

#### Formal Definitions



- Process is made up of atoms and objects
- Atom: immutable, identified by content, lower case
- Object: mutable, replicated, Capitalized
- Clients perform operations by querying and modifying object state



# State Based vs Operation Based



- Updates happen at source
- Modified payload is sent to replicas
- The system transmits state to other replicas

- Payload and initial operations are state based
- Updates cannot have side effects but can compute results
- They are communicated to replicas asynchronously
- Executed locally at source





#### CVRDTs vs CmRDTs



- State Based CRDT
- Object takes values in a semilattice, after merge(x,y)
- Merge is idempotent and commutative
- The set advances towards a Least Upper Bound (LUB)

- Needs reliable update mechanism that delivers all updates at every replica
- Order of operations can be specified
- If updates are concurrent, they have to commute
- Causal delivery is sufficient





#### Basic CRDTs



- Counters
- Registers
- Sets
- Graphs





# Counters: Op-based



#### Specification 5 op-based Counter

```
1: payload integer i
```

2: initial 0

3: query value(): integer j

4: let j = i

5: update increment ()

6: downstream ()

7: i := i + 1

8: update decrement ()

9: downstream ()

10: i := i - 1

▶ No precond: delivery order is empty

▷ No precond: delivery order is empty



### Counters: State Based Increment Only



- G-Counter: increment only counter
- State based counters can have issues
- Merge can converge on the wrong value, or not be idempotent
- Solution, vector clocks!
- Payload is a vector of integers, with each replica having an entry
- Count is the sum of all entries





### Modifications to Counters



- PN Counter:
- Combines 2 G-Counters as a P (increment) and N (decrement)

- Non Negative Counter:
- A client cannot create more decrements than increments (strong condition)





## Registers



- Register is a memory cell storing an opaque atom or object
- Assign to update
- Value to query
- Non concurrent assigns have to be sequentially ordered
- If non concurrent updates don't commute, we need to resolve it
- We pick a winner (LWW) or both get to stay (MV)



# Last-Writer-Wins Register



- The total order is decided by the timestamp
- Timestamps are unique, totally ordered and causal
- Assign generates a new timestamp
- Operation happens only if downstream timestamp is lower than the operation

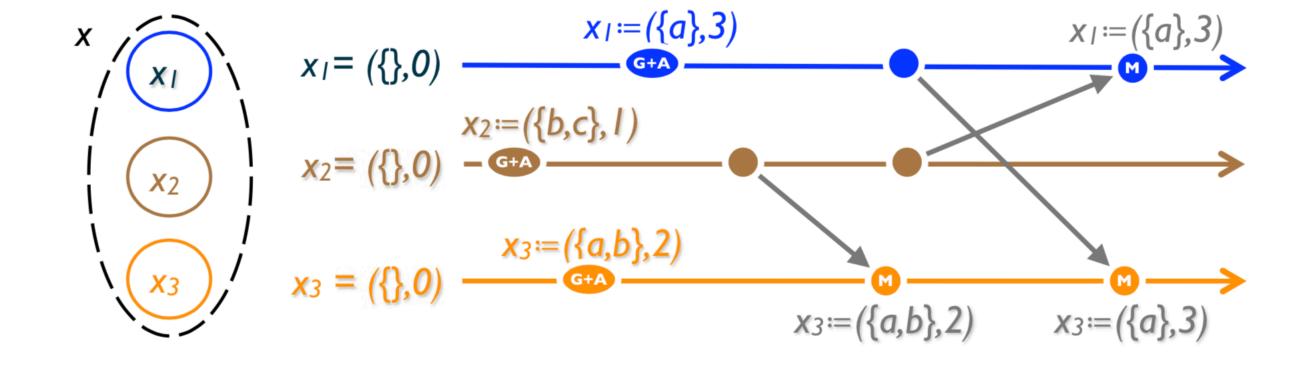


Figure 8: LWW-Set (state-based). Payload is a pair (set, timestamp)



## MV Register



- Merges values by taking a union
- Clients can then reduce them to a single value if required
- Payload is a set of (X, versionVector)
- Assign computes a version vector that dominates other version vectors

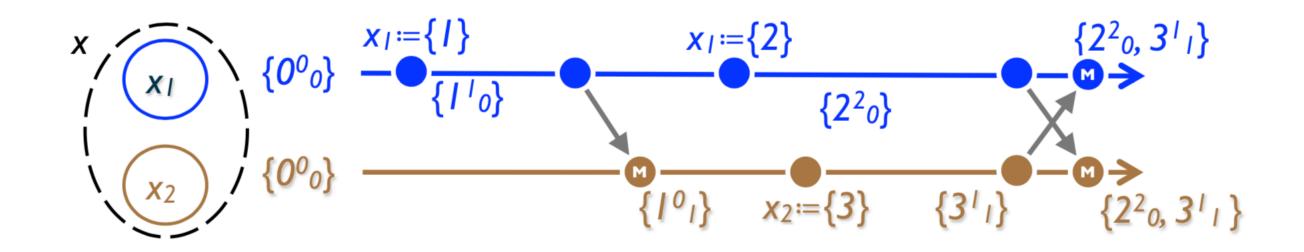


Figure 9: MV-Register (state-based)



# Growth Only Set



Supports add and lookup operations

```
Specification 11 State-based grow-only Set (G-Set)
```

```
1: payload set A

2: initial \varnothing

3: update add (element e)

4: A := A \cup \{e\}

5: query lookup (element e) : boolean b

6: let b = (e \in A)

7: compare (S, T) : boolean b

8: let b = (S.A \subseteq T.A)

9: merge (S, T) : payload U

10: let U.A = S.A \cup T.A
```



### Two Phase - 2P Set



- Element can be added and removed, but cannot be added again
- Combines 2 G-sets for adding and removing
- Since G-Sets don't support deletion, object cannot be re-added



## 2P Set Implementations



#### State Based

- Payload is composed of two sets, A and R.
- LUB is the union

#### Op-Based

Concurrent adds and removes commute

#### U Set

 U enforces every element to be unique, and ensures that add(e) is delivered before remove (e)



#### LWW-element-set



- LWW attaches a timestamp to each element
- Add set A and Remove set R contain (element, timestamp) pairs
- Merging takes unions of add and remove sets



#### PN-Set



- Similarly, we can associate a counter to each element
- Adding an element increments the associated counter
- Removing decrements it
- Element is in the set if the counter is positive
- While PN-set converges, there can be cases where add has no effect



### Observe-Remove Set



- Supports adding and removing elements
- Each added element is tagged uniquely
- While removing, unique tags observed at source are removed

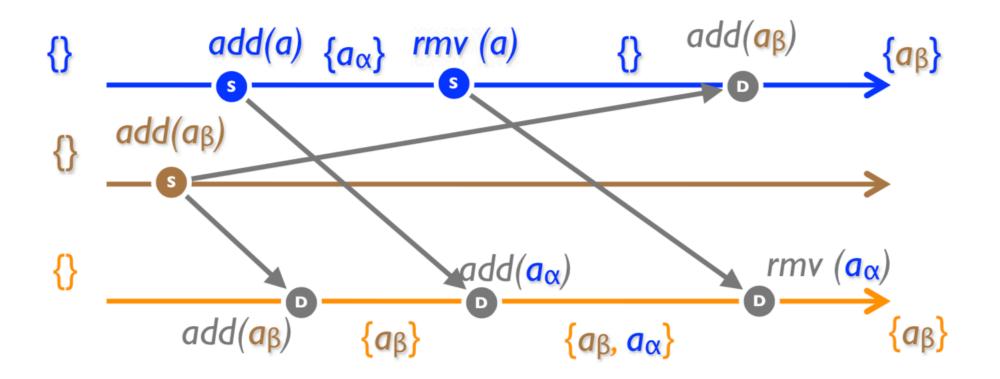


Figure 14: Observed-Remove Set (op-based)



# Graphs



• Maintaining a tree or a DAG cannot be done by a CRDT since it requires a global invariant, which is impossible to determine without synchronization

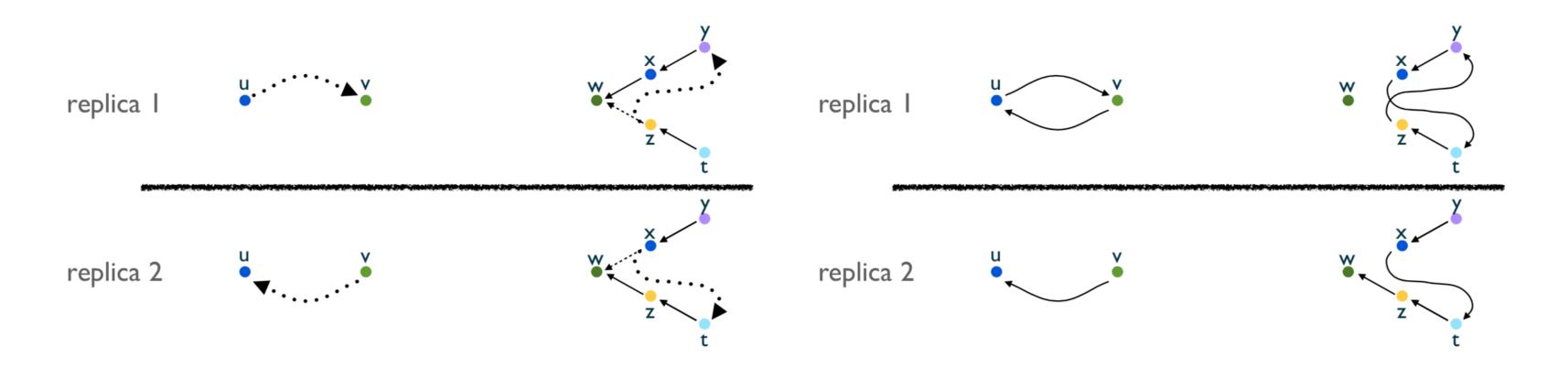


Figure 15: Maintaining strong properties in a graph (counter-example). Left: initial state and update (dashed edges removed, dotted edges added); right: final state.



#### Monotonic DAGs



- However, certain local conditions can be laid out, such as directionality and monotonicity to allow CRDTs
- DAG is initialised with left and right sentinels  $\vdash$  and  $\dashv$  and edge ( $\vdash$ ,  $\dashv$ ).
- Concurrent addEdge is either independent or idempotent
- However removals can be problematic



# Cooperative Text Editing and RGAs



- Users often collaborate on text editing: google docs
- Users can add/remove text
- Users can go offline
- CRDTs can ensure that edits always converge and never conflict, even if users are offline



# Replicated Growable Array



- Implements a sequence as a linked list
- AddRight(a,v) adds an element to the right of v
- If inserts happen twice at the same position
  - AddRight(a,v);AddRight(v,b) it adds b to the left of a with higher timestamp
- This is a subclass of Add-Remove partial order



## Class Questions and Conclusions



- What is the cost of implementing CRDTs? Are they too expensive?
- Are the few implementations of counters/ registers and sets exhaustive?
- Have better CRDTs been explored?
- What are some practical examples?

