Limits

Intuition of a limit

Getting ”close” and motivation

”Definition”

Let $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$. Let $a \in A$ we write

$$\lim_{x \to a^-} f(x) = L_-$$

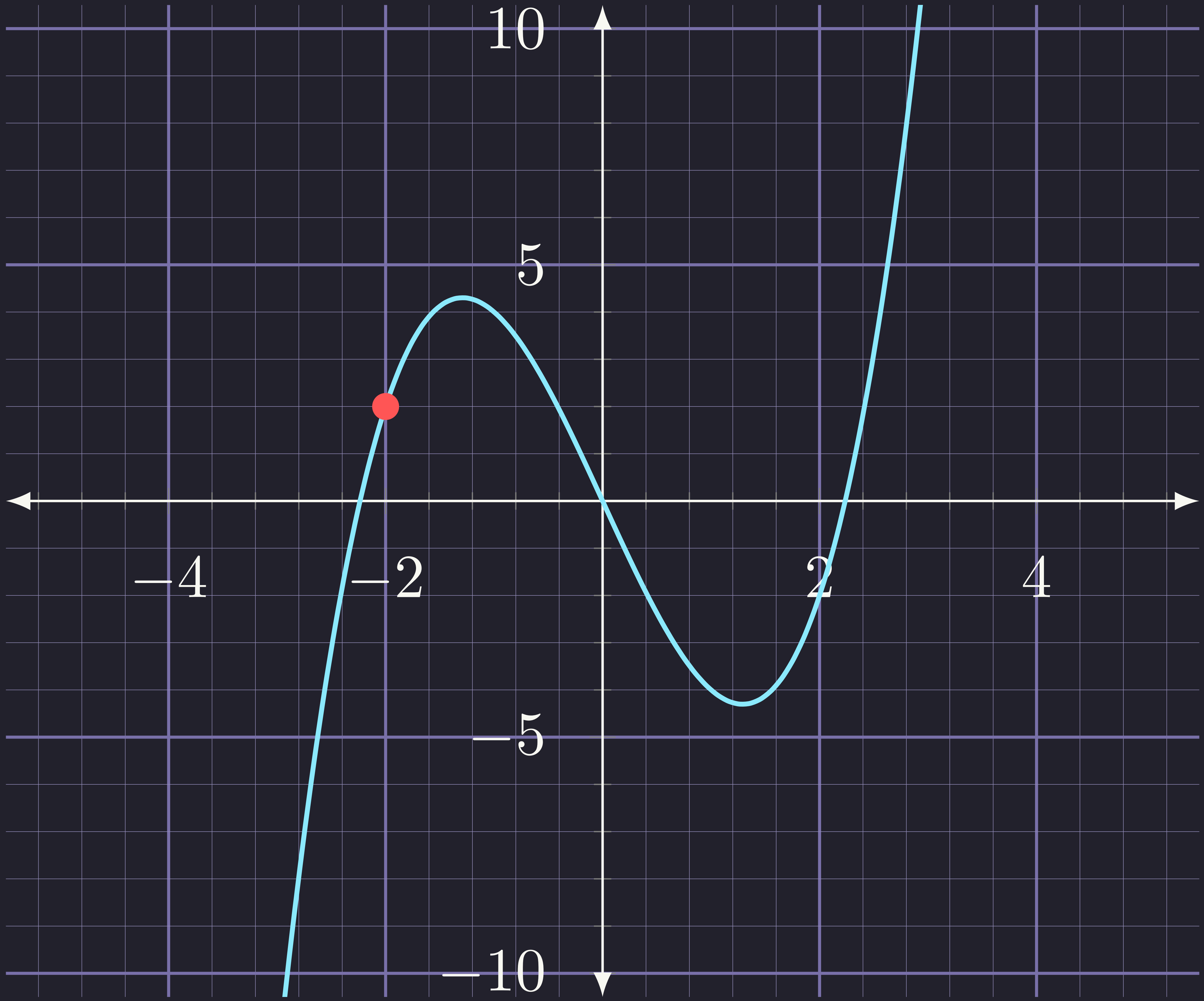
$$\lim_{x \to a^+} f(x) = L_+$$

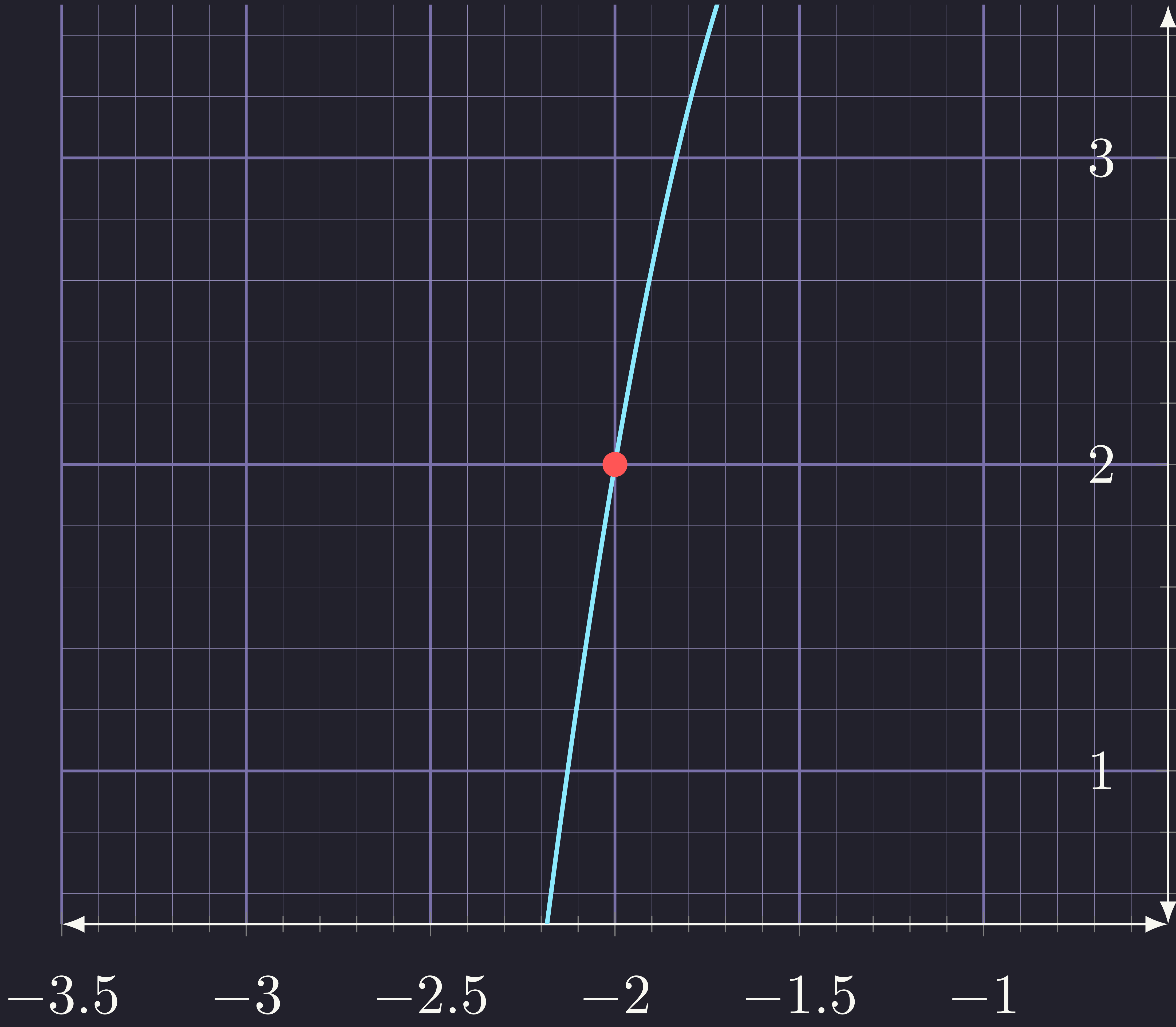
If $L_+ = L_-$ we can write

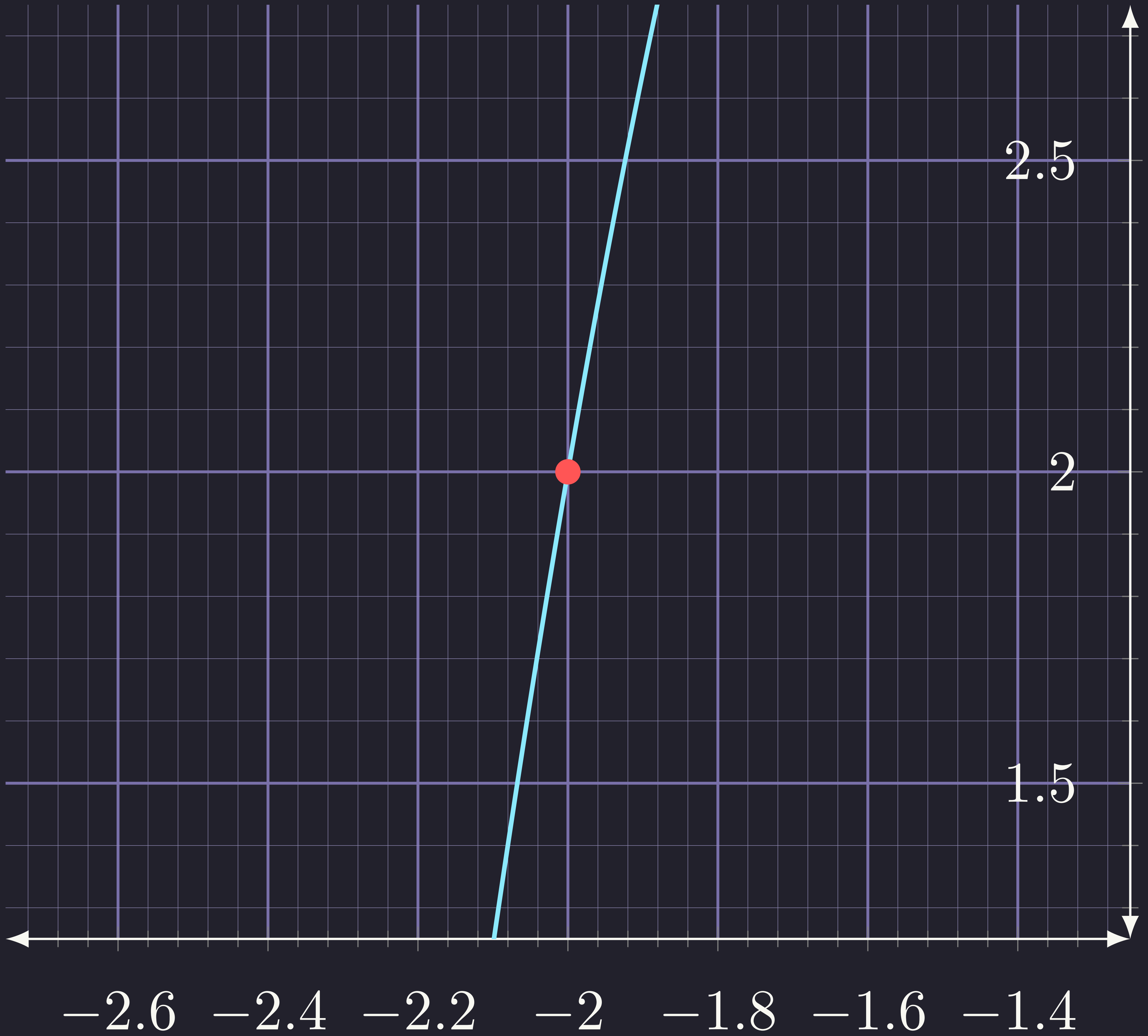
$$\lim_{x \to a} f(x) = L$$

Example:

Find: $\lim_{x \to a^-} f(x) = L_-$ $\quad \lim_{x \to a^+} f(x) = L_+$ $\quad \lim_{x \to a} f(x) = L$

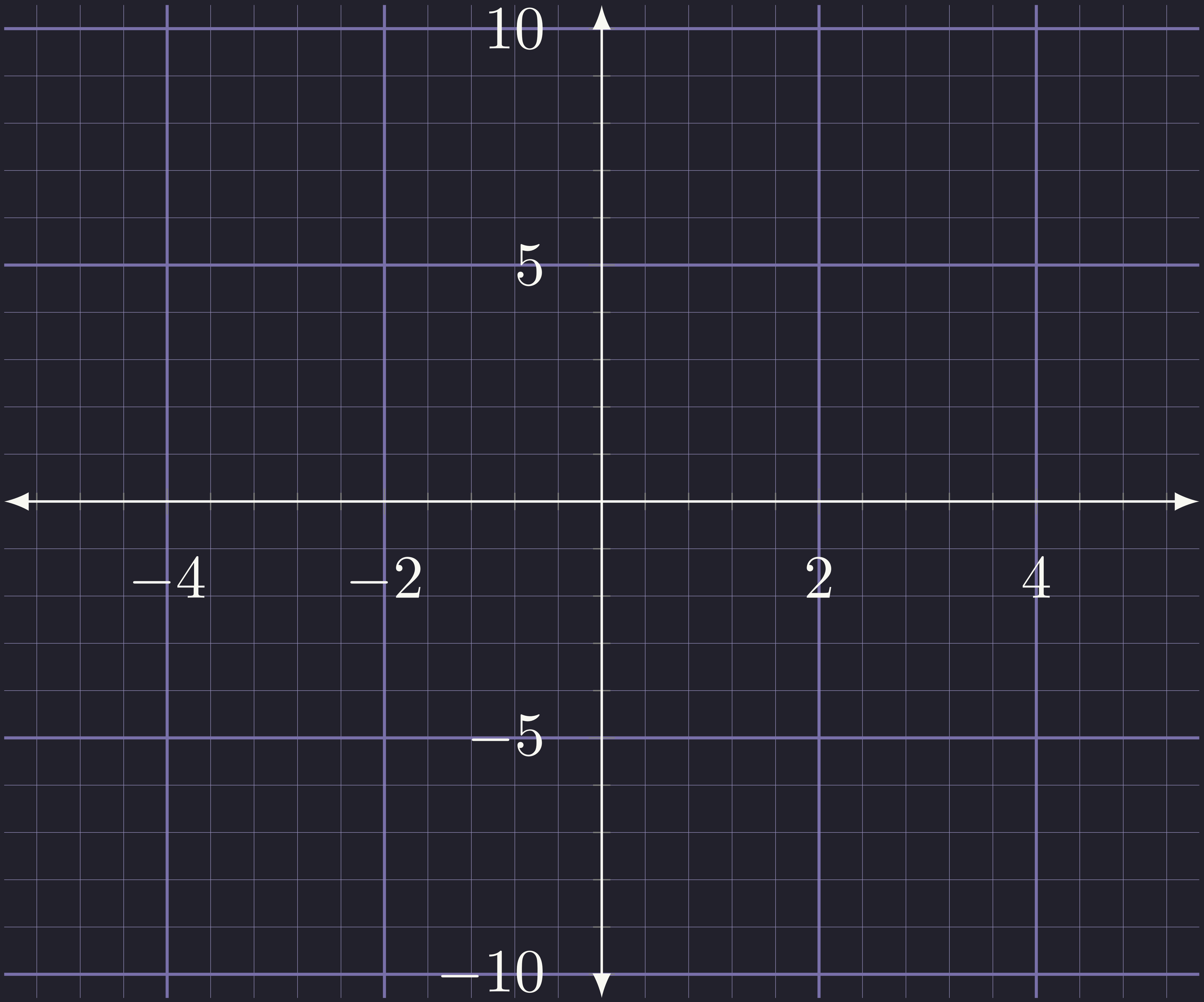



3
2
1
−3.5
−3
−2.5
−2
−1.5
−1


2.5
2
1.5
−2.6
−2.4
−2.2
−2
−1.8
−1.6
−1.4

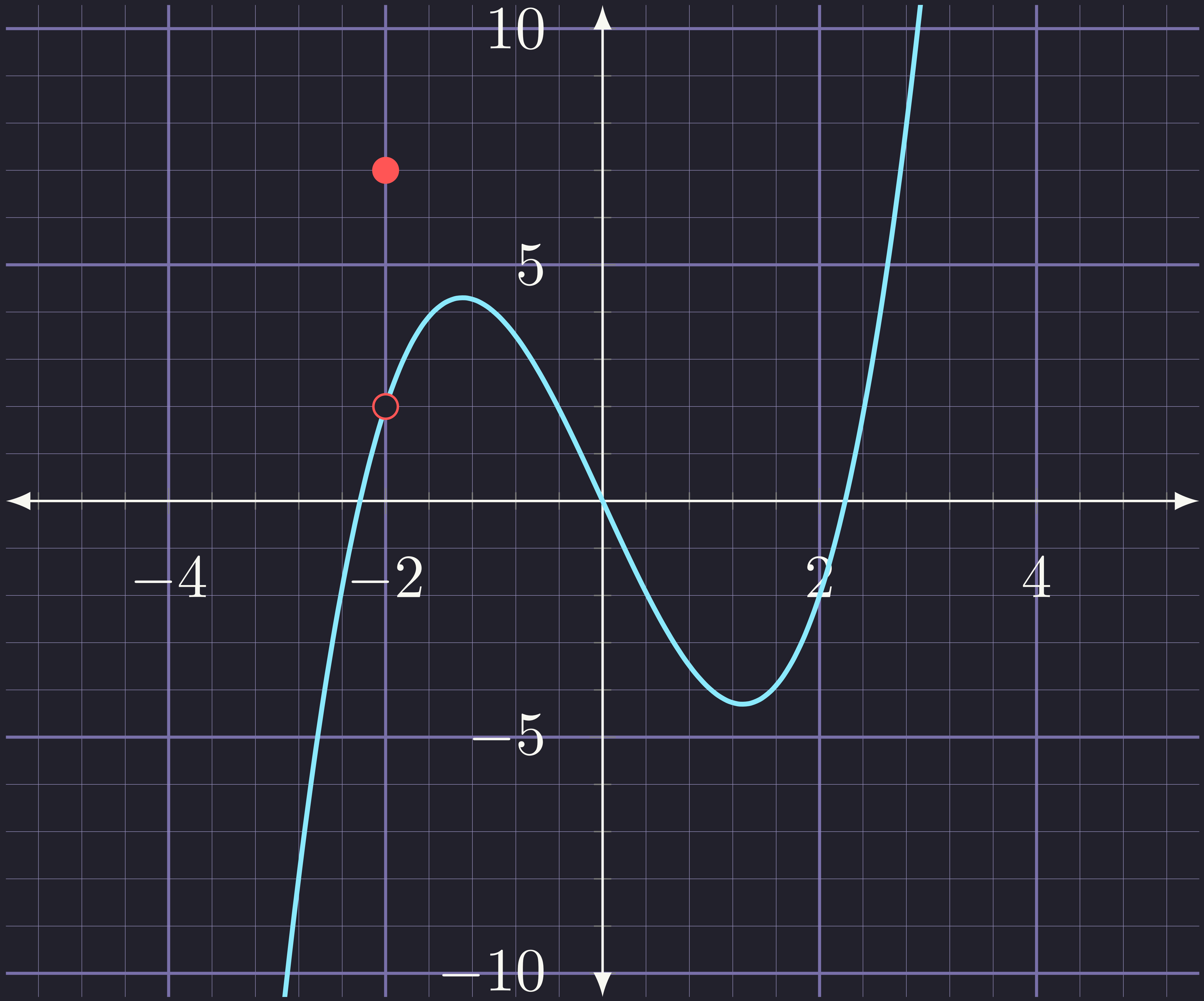
Example:

Give me an example of a function/graph where $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$



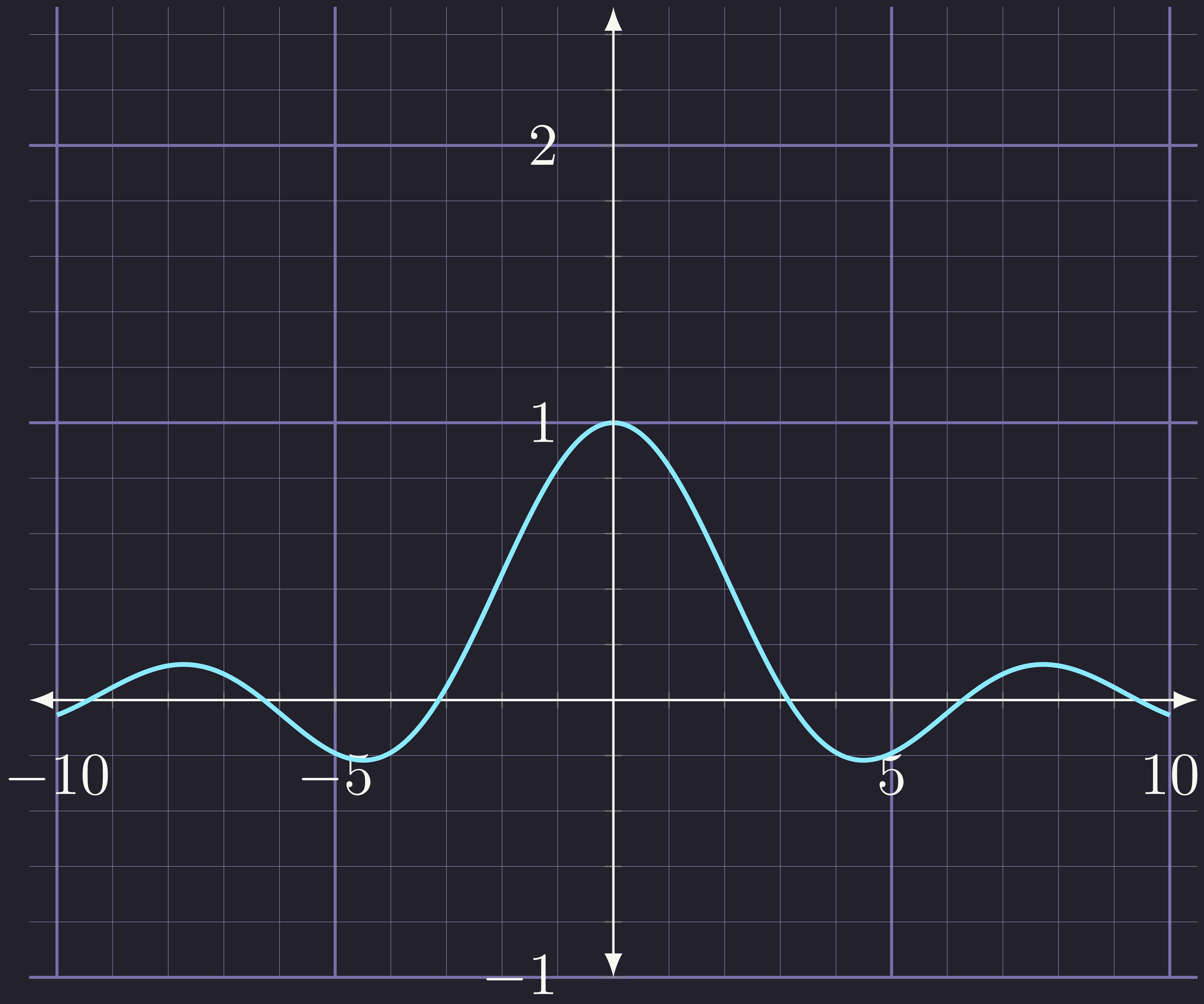
Example:

Find:$\lim_{x\to a^-} f(x) = L_-$ $\quad \lim_{x\to a^+} f(x) = L_+$ $\quad \lim_{x\to a} f(x) = L$



Estimateing limits

Graphically



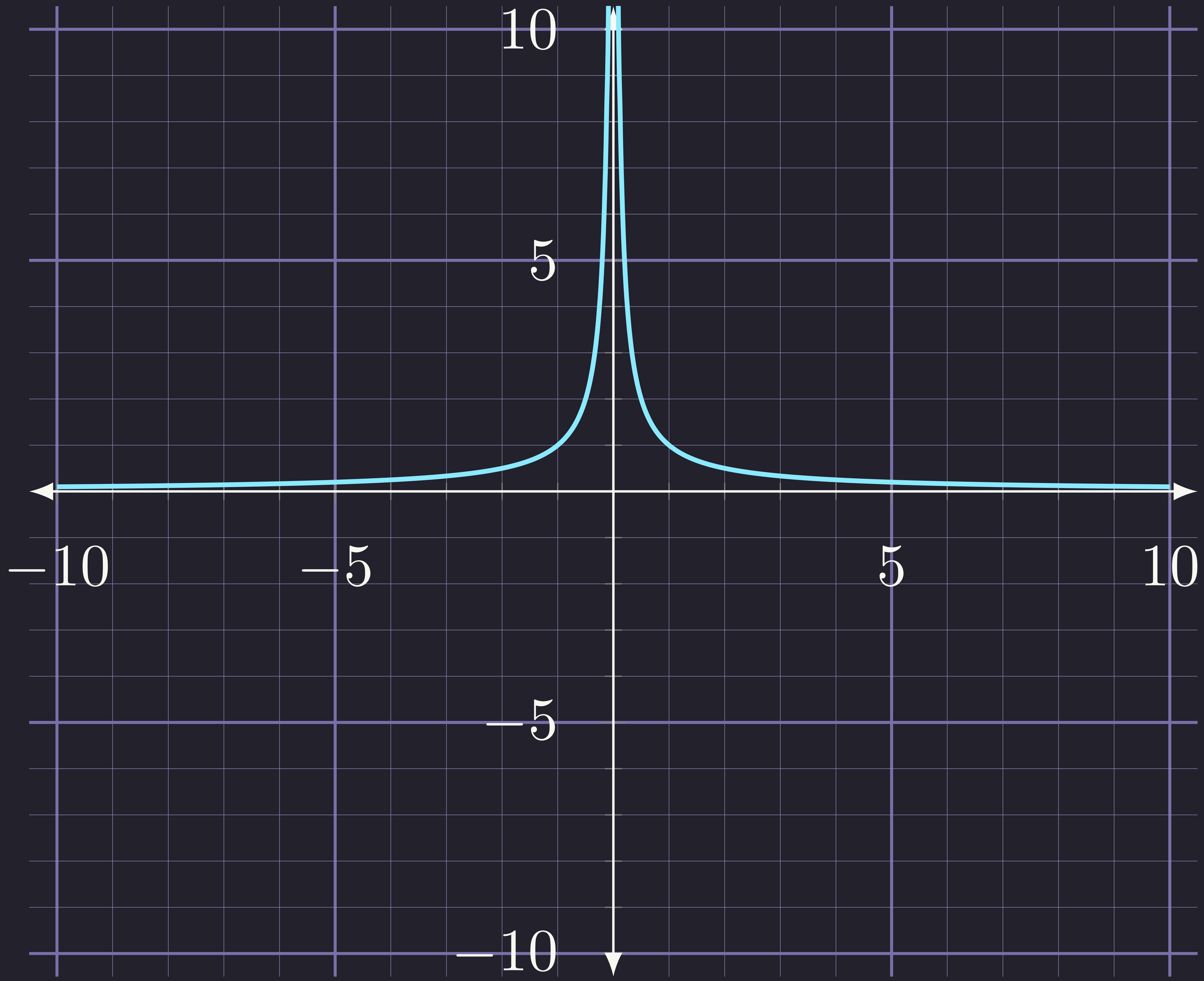
Tabular

x	$\frac{\sin x}{x}$		x	$\frac{\sin x}{x}$
π	0		$-\pi$	0
$\frac{\pi}{2}$	$\frac{2}{\pi}$		$-\frac{\pi}{2}$	$\frac{2}{\pi}$
$\frac{\pi}{4}$	$\frac{2\sqrt{2}}{\pi}$		$-\frac{\pi}{4}$	$\frac{2\sqrt{2}}{\pi}$
$\frac{\pi}{6}$	$\frac{3}{\pi}$		$-\frac{\pi}{6}$	$\frac{3}{\pi}$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
0	1		0	1

Algebraically

Tomorrow

Divergence



Definition

Let $\lim_{x \to a} f(x)$ exist. The limit is said to diverge if $|f(x)|$ gets arbitrarily large as $x \to a$.

There are a few cases:

1. $\lim_{x \to a^-} -f(x) = \infty$
2. $\lim_{x \to a^-} f(x) = -\infty$
3. $\lim_{x \to a^+} -f(x) = \infty$
4. $\lim_{x \to a^+} f(x) = -\infty$
5. $\lim_{x \to a} -f(x) = \infty$
6. $\lim_{x \to a} f(x) = -\infty$

These are culled vertical asymptotes.

Question:

What are some examples for each of the above?