminf_{n\to\infty} A_n := \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

$x \in \liminf_{n\to\infty} A_n \Leftrightarrow \exists n \in \mathbb{N}^+, \forall k \geq n, x \in A_k$

$\Leftrightarrow$ not finitely happen

It is obvious that $\liminf_{n\to\infty} A_n \subseteq \limsup_{n\to\infty} A_n$

Personal IDEA

'Global Convergence'

a.s. convergence

sample space Ω

convergence smoothly, in the sense of almost all point $\omega \in \Omega$ can converge

$\Rightarrow$ something like 'pointwise convergence' in sample space

Ω

$\mathbb{P}(\cdot)$

$\mathbb{P}: \Omega \longrightarrow \mathbb{R}$

$X: \Omega \longrightarrow \mathbb{R}$

可列个聚点 in sample space

$\Downarrow$

the length must $\to 0$

$\Downarrow$

$\boxed{P \to 0}$

'Local Convergence'

P convergence

converge in the sense probability

measure the difference between $X_n(\omega)$ & $X(\omega)$ with probability measure

$\boxed{\mathbb{P}(\{\omega : |X_n(\omega) - X(\omega)| \geq \varepsilon\}) \longrightarrow 0}$

interpretable in real-life

$\begin{cases} X: \Omega \longrightarrow \mathbb{R} \\ \mathbb{P}: \Omega \longrightarrow \mathbb{R} \end{cases}$

just need to guarantee the length must $\to 0$

$\Downarrow$

can be 不可列个聚点