Extra Session 7 Material

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Points you, at least, should be able to do / and try to practice for today's session parts

1. Consumer Optimal Choice part:
   (a) Draw budget constraint line with given P_A, P_B, I
   (b) Be able to derive new budget constraint line after changes in price or income level
   (c) Based on the properties of goods (Income change: normal or inferior good / Price change: substitute or complement good), be able to draw new optimal points with new budget constraint and indifference curves
   (d) And express the idea that why the new optimal point is derived like that in regard with the properties of goods
   (e) Link to demand curve derivation
2. Externality part:
   (a) Derive socially optimal quantity with given demand curves, supply curves and social cost / benefit
   (b) Solve tax / subsidy problem by putting the same amount of social cost / benefit

1 Consumer Optimal Choice

1.1 Budget Constraint

The **main goals** are

(1) Draw budget constraint line

(2) Draw new budget constraint line after some changes (change in price or income)

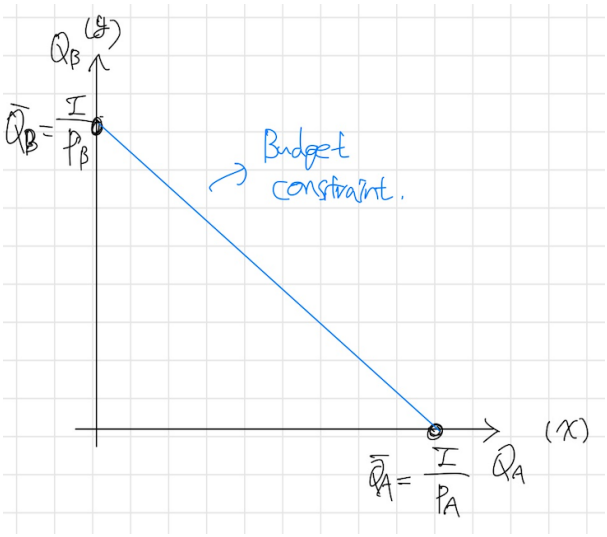
1. The **simplest way you can derive budget constraint** is,

   (i) Start with $P_A \times Q_A + P_B \times Q_B = I$. Then,

   (ii) Calculate two intercepts on x-axis and y-axis (maximum amount of good A or good B with given income I)

$$\text{When } Q_A = 0: \quad P_B \times Q_B = I \Rightarrow \bar{Q}_B = \frac{I}{P_B}$$
$$\text{When } Q_B = 0: \quad P_A \times Q_A = I \Rightarrow \bar{Q}_A = \frac{I}{P_A}$$

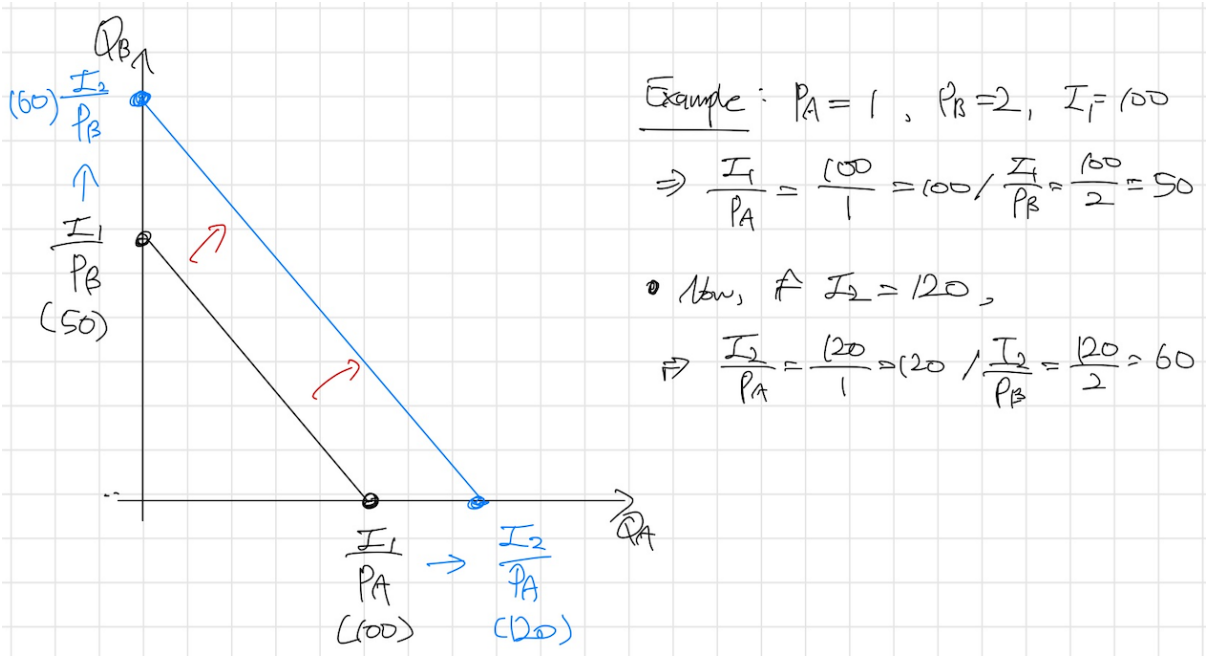
   (iii) Then, with two points, you can simply connect two points to draw budget constraint line.



2. Now, **derive new budget constraint line** after some changes (change in price or income). You can simply do this **only working with intercepts**.

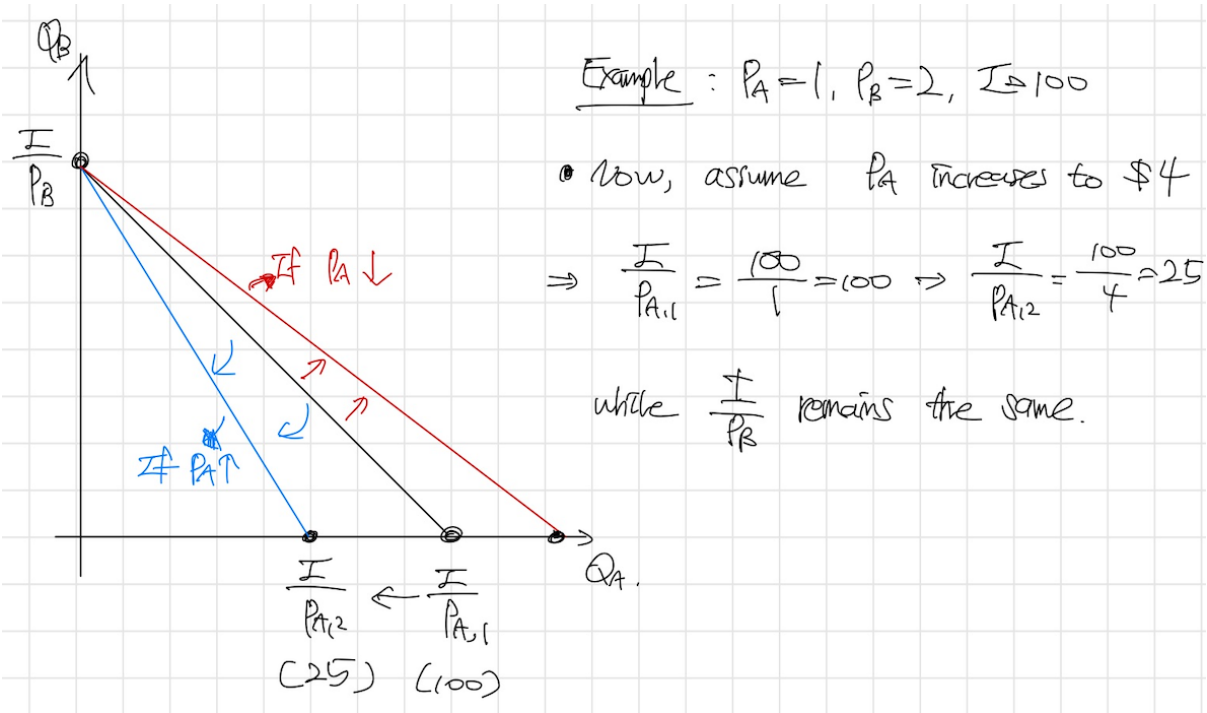
   1. When **income increases (decreases)** to I_2 from I_1, **both intercepts** increase (decrease) proportionally. So, you can just shift the line outward (inward) parallel.

$$\text{Old } \bar{Q}_B = \frac{I_1}{P_B} \quad \Rightarrow \quad \text{New } \bar{Q}_B = \frac{I_2}{P_B}$$
$$\text{Old } \bar{Q}_A = \frac{I_1}{P_A} \quad \Rightarrow \quad \text{New } \bar{Q}_A = \frac{I_2}{P_A}$$



2. When **price of good A increases (decreases)** to $P_{A,2}$ from $P_{A,1}$, only **A-intercept** decreases (increases) while **B-intercept** remains the same. So, you can just rotate the line inward (outward) keeping B-intercept point.

$$\text{Old } \bar{Q}_A = \frac{I}{P_{A,1}} \quad \Rightarrow \quad \text{New } \bar{Q}_A = \frac{I}{P_{A,2}}$$
$$\text{Old } \bar{Q}_B = \frac{I}{P_B} \quad \Rightarrow \quad \text{New } \bar{Q}_B = \frac{I}{P_B} \quad \text{(So, it remains the same)}$$



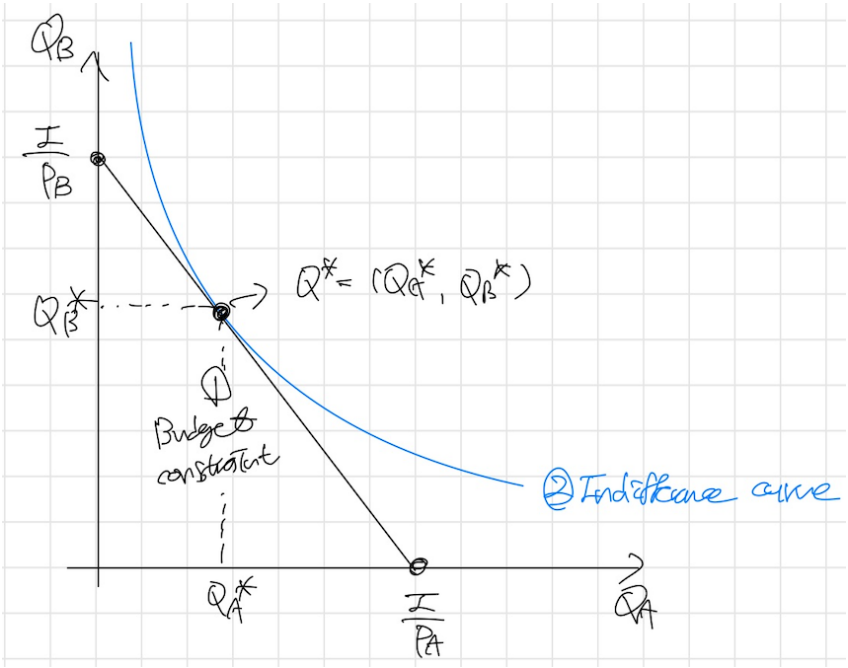
1.2 Optimal Choice with Indifference Curve

Now, you should be able to derive optimal choice point. The one thing you should remember is,

- **Optimal choice point is where budget constraint line is tangent to the highest possible indifference curve.**

The **procedure** is like this:

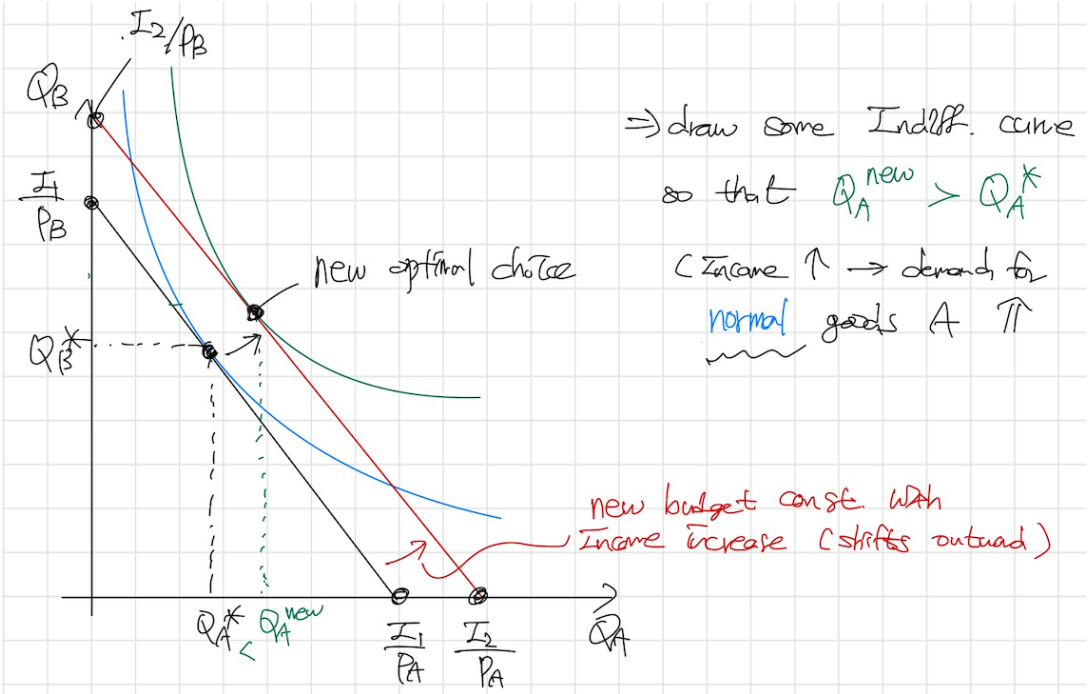
1. First, draw budget constraint line with given P_A, P_B, I (using intercept method above)
2. Second, just arbitrarily draw an indifference curve which is tangent to the budget constraint line (just make it sure it meets budget constraint line only at one point - tangent). And say that is optimal choice point.



3. Third, derive new budget constraint line after some changes (change in price or income) using intercept method above.
4. Fourth, based on the properties of goods (Income change: normal or inferior good / Price change: substitute or complement good), derive new optimal points **which can represent the properties of goods** by arbitrarily drawing new indifference curves tangent to the new budget constraint line.
5. Fifth, link the changes in optimal quantities of good A and good B with demand curves of each of goods A and B (shifts or along the demand curve itself)

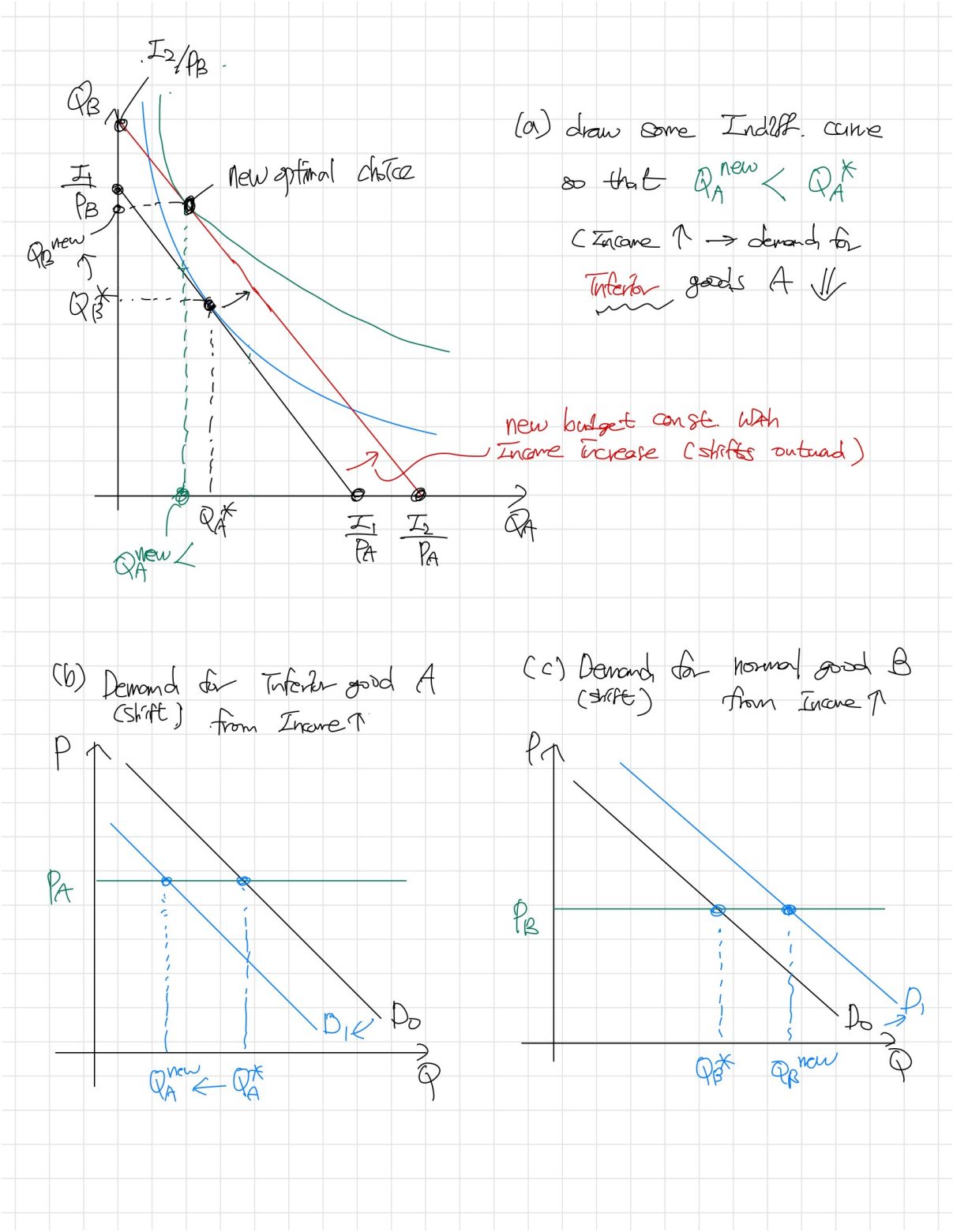
Some exercises for each case with income change / price changes:

(1) If the good A is a **normal good**, an **increase in income** will shift the budget constraint outward and the new optimal quantity for good A should be on the right side to the original Q_A
⇒ Demand for good A shifts right from increase in income



(2) If the good A is an **inferior good**, an **increase in income** will shift the budget constraint outward, but the optimal quantity for good A should be on the left side to the original Q_A

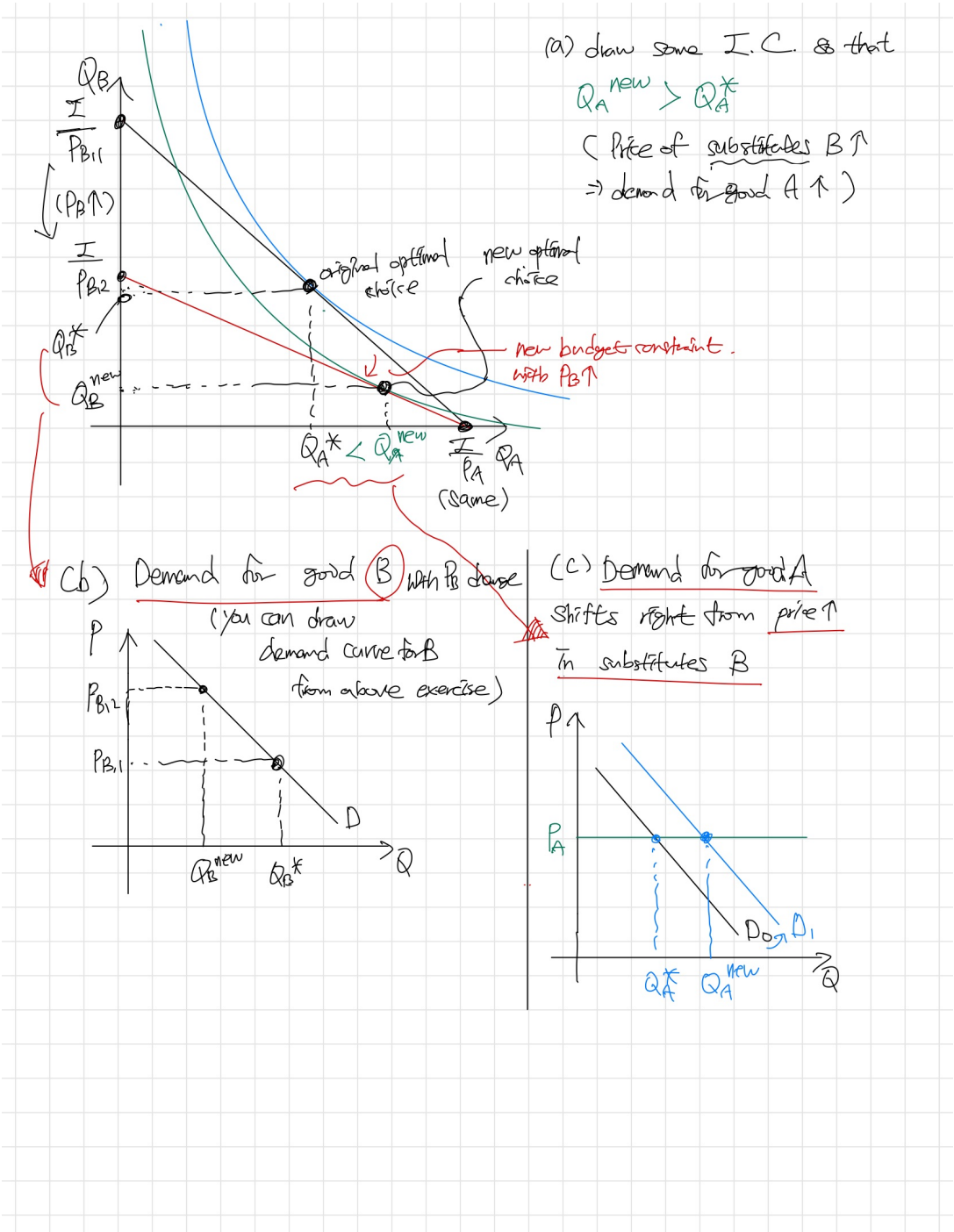
$\Rightarrow$ Demand for good A shifts left from increase in income



(3) If the goods A and B are **substitutes**, an **increase in the price of good B** will change the budget constraint and the new optimal quantity for good A should be on the right side to the original Q_A

$\Rightarrow$ For good A, demand for good A shifts right from increase in price of 'substitutes' good B

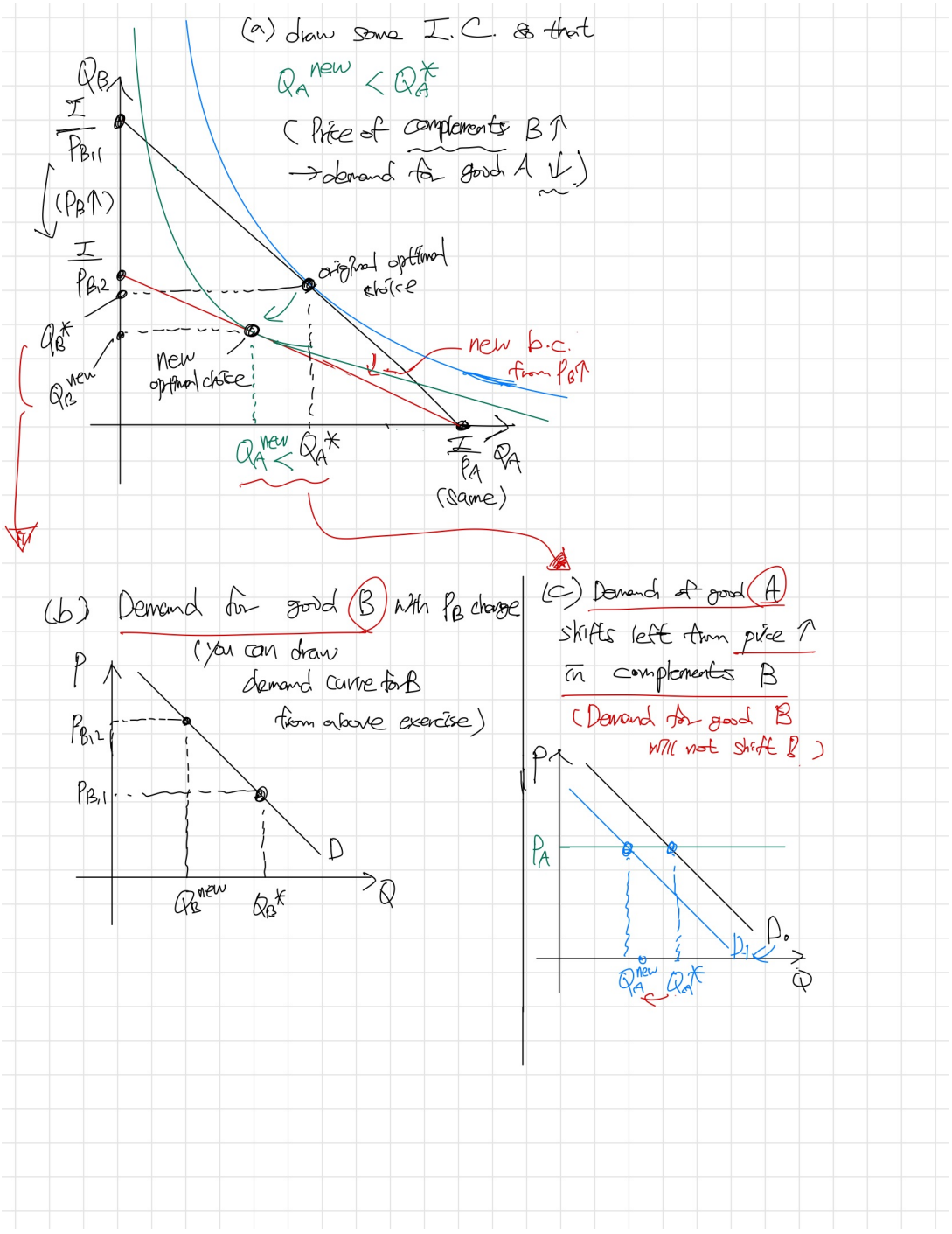
$\Rightarrow$ For good B, with this change in price B, you can just derive the demand curve itself of good B (it doesn't shift because it's change in its OWN price)



(4) If the goods A and B are **complements**, an **increase in the price of good B** will change the budget constraint and the new optimal quantity for good A should be on the left side to the original Q_A

$\Rightarrow$ For good A, demand for good A shifts left from increase in price of 'complements' good B

$\Rightarrow$ For good B, with this change in price B, you can just derive the demand curve itself of good B (it doesn't shift because it's change in its OWN price)



2 Externality

Basic logic:

- From transaction between buyers and sellers, there exists some externalities which affect third party not involved in the transaction.
- In socially optimal view point (**when considering third party**), the externality should be considered while both buyers and sellers do not consider it for market equilibrium
- Therefore, **government intervenes** to **make market participants to consider it** by taxing or subsidizing the transaction:
- It's like, "Because you don't consider it, now, I will let you do so by taxing or subsidizing you.". Then, because of explicit additional cost from tax or benefit from subsidy, buyers and sellers will consider the externality in their decision making.

2.1 Derivation of Socially Optimal Quantity

Let's say, the current demand and supply curves are like this (same with exercise)

$$Q^D = 20 - P$$
$$Q^S = P - 2$$

And let's say **social cost (negative externality)** or **social benefit (positive externality)** is \$4 **per unit** (different with 'Total Social Cost/Benefit' $= Q^* \times 4$).

Then, the simplest way is to set socially optimal supply curve as

$$\text{(Negative Ext):} \quad Q^{S,social} = (P - (\text{social cost})) - 2$$
$$\text{(Positive Ext):} \quad Q^{S,social} = (P + (\text{social benefit})) - 2$$

Then in this case,

$$\text{(Negative Ext):} \quad Q^{S,social} = (P - 4) - 2 = P - 6$$
$$\text{(Positive Ext):} \quad Q^{S,social} = (P + 4) - 2 = P + 2$$

The logic is that, in the view of socially optimal choice, suppliers should have considered additional cost with social cost or additional benefit with social benefit. Therefore, it's the same with increase or decrease in production cost which shifts supply curve to the left or right. So, you can derive socially optimal supply curve

But it's not the shift of original firms' supply curve. It's to derive artificial 'Social Supply Curve' from firms' supply curve. The only purpose doing this

is to calculate socially optimal level of quantity Q^{social}!

$$
\begin{aligned}
Q^{S,private} &= P - 2 \\
\text{(Negative Ext):} \quad Q^{S,social} &= P - 6 \Rightarrow \quad : \text{Left shifted shape} \\
\text{(Positive Ext):} \quad Q^{S,social} &= P + 2 \Rightarrow \quad : \text{Right shifted shape}
\end{aligned}
$$

With social supply curve, you can derive socially optimal quantity. **The focus here is to derive socially optimal quantity**. So, you don't need to think about what's socially optimal price.

(1) First, for the negative case

$$
\begin{aligned}
Q^D &= Q^{S,social} \\
20 - P &= (P - 4) - 2 \\
2P &= 26 \\
P &= 13 \\
Q^{social} &= 20 - 13 = 7 < Q^* = 9
\end{aligned}
$$

So, in negative externality case, socially optimal quantity should be lower than current market equilibrium quantity. So, from current market equilibrium quantity, it should be discouraged - tax.

(2) For positive externality case,

$$
\begin{aligned}
Q^D &= Q^{S,social} \\
20 - P &= (P + 4) - 2 \\
2P &= 18 \\
P &= 9 \\
Q^{social} &= 20 - 9 = 11 > Q^* = 9
\end{aligned}
$$

So, in positive externality case, socially optimal quantity is higher than current market equilibrium quantity. So, from current market equilibrium quantity, it should be encouraged - subsidy

2.2 Government Intervention to Achieve Socially Optimal Quantity

(1) Now, how can the government intervene to achieve socially optimal quantity in the case of **negative externality**?

- The government can impose tax on **suppliers or buyers** (since two have the same

effect of tax as we learned)

- Set the tax with the **same amount of social cost** to achieve **socially optimal quantity**.

Let's say, the government imposes tax on suppliers. Then, it's the same with previous tax problem.

$$\begin{aligned}
\text{(Tax on Supplier case)} \quad P^S &= P^D - \text{tax} = P^D - 4 \\
Q^D(P^D) &= Q^S(P^S) \\
20 - P^D &= P^S - 2 \\
20 - P^D &= (P^D - 4) - 2 \\
2P^D &= 26 \\
P^D = 13 \;&\Rightarrow\; P^S = P^D - 4 = 9
\end{aligned}$$

$$Q^{tax} = Q^D(P^D) = 20 - P^D = 7 \quad \text{or,} \quad Q^{tax} = Q^S(P^S) = P^S - 2 = 7 \quad \text{(Same)}$$

Then, you can check that with the **same amount of tax as social cost**, the government can achieve socially optimal quantity.

$$Q^{social} = Q^{tax} = 7$$

(2) Now, how can the government intervene to achieve socially optimal quantity in the case of **positive externality**?

- The government can give subsidy on **suppliers or buyers** (since two have the same effect of subsidy as we learned)
- Set the subsidy with the **same amount of social benefit** to achieve **socially optimal quantity**.

Let's say, the government gives subsidy to suppliers. Then, it's the same with previous subsidy problem.

$$\begin{aligned}
\text{(Subsidy on Supplier case)} \quad P^S &= P^D + \text{subsidy} = P^D + 4 \\
Q^D(P^D) &= Q^S(P^S) \\
20 - P^D &= P^S - 2 \\
20 - P^D &= (P^D + 4) - 2 \\
2P^D &= 18 \\
P^D = 9 \;&\Rightarrow\; P^S = P^D + 4 = 13
\end{aligned}$$

$$Q^{tax} = 20 - P^D = 11 \quad \text{or,} \quad Q^{tax} = P^S - 2 = 11 \quad \text{(Same)}$$

Then, you can check that with the **same amount of subsidy as social benefit**, the government can achieve socially optimal quantity.

$$Q^{social} = Q^{tax} = 11$$