Additional Problem Set for Lecture 2: Truth

(1) What is the truth equivalence for the sentence 'The MCMP is a center for mathematical philosophy'?

(2) Is it possible to replace the sentence 'For every sentence x, if x is true then also the double negation of x is true' by a sentence which does **not** include the truth predicate, but which says the same as the given sentence?

(3) By applying the grammatical rules of L_{simple}, show that 'It is not the case that it is not case that it is not the case that Socrates is a teacher of Tarski' is a sentence of L_{simple}.

(4) Can we also derive the truth equivalence

'Socrates is a teacher of Tarski and it is not the case that Socrates is a teacher of Tarski' is true if and only if Socrates is a teacher of Tarski and it is not the case that Socrates is a teacher of Tarski

(that is, the truth equivalence for the contradictory sentence 'Socrates is a teacher of Tarski and it is not the case that Socrates is a teacher of Tarski') from our materially adequate definition of truth for L_{simple}?

(5) Fill in the missing part of the inductive proof of the theorem in the lecture as far as 'or'-sentences are concerned. That is, prove:

if (y is true or \neg y is true) and (z is true or \neg z is true), then y \or z is true or \neg (y \or z) is true.

(I write 'y \or z' here as a shorthand for: the result of putting y together with 'or' and with z. By '\neg' I mean negation ('not').)

(6) Think about the following sentence:

(#) If the sentence that is introduced by a hash sign is true, then the moon is made of green cheese.

Does that sentence lead to similar trouble as the Liar sentence?

Solutions:

(1) The truth equivalence for the sentence 'The MCMP is a center for mathematical philosophy' is the following sentence:

'The MCMP is a center for mathematical philosophy' is true if and only if the MCMP is a center for mathematical philosophy.

(2) No (or at least not easily).

What should we say instead? 'For every sentence x, if x then the double negation of x' ('For every sentence, if it then the double negation of it')? It would need work to come up with a formal language in which this is well-formed. Or shoulw we use the scheme 'If A then not not A'? But that is not a concrete sentence.

(3) By applying the rule 'If we put a name (in L_{simple}) before 'is a teacher of' and another one after it, we end up with a sentence of L_{simple}', we get: (1) 'Socrates is a teacher of Tarski' is a sentence of L_{simple}.

Now we apply the rule 'If we put 'it is not the case that' in front of a sentence of L_{simple}, we get a sentence of L_{simple} again' three times: first to the sentence mentioned in (1), which gives us (2) 'It is not the case that Socrates is a teacher of Tarski' is a sentence of L_{simple}. Then to the sentence mentioned in (2), which gives us (3) 'It is not the case that it is not the case that Socrates is a teacher of Tarski' is a sentence of L_{simple}. And finally to the sentence mentioned in (3), which gives us (4) 'It is not the case that it is not the case that it is not the case that Socrates is a teacher of Tarski' is a sentence of L_{simple}.

(By the way: this also tell us how to prove that L_{simple} includes infinitely many sentences--just keep adding negations.)

(4) Yes. 'Materially adequate' means that one can derive truth equivalences for all the sentences of L_{simple}, and 'Socrates is a teacher of Tarski and it is not the case that Socrates is a teacher of Tarski' is a sentence of L_{simple}.

Of course, the right-hand side of the relevant truth equivalence is false (even logically false), which is why it follows immediately that the sentence 'Socrates is a teacher of Tarski and it is not the case that Socrates is a teacher of Tarski' is not true.

(5) Assume that (y is true or \neg y is true) and (z is true or \neg z is true).

This implies that there are four possible cases really:

(i) y is true, z is true: by our definition of truth it holds that y \or z is true if and only if y is true or z is true (that is, at least one of y and z is true). But it is the case that at least one of y and z is true; in fact even both are true. So it follows: y \or z is true.

And by logic this entails: y \or z is true or \neg (y \or z) is true.

(ii) y is true, \neg z is true: by our definition of truth it holds that y \or z is true if and only if y is true or z is true (that is, at least one of y and z is true). But it is the case that at least one of y and z is true; for y is true. So it follows: y \or z is true. And by logic this entails: y \or z is true or \neg (y \or z) is true.

(iii) \neg y is true, z is true: by our definition of truth it holds that y \or z is true if and only if y is true or z is true (that is, at least one of y and z is true). But it is the case that at least one of y and z is true; for z true. So it follows: y \or z is true. And by logic this entails: y \or z is true or \neg (y \or z) is true.

(iv) \neg y is true, \neg z is true: by our definition of truth, \neg y is true if and only if y is not true; so we have: y is not true. Analogously, \neg z is true if and only if z is not true; so we have: z is not true. Furhermore, by our definition of truth, it holds that y \or z is true if and only if y is true or z is true (that is, at least one of y and z is true). But it is not the case that at least one of y and z is true; for both of them are not true. So it follows: y \or z is not true. From this we can conclude, since once again by our definition of truth it holds that \neg (y \or z) is true if and only if y \or z is not true: \neg (y \or z) is true. And by logic this entails again: y \or z is true or \neg (y \or z) is true. Which is what we were supposed to prove.

(6) Yes, it does. The corresponding paradox is called 'Curry's Paradox'. If you want to know more about it, and about how to derive a contradiction from the truth equivalence for the hash sentence, please take a look at <http://plato.stanford.edu/entries/curry-paradox/>.