Additional Problem Set for Lecture 4: If-Then

(1) Compare (this is an example from Ernest Adams): (i) If Oswald hadn't shot Kennedy in Dallas, Kennedy would be alive today. (ii) If Oswald didn't shoot Kennedy in Dallas, then Kennedy is alive today.

Which one is subjunctive, which one is indicative?
Which one sounds acceptable to you?

(2) Please show that $\neg A \vee B$ is logically equivalent to $\neg(A \wedge \neg B)$. What you have to do is: First assume that $\neg A \vee B$ is true, and then prove on that basis that also $\neg(A \wedge \neg B)$ is true. Secondly, the other way around: assume that $\neg(A \wedge \neg B)$ is true, and then prove on that basis that also $\neg A \vee B$ is true.

(3) When e.g. a philosopher puts forward an argument of the form

(P1) If A then it holds that B and not B. Therefore: (C) Not A.

we regard the argument as logically valid. Prove that indeed this argument form turns out to be valid, if we understand 'If C then D' as 'Not C or D' (that is, as $\neg C \vee D$).

(4) Let $W = \{w_1, w_2, w_3\}$. Let a probability measure P be given by the following B-assignment to worlds in W: $B(w_1) = 1/4$, $B(w_2) = 1/4$, $B(w_3) = 1/2$.
Determine (i) $P(\{w_1\}|\{w_1, w_2\})$, (ii) $P(\{w_1, w_2\}|\{w_1\})$, (iii) $P(\{w_2\}|\{w_2, w_3\})$.

(5) If W is the given set of all possible worlds again, and if X is an arbitrary proposition (subset of W), what is the conditional probability $P(X|W)$?

(6) Assume thesis 2 from the lecture to be true again; rational degrees of acceptability for indicative conditionals are measured in terms of the corresponding conditional probabilities. Now consider a fair six-sided die; say, your degree of belief function P reflects the fairness of the six-sided die: what is then your degree of acceptability for the indicative conditional 'If in the next throw the die rolls a 2, then it will roll an even number'?

Solutions:

(1) (i) is subjunctive, (ii) is indicative. Both are not acceptable, but for different reasons: (i) If Oswald hadn't shot Kennedy in Dallas, then Kennedy would probably not have been shot at all; but since John F. Kennedy was born in 1917, it is nevertheless unlikely that he would still be alive today. (ii) If Oswald didn't shoot Kennedy in Dallas, then someone else shot him (for I do believe that he was shot back then); in any case, Kennedy is not alive today.

(2) Assume $\neg A \vee B$ is true. Since we understand 'or' inclusively, so that it leaves open the possibility that both $\neg A$ and B are the case, precisely one of the following three possibilities must be the case: (i) $\neg A$ is true, B is false; (ii) $\neg A$ is false, B is true; (iii) $\neg A$ is true, B is true. In case (i), $A \wedge \neg B$ is false, since A is false (because $\neg A$ is true). Hence, $\neg(A \wedge \neg B)$ is true. In case (ii), $A \wedge \neg B$ is false again, since $\neg B$ is false (because B is true). Hence, $\neg(A \wedge \neg B)$ is true. In case (iii), $A \wedge \neg B$ is false, since A is false again (and also since B is true again); so once again $\neg(A \wedge \neg B)$ is true. We find that in every possible case in which $\neg A \vee B$ is true, also $\neg(A \wedge \neg B)$ is true.

Now for the other direction: Assume $\neg(A \wedge \neg B)$ is true. Therefore, $A \wedge \neg B$ is false. This leaves us with three possible cases: (i) A is true, $\neg B$ is false. (ii) A is false, $\neg B$ is true. (iii) A is false, $\neg B$ is false. In case (i), B is true because $\neg B$ is false; hence $\neg A \vee B$ is true. In case (ii), $\neg A \vee B$ is true since $\neg A$ is true (as A is false). In case (iii), $\neg A \vee B$ is true because A is false (and also because $\neg B$ is false). Thus, in every possible case in which $\neg(A \wedge \neg B)$ is true, also $\neg A \vee B$ is true.

Taking both directions together we have: $\neg A \vee B$ is logically equivalent to $\neg(A \wedge \neg B)$.

(3) So we are considering an argument of the form

(P1) Not A or (B and not B). Therefore: (C) Not A.

Assume P1 to be true: So it is true that not A or it is true that (B and not B). Since the conjunction (B and not B) cannot be true, it must be true that not A. This is exactly what the conclusion C says.

The argument form is therefore logically valid.

(4) By the definition of conditional probability, we have:

(i) $P(\{w_1\}|\{w_1, w_2\}) = \frac{P(\{w_1,w_2\}\cap\{w_1\})}{P(\{w_1,w_2\})} = \frac{P(\{w_1\})}{P(\{w_1,w_2\})} = \frac{1/4}{1/4+1/4} = \frac{1}{2}$.

(ii) $P(\{w_1, w_2\}|\{w_1\}) = \frac{P(\{w_1\}\cap\{w_1,w_2\})}{P(\{w_1\})} = \frac{P(\{w_1\})}{P(\{w_1\})} = \frac{1/4}{1/4} = 1$.

(iii) $P(\{w_2\}|\{w_2, w_3\}) = \frac{P(\{w_2,w_3\} \cap \{w_2\})}{P(\{w_2,w_3\})} = \frac{P(\{w_2\})}{P(\{w_2,w_3\})} = \frac{1/4}{1/4+1/2} = \frac{1/4}{3/4} = \frac{1}{3}$.

(5) $P(X|W) = \frac{P(W \cap X)}{P(W)} = \frac{P(X)}{P(W)} =$, since $P(W) = 1$ by the laws of probability, $= \frac{P(X)}{1} = P(X)$.

This means that every (unconditional) probability $P(X)$ can be regarded as a conditional probability $P(X|W)$, that is, as a conditional probability of X given the total set W of worlds. Or in other words: supposing that the proposition W is true is as good as not supposing anything at all; or as good as making a supposition of the form: 'Suppose A or not A' – such a supposition is irrelevant, as 'A or not A' is true anyway and hence does not need to be supposed. (By the way: in the standard terminology of probability theory, W would be called the sample space of P.)

(6) Since the die is six-sided, we may assume the relevant set W of possible worlds to consist of precisely six worlds that correspond to: die rolls 1; die rolls 2;...; die rolls 6. Since the six-sided die is also fair, your degree of belief function P ought to be have the following property: $P(\{\text{die rolls 1}\}) = P(\{\text{die rolls 2}\}) = \ldots = P(\{\text{die rolls 6}\}) = 1/6$. By thesis 2, your degree of acceptability for 'If in the next throw the die rolls a 2, then it will roll an even number' must be your conditional probability of the die rolling an even number in the next throw given that the die rolls a 2 in the next throw. That conditional probability is $P(\{\text{die rolls 2}, \text{die rolls 4}, \text{die rolls 6}\}|\{\text{die rolls 2}\})$ $= \frac{P(\{\text{die rolls 2}\} \cap \{\text{die rolls 2,die rolls 4,die rolls 6}\})}{P(\{\text{die rolls 2}\})} = \frac{P(\{\text{die rolls 2}\})}{P(\{\text{die rolls 2}\})} = \frac{1/6}{1/6} = 1$.

And that is exactly as it should be the case, right?