Kalman Filtered Compressive Sensing Using Pseudo-Measurements

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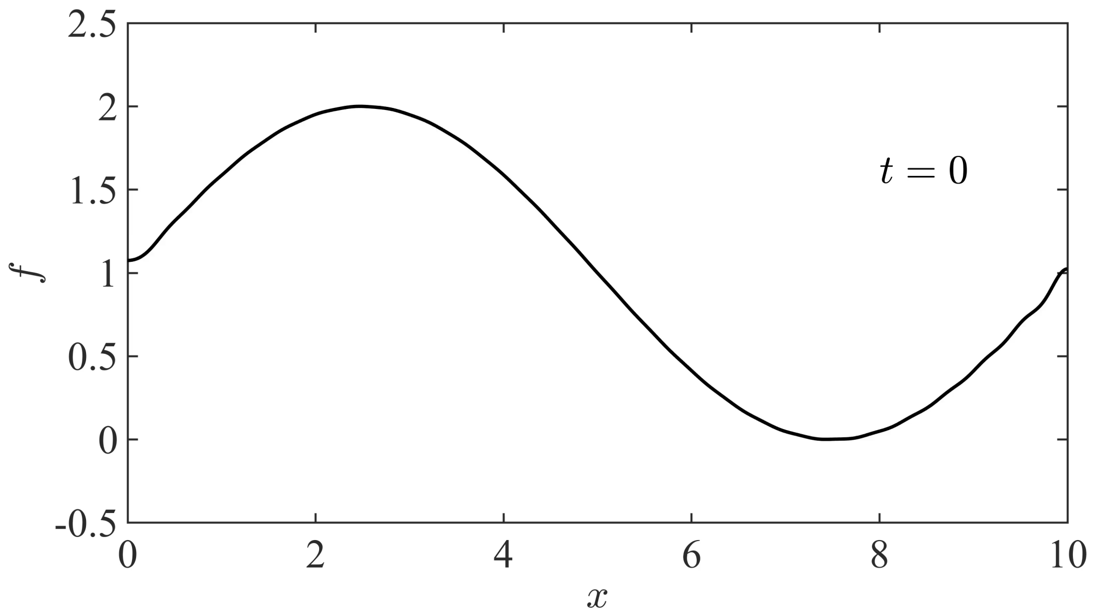

Temperature Distribution

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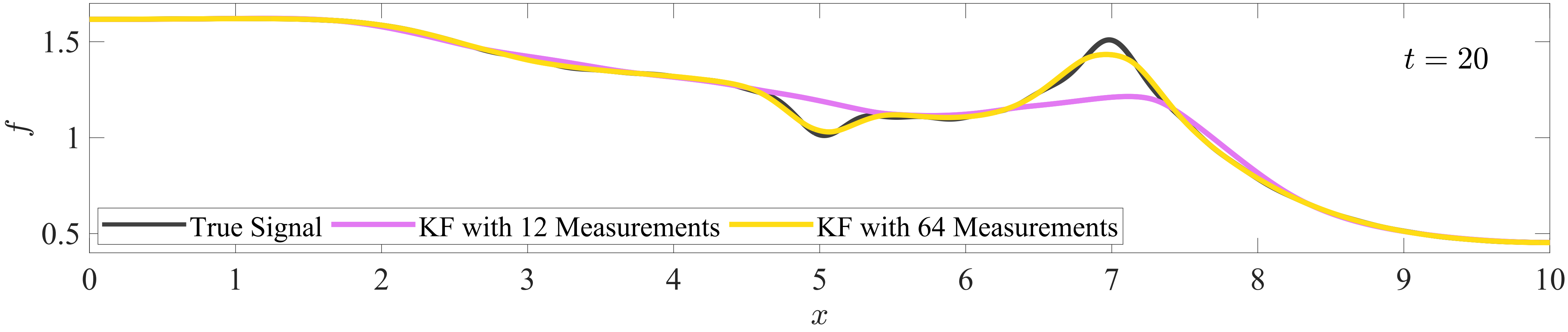
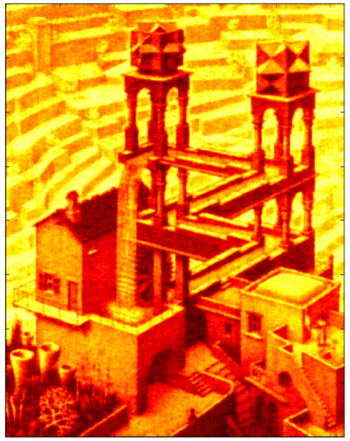


The number of measurements required by Kalman filter is large.

Compressive Sensing



http://www.pyrunner.com/weblog/2016/05/26/compressed-sensing-python/

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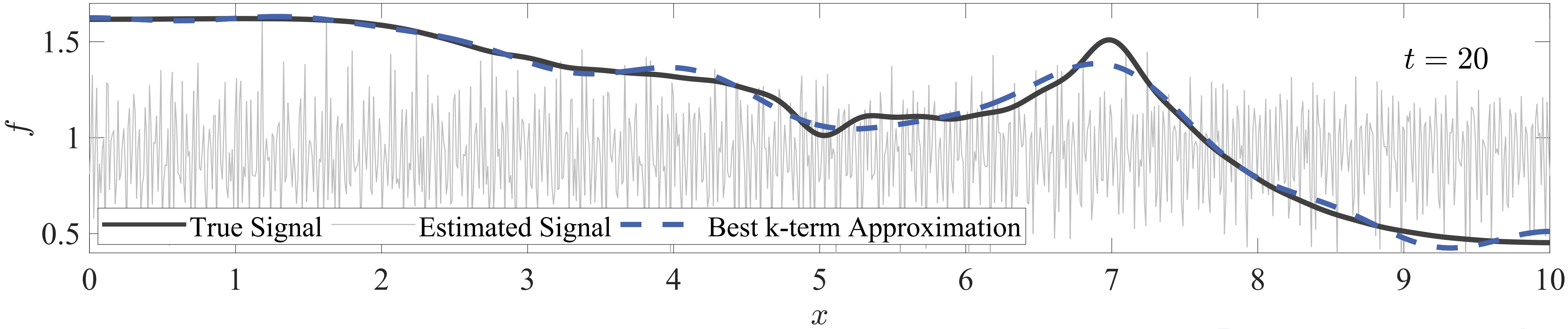

Representation of signal:

$$\underline{f}_n = \Theta \cdot \underline{z}_n$$

Representation of sensing:

$$\underline{\tilde{y}}_n = \Phi_n \cdot \underline{f}_n = \Phi_n \cdot \Theta \cdot \underline{z}_n$$

$$:= \widetilde{\Theta} \cdot \underline{z}_n$$

$$\hat{\underline{z}}_n = \arg\min \left\| \underline{z}_n \right\|_1$$

$$\text{s.t.} \ \left\| \underline{\tilde{y}}_n - \widetilde{\Theta}_n \cdot \hat{\underline{z}}_n \right\|_2 \leq \varepsilon$$

[D. L. Donoho et al. 2003]

Best k-term approximation

$$z' = \arg\min_{v \in \Sigma_k} \left\| \underline{f} - \Theta v \right\|_2$$

$$f' = \Theta \cdot z'$$

Prerequisites for CS:

1. Signal should be sparse/compressible
2. Sensing matrix must satisfy Restricted Isometry Property (RIP)

Example: $\Phi_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \rightarrow \Phi_n \cdot f = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix}$

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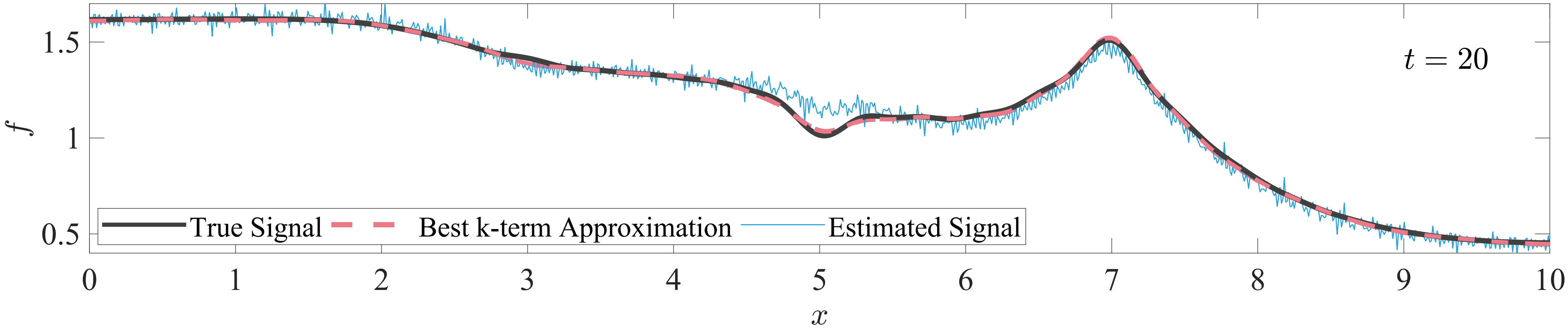

Sparse Changes ⇒ Compressibility

$\underline{f}_n = \Phi \cdot \underline{z}_n$ $\quad\quad$ $\boldsymbol{\underline{f}_n = \Phi \cdot \left(\hat{\underline{z}}_{n-1} + \Delta \underline{z}_{n-1}\right)}$

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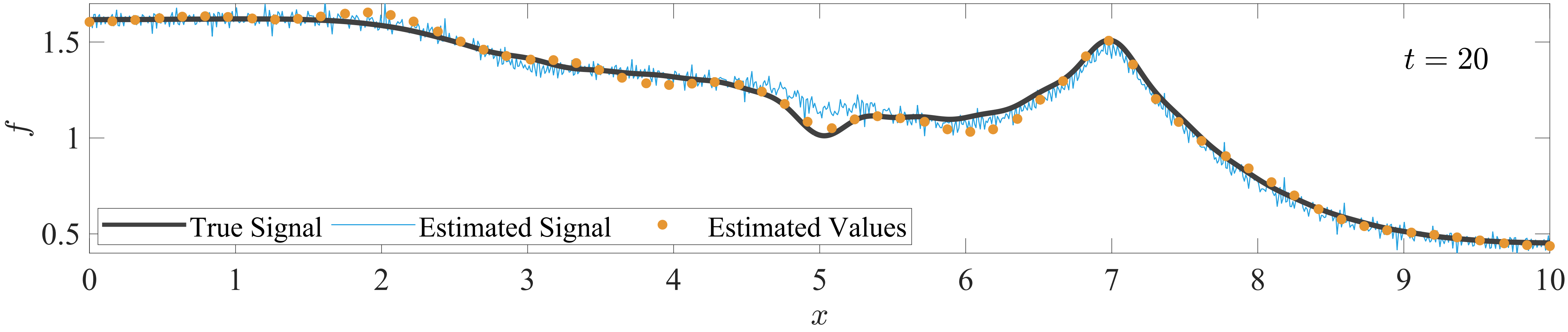

Restricted Isometry Property (RIP)

$$(1-\delta)\|\underline{z}\|_2^2 \leq \|\widetilde{\Theta} \cdot \underline{z}\|_2^2 \leq (1+\delta)\|\underline{z}\|_2^2$$

Orthogonality

Dimensionality Reduction:

$$\mathbf{1024 \rightarrow 64}$$

Dimensionality	Mean Square Error (MSE)
1024	1.918
64	0.843

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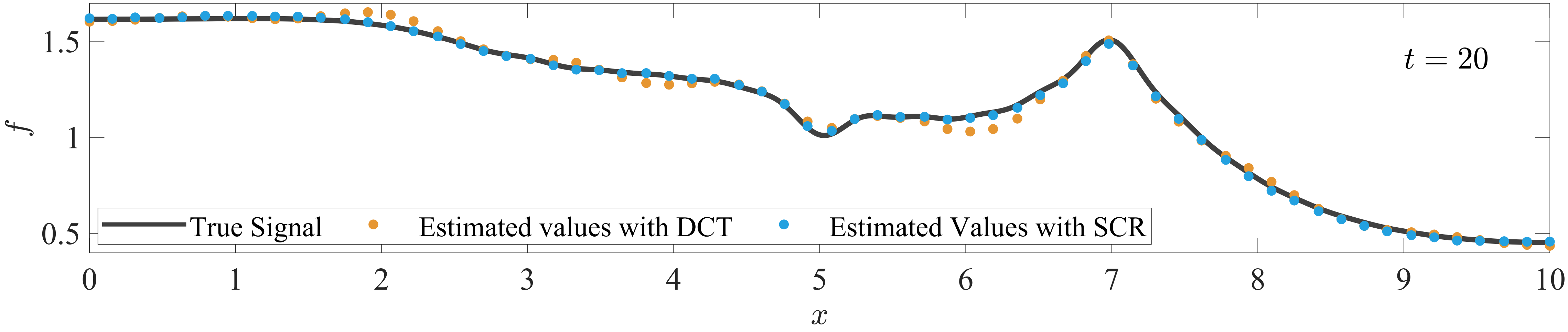

Sparse Coding

$$\min_{\alpha_n, \Theta} \sum_{n=1}^{N} \left\{ \left\| \underline{y}_n - \Theta \cdot \alpha_n \right\|_2^2 + \lambda \left\| \underline{\alpha}_n \right\|_0 \right\}$$

$$\text{s.t.} \quad \Theta^{\mathrm{T}} \Theta = I$$

Basis	Mean Square Error (MSE)
DCT	0.843
SCR	0.141

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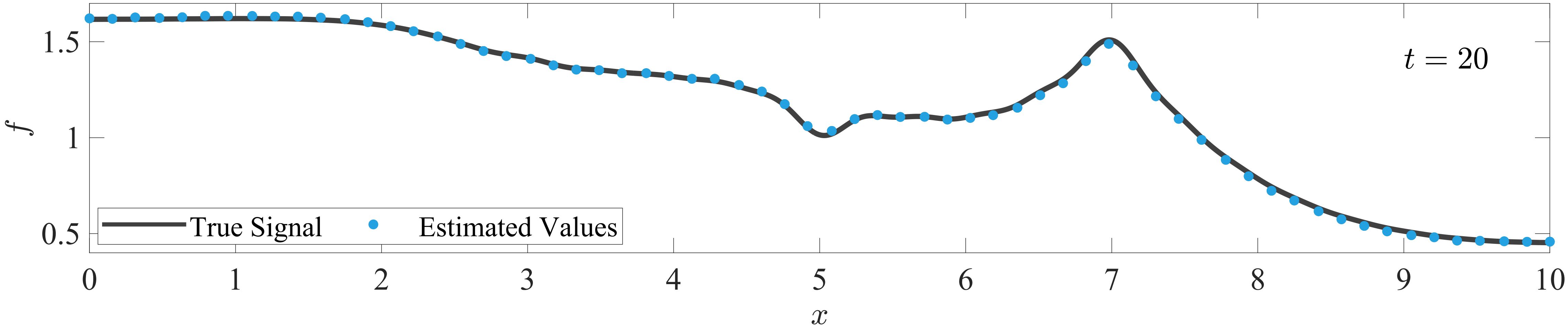

Problems:

1. Discrete points instead of a signal
2. How to integrate CS with KF

Pseudo-Measurements

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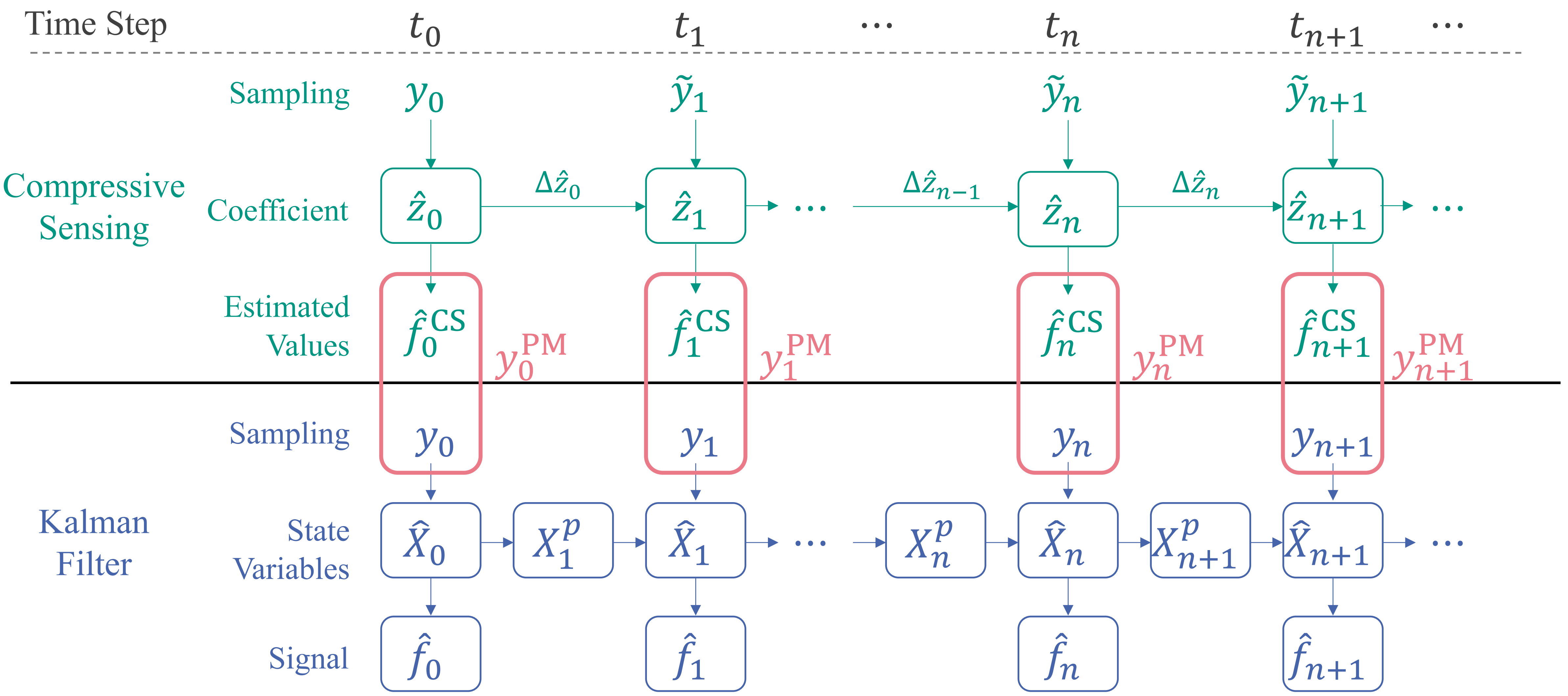

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Dynamic Weighting

$$\|e\|_2 = \left\|\underline{\hat{z}} - \underline{z}\right\|_2 \leq C\,\frac{\min_{\underline{v}_K \in \Sigma_K} \left\|\underline{z} - \underline{v}_K\right\|_1}{\sqrt{K}} \approx C\,\frac{\text{Best } K - \text{term Approximiation of } \hat{z}}{\sqrt{K}}$$

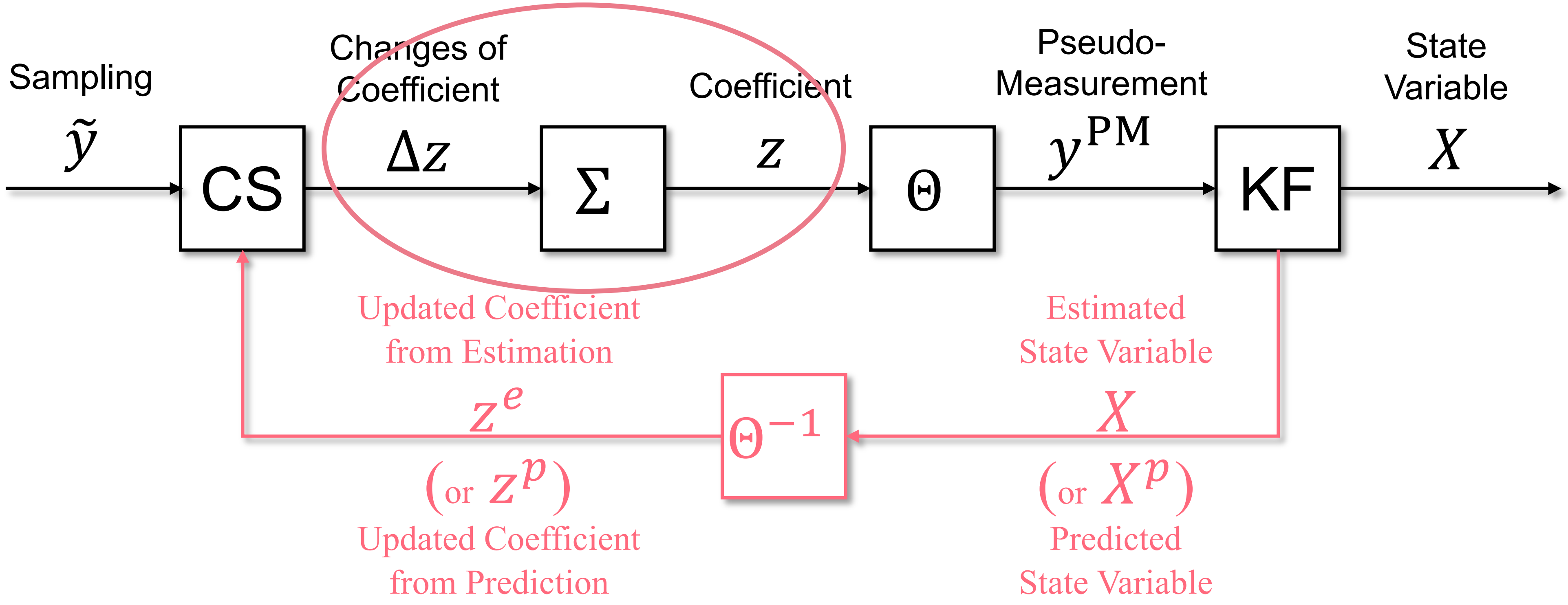
$\mathbb{E}\{\hat{z}\} = \mathbb{E}\{z\}$

3σ bound of the estimation error

Measurements from sensors and Pseudo-Measurements have different uncertainty

Uncertainty of Pseudo-Measurements changes from time to time

Coefficients Updating



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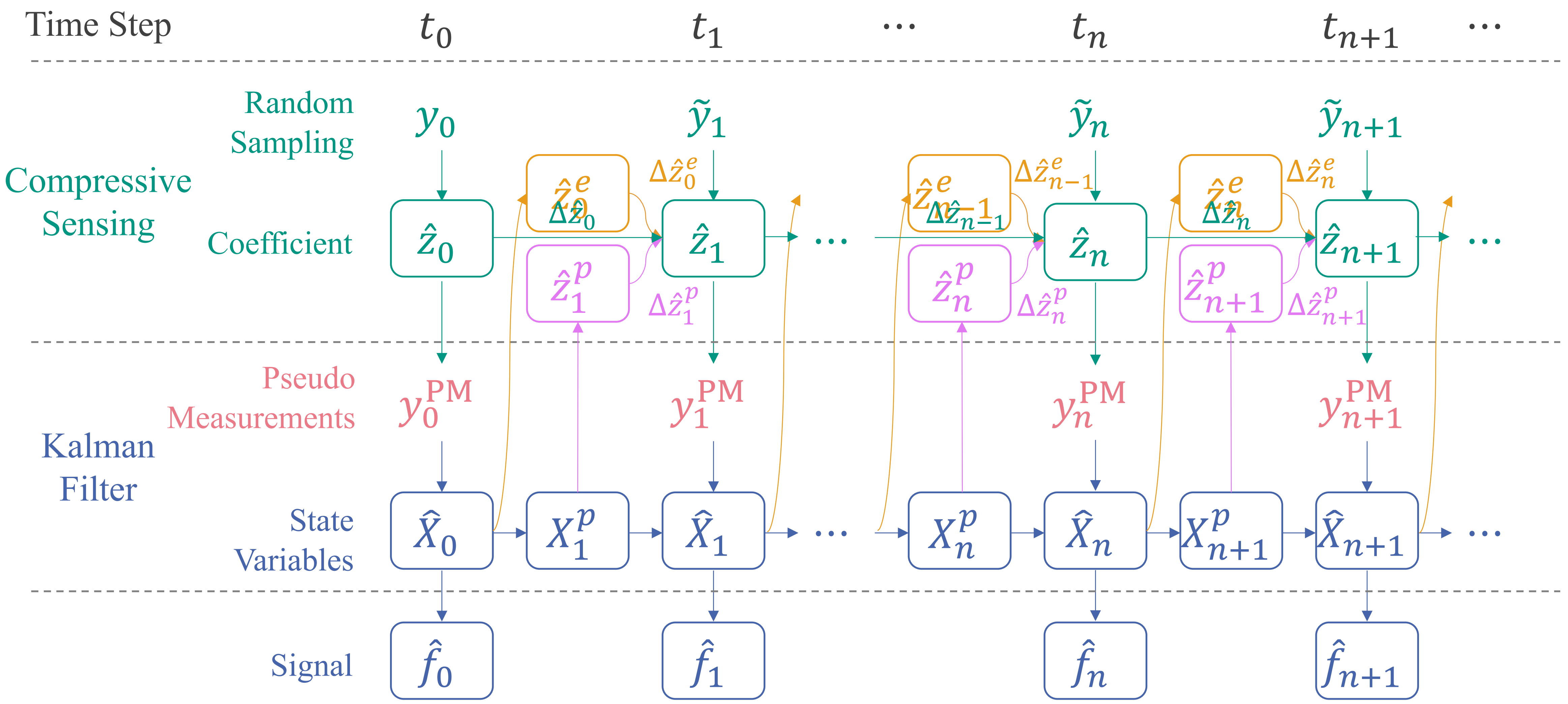

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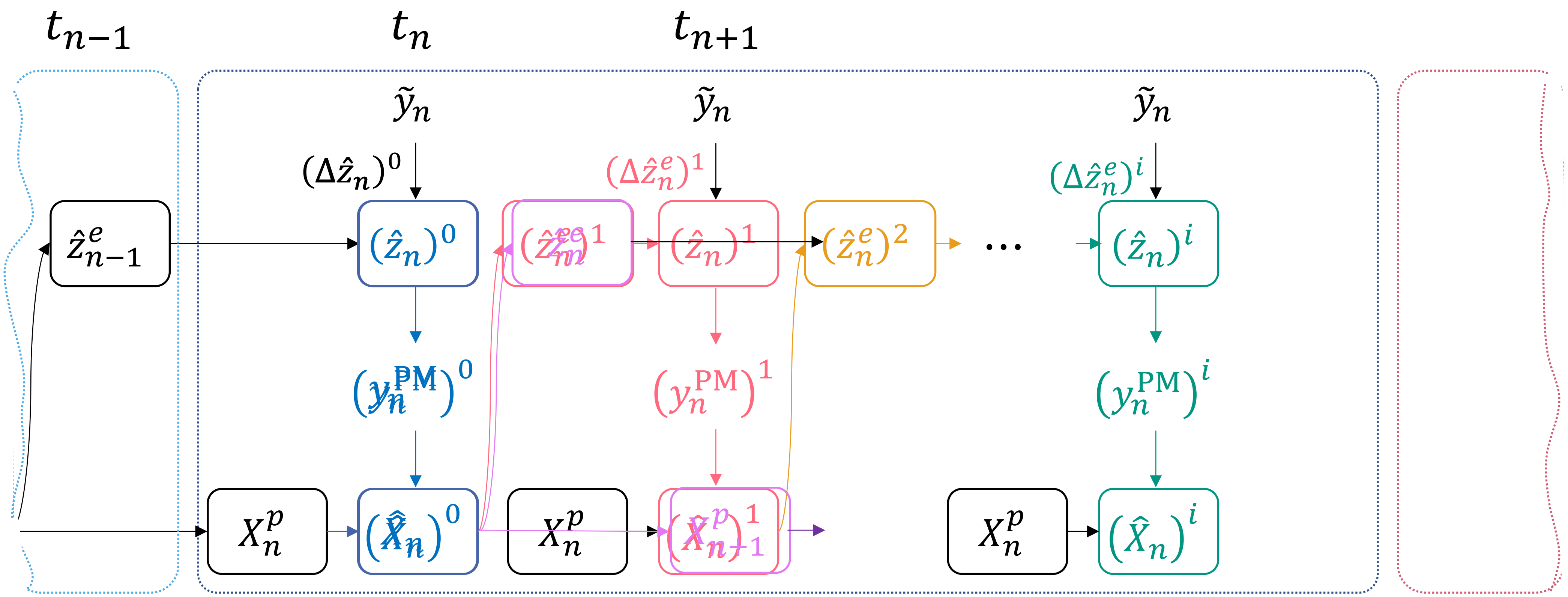
Iterated Updating



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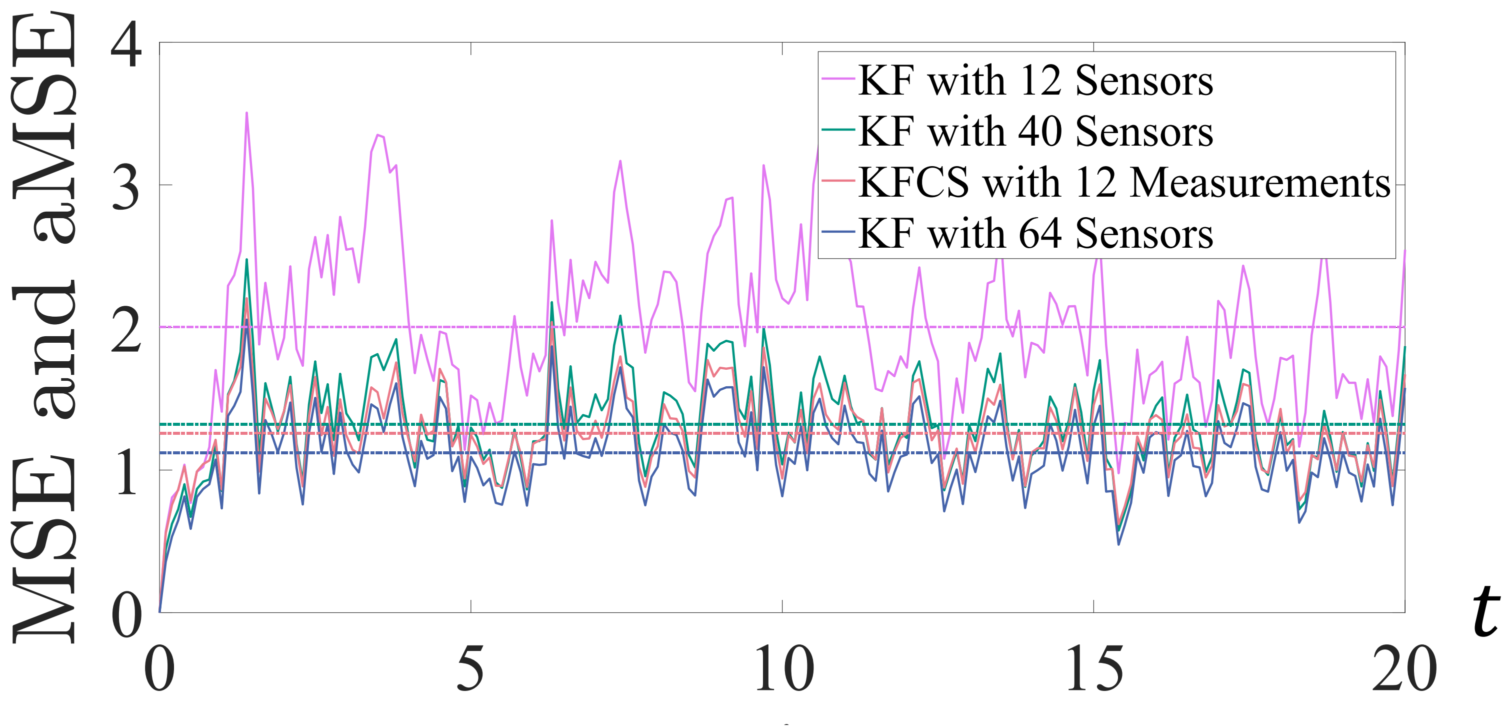
Evaluation



$$\mathrm{MSE}_n = \frac{1}{N_S}\sum_{s=1}^{N_S}\left\|\hat{f}_n^{(s)} - f_n^{(s)}\right\|_2^2$$

$$\mathrm{aMSE} = \frac{1}{N_t}\sum_{n=1}^{N_t}\mathrm{MSE}_n$$

	KFCS 12
KF 12	37%
KF 40	5%
KF 64	-11%

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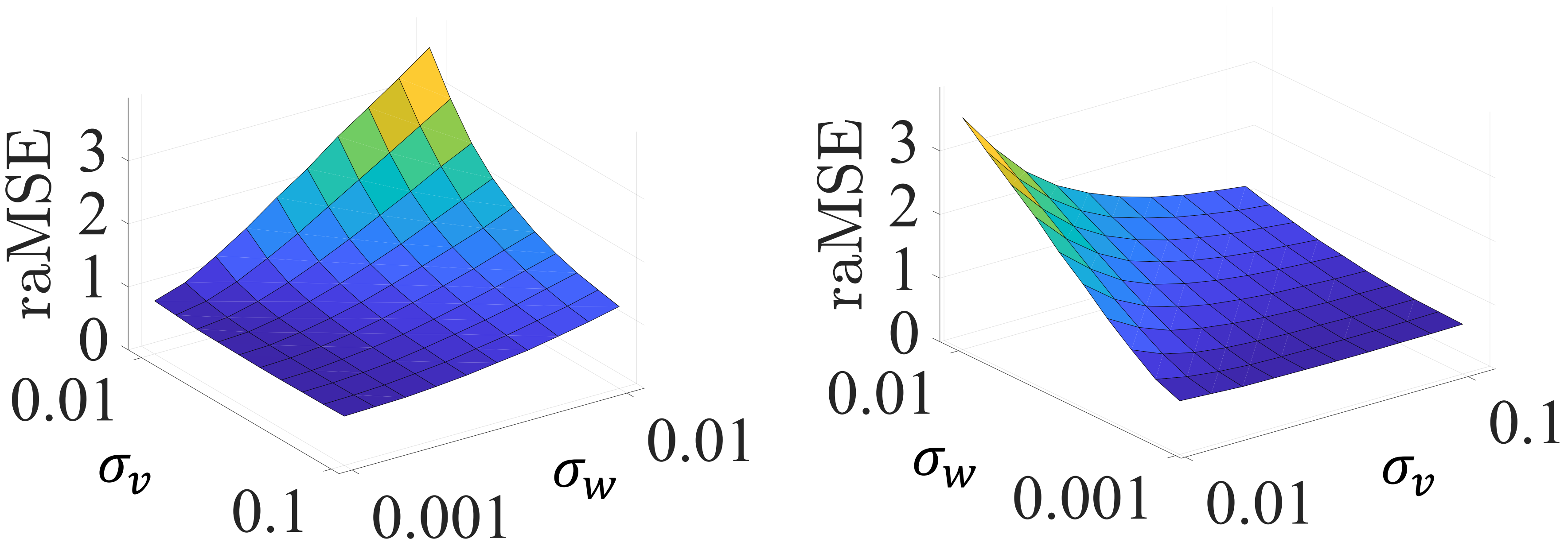

$$\mathrm{raMSE} = \frac{\mathrm{aMSE}_{\mathrm{KFCS}}}{\mathrm{aMSE}_{\mathrm{KF}}}$$

σ_w: System noise
σ_v : Measurement noise

Our proposal is more robust against measurement noise

Precise system model + Low sensor quality

Thank you for your attention