Principles of Economics

Discussion Session 5: Externalities

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Externalities

- When a market transaction effects a third party.
- **Negative Externality:** The third party is negatively effected.
  - Quantity produced is too high.
  - Policy response is a tax.
- **Positive Externality:** The third party is positively effected.
  - Quantity produced is too low.
  - Policy response is a subsidy.

The Optimal Policy Response

- The policymaker has to strike a balance.
- Take carbon emissions for example:
  - Carbon emissions are a negative externality.
  - A complete ban on carbon emissions would completely eliminate the harm.
  - But surplus would be so reduced that the costs outweigh the benefits.
- If the policymaker's goal is to maximize total surplus, then it turns out the optimal policy response is a per-unit tax exactly equal to the size of the per-unit externality!

Solving for the Optimal Tax

To see why, consider

- $Q^D = 20 - P$
- $Q^S = P - 2$

- Suppose there is a per-unit externality of size ε
- And the policymaker seeks to maximize $TS = CS + PS + TR - (Q \times \varepsilon)$

We wish to solve for the optimal tax:

- We know that the imposition of a tax implies $P^D = P^S + t$.
- Solving the tax problem as usual results in

$$P^S = 11 - \frac{1}{2}t$$

$$P^D = 11 + \frac{1}{2}t$$

$$Q^{\text{tax}} = 9 - \frac{1}{2}t$$

Solving for the Optimal Tax, continued...

$P^S = 11 - \frac{1}{2}t, \quad P^D = 11 + \frac{1}{2}t, \quad Q^{\text{tax}} = 9 - \frac{1}{2}t$

- $CS = \frac{1}{2}bh = \frac{1}{2}Q(20 - P^D) = \boxed{\frac{1}{2}(9 - \frac{1}{2}t)^2}$
- $PS = \frac{1}{2}bh = \frac{1}{2}Q(P^S - 2) = \boxed{\frac{1}{2}(9 - \frac{1}{2}t)^2}$
- $TR = Q \times t = \boxed{(9 - \frac{1}{2}t)t}$
- $Q \times \varepsilon = \boxed{(9 - \frac{1}{2}t)\varepsilon}$

Then we can find $TS = CS + PS + TR - (Q \times \varepsilon)$ and maximize it!

- $TS = -\frac{1}{4}t^2 + \frac{1}{2}\varepsilon\, t + 81 - 9\,\varepsilon$
- The input which maximizes a quadratic function is equal to $\frac{-b}{2a}$
- So

$$t^{\text{optimal}} = \frac{-(\varepsilon/2)}{2(-1/4)} = \varepsilon$$

Exercise 1: Crackin' Open a Cold One

Q1: Consider the following market for beers:

- $Q^D = 20 - P$
- $Q^S = P - 2$

Suppose that drinking beers at a party makes the party more fun for everybody, including non-drinkers, to the tune of \$2 of fun per beer.

1. Find the market equilibrium quantity.
2. What kind of externality is this?
3. What is the appropriate policy response?
4. How big should the response be?
5. Assume the policy is implemented. Find the socially optimal quantity.

Exercise 1: Crackin' Open a Cold One

Solution:

1. $P = 11,\ Q = 9$
2. Positive
3. The government should subsidize beer
4. The optimal subsidy is equal to the size of the externality: $\sigma^{\text{optimal}} = \$2$
5. $P^D = P^S - \sigma = P^S - 2$, so

$$
\begin{aligned}
Q^D(P^D) &= Q^S(P^S) \\
20 - P^D &= P^S - 2 \\
20 - (P^S - 2) &= P^S - 2 \\
\implies \quad P^S &= 12, \\
P^D &= 10, \\
Q^{\text{optimal}} &= 10.
\end{aligned}
$$

Exercise 2: Sugar Consumption

Consider the market for sugar:

- Demand: $Q^D = 400 - 10P$
- Supply: $Q^S = 30P - 800$
- Suppose sugar consumption negatively affects one's health, which adds \$4 of cost to the Medicare and Medicaid systems per unit of sugar.

1. Find the market equilibrium quantity.
2. What kind of externality is this?
3. What is the appropriate policy response?
4. How big should the response be?
5. Assume the policy is implemented. Find the socially optimal quantity.

Exercise 2: Sugar Consumption

Solution:

1. $P = 30,\ Q = 100$
2. Negative
3. The government should tax sugar consumption
4. The optimal tax is equal to the size of the externality: $t^{\text{optimal}} = \$4$
5. $P^D = P^S + t = P^S + 4$, so

$$\begin{aligned} Q^D(P^D) &= Q^S(P^S) \\ Q^D(P^S + 4) &= Q^S(P^S) \\ 400 - 10(P^S + 4) &= 30P^S - 800 \\ P^S = 29,\ P^D &= 33,\ Q^{\text{optimal}} = 70 \end{aligned}$$

Food for Thought

- Suppose a nuclear power plant is leaking radioactive waste into a nearby river.
- Some economists estimate in dollar terms the harm befalling the people downstream and the river's ecosystem.
- The government imposes the optimal tax. Are we satisfied with this solution?