Principles of Economics

Discussion Session 10: Productivity

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The Production Function

- A model of the macroeconomy requires a **production function**.
  - Model of how different inputs (A, L, K, H, N) interact to create output Y.
- Generally,

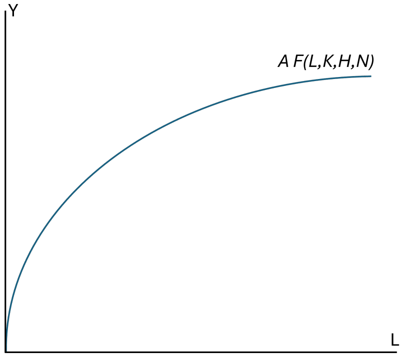
$$Y = A \times F(L, K, H, N)$$

  where
  - $A :=$ technology
  - $L :=$ labor
  - $K :=$ capital
  - $H :=$ human capital
  - $N :=$ natural resources
- A commonly used basic production function is

$$Y = A\, L^{\alpha}\, K^{1-\alpha}, \quad \text{where } 0 < \alpha < 1$$

Diminishing Marginal Product

- The production function exhibits **diminishing marginal product**:
  - The increase in output due to increasing an input is decreasing, holding other inputs constant.
  - $\partial F(L, K, H, N)/\partial L > 0$ and $\partial^2 F(L, K, H, N)/\partial L^2 < 0$



$\Longrightarrow$ A capital-rich country gains more output from an increase in labor than does a labor-rich country.

Productivity

- The ratio of capital to labor, K/L, is determinative of the productivity of each.
- The ratio of output to labor, Y/L, tells us the productivity of labor.
- Higher capital-to-labor ratio begets greater productivity of labor:

$$\uparrow \frac{K}{L} \implies \uparrow \frac{Y}{L}$$

Exercise 1: Productivity

Suppose

- Country A has a labor force of 50 million people and \$10 billion of capital.
- Country B has a labor force of 25 million people and \$20 billion of capital.

Question: Assuming technology and other inputs are identical,

1. Which country has the higher capital-to-labor ratio?
2. Which country has the higher productivity of labor?
3. Which country's GDP (Y) would benefit more from an influx of labor through immigration?
4. Which country's GDP would benefit more from an influx of capital through foreign investment?

Exercise 1: Productivity

Solution:

1. Country B. $\frac{K_B}{L_B} = \frac{20\times10^9}{25\times10^6} = 800 > 200 = \frac{10\times10^9}{50\times10^6} = \frac{K_A}{L_A}$
2. Country B. Higher $\frac{K}{L}$ implies higher $\frac{Y}{L}$.
3. Country B. Higher productivity of labor implies greater benefit to GDP from additional labor.
4. Country A. Lower $\frac{K}{L}$ implies higher productivity of capital, which implies greater benefit to GDP from additional capital.

International Flows

- Suppose we have a capital-rich country and a capital-poor country.
- Given our answers on the last slide, what do we expect to happen if labor and capital are allowed to move between the two countries?
- Does this happen in practice? (Lucas Paradox, colonial America)

Exercise 2: GDP per Capita

Suppose

- Country A has a population of 240 million, and one quarter of its population is in the labor force. Its GDP is $12 trillion.
- Country B has a population of 60 million, and one fifth of its population is in the labor force. Its GDP is $2.4 trillion.

Question:

1. Which country has the higher GDP per capita?
2. Which has the higher productivity of labor?

Exercise 2: GDP per Capita

1. Country A.

$$\frac{Y_A}{\text{population A}} = \frac{12 \times 10^{12}}{240 \times 10^6} = 50,000 > 40,000 = \frac{2.4 \times 10^{12}}{60 \times 10^6} = \frac{Y_B}{\text{population B}}$$

2. They are equal!

$$\frac{Y_A}{L_A} = \frac{12 \times 10^{12}}{60 \times 10^6} = 200,000 = \frac{2.4 \times 10^{12}}{12 \times 10^6} = \frac{Y_B}{L_B}$$

Exercise 3: Very Fun Algebra Problems

1. If labor productivity in the United States increases by 5% and the labor force grows by 4%, what is its GDP growth rate?

2. If the labor force falls by 8%, by how much must labor productivity increase for GDP growth to be positive?

Exercise 3: Very Fun Algebra Problems

1. Labor productivity increase by 5% $\implies$ $\frac{Y_2}{L_2} = 1.05\,\frac{Y_1}{L_1}$.

   Labor force increase by 4% $\implies$ $L_2 = 1.04\,L_1$.

   Substitute $1.04\,L_1$ in for L_2 in the labor productivity growth equation and solve for Y_2 to find that $Y_2 = 1.092\,Y_1$.

   $\implies$ GDP grows by 9.2%.

2. Labor force falls by 8% $\implies$ $L_2 = 0.92\,L_1$.

   Let g represent the growth rate of labor productivity, so $\frac{Y_2}{L_2} = (1+g)\frac{Y_1}{L_1}$.

   Substitute $0.92\,L_1$ in for L_2 in the labor productivity growth equation, to get

   $$\frac{Y_2}{0.92\,L_1} = (1+g)\frac{Y_1}{L_1}$$

   Rearrange terms to find that the GDP growth rate is

   $$\frac{Y_2 - Y_1}{Y_1} = 0.92(g+1) - 1$$

   We want to find the g such that $(Y_2 - Y_1)/Y_1 > 0$, so set $0.92(g+1) - 1 > 0$ and solve for g.

   $\implies$ $g > 8.7\%$