CSCI 170 Homework #5

Due Date: June 22, 2018 at 11:59 P.M.

1. You may submit without any penalty until June 25, 2018 at 11:59 P.M.

2. *Scheduling classes*

   (a) A math professor is in a bind to offer an introductory math class to a set of students. Here's why: Alice won't take a math class with Bob, Bob won't take a math class with Charlie, Charlie won't take a math class with David, David will not take a math class with Edward and Edward will not take a math class with Alice. To make matters worse, Fiona will not take take a math class with David or Alice. At least how many times must this math professor offer this class to accommodate this aforementioned set of students?

   (b) A new extremely accommodating school first asks students what classes they would like to take for the term. Then once the registrar has this list of desired classes, the registrar determines the minimum number of time slots necessary for the courses such that no students have any scheduling conflicts. First, translate the registrar's problem into graph theoretic terms and then determine how the registrar can determine the minimum number of slots needed.

3. Show that any graph with at least two vertices has at least two vertices of the same degree.

4. In class we discussed Kruskal's algorithm for finding a minimum spanning tree (MST) of a weighted connected graph. In this problem we will discuss Prim's algorithm to find the MST of a weighted graph. Prim's is also a greedy algorithm and works as follows: Choose any arbitrary vertex, s, to be the starting vertex. Then at each iteration of the algorithm add the lowest cost edge to the MST such that the edge does not create a cycle in the MST and the edge connects a vertex not in the MST with a vertex that is already in the MST. This is how the algorithm expands the MST at each iteration. The algorithm ends when all vertices are included in the MST.

   (a) First, write the cut property in predicate logic: Let $G = (V, E)$ be a connected weighted graph. For any subset of vertices $S \subseteq V$, let e be the minimum cost edge such that exactly one endpoint of e is in S and the other endpoint is in $V - S$, then the MST of G must contain e.

   (b) Prove the cut property by contradiction. You may assume without loss of generality that the edge weights of the graph are distinct.

   (c) Prove Prim's algorithm by induction over the number of vertices in the MST, $n \geq 1$. You may assume without loss of generality that the edge weights are distinct and you may use the cut property directly.

5. *Surveying first-year computer science students*

   (a) A computer science department creates a survey to learn how many incoming first-year students have already programmed in specific languages. The first question asks students if they have ever written a program in Java. To this question, 18 students reply yes. The second question asks students if they have ever written a program in Python. To this question, 12 students reply yes. The third question ask students if they have written a program in C/C++. To this question, 10 students reply yes. How many students have written a program in Java, Python, C, or C++? What additional information would you need to be certain of the number of students?

   (b) Show that for any sets A and B, $|A \cup B| = |A - B| + |B - A| + |A \cap B|$. (Hint: consider the inclusion-exclusion principle from set theory.)

6. *Arranging packages for delivery*

   (a) A postal worker must deliver 5 packages to 5 different customers in the same apartment complex. In how many ways may the postal worker deliver all the packages?

   (b) The same postal worker has 5 distinct mailbox keys on his keyring. How many ways are there to arrange his keys on the keyring? (An arrangement is the same if a key is between the same two keys regardless of their position on the keyring.)

   (c) The postal worker now has 3 packages for each of the 4 different customers. He has color coded each package to the same customer the same color, so he cannot distinguish the packages to a single customer any longer. When he looks in his bag he sees 12 packages, 3 each of 4 distinct colors. In how many distinct ways can the postal worker pull all the packages from his bag?