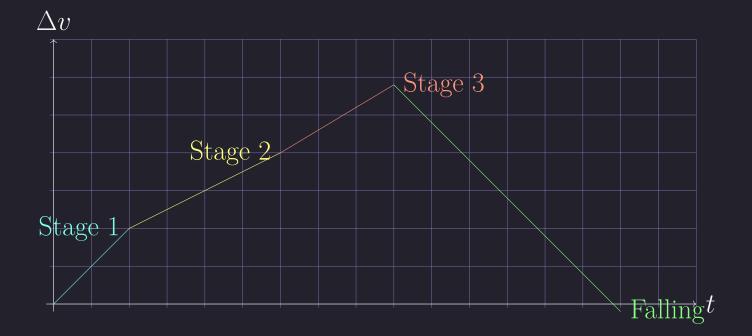
## Types of Functions

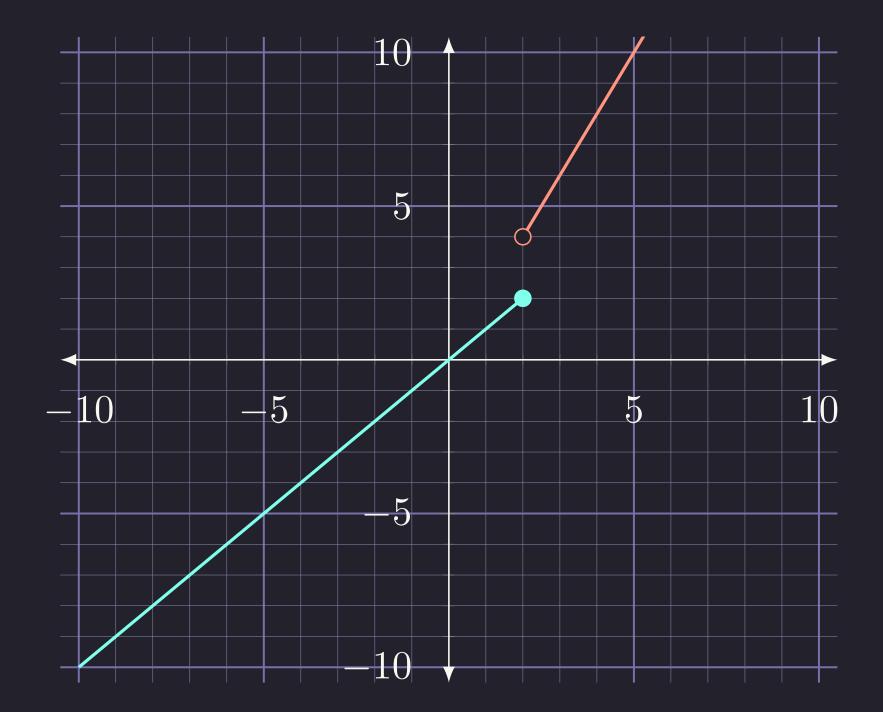
#### Piecewise Functions

### Notation

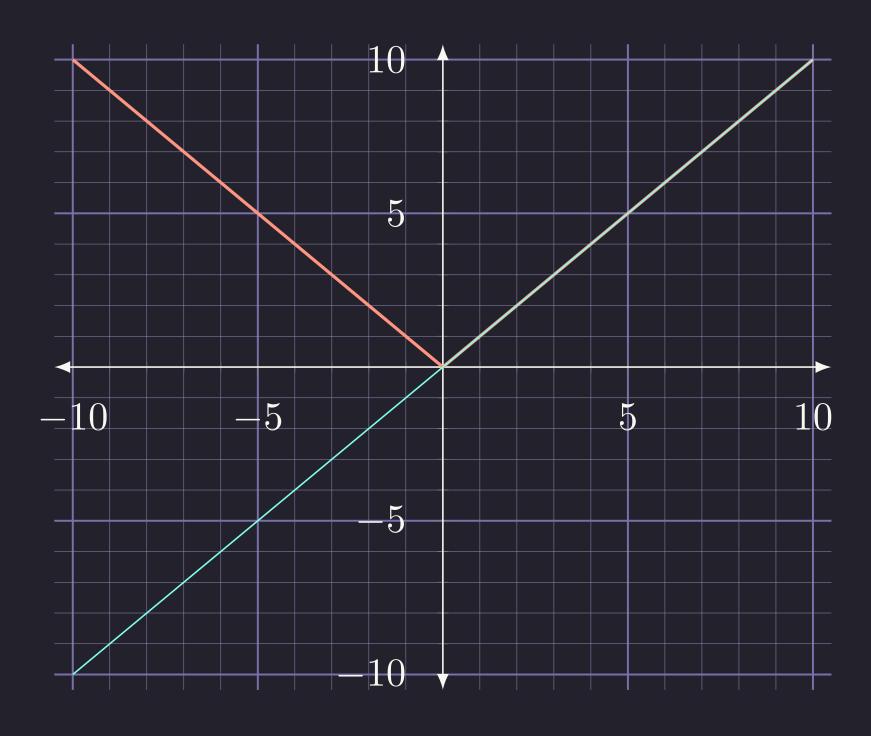
$$f(x) = \begin{cases} \text{function, condition} \\ \vdots \\ \text{function, condition} \end{cases}$$

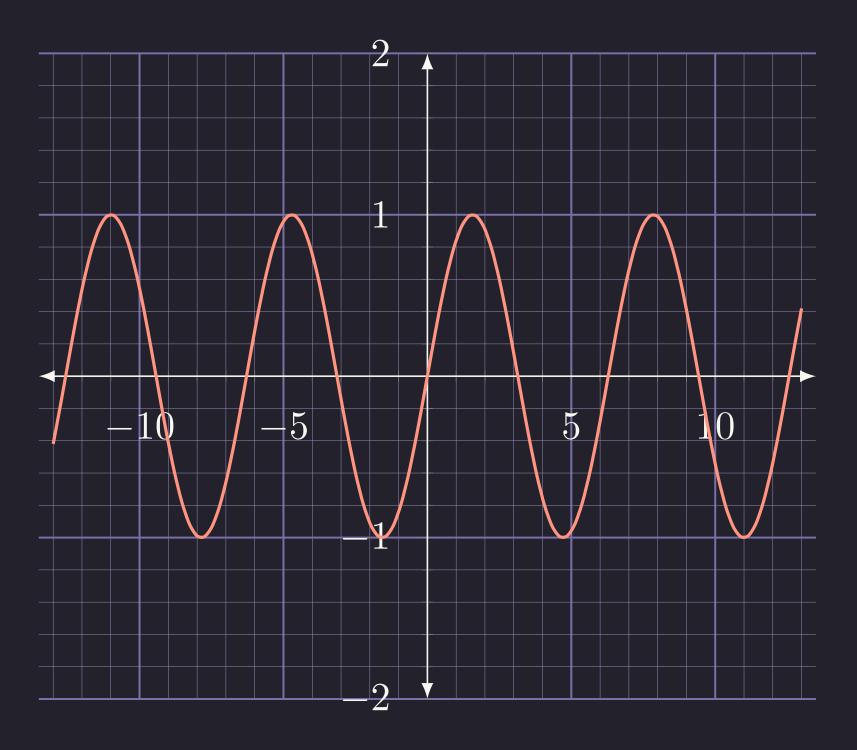


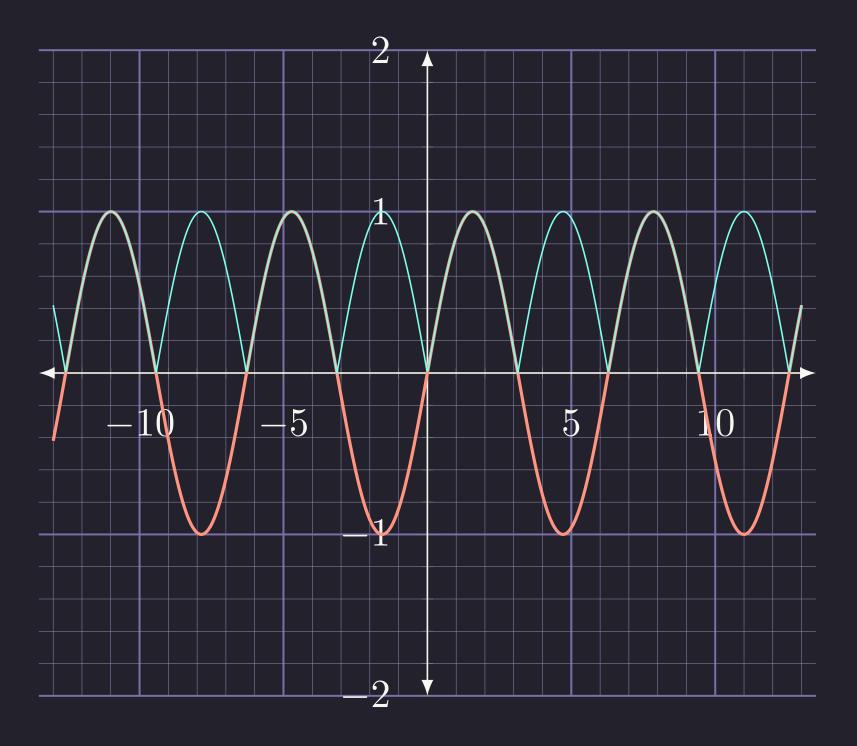
$$f(x) = \begin{cases} x, & [0,2) \\ \frac{1}{2}x, & (2,6] \\ \frac{3}{5}x, & (6,9] \\ -x, & (9,15] \end{cases}$$



## Absolute Value







## Even and Odd



A function f(x) is called:

Even: If  $\forall x \in D$  we have f(-x) = f(x)

Odd: If  $\forall x \in D$  we have f(-x) = -f(x)

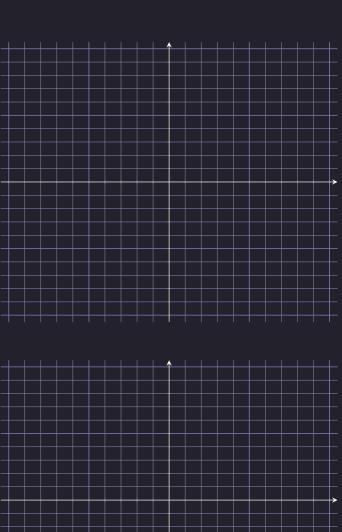
Neither:

### Symmetries:

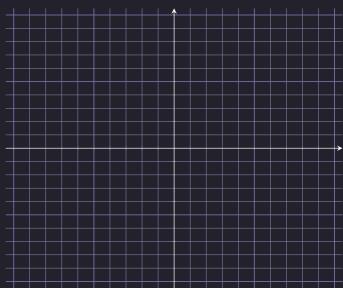
Even:

Odd:

Neither:







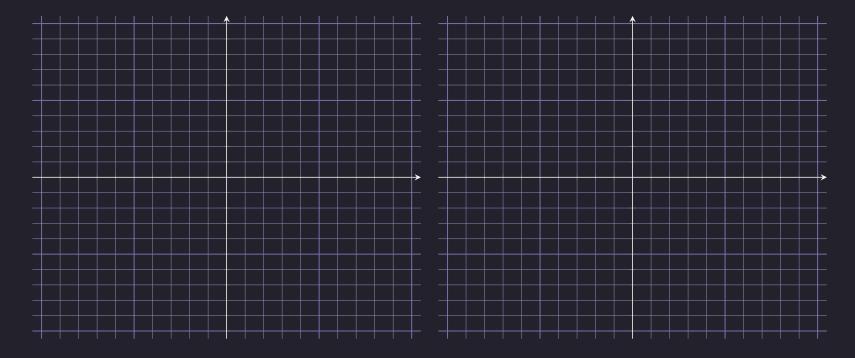
## Increasing Decreasing

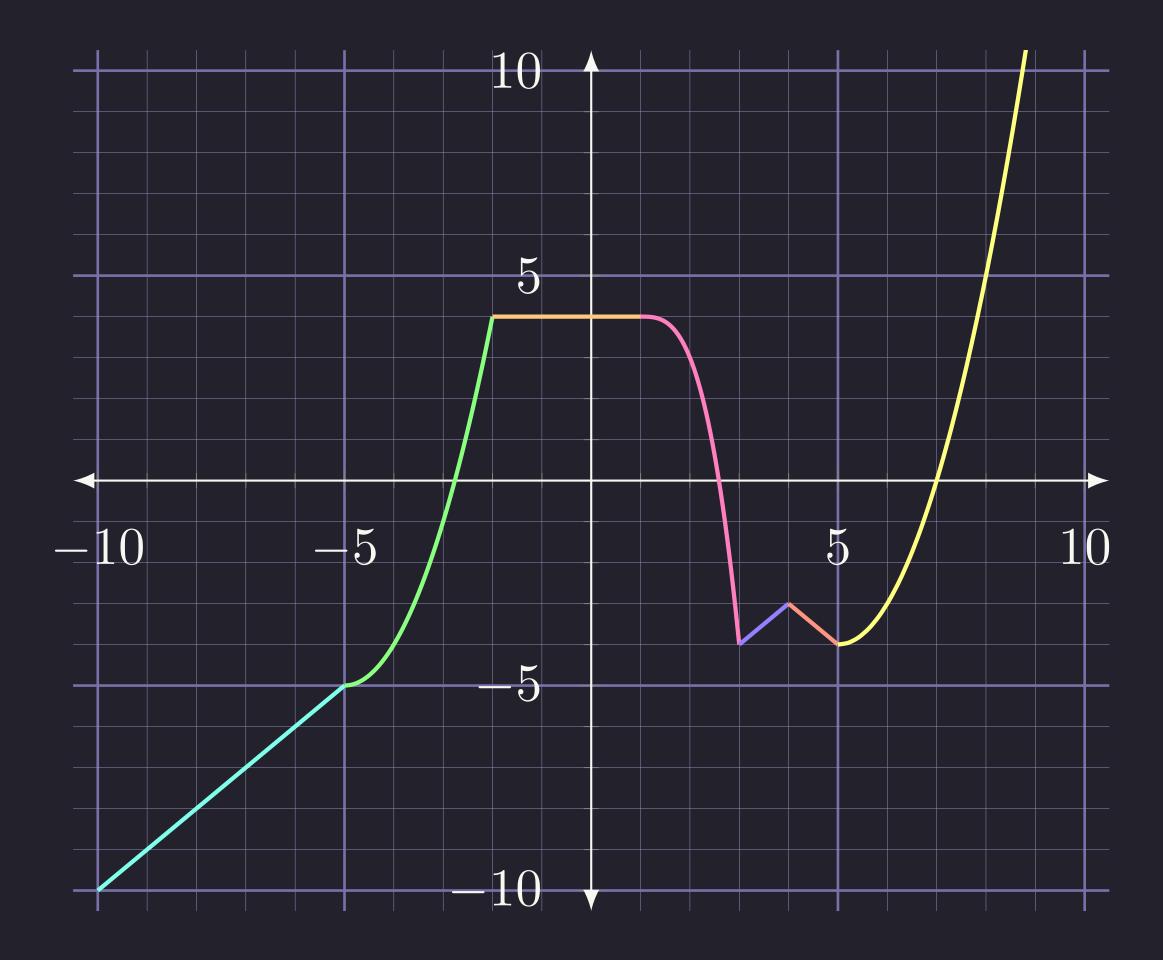
#### Definition:

A function f(x) on an inteval I is called:

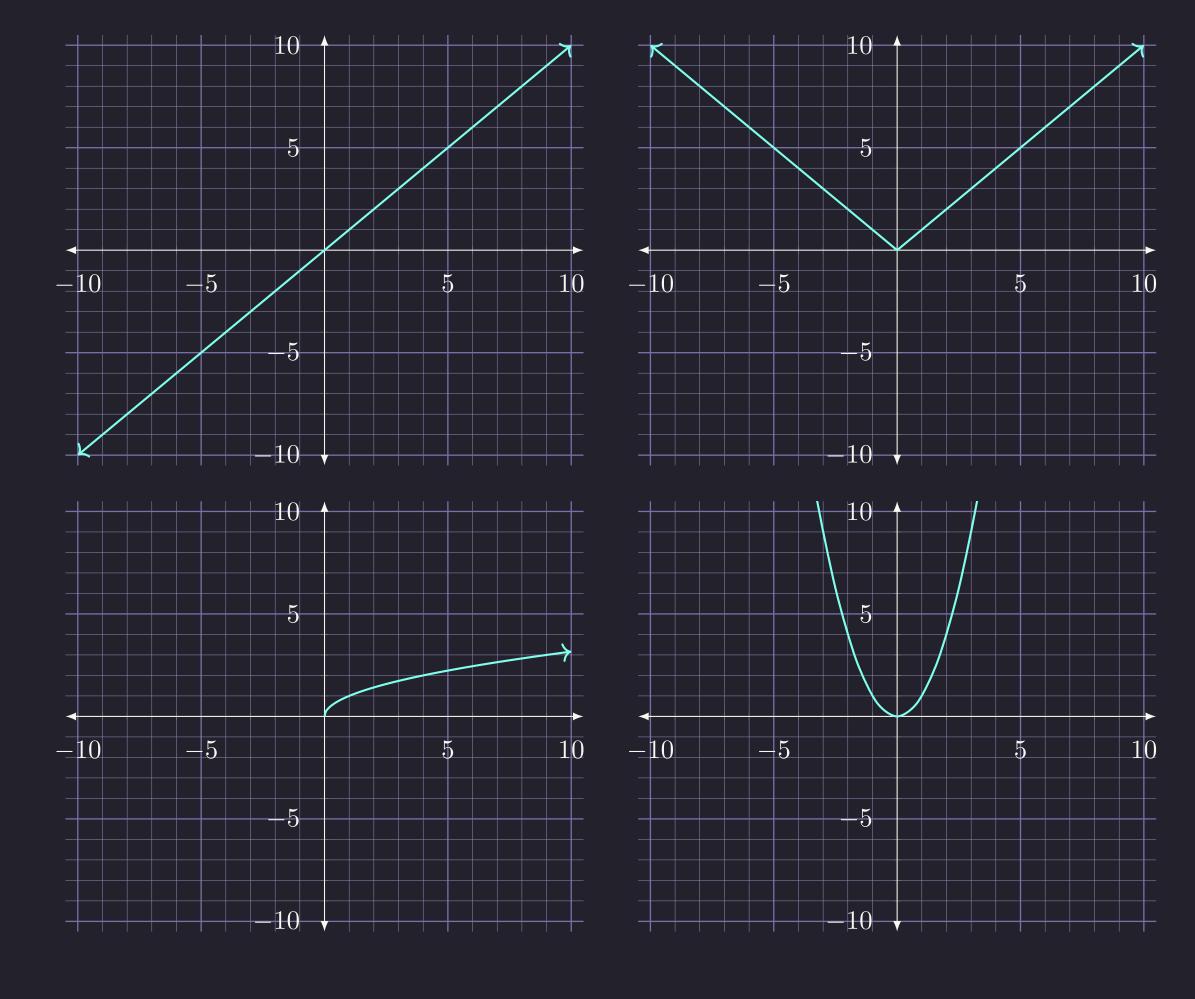
Increasing:  $\forall a, b \in I \text{ if } a < b \text{ then } f(a) < f(b)$ 

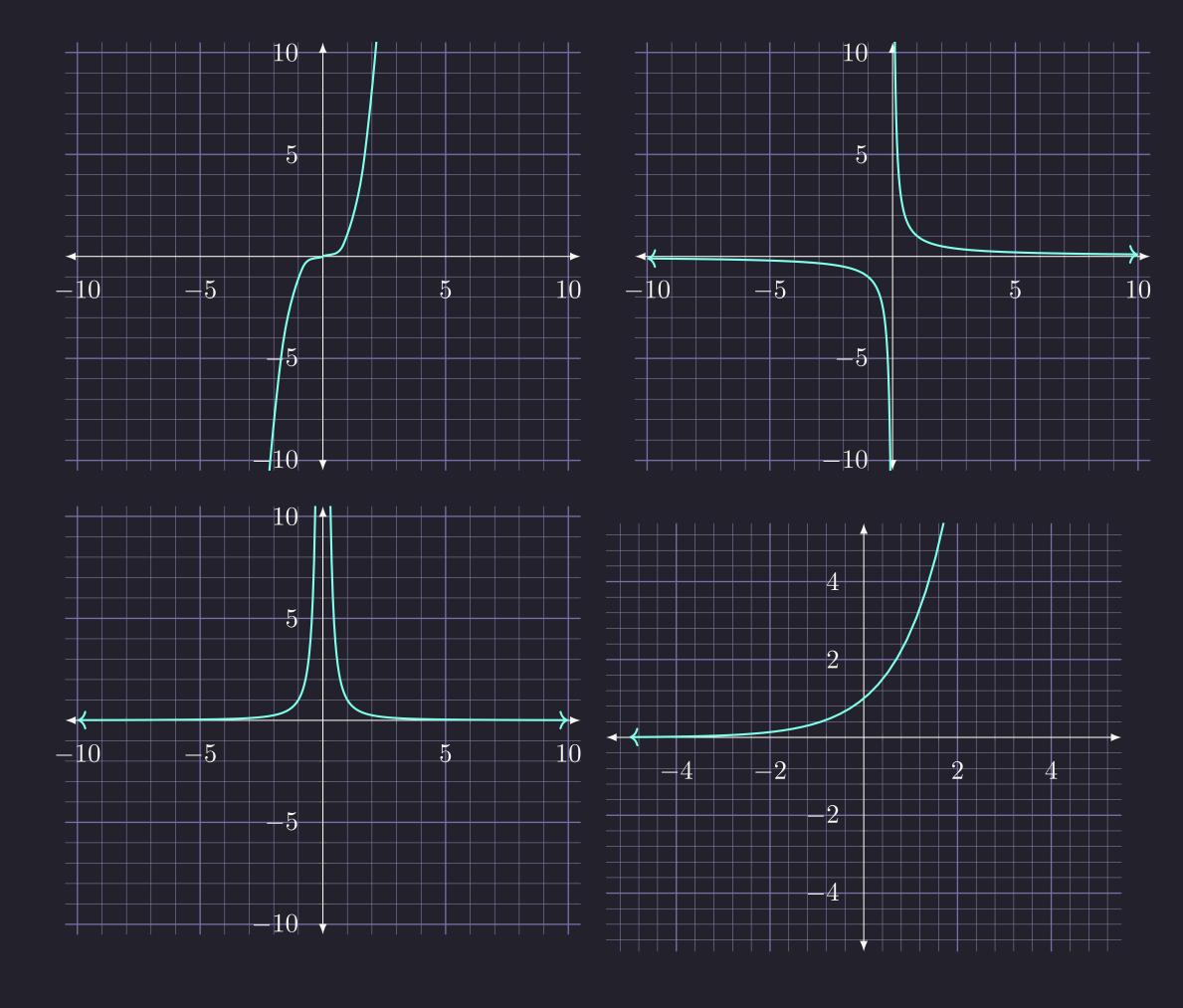
Decreasing:  $\forall a, b \in I \text{ if } a < b \text{ then } f(a) > f(b)$ 





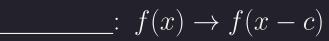
## "Parent" Functions

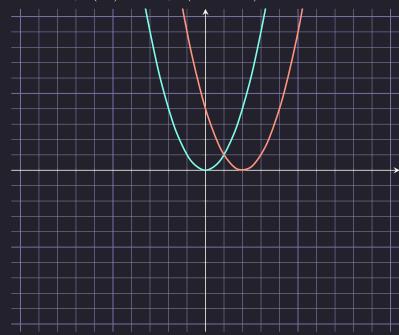




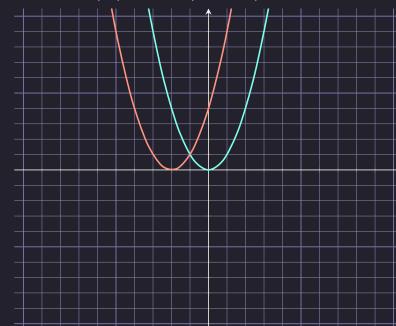
# Translation of functions

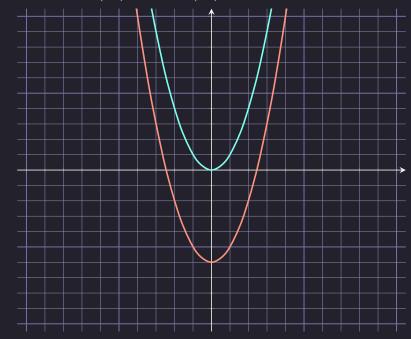
Shifts:



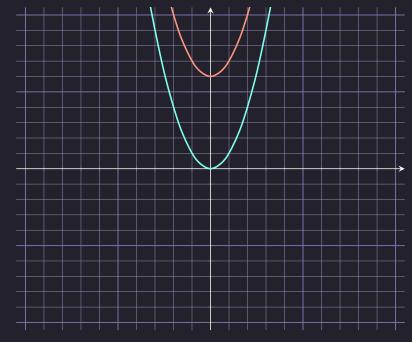


$$\underline{\qquad}: f(x) \to f(x+c)$$





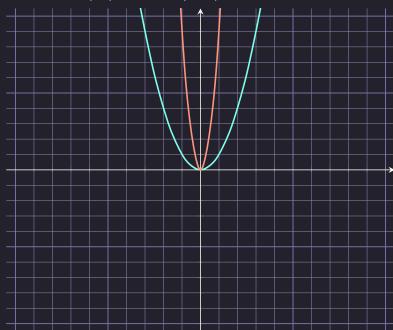
 $: f(x) \to f(x) + c$ 



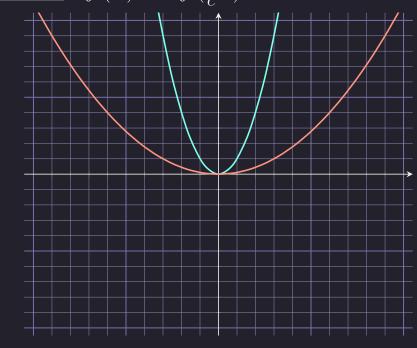
## Stretching and reflection:

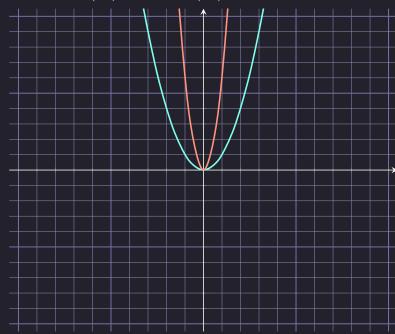
For 1 < c

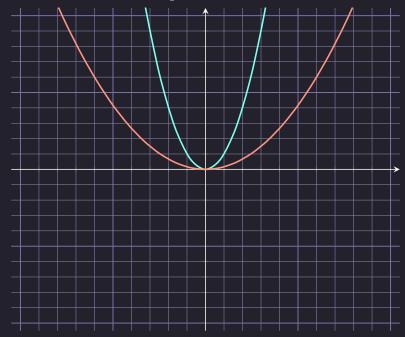


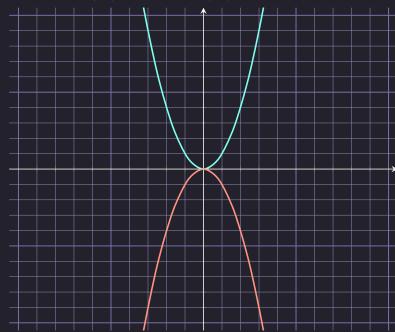


 $\underline{\qquad}: f(x) \to f(\frac{1}{c}x)$ 

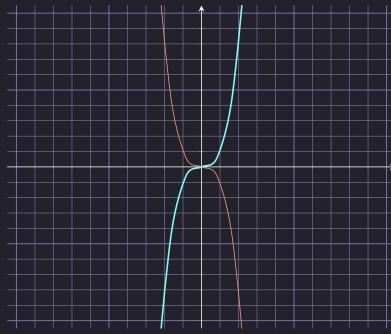








 $\underline{\qquad} : f(x) \to f(-x)$ 



## Function Arithmetic

$$(f+g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

If we let 
$$f(x) = x^2$$
 and  $g(x) = 4x^3$ 

$$(f+g)(x) = \underline{\hspace{1cm}}$$

$$(f-g)(x) = \underline{\hspace{1cm}}$$

$$(f \cdot g)(x) = \underline{\hspace{1cm}}$$

$$\left(\frac{f}{g}\right)(x) = \underline{\qquad}$$

#### Domain

Addition, multiplication  $+ - \cdot$ 

The domain of f+g, f-g, and  $f\cdot g$  is the intersection of the domains of f and g. In other words, f+g, f-g, and  $f\cdot g$  are defined wherever both f and g are defined.

#### Division

For  $\frac{f}{g}$ , the domain is the intersection of the domains of f and g excluding x where g(x) is 0.



## Function Composition

We let f(x) and g(x) be two functions with "compatable" domain and codomain

the compostion of f(x) and g(x) written

$$(f \circ g)(x)$$

is defined to be

$$f\left(g\left(x\right)\right)$$



Let 
$$f(x) = x^2$$
 and  $g(x) = \sqrt{x} = \underline{\hspace{1cm}}$ 

$$(f \circ g)(x) =$$

$$(g \circ f)(x) =$$

Let 
$$f(x) = x^2$$
 and  $g(x) = x + 1$ 

$$(f \circ g)(x) =$$

$$\left(g\circ f\right)\left(x\right)=$$

#### Domain

The domain of  $f \circ g$  is all x in the domain of g so that g(x) is in the domain of f.

#### Note:

The easiest way to find the domain is usually to write an expression for  $(f \circ g)(x)$  and find its domain without simplifying.

Let  $f: \mathbb{R} \to \mathbb{R}$  where  $f: x \mapsto \frac{1}{x}$ , and  $g: (0, \infty) \to \mathbb{R}$  where  $g: x \mapsto x^2$ .

Find the domain of  $(f \circ g)(x)$ :