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## **Algorithms Lab**

## Exercise - Cantonal Courier

To earn money for the journey of your dreams (Barcelona, Gothenburg and London) you come up with an idea for a shipping company. You are going to have an employee (a courier) in each canton of Algoland. The caveat that is going to let you offer competitive rates, while promoting sustainability, is that your couriers are going to use public transport. The distinctive feature of Algoland's public transport system is that each canton consists of several zones – depending on the route you need to buy tickets for some subset of those zones.

For each canton you are given a list of possible assignments and the reward for each of them that a customer will pay if you agree to take it. For each assignment you are also given the list of zones for which you need a ticket in case you take it. Finally, you are given the price of the ticket for each zone. All the tickets are day passes and your couriers are truly excellent, so you can be sure that once bought, a single ticket can be reused for several jobs.

Find the optimal profit (payments for selected jobs minus costs of needed tickets) you can achieve for each canton.

**Input** The first line of the input contains  $1 \le T \le 100$ , the number of cantons. Each canton is described as follows:

- It starts with a line holding two integers Z J, separated by a space. They denote
  - $1 \le Z \le 100$ , the number of zones in the canton, and
  - $1 \le J \le 100$ , the number of jobs available in this canton.
- In the next line there are Z integers  $c_0, \ldots, c_{Z-1}, 1 \le c_i \le 5000$ , where  $c_i$  is the cost of the ticket for zone i.
- The third line of each canton contains J integers  $p_0, \ldots, p_{J-1}, 1 \le p_j \le 5000$ , where  $p_j$  is the reward for job j.
- J lines follow: the ith line describes the tickets needed for job i (0-based). Each of those lines starts with  $0 \le N_i \le Z$ , followed by a strictly increasing sequence of  $N_i$  zones (0-based) for which the tickets are needed.

All consecutive numbers in a line are single-space separated.

**Output** For each testcase output a single line with an integer: your maximum achievable profit, i.e. the payments minus the ticket costs in an optimum choice of jobs.

**Points** There are 3 groups of test sets, worth 100 points in total.

- 1. For the first group of test sets, worth 20 points, you may assume that each job requires the purchase of at most one ticket (i.e.  $\forall i : N_i \leq 1$ ).
- 2. For the second group of test sets, worth 20 points, you may assume that  $\forall i < j < k$ , if a job requires the zones i and k, it also requires zone j.
- 3. For the third group of test sets, worth 60 points, there are no additional assumptions.

Corresponding sample test sets are contained in test i. in/out, for  $i \in \{1, 2, 3\}$ .

## **Sample Input** Sample Output 4 3 2 5 2 3 10 1 1 2 3 4 5 3 0 1 0 1 0 1 1 1 1 3 5 3 1 9 1 2 3 4 5 1 0 2 0 1 2 0 1 2 1 2 2 1 2 4 3 1 5 6 7 3 4 10 2 0 1 1 1 2 2 3 3 3 3 3 3 4 4 4 3 0 1 2 3 0 1 2 3 0 1 2