Additional Problem Set for Lecture 1: Infinity

(1) Do you think the following argument is logically valid, too?

(P1) Jacques does not put forward arguments. (P2) If Jacques is a philosopher, then he puts forward arguments. (C) Jacques is not a philosopher.

(2) Show that for all positive natural numbers n (all natural numbers n greater than 0):

$1/2 + 1/4 + ... + 1/2^n = 1 - 1/2^n$.

(Hint: Assume that $1/2 + 1/4 + ... + 1/2^n = x$; then divide both sides of the equation by 2 and determine x by calculation.)

By the way, this also proves that for all positive natural numbers n: $1/2 + 1/4 + ... + 1/2^n = 1 - 1/2^n < 1$.
That is, however large the n, the sum $1/2 + 1/4 + ... + 1/2^n$ is bounded by 1.

(3) Is the set {9,3,1} identical to the set {3,9,1}?

(4) Is the set {9,3,1} a subset of the set {3,9,1}? Is it a proper subset?

(5) Show that the set {2,4,6,8,...} of even natural numbers is infinite (in the sense of having equally many members as some of its proper subsets, or, in other words, in the sense that there is a pairing off between {2,4,6,8,...} and some of its proper subsets).

Solution:

(1) Yes, the argument is logically valid.

We need to show that if P1 and P2 are true, then necessarily also C is true.

Suppose P1 and P2 to be true. Now assume for reductio that C is false: so Jacques is a philosopher. By P2, he also puts forward arguments then. But this contradicts P1. So C must be true.

The logical rule that corresponds to this argument is called 'Modus Tollens' in the philosophical tradition.

(2) $1/2 + 1/4 + ... + 1/2^n = x$

By dividing both sides by 2:

$$1/4 + 1/8 + ... + 1/2^{(n+1)} = x/2$$

Now subtract $1/4 + 1/8 + ... + 1/2^{(n+1)}$ from $1/2 + 1/4 + ... + 1/2^n$ on the left, and subtract $x/2$ from x on the right. This leaves us with:

$$1/2 - 1/2^{(n+1)} = x/2$$

That is:

$$1 - 1/2^n = x$$

(3) Yes, as they have the same members.

(4) Yes; in fact, every set is a subset of itself. And no; no set is a proper subset of itself.

(5) For instance, the set {4,8,12,16,...} of positive natural numbers that are multiples of 4 is a proper subset of {2,4,6,8,...}, and there exists a pairing off between {2,4,6,8,...} and {4,8,12,16,...}:

2 <-> 4
4 <-> 8
6 <-> 12
8 <-> 16
and so on.

In general (for n being an arbitrary positive natural number):

2n <-> 4n