Try

(a) Show that the map $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $f(x,y) = (x+y, x)$ is a linear transformation

(b) Let T be a linear operator on a real vector space V and let $V_1, V_2, V_3 \in V$ be such that

$T(V_1) = V_1 + V_2$

$T(V_2) = -V_1 + 2V_2 + V_3$

and ~~$T(V_3)$~~

$T(V_2 + V_3) = -2V_1 + V_2 - 3V_3$

Find $T(-3V_1 + V_2 + 2V_3)$ in terms of V_1, V_2, V_3

(c) Show that the set

$$S = \{(x,y) \in \mathbb{R}^2 \mid 2x - y = 0\}$$

is a subspace of the vector space $\mathbb{R}^2$

d

let V be a vector space over $\mathbb{R}$
and let $a, b, c \in V$.
show that if a, b, c are
linearly independent then

$a+b$, $b+c$ and $c+a$
are linearly independent

E

consider the transformation
$T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right] = \begin{pmatrix} x+y+z \\ 2x-3y+4z \end{pmatrix}$$

Prove that T is a linear transformation
find the null space of T and
determine whether T is injective

F

Verify whether the following are subspaces of $\mathbb{R}^3$

(i) $S = \{(w_1, w_2, w_3) \in \mathbb{R}^3 \mid w_1 + w_2 + w_3 = 0\}$

(ii) $T = \{(a+2b, a+1, a) \mid a, b \in \mathbb{R}\}$

F

(i) Let $M_2(\mathbb{R})$ be the vector space of 2×2 matrices.
Define $T: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) \to \begin{bmatrix} a+2b+c & b-c+d \\ -a-3c+2d & a+3b+d \end{bmatrix}$$

show that T is a linear transformation

(ii) if T is a linear transformation find the kernel or null space of T