SOLUTION

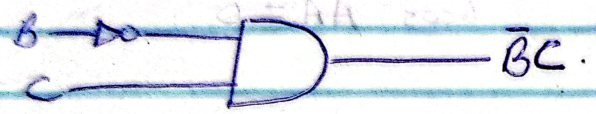
There are two 1s in the output column of the given truth table. The corresponding binary are 001 and 101. These values are converted into product terms as follows

$001 \rightarrow \bar{A}\bar{B}C$ and $101 \rightarrow A\bar{B}C$

$\therefore Y = \bar{A}\bar{B}C + A\bar{B}C$

$= \bar{B}C(\bar{A}+A)$ { NOTE $\bar{A}+A = 1$

$= \underline{\bar{B}C}$



EXAMPLE THREE

Prove the following Boolean identity: $(A+B)(A+C) = A+BC$

SOLUTION

$Y = (A+B)(A+C)$

$= AA + AC + BA + BC$

$= A + AC + AB + BC$

$= A(1+C) + AB + BC$ Recall $(1+C) = 1$

$= A + AB + BC$

$= A(1+B) + BC$ Recall $(1+B) = 1$

$= \underline{A + BC}$

EXAMPLE FOUR

Prove the following identity $A + \bar{A}B = A + B$

SOLUTION

$Y = A + \bar{A}B = A \cdot 1 + \bar{A}B$

$= A(1+B) + \bar{A}B$

$= A \cdot 1 + AB + \bar{A}B$

$= A + BA + \bar{A}B$

$= A + B(A + \bar{A}) = A + B \cdot 1$

$= \underline{A + B}$

EXAMPLE FIVE

Prove the following identity:

(a) $(A+B)(A+\bar{B})(\bar{A}+C) = AC$

(b) $ABC + A\bar{B}C + AB\bar{C} = A(B+C)$

SOLUTION

(a) $Y = (A+B)(A+\bar{B})(\bar{A}+C) = (AA + A\bar{B} + BA + B\bar{B})(\bar{A}+C)$

$= (A + AB + A\bar{B})(\bar{A}+C) = [A(1+B) + A\bar{B}](\bar{A}+C)$ bcos

$= (A + A\bar{B})(\bar{A}+C)$

$= A(1+\bar{B})(\bar{A}+C)$

$= A(\bar{A}+C) = A\bar{A} + AC$ bcos $A\bar{A} = 0$

$= AC$

(b) $Y = ABC + A\bar{B}C + AB\bar{C}$

$= AC(B+\bar{B}) + AB\bar{C}$

$= AC + AB\bar{C}$

$= A(C + B\bar{C})$

$= A(C+B)$

$= AC + AB$

EXAMPLE SIX

Simplify the following Boolean expression:

$AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}C$

SOLUTION

$Y = AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}C$

$Y = ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$

$Y = AB(C+\bar{C}) + A\bar{B}(\bar{C}+C) + \bar{A}BC$

$Y = AB + A\bar{B} + \bar{A}BC$

$Y = A(B+\bar{B}) + \bar{A}BC$

$Y = A + \bar{A}BC$

$Y = A + BC$

E SEVEN

Design a logic Circuit whose output is HIGH only when majority of the inputs A, B and C are HIGH

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

SOLUTION

Since there are three inputs, A, B and C, therefore whenever two or more than two (i.e. a majority) inputs are HIGH, the output is HIGH. This situation can be represented in the form of a table as shown above.

There are four 1s in the output column of the truth table. The corresponding binary values are 011, 101, 110 and 111 respectively. Converting these values into product term and summing up all the terms, we get

$$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$$

For easy simplification, we add the term ABC two times to the Boolean expression for outputs.

$$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC$$

NOTE
$ABC + ABC + ABC = ABC$

Bringing together those terms which have two common letters, we get,

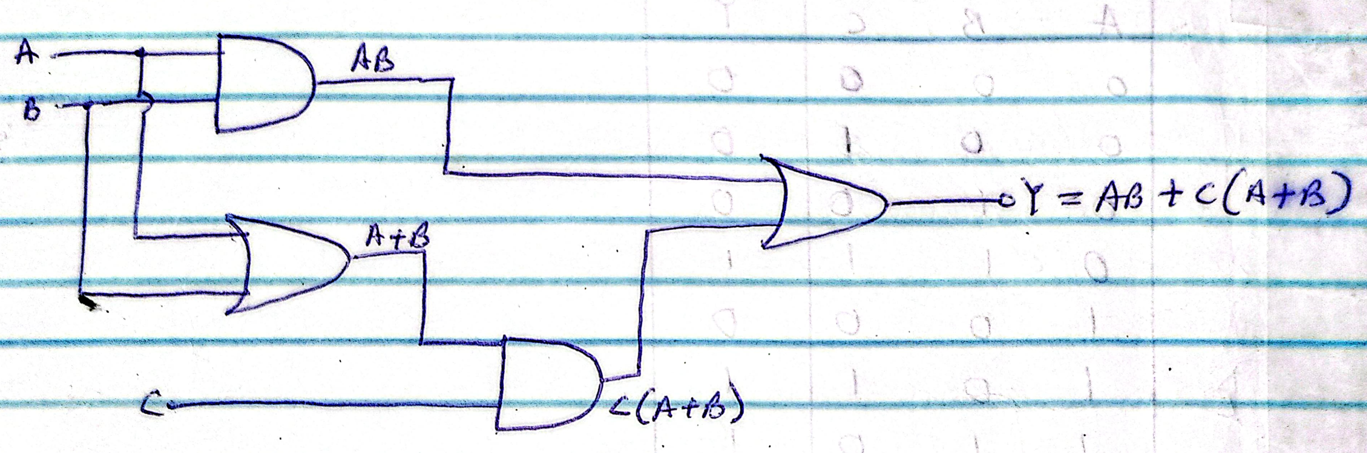
$$Y = AB\bar{C} + ABC + A\bar{B}C + ABC + \bar{A}BC + ABC$$

$$Y = AB(\bar{C}+C) + AC(\bar{B}+B) + BC(\bar{A}+A)$$

$$Y = AB + AC + BC$$

$$Y = AB + C(A+B)$$

The logic circuit that produces the output $Y = AB + C(A+B)$ is shown below



DE MORGAN'S THEOREM

These two theorems (or rules) are a great help in simplifying Complicated logical expressions.

The theorems can be stated as follows:

(1) $\overline{A+B} = \bar{A}\cdot\bar{B}$

(2) $\overline{A \cdot B} = \bar{A}+\bar{B}$

The first statement says that the complement of a sum is equal the product of complements. In fact, it allows transformation from a sum-of-product form to a product-of-sum form.

As seen from the above two laws, the procedure required for taking out an expression from under a NOT sign is as follows:

1. Complement the given expression i.e., remove the overall NOT sign
2. Change all the AND's to OR's and all the OR's to AND's,
3. Complement or negate all individual variables.

As an illustration, take the following example

$$\overline{A+BC} = \overline{A+BC}$$
$$= \overline{A\cdot(B+C)}$$
$$= \bar{A}(\bar{B}+\bar{C})$$

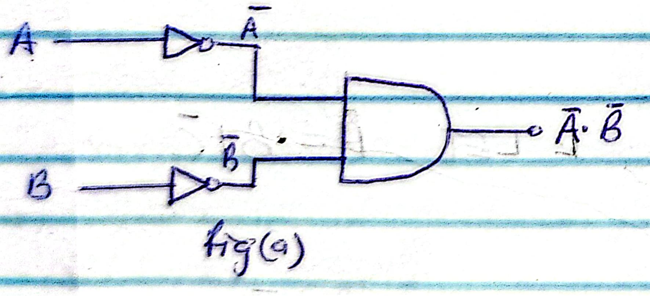
Next, Consider this example

$$\overline{(\bar{A}+B+\bar{C})(\bar{A}+B+C)} = \overline{(\bar{A}+B+\bar{C})}\,\overline{(\bar{A}+B+C)}$$
$$= (\bar{\bar{A}}\bar{B}\bar{\bar{C}})+(\bar{\bar{A}}\bar{B}\bar{C})$$
$$= (\bar{\bar{A}}\bar{B}\bar{\bar{C}})+(\bar{\bar{A}}\bar{B}\bar{C})$$
$$= (A\bar{B}C)+(A\bar{B}\bar{C})$$

This process is called demorganization. It should, however, be noted that opposite procedure would be followed in bring an expression under the NOT sign. Let us bring the expression $\bar{A}+\bar{B}+\bar{C}$ under the not sign.

$$\bar{A}+\bar{B}+\bar{C} = \bar{\bar{\bar{A}}}+\bar{\bar{\bar{B}}}+\bar{\bar{C}} \longrightarrow \text{Step 3}$$
$$= \overline{A+B+C}$$
$$= \overline{\overline{ABC}} \longrightarrow \text{Step 2}$$
$$= \overline{ABC} \longrightarrow \text{Step 1}$$

The figures below show the circuits to illustrates De Morgan's theorems.



fig(a)

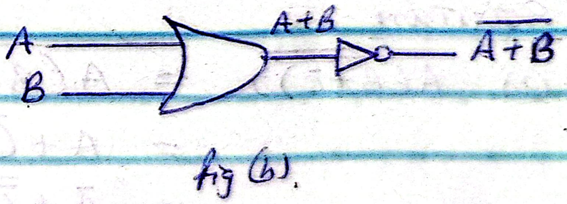


fig (b).

As seen, basic Logic function can be either OR gate or AN gate.

EXAMPLE EIGHT

Demorganize the expression: $\overline{(A+B)(C+D)}$

SOLUTION

$$\overline{(A+B)(C+D)} = \overline{(A+B)}\,\overline{(C+D)}$$
$$= \overline{\overline{(AB)}+\overline{(CD)}}$$
$$= (\bar{A}\bar{B})+(\bar{C}\bar{D}) = (\overline{AB})+(\overline{CD})$$

EXAMPLE NINE

Simplify the expression: $(AB+C)(AB+D)$

SOLUTION.

Let $Y = (AB+C)(AB+D)$

$= ABAB + ABD + ABC + CD$

$= AABB + ABD + ABC + CD$

RECALL, $A \cdot A = A$, $B \cdot B = B$

$= AB + ABD + ABC + CD$

$= AB(1+D) + ABC + CD$

RECALL, $1+D = 1$

$= AB + ABC + CD$

$= AB(1+C) + CD$

$= AB + CD$

$\therefore (AB+C)(AB+D) = AB + CD.$

EXAMPLE TEN

Simplify each of the following expression using Demorgan's theorem (a) $A\overline{(B+\bar{C})}D$ (b) $\overline{(M+\bar{N})(\bar{M}+N)}$ (c) $\overline{\overline{AB}}\bar{C}D$

SOLUTION

(a) $\overline{A\overline{(B+\bar{C})}D} = A\overline{(B+\bar{C})}D$ LET $P = B+\bar{C}$

$= A + \overline{(P)} + D$

$= \bar{A} + \overline{\overline{(P)}} + \bar{D}$

$= \bar{A} + P + \bar{D} = \bar{A} + B + \bar{C} + \bar{D}$

(b) $\overline{(M+\bar{N})(\bar{M}+N)} = (M+\bar{N})(\bar{M}+N)$

$= (M\bar{N}) + (\bar{M}N)$

$= (\bar{M}\bar{\bar{N}}) + (\bar{\bar{M}}\bar{N})$

$= (\bar{M}N) + (M\bar{N})$

$= \underline{\bar{M}N + M\bar{N}}$

(c) $\overline{\overline{\overline{AB}\,\bar{C}}D} = \overline{\overline{AB}\,\bar{C}D}$

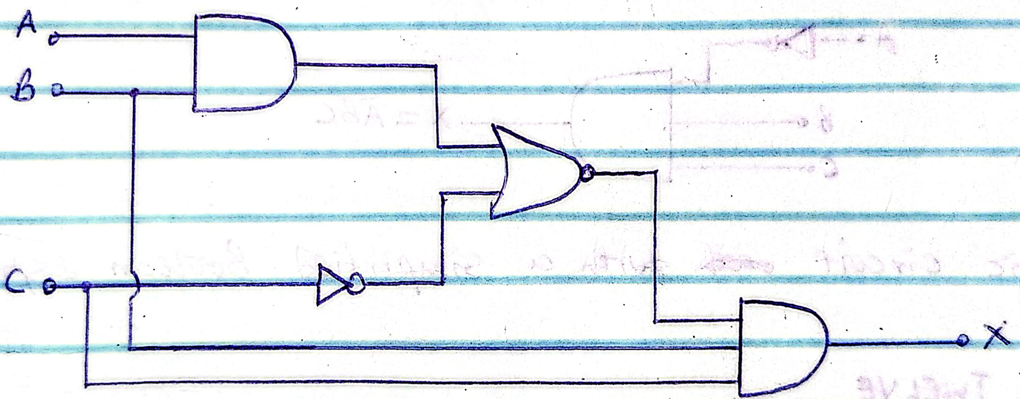
$= \overline{\overline{AB}\,\bar{C}} + D$

$= \overline{\overline{\overline{AB}\,\bar{C}}} + \bar{D}$

$= \overline{AB}\,\bar{C} + \bar{D}$

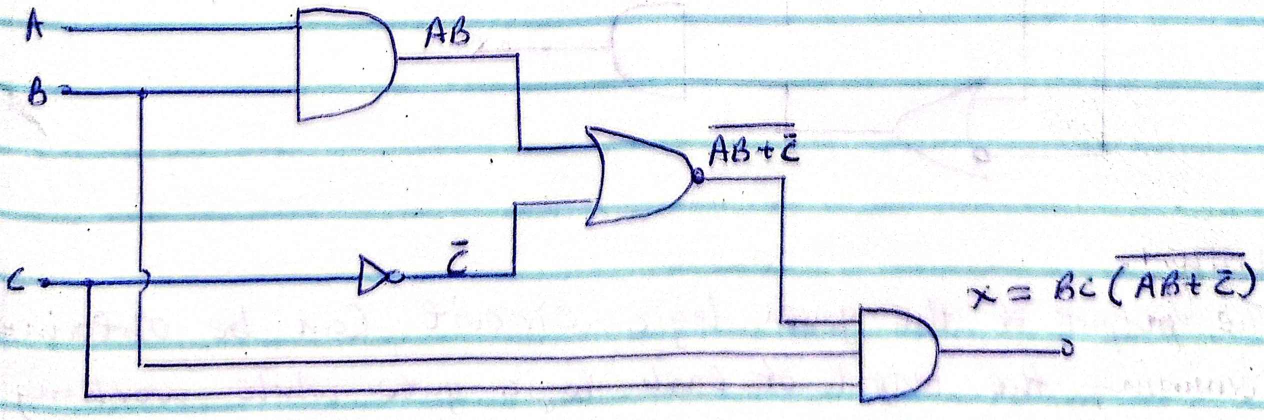
EXAMPLE ELEVEN

Determine the Boolean expression for the logic circuit shown below. Simplify the Boolean expression using Boolean law and De-morgan's theorem. Redraw the Logic circuit using simplified Boolean expression.



SOLUTION

The output of the given circuit can be obtained by determining the output of each logic gate ~~which~~ while working from left to right.



Simplifying the output using De-Morgan's theorem.

$$\overline{BC(\overline{AB} + \bar{C})} \;=\; \overline{BC}\,(\overline{AB}+\bar{C})$$

$$= \overline{BC}\,(\overline{A+B})\cdot\bar{C}$$

$$= \overline{BC}\,(\bar{A}+\bar{B})\cdot\bar{\bar{C}}$$

$$= \overline{BC}(\bar{A}+\bar{B})\cdot C$$

$$= \overline{BC}\cdot C\,(\bar{A}+\bar{B})$$

RECALL, $C\cdot C = C$

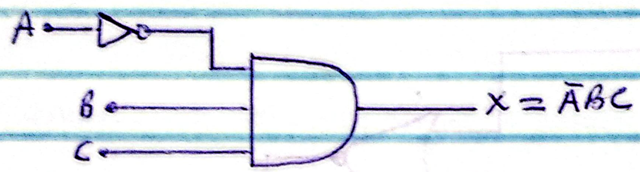
$$= BC(\bar{A}+\bar{B})$$

$$= \bar{A}BC + BC\bar{B}$$

$$= \bar{A}BC + CB\cdot\bar{B}$$

RECAL, $B\cdot\bar{B} = 0$

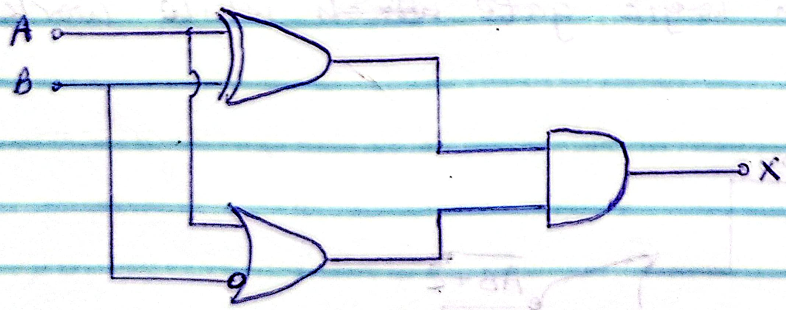
$$= \underline{\bar{A}BC}$$



The logic circuit ~~with~~ with a simplified Boolean expression.

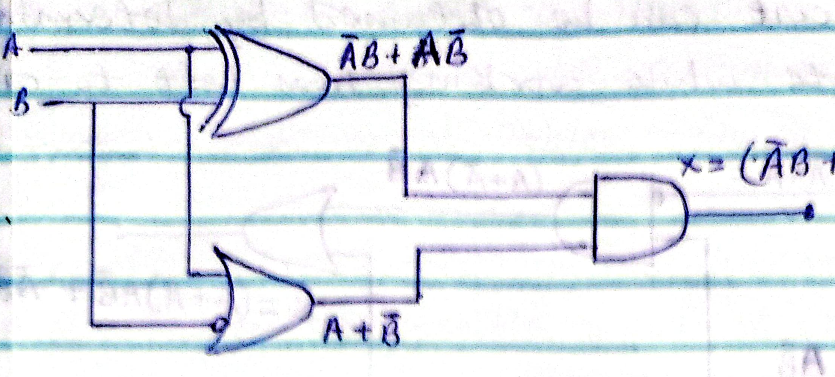
EXAMPLE TWELVE

Determine the output X of the logic circuit shown below. Using Boolean Laws and theorems simplify the output. Redraw the logic circuit with the simplified expression.



SOLUTION

The output of the given logic circuit can be obtained by determining the output of each logic gate while working from left to right.



As seen from the figure above, output $x = (\bar{A}B + A\bar{B})(A + \bar{B})$

$= A\bar{A}B + \bar{A}B\bar{B} + AA\bar{B} + A\bar{B}\bar{B}$

RECALL, $A \cdot \bar{A} = 0$, $B \cdot \bar{B} = 0$, $A \cdot A = A$, $\bar{B} \cdot \bar{B} = \bar{B}$

$= 0 + 0 + A\bar{B} + A\bar{B}$

$= A\bar{B} + A\bar{B}$, RECALL, $\bar{B} + \bar{B} = \bar{B}$

$= \cancel{A\bar{B}}\ A(\bar{B} + \bar{B})$

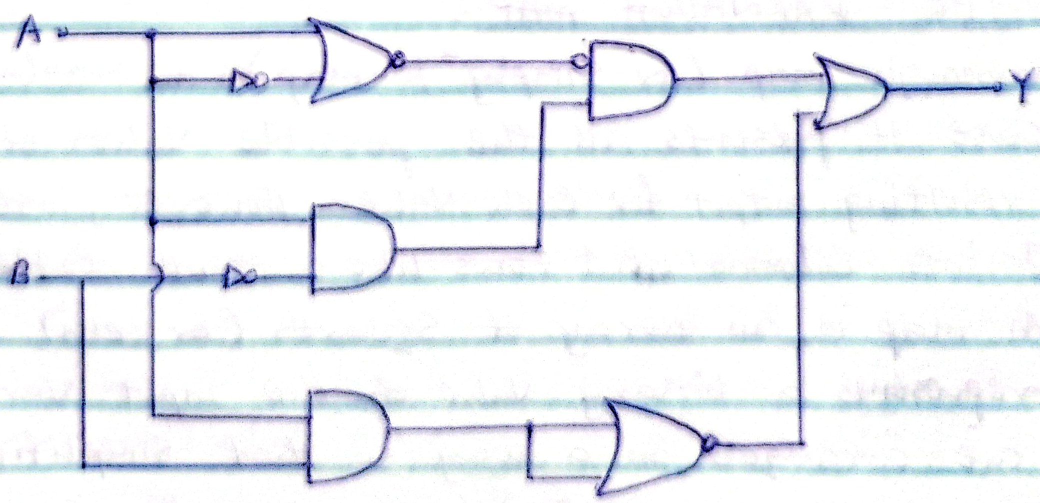
$= \underline{\underline{A\bar{B}}}$ Ans

EXAMPLE THIRTEEN

Determine the output of the logic circuit ~~using~~ shown below. Simplify the output Boolean expression and sketch the logic circuit.

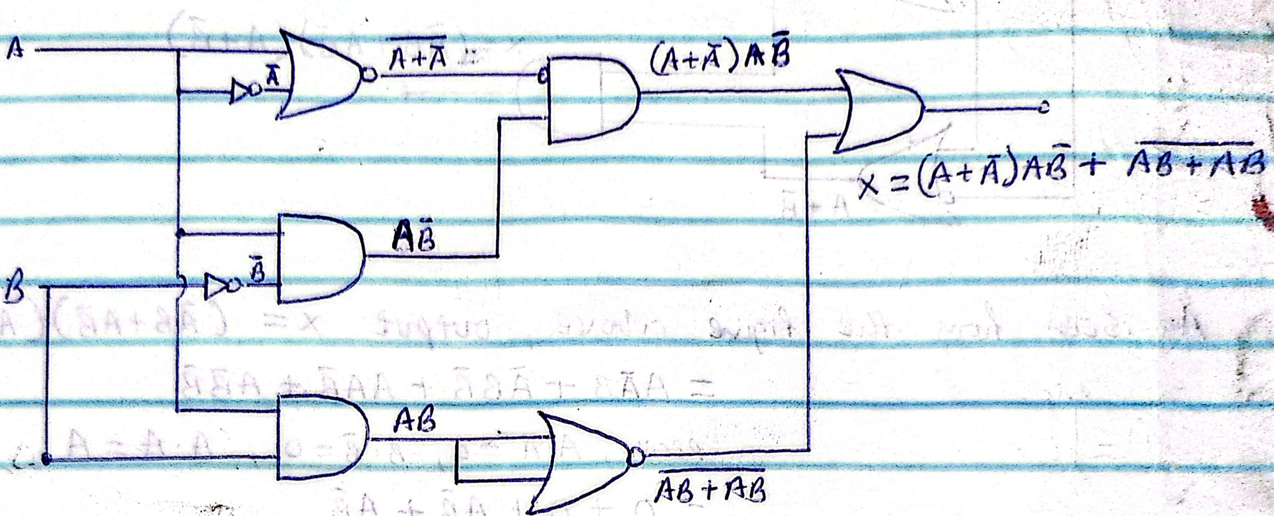
~~SOLUTION~~

~~The output of the ci~~



SOLUTION

The output of the circuit can be obtained by determing the output of each logic gate while working from left to right.



Using de-morgan's on $\overline{AB+AB} = AB+AB$

$= (A+B)\cdot(A+B)$

$= (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{B})$

$= \bar{A}\bar{A}+\bar{A}\bar{B}+\bar{A}\bar{B}+\bar{B}\bar{B}$

$= \bar{A}(1+\bar{B})+\bar{B}(1+\bar{A})$

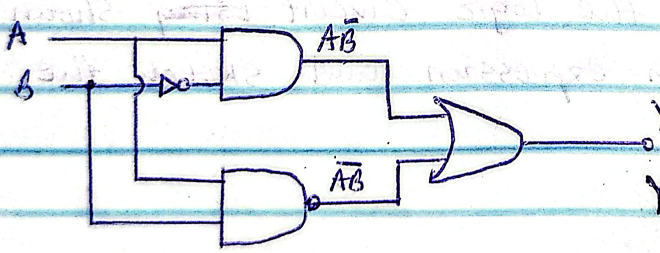
RECALL $\bar{A}\bar{A} = \bar{A}$, $(1+\bar{B}) = 1$

$= \bar{A}+\bar{B} = \overline{AB}$

RECALL $\overline{A+B} = \bar{A}\cdot\bar{B}$, $\overline{A\cdot B} = \bar{A}+\bar{B}$

$\therefore X = (A+\bar{A})A\bar{B} + \overline{AB}$ RECALL $A+\bar{A} = 1$

$X = A\bar{B} + \overline{AB}$



$Y = A\bar{B} + \overline{AB}$

$Y = A\bar{B} + \overline{AB}$

THE KARNAUGH MAP

The karnaugh map (or simply a K-map) is similar to a truth table because it preserves all the possible values of input variable and the resulting output for each value. However, instead of being organised into columns and rows like a truth table, the karnaugh map is an array of squares (or cells) in which each square represents a binary value of the input variables. The squares are arranged in a way so that simplification of given expression is simply a matter of grouping the squares. Karnaugh maps can be used for expression with two, three, four, and five variable Karnaugh maps to illustrate the principles. Karnaugh map with five-variable is beyond the scope of this writeup. For higher number of variables, Quine-McClusky method can be used.

The number of squares in a Karnaugh map is equal to the total number of possible input variable combinations (as is the number of rows in a truth table). For two variable, the number of square is $2^2 = 4$, for three variables, the number of squares is $2^3 = 8$ and for four variables, the number of squares is $2^4 = 16$.

The Two-Variable Karnaugh Map

The figure below shows a two-variable Karnaugh map.

B \ A	$\bar{A}$	A
$\bar{B}$	$\bar{A}\bar{B}$	$A\bar{B}$
B	$\bar{A}B$	AB

(a)

As seen, it is an array of four squares. In this case, A and B are used for two variables although any other two letters could be used. The binary values of A (i.e. 0 and 1) are indicated across the top as A and $\bar{A}$ and the binary values of B are indicated along the left side as A and $\bar{A}$. The value of a given square is the value of "A" at the top in the same column combine with the value of B at the left in the same row.

The Three-Variable Karnaugh Map

The figure below shows a three-variable Karnaugh map.

C \ AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
C	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

As seen it ~~has~~ is an array of eight squares. In this case, A, B and C are use for the variables although any other three letters could be used. The values of "A and B" are across the top and the value of "C" along the left side.

The value of a given square is the values of A and B at the top on the same column combined with the value of "C" at the left on the same row.

The Four-Variable Karnaugh Map

The figure below shows a four-variable Karnaugh Map.

$CD \backslash AB$	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
$\bar{C}D$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
CD	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
$C\bar{D}$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

As seen, it is an array of sixteen squares. In this case A, B, C and D are used for the variables. The values of A and B are across the top and the values of C and D are along the left side. The sequence of variable values may be noted carefully.

The value of a given square is the values of A and B at the top in the same column combined with the values of C and D at the left in the same row.

EXAMPLE FOURTEEN

Implement the following Boolean expression.

(a) $F(A,B,C) = \Sigma\{0, 2, 3, 7\}$

SOLUTION

Decimal	Inputs A	B	C	Output	
0	0	0	0	1	$\rightarrow \bar{A}\bar{B}\bar{C}$
1	0	0	1	0	
2	0	1	0	1	$\rightarrow \bar{A}B\bar{C}$
3	0	1	1	1	$\rightarrow \bar{A}BC$
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	0	
7	1	1	1	1	$\rightarrow ABC$

$C \backslash AB$	00 ($\bar{A}\bar{B}$)	01 ($\bar{A}B$)	11 (AB)	10 ($A\bar{B}$)
$\bar{C}$ 0	1	1		
C 1		1	1	

$$F = \bar{A}\bar{C} + BC$$

AB	$\bar{A}$		A	
$\bar{C}$	1	1		
C		1	1	

$F = \bar{A}\bar{C} + BC + \bar{A}B$

KARNAUGH MAP

SHORT WAY OF INPUTTING OUTPUTS IN MAP

RECAP EXAMPLE FIFTEEN: $F(A,B,C) = \Sigma(1,4,5,6)$

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

$\Rightarrow$

C \ AB	00 ($\bar{A}$, $\bar{B}$)	01 ($\bar{A}$, B)	11 (A, B)	10 (A, $\bar{B}$)
0			1	1
1	1			1

RECAP EXAMPLE SIXTEEN

Implement the following Boolean expression
$F(A,B,C,D) = \Sigma\{1,5,6,7,12,13,14,15\}$

CD \ AB	00 ($\bar{A}$, $\bar{B}$)	01 ($\bar{A}$, B)	11 (A, B)	10 (A, $\bar{B}$)	
00 ($\bar{C}$)	0	4	12	8	$\bar{D}$
01 ($\bar{C}$)	1	5	13	9	D
11 (C)	3	7	15	11	D
10 (C)	2	6	14	10	$\bar{D}$

$\Rightarrow$

CD \ AB	00	01	11	10
00			1	
01	1	1	1	
11		1	1	
10		1	1	

RECAP EXAMPLE SEVENTEEN

Implement the following boolean expression and draw the output circuit. $F(A,B,C,D) = \Sigma\{0,1,2,3,5,7,8,9,10,13\}$

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$\Rightarrow$

CD \ AB	00 ($\bar{A}$, $\bar{B}$)	01 ($\bar{A}$, B)	11 (A, B)	10 (A, $\bar{B}$)	
00 ($\bar{C}$)	1			1	$\bar{D}$
01 ($\bar{C}$)	1	1	1	1	D
11 (C)	1	1			D
10 (C)	1			1	$\bar{D}$

EXAMPLE FIFTEEN

Implement the following Boolean expression.

$F(A, B, C) = \Sigma\{1, 4, 5, 6\}$

NOTE

USING BOOLEAN ALGEBRA

$Y = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$

$Y = A\bar{B}\bar{C} + AB\bar{C} + \bar{A}\bar{B}C + A\bar{B}C$

$\underline{Y = A\bar{C} + \bar{B}C}$

SOLUTION

Decimal	Inputs A	B	C	Output F	
0	0	0	0	0	
1	0	0	1	1	$\bar{A}\bar{B}C$
2	0	1	0	0	
3	0	1	1	0	
4	1	0	0	1	$A\bar{B}\bar{C}$
5	1	0	1	1	$A\bar{B}C$
6	1	1	0	1	$AB\bar{C}$
7	1	1	1	0	

C \ AB	00 ($\bar{A}$)	01 ($\bar{A}$)	11 (A)	10 (A)
$\bar{C}$ 0			1	1
C 1	1			1
	$\bar{B}$	B	B	$\bar{B}$

$F = A\bar{C} + \bar{B}C$

EXAMPLE SIXTEEN

Implement the following Boolean expression

$F\{A, B, C, D\} = \Sigma\{1, 5, 6, 7, 12, 13, 14, 15\}$

SOLUTION

Decimal	Inputs A	B	C	D	F output	
0	0	0	0	0	0	
1	0	0	0	1	1	$\bar{A}\bar{B}\bar{C}D$
2	0	0	1	0	0	
3	0	0	1	1	0	
4	0	1	0	0	0	
5	0	1	0	1	1	$\bar{A}B\bar{C}D$
6	0	1	1	0	1	$\bar{A}BC\bar{D}$
7	0	1	1	1	1	$\bar{A}BCD$
8	1	0	0	0	0	
9	1	0	0	1	0	
10	1	0	1	0	0	
11	1	0	1	1	0	
12	1	1	0	0	1	$AB\bar{C}\bar{D}$
13	1	1	0	1	1	$AB\bar{C}D$
14	1	1	1	0	1	
15	1	1	1	1	[illegible]	

CD \ AB	00 ($\bar{A}$)	01 ($\bar{A}$)	11 (A)	10 (A)
$\bar{C}$ 00			1	
$\bar{C}$ 01	1	1	1	
C 11		1	1	
C 10		1	1	
	$\bar{B}$	B	B	$\bar{B}$

$F = \bar{A}\bar{C}D + BC + AB$

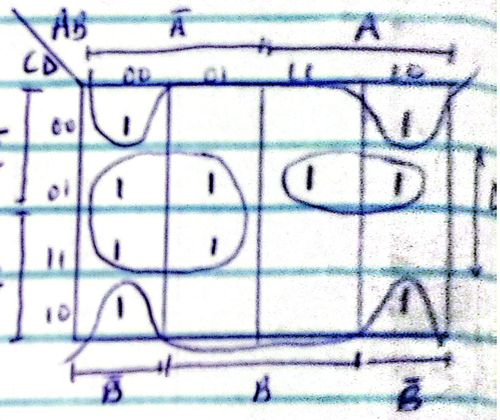
on. p (17) →

EXAMPLE SEVENTEEN

Implement the following boolean expression and draw the output circuit.

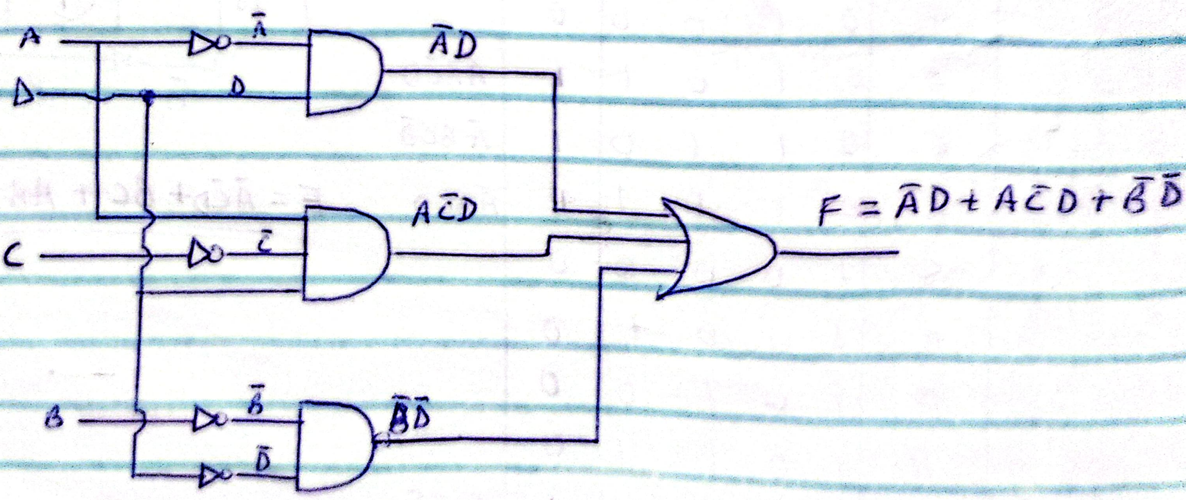
$F(A, B, C, D) = \Sigma\{0, 1, 2, 3, 5, 7, 8, 9, 10, 13\}$

DECIMAL	INPUTS A	B	C	D	OUTPUTS F	
0	0	0	0	0	1	$\bar{A}\bar{B}\bar{C}\bar{D}$
1	0	0	0	1	1	$\bar{A}\bar{B}\bar{C}D$
2	0	0	1	0	1	$\bar{A}\bar{B}C\bar{D}$
3	0	0	1	1	1	$\bar{A}\bar{B}CD$
4	0	1	0	0	0	
5	0	1	0	1	1	$\bar{A}B\bar{C}D$
6	0	1	1	0	0	
7	0	1	1	1	1	$\bar{A}BCD$
8	1	0	0	0	1	$A\bar{B}\bar{C}\bar{D}$
9	1	0	0	1	1	$A\bar{B}\bar{C}D$
10	1	0	1	0	1	$A\bar{B}C\bar{D}$
11	1	0	1	1	0	
12	1	1	0	0	0	
13	1	1	0	1	1	$AB\bar{C}D$
14	1	1	1	0	0	
15	1	1	1	1	0	



$F = \bar{A}D + A\bar{C}D + \bar{B}\bar{D}$

CIRCUIT



ASSIGNMENT

Implement the following Boolean expression ~~using~~ and draw the circuits using only NAND gates.

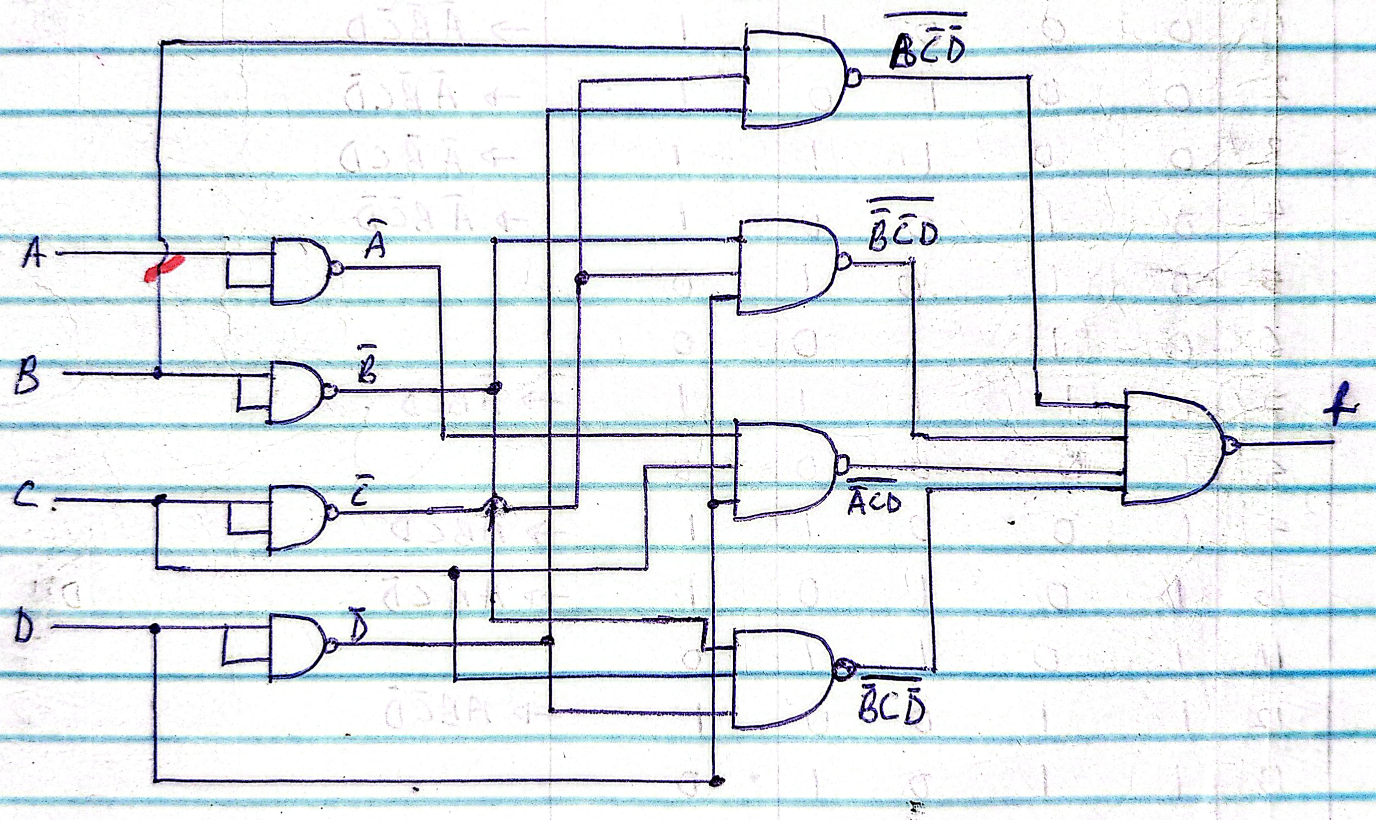
$$F(A, B, C, D) = \Sigma(1, 2, 3, 4, 7, 9, 10, 12)$$

SOLUTION

DECIMAL	A	B	C	D	OUTPUT	
0	0	0	0	0	0	
1	0	0	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}D$
2	0	0	1	0	1	$\rightarrow \bar{A}\bar{B}C\bar{D}$
3	0	0	1	1	1	$\rightarrow \bar{A}\bar{B}CD$
4	0	1	0	0	1	$\rightarrow \bar{A}B\bar{C}\bar{D}$
5	0	1	0	1	0	
6	0	1	1	0	0	
7	0	1	1	1	1	$\rightarrow \bar{A}BCD$
8	1	0	0	0	0	
9	1	0	0	1	1	$\rightarrow A\bar{B}\bar{C}D$
10	1	0	1	0	1	$\rightarrow A\bar{B}C\bar{D}$
11	1	0	1	1	0	
12	1	1	0	0	1	$\rightarrow AB\bar{C}\bar{D}$
13	1	1	0	1	0	
14	1	1	1	0	0	
15	1	1	1	1	0	

CD \ AB	00	01	11	10
00		1	1	
01	1			1
11	1	1		
10	1			1

$f(A,B,C,D) = B\bar{C}\bar{D} + \bar{B}\bar{C}D + \bar{A}CD + \bar{B}C\bar{D}$



CLASS WORK

(a) $Y = \Sigma m(1, 3, 5, 9, 11, 13)$

(b) $Y = \Sigma m(1, 2, 3, 4, 5, 7, 9, 11, 13, 15)$

(c) $Y = \Sigma m(1, 2, 9, 10, 11, 14, 15)$

USE FOR GENERAL LECTURE 312 516 & 729

ESSENTIAL ELECTRONICS AND BOOLEAN ALGEBRA

Digital representation of information is the representati of data using discrete (discontinous) values; i.e numbers in Binary, decimal, hexadecimal etc.

CONVERSION FROM ONE BASE TO ANOTHER

It is possible to use any number as a base in building numerical system. The number of digits used in the system is always to the base.

Decimal or Denary Base-10-Digits	Quinary Base-5-Digits	Binary Base-2-Digits	Octal Base-8-Digits	Hexadecimal Base-16-Digits
0	0	0000	0	0
1	1	0001	1	1
2	2	0010	2	2
3	3	0011	3	3
4	4	0100	4	4
5		0101	5	5
6		0110	6	6
7		0111	7	7
8		1000	10	8
9		1001	11	9
10		1010	12	10(A) These are the Currents
11		1011	13	11(B)
12		1100	14	12(C) Conv-entional notations
13		1101	15	13(D)
14		1110	16	14(E)
15		1111	17	15(F)

EXAMPLE ONE

Convert the following numbers to base 10

(a) 1246_7 (b) 134_{16} (c) 10111_2 (d) 6.4_7.

SOLUTION

(a) $1246_7 = \overset{3}{1}\overset{2}{2}\overset{1}{4}\overset{0}{6}_7 = 1\times7^3 + 2\times7^2 + 4\times7^1 + 6\times7^0$

$= (1\times343) + (2\times49) + (4\times7) + (6\times1)$

$= \underline{\underline{475_{10}}}$

(b) $134_{16} = \overset{2}{1}\overset{1}{3}\overset{0}{4}_{16} = (1\times16^2) + (3\times16^1) + (4\times16^0)$

$= (1\times256) + (3\times16) + (4\times1)$

$= 256 + 48 + 4$

$= \underline{\underline{308_{10}}}$

(c) $10111_2 = \overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{1}\overset{0}{1}_2 = (1\times2^4) + (0\times2^3) + (1\times2^2) + (1\times2^1) + ($

$= (1\times16) + (0) + (1\times4) + (1\times2) + (1\times$

$= 16 + 0 + 4 + 2 + 1$

$= \underline{\underline{23_{10}}}$

(d) $6.4_7 = \overset{0}{6}.\overset{-1}{4}_7 = 6\times7^0 + 4\times7^{-1}$

$= 6\times1 + \frac{4}{7}$

$= \frac{6}{1} + \frac{4}{7} = \frac{42+4}{7} = \frac{46}{7} = \underline{\underline{6\frac{4}{7}}}$

EXAMPLE TWO

Convert to base 2 the following 106_{10}.

SOLUTION

EXAMPLE ONE

Convert the following numbers to base 10

(a) 1246_7 (b) 134_{16} (c) 10111_2 (d) 6.4_7

SOLUTION

(a) $1246_7 = \overset{3}{1}\overset{2}{2}\overset{1}{4}\overset{0}{6}_7 = 1\times7^3 + 2\times7^2 + 4\times7^1 + 6\times7^0$

$= (1\times343) + (2\times49) + (4\times7) + (6\times1)$

$= \underline{\underline{475_{10}}}$

(b) $134_{16} = \overset{2}{1}\overset{1}{3}\overset{0}{4}_{16} = (1\times16^2) + (3\times16^1) + (4\times16^0)$

$= (1\times256) + (3\times16) + (4\times1)$

$= 256 + 48 + 4$

$= \underline{\underline{308_{10}}}$

(c) $10111_2 = \overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{1}\overset{0}{1}_2 = (1\times2^4) + (0\times2^3) + (1\times2^2) + (1\times2^1) + (1\times2^0)$

$= (1\times16) + (0) + (1\times4) + (1\times2) + (1\times1)$

$= 16 + 0 + 4 + 2 + 1$

$= \underline{\underline{23_{10}}}$

(d) $6.4_7 = \overset{0}{6}.\overset{-1}{4}_7 = 6\times7^0 + 4\times7^{-1}$

$= 6\times1 + \frac{4}{7}$

$= \frac{6}{1} + \frac{4}{7} = \frac{42+4}{7} = \frac{46}{7} = \underline{\underline{6\frac{4}{7}}}$

EXAMPLE TWO

Convert to base 2 the following 106_{10}.

SOLUTION

2	106
2	53 R 0
2	26 R 1
2	13 R 0
2	6 R 1
2	3 R 0
2	1 R 1

1101010_2

EXAMPLE THREE

Convert 529_{10} to octal (b) Convert 1004_{10} to base 16

SOLUTION

8	529
8	66 R 1
8	8 R 2
8	1 R 0
	0 R 1 ↑ → 1021_8

(b) SOLUTION

16	1004
16	62 R 12 − C
16	3 R 14 − E
16	0 R 3 − 3 ↑

$1004_{10} = 3EC_{16}$

Binary – Octal – Hexadecimal Conversion

Applying the knowledge of indices, it is easier to convert numbers to octal and hexadecimal when the initial conversion had ~~being~~ been done in binary.

EXAMPLE FOUR

Convert 106_{10} (a) Base Eight (b) Hexadecimal.

SOLUTION

(a)

2	106
2	53 R 0
2	26 R 1
2	13 R 0
2	6 R 1
2	3 R 0
2	1 R 1 ↑ → 1101010_2.
2	0 R 1

Now since octal is base 8 and 8 is 2^3, we group the binary numbers in three's starting from the right hand moving towards the left of the reader.

Hence 1101010_2 group in three will be 1,101,010. Then each group is converted to denary.

$$1,\ 101,\ 010 = 1,\ 5,\ 2 = 152_8$$

(b) 1101010_2 to hexadecimal.

Now to hexadecimal. Again $16 = 2^4$, hence we group in fours starting from the right hand and moving towards the left of the reader, so it is 110,1010 and each group is

Converted to the decimal scale accordingly:

110, 1010

6 A(10)

It is important that each reader be very conversant the conversion of numbers from 0 to 15 in decimal to binary before using the method.

EXAMPLE FIVE

Convert $(14B2)_{16}$ to binary

SOLUTION

1	4	B	2
0001	0100	1011	0010

0001010010110010_2

1010010110010_2

EXAMPLE SIX

Convert $(14B2)_{16}$ to Decimal

SOLUTION

$14B2_{16} = 1 \times 16^3 + 4 \times 16^2 + B \times 16^1 + 2 \times 16^0$

$= 4096 + 1024 + (11 \times 16) + 2$

$= 5298_{10}$

EXAMPLE SEVEN

Convert ~~76~~ $(74632)_8$ to ~~decimal~~ hexadecimal

SOLUTION

$(74632)_8 \rightarrow$ 7 4 6 3 2

111 100 110 011 010

111100110011010_2 {Combining Together}

Grouping in Four's $\underset{7}{111}\,\underset{9}{1001}\,\underset{9}{1001}\,\underset{A}{1010} = 799A_{16}$

EXAMPLE EIGHT

Convert $98BF_{16}$ to Octal

SOLUTION

9	8	B	F
1001	1000	1011	1111

$\rightarrow 1,001,100,010,111,111_2$

1	001	100	010	111	111
1	1	4	2	7	7

$= \underline{114277_8}$

EXAMPLE NINE

Convert 0.111010_2 to (a) Octal (b) Hexadecimal

SOLUTION

(a) $0.\overrightarrow{111},\overrightarrow{010}$ { NOTE: After the decimal point, move from left to right

$0.\;7\;\;2 = \underline{\underline{0.72_8}}$

(b) $0.111010_2 = 0.\overrightarrow{1110},\overrightarrow{1000}$

$0.\;E\;\;8 = \underline{\underline{0.E8_{16}}}$

EXAMPLE TEN

Convert to Hexadecimal 0.101101111_2.

SOLUTION

$0.\overrightarrow{1011},\overrightarrow{0111},\overrightarrow{1000}$

$0.\;B\;\;7\;\;8 \rightarrow \underline{\underline{0.B78_{16}}}$

ADDITION AND SUBTRACTION OF BASES

Addition and subtraction in any base other than ten is the same, the only exception is that the number/digit would not exceed the highest digit of the base.

EXAMPLE ELEVEN

Add the following $72_8 + 643_8$.

SOLUTION

$$\begin{array}{r} 72_8 \\ +\ 643_8 \\ \hline 735_8 \end{array}$$

NOTE

we write down the remainder and take away the number of time and add to the next number (addition).

EXAMPLE TWELVE

Subtract the following $2311_4 - 213_4$

SOLUTION

$$\begin{array}{r} 2311_4 \\ -\ 213_4 \\ \hline 2032_4 \end{array}$$

MULTIPLICATION AND DIVISION

In the case of multiplication, it follows the same pattern as that of multiplication in base ten, the only difference here is that instead of thinking in tens we go along in the given base thus:

EXAMPLE THIRTEEN

Simplify (a) $121_3 \times 2_3$ (b) $817_9 \times 36_9$

SOLUTION

(a)
$$\begin{array}{r} 121_3 \\ \times\ 2_3 \\ \hline 1012_3 \end{array}$$

(b)
$$\begin{array}{r} 817_9 \\ \times\ 36_9 \\ \hline 5416_9 \\ +\ 2653_9 \\ \hline 33046_9 \end{array}$$

DIVISION

Division in bases other than base 10 is the same, the only difference is when the first digit of the division is less than the divisor, we convert the digit (dividend) to the next place value.

EXAMPLE FOURTEEN

Divide (a) 1211_3 by 21_3 (b) 1110101_2 by 1001_2 (3) $204_5 \div 3_5$
(d) $334_5 \div 2_5$ (e) $1022_3 \div 12_3$

SOLUTION

(a)

$$
\begin{array}{r}
21 \\
21 \overline{)\,1211_3} \\
-112_3 \\
\hline
021_3 \\
21_3 \\
\hline
0
\end{array}
$$

$= \underline{\underline{21_3}}$.

(b)

$$
\begin{array}{r}
1101 \\
1001 \overline{)\,1110101_2} \\
-1001_2 \\
\hline
1011_2 \\
-1001_2 \\
\hline
-1001_2 \\
1001_2 \\
\hline
0
\end{array}
$$

$= \underline{\underline{1101_2}}$

NOTE
we multiply in the given base

(c)

$$
\begin{array}{r}
33 \\
3 \overline{)\,204_5} \\
-14 \\
\hline
14 \\
-14 \\
\hline
0
\end{array}
$$

$= \underline{\underline{33_5}}$

(d)

$$
\begin{array}{r}
142 \\
2 \overline{)\,334_5} \\
-2 \\
\hline
13 \\
13 \\
\hline
04 \\
-04 \\
\hline
0
\end{array}
$$

$= \underline{\underline{142_5}}$

(e)

$$
\begin{array}{r}
21 \\
12_3 \overline{)\,1022_3} \\
-101_3 \\
\hline
12_3 \\
-12_3 \\
\hline
0
\end{array}
$$

$= \underline{\underline{21_3}}$

Having learnt the Conversion of decimal numbers to binary th
Can Communicate with each other. For example we Can assig
Values to the English alphabet.

Letters	NUMERICAL CODES	Binary Equivalent Codes
A	01	00001
B	02	00010
C	03	00011
D	04	00100
E	05	00101
F	06	00110
G	07	00111
H	08	01000
I	09	01001
J	10	01010
K	11	01011
L	12	01100
M	13	01101
N	14	01110
O	15	01111
P	16	10000
Q	17	10001
R	18	10010
S	19	10011
T	20	10100
U	21	10101
V	22	10110
W	23	10111
X	24	11000
Y	25	11001
Z	26	11010

Using the above binary codes, the following two messages can be sent as shown:

1) I am afraid of Mathematics

MESSAGE	Numerical Codes	Binary Equivalent
_I	_09	_01001
A	01	00001
_M	_13	_01101
A	01	00001
F	06	00110
R	18	10010
A	01	00001
I	09	01001
_D	_04	_00100
O	15	01111
_F	_06	_00110
M	13	01101
A	01	00001
T	20	10100
H	08	01000
E	05	00101
M	13	01101
A	01	00001
T	20	10100
I	09	01001
C	03	00011
S	19	10011

The dashes represents the separation of the words "I, am, afraid, of, Mathematics", for easy reading.

ASSIGNMENT

(1) Convert 7123.263_{10} to base 2

(2) $1101101101110110.10100011_2$ to base 16

(3) 10101110.10001_2 to base 8

(4) 713.34_8 to base 2

(5) Convert 1004.546875_{10} to base 16.

(6) 1101011100.10111_2 to base 16.