

Principles of Economics

Discussion Session 10: Productivity

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The Production Function

- A model of the macroeconomy requires a **production function**.
 - Model of how different inputs (A, L, K, H, N) interact to create output Y .
- Generally,

$$Y = A \times F(L, K, H, N)$$

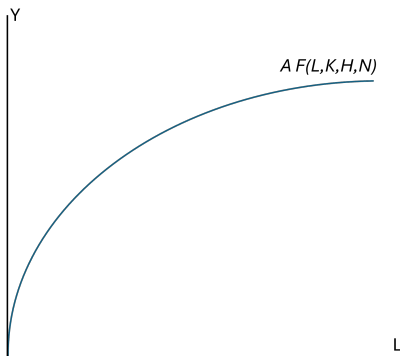
where

- A := technology
 - L := labor
 - K := capital
 - H := human capital
 - N := natural resources
- A commonly used basic production function is

$$Y = A L^{\alpha} K^{1-\alpha}, \quad \text{where } 0 < \alpha < 1$$

Diminishing Marginal Product

- The production function exhibits **diminishing marginal product**:
 - The increase in output due to increasing an input is decreasing, holding other inputs constant.
 - $\partial F(L, K, H, N) / \partial L > 0$ and $\partial^2 F(L, K, H, N) / \partial L^2 < 0$



⇒ A capital-rich country gains more output from an increase in labor than does a labor-rich country.

- The ratio of capital to labor, K/L , is determinative of the productivity of each.
- The ratio of output to labor, Y/L , tells us the productivity of labor.
- Higher capital-to-labor ratio begets greater productivity of labor:

$$\uparrow \frac{K}{L} \implies \uparrow \frac{Y}{L}$$

Exercise 1: Productivity

Suppose

- Country A has a labor force of 50 million people and \$10 billion of capital.
- Country B has a labor force of 25 million people and \$20 billion of capital.

Question: Assuming technology and other inputs are identical,

- 1 Which country has the higher capital-to-labor ratio?
- 2 Which country has the higher productivity of labor?
- 3 Which country's GDP (Y) would benefit more from an influx of labor through immigration?
- 4 Which country's GDP would benefit more from an influx of capital through foreign investment?

Exercise 1: Productivity

Solution:

- ① Country B. $\frac{K_B}{L_B} = \frac{20 \times 10^9}{25 \times 10^6} = 800 > 200 = \frac{10 \times 10^9}{50 \times 10^6} = \frac{K_A}{L_A}$
- ② Country B. Higher $\frac{K}{L}$ implies higher $\frac{Y}{L}$.
- ③ Country B. Higher productivity of labor implies greater benefit to GDP from additional labor.
- ④ Country A. Lower $\frac{K}{L}$ implies higher productivity of capital, which implies greater benefit to GDP from additional capital.

Exercise 2: GDP per Capita

Suppose

- Country A has a population of 240 million, and one quarter of its population is in the labor force. Its GDP is \$12 trillion.
- Country B has a population of 60 million, and one fifth of its population is in the labor force. Its GDP is \$2.4 trillion.

Question:

- 1 Which country has the higher GDP per capita?
- 2 Which has the higher productivity of labor?

Exercise 2: GDP per Capita

① Country A.

$$\frac{Y_A}{\text{population A}} = \frac{12 \times 10^{12}}{240 \times 10^6} = 50,000 > 40,000 = \frac{2.4 \times 10^{12}}{60 \times 10^6} = \frac{Y_B}{\text{population B}}$$

② They are equal!

$$\frac{Y_A}{L_A} = \frac{12 \times 10^{12}}{60 \times 10^6} = 200,000 = \frac{2.4 \times 10^{12}}{12 \times 10^6} = \frac{Y_B}{L_B}$$

Exercise 3: Very Fun Algebra Problems

- 1 If labor productivity in the United States increases by 5% and the labor force grows by 4%, what is its GDP growth rate?
- 2 If the labor force falls by 8%, by how much must labor productivity increase for GDP growth to be positive?

Exercise 3: Very Fun Algebra Problems

① Labor productivity increase by 5% $\implies \frac{Y_2}{L_2} = 1.05 \frac{Y_1}{L_1}$.

Labor force increase by 4% $\implies L_2 = 1.04 L_1$.

Substitute $1.04 L_1$ in for L_2 in the labor productivity growth equation and solve for Y_2 to find that $Y_2 = 1.092 Y_1$.

\implies GDP grows by 9.2%.

② Labor force falls by 8% $\implies L_2 = 0.92 L_1$.

Let g represent the growth rate of labor productivity, so $\frac{Y_2}{L_2} = (1 + g) \frac{Y_1}{L_1}$.

Substitute $0.92 L_1$ in for L_2 in the labor productivity growth equation, to get

$$\frac{Y_2}{0.92 L_1} = (1 + g) \frac{Y_1}{L_1}$$

Rearrange terms to find that the GDP growth rate is

$$\frac{Y_2 - Y_1}{Y_1} = 0.92(g + 1) - 1$$

We want to find the g such that $(Y_2 - Y_1)/Y_1 > 0$, so set $0.92(g + 1) - 1 > 0$ and solve for g .

$\implies g > 8.7\%$