Stage 1 – Desired Results

ESTABLISHED GOALS

Math is an important but much maligned subject of study. Students often say, "I'm just bad at math", in contrast you rarely hear of students saying, "just bad at science" or "just bad at history". One of the, as I see it, flaws of early math education is the reliance on computation and "numbers" over abstract reasoning skills. A key skill in math as in the "real world" is the concept of equality. We use different concepts of equality daily but often ignore its deep mathematical importance, relegating mathematical equivalence to things of the form 2+2=4. Our primary goal is to show:

Numbers have nothing to do with it.

Transfer

Students will be able to independently use their learning to…

Students will be able to identify the concept of mathematical equivalence in non "mathematical" contexts.

Meaning

UNDERSTANDINGS

Students will understand that…

- Knots are mathematical
- Knots have "hard science" applications
- Knot equivalence
- Determining knot equivalence

ESSENTIAL QUESTIONS

- How are pictures math?
- How are pictures the same?
- How do we tell if two pictures are not the same?
- How do we use these pictures in "real life"?

Acquisition

Students will know…

- **DNA can form knots.**
- **Knots are "wiggled" ropes in space.**
- **Reidemeister moves give knot equivalence.**
- **Knots have polynomials**
- **Polynomials tell non-equivalence.**

Students will be skilled at…

Students will be able to take two knot diagrams and compute their polynomials to determine non-equivalence.

Stage 2 – Evidence and Assessment

Evaluative Criteria

- Can apply bracket relations to a knot diagram
- Can compute writhe
- Can show knot non-equivalence

Assessment Evidence

PERFORMANCE TASK(S):

Students will complete an in-class group worksheet covering each essential question.
Questions will ask for students to apply content shown by the instructor to a novel situation.

OTHER EVIDENCE:

Students will be periodically asked to reflect and discuss content while it is being presented.
Students will be given physical models demonstrating core concepts and asked to investigate them during the lesson.

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

This lesson is designed to be completed in one 50min section. The students are expected to have at least college algebra experience (combining like terms, polynomials, multiplicative inverses, and additive inverses). In an effort to increase engagement and retention the lesson will be couched in a narrative about the zombie apocalypse. Students are cast as researchers at the CDC tasked with confirming a vaccine contains an anti-virus.

Lesson outline:

1. Introduce the story
2. Describe DNA knot theory
    a. Motivational DNA knot picture
    b. Definition of DNA
    c. Circular DNA
    d. Recombination of DNA
3. Define Anti-knots
4. Mathematical knots
    a. Definition
    b. Diagrams
    c. Equivalence
        i. Type I
        ii. Type II
        iii. Type III
5. Class Poll: Same knot or different knot?
6. Class Poll: What's the important information inside a knot diagram?
7. In math we're lazy. We want to make our life easier. If crossings are the "important" information, how do we change them to make our life easier?
8. Smoothing clockwise
9. Smoothing anticlockwise
10. Switch to algebra
    a. We know algebra well, we've been using it since we were kids. Pictures are hard, so what if we try to get rid of them?
11. Skein relation
12. Remind students what the goal is and put pieces together
    a. How do we tell two knots apart?
    b. How can we use that and our bracket to build our polynomial?
13. Check bracket on Type II move
14. Remind students of goal
15. Class poll: students for coefficients?
16. Put pieces together, rules and coefficients for type II
17. **Group activity:** Type III
18. Demo Type I
19. Remind students of goal
20. + and – for Type I
21. Define + and – crossings
22. Define writhe
23. Demo writhe
24. **Group activity:** Compute writhe
25. Fix Type I
26. **Group activity**: Verify Type I.2 (time allowing)

27. Define Jones polynomial
28. Compute for virus
29. **Group activity**: Compute for anti-virus
30. Compare findings