1. Even proof

Proposition: Suppose a,b $\in Z$, If a and b are even, then ab is even.

Proof: Suppose a,b$\in Z$, by definition of even there exists integers K and Q such that a = 2K and b = 2Q. From this $ab = 2k2$Q which we can then simplify into 4KQ = 2(2KQ). Let M = 2KQ. Since $Z$ is closed under multiplication and 2,K,Q$\in Z$, 2KQ$\in Z$. Thus ab = 2M where 2M is an integer and so by definition, ab is even. ■

2. Divides

Definition: Suppose that a,b$\in Z$. We say that b divides a, denoted b|a, when there exists an integer k such that a = bk.

Does 2|10? Yes 2|10 because 10 = 2*5.
Does 4|10? 4 does not divide 10 because if 10 = 4k then k = 2.5 /$\in Z$

Synonyms:

- b is a divisor of a
- b is a factor of a
- a is a multiple of b
- $\frac{a}{b}$ is an integer. [k = $\frac{a}{b}$]

3. Proofs

Prove the following:

Proposition: Suppose that a,b,c$\in Z$. If b|a and b|c then b|(a+c).

Proof: Suppose that a,b,c$\in Z$. By definition of divides there exists an integer k where a = bk and an integer $k_1$ where c = $bk_1$. So from this we get (a+c) = $b(k+k_1)$. Since the integers are closed under addition $k+k_1$ must be an integer. Because of that by definition of divides b|(a+c). ■

Proposition: Suppose that x,y,z$\in Z$. If x|y and y|z, then x|z.

Proof: Suppose that x,y,z∈$Z$. By definition of divides there exists an integer g where y = xg and an integer f where z = yf. From this we can get z = xgf, simplifying it would get $\frac{z}{xg}$ = f. Because $\frac{z}{y}$ is an integer, $\frac{x}{zg}$ must be an integer. (Or Since $Z$ is closed under multiplication, $\frac{x}{zg}$∈$Z$). Therefore x|z. ■

- From this we can get z = xgf. Here I can actually just set z = x(gf) and then follow it with Since $Z$ is closed under multiplication x(gf)∈$Z$. Therefore x|z. ■
  I just made it a little more complicated then needed.

My professors proof for the second:
Proposition: Suppose that x,y,z∈$Z$. If x|y and y|z, then x|z.
Proof: Suppose x,y,z∈$Z$ and x|y and y|z. Then by definition, there exists integers k and m such that y=xk and z=ym. So combining these equations we see that z=xkm. Let n = km, Since $Z$ is closed under multiplication n=km∈$Z$. Therefore sinc z=xn and n∈$Z$, x|z.