Definition: Suppose that a is an integer, then a is even when there exists $k \in \mathbb{Z}$ such that $a = 2k$. Also a is odd when there exists $n \in \mathbb{Z}$ such that
$a = 2n+1$

Propositions: Suppose that $a,b \in \mathbb{Z}$. If a is odd and b is odd then a*b is odd.*
Proof: Suppose a and b are both $\mathbb{Z}$. By definition of odd this means that there exists integers n_1, n_2 such that $a = 2n_1 + 1$ and $b = 2n_2 + 1$. From this we have $ab = (2n_1 + 1)(2n_2 + 1)$ which equals $(4n_1n_2 + 2n_1 + 2n_2 + 1) = 2(2_1n_2 + n_1 + n_2) + 1$. Let n_3 be equal to $(2_1n_2 + n_1 + n_2)$. Since $\mathbb{Z}$ is closed under addition and multiplication and $2, n_1, n_2 \in \mathbb{Z}$, $n_3 \in \mathbb{Z}$. Thus, $ab = 2n_3 + 1$ where n_3 is an integer, and so by defintion ab is odd. ■