Truth as a property of

- descriptive sentences:

  The descriptive sentence ‘Snow is white’ is true.

Truth as a property of

- descriptive sentences:

  The descriptive sentence ‘Snow is white’ is true.

- propositions:

  What ‘snow is white’ expresses is true.
  That snow is white is true.
  The proposition that snow is white is true

- I believe A, and A is true.
- I believe A, and A is false.
- I do not believe A, and A is true.
- I do not believe A, and A is false.

- ‘Peter is at home right now’ is true
  or ‘Peter is not at home right now’ is true.

- ‘Peter is at home right now’ is true
  or ‘Peter is not at home right now’ is true.
- But it is possible that:

  Neither ‘Peter is at home right now’ is believed by me
  nor ‘Peter is not at home right now’ is believed by me.

Grandfatherhood:

For all x, for all z: x is a grandfather of z if and only if

(i) there is a y, such that x is the father of y and y is the father of z,

or

(ii) there is a y, such that x is the father of y and y is the mother of z.

A descriptive sentence or a proposition is true if and only if it corresponds to reality. [???]

(Image source:
Mathematisches Forschungsinstitut Oberwolfach gGmbH

In order to be satisfactory, a definition of truth has to be *formally correct* and *materially adequate*.

(1) ‘Snow is white’ is true if and only if snow is white.

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Left-hand side of (1):

(2) ‘Snow is white’ is true.

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Left-hand side of (1):

(2) ‘Snow is white’ is true.

Right-hand side of (1):

(3) Snow is white.

(1) ‘Snow is white’ is true if and only if snow is white.

Left-hand side of (1):

(2) ‘Snow is white’ is true.

Right-hand side of (1):

(3) Snow is white.

Sentence (1) is of the form:

(T) ‘A’ is true if and only if A.

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Other instances of (T):

‘Tarski is a philosopher’ is true if and only if Tarski is a philosopher.

‘Munich is in Germany’ is true if and only if Munich is in Germany.

‘2+2=4’ is true if and only if 2+2=4.

$\vdots$

(1) ‘Snow is white’ is true if and only if snow is white.

Left-hand side of (1):

(2) ‘Snow is white’ is true.

Right-hand side of (1):

(3) Snow is white.

Sentence (1) is of the form:

(T) ‘A’ is true if and only if A.

Material Adequacy:

A definition of truth for the descriptive sentences of a language L is materially adequate if and only if the definition implies all truth equivalences, that is, all instances of the truth scheme

(T) 'A' is true if and only if A

in which the place-holder 'A' is replaced by an arbitrary descriptive sentence in the language L.

(1) ‘Snow is white’ is true if and only if snow is white.

Aristotle, *Metaphysics*:

> "to say of what is that it is, or of what is not that it is not, is true"

Aristotle, *Metaphysics*:

> "to say of what is that it is, or of what is not that it is not, is true"

(T) 'A' is true if and only if A.

Aristotle, *Metaphysics*:

> "to say of what is that it is, or of what is not that it is not, is true"

(T) 'A' is true if and only if A.

(A and 'A' is true) or (not A and 'A' is not true).

- The last sentence asserted by Caesar is true.
- Every sentence that is provable in mathematics is true.

(T) ‘A’ is true if and only if A.

(T) ‘A’ is true if and only if A.

For all x: x is true if and only if . . .

(T) ‘A’ is true if and only if A.

For all x: x is true if and only if . . .

For all x, for all z: x is a grandfather of z if and only if . . .

Grandfatherhood:

For every two persons: *one* is a grandfather of *the other* if and only if

(i) there is a y, such that *the one* person is the father of y and y is the father of *the other* person,

or

(ii) there is a y, such that *the one* person is the father of y and y is the mother of *the other* person.

For all A: 'A' is true if and only if A.

For all A: 'A' is true if and only if A.

For every sentence/proposition, *it* (in quotation marks?) is true if and only if *it*.

Material Adequacy:

A definition of truth for the descriptive sentences of a language L is materially adequate if and only if the definition implies all truth equivalences, that is, all instances of the truth scheme

(T) 'A' is true if and only if A

in which the place-holder 'A' is replaced by an arbitrary descriptive sentence in the language L.

Vocabulary of L_{simple}*:*

- Names: ‘Socrates’, ‘Plato’, ’Aristotle’, ‘Tarski’.
- Predicates: ‘is a philosopher’, ‘is a teacher of’.
- Logical Symbols: ‘it is not the case that’ (‘not’), ‘and’, ‘or’.

Grammar of L_{simple}*:*

- If we put a name before ‘is a philosopher’ we get a sentence of L_{simple}.

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- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.

'Socrates is a philosopher' is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before ‘is a philosopher’ we get a sentence of L_{simple}.
- If we put a name before ‘is a teacher of’ and another one after it, we get a sentence of L_{simple}.

‘Plato is a teacher of Aristotle’ is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.

'Plato is a teacher of Aristotle' is a sentence of L_{simple}.

'Aristotle is a teacher of Plato' is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.

'Plato is a teacher of Aristotle' is a sentence of L_{simple}.

'Aristotle is a teacher of Plato' is a sentence of L_{simple}.

'Socrates is a teacher of Socrates' is a sentence of L_{simple}.

Grammar of L_{simple}:

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.

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- If we put a name before ‘is a philosopher’ we get a sentence of L_{simple}.
- If we put a name before ‘is a teacher of’ and another one after it, we get a sentence of L_{simple}.
- If we put ‘it is not the case that’ (‘not’) in front of a sentence of L_{simple}, we get a sentence of L_{simple}.

‘It is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
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- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.

'It is not the case that Aristotle is a teacher of Plato' is a sentence of L_{simple}.

'Aristotle is not a teacher of Plato' is a sentence of L_{simple}.

Grammar of L_{simple}:

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.

'It is not the case that Aristotle is a teacher of Plato' is a sentence of L_{simple}.

'Aristotle is not a teacher of Plato' is a sentence of L_{simple}.

'It is not the case that it is not the case that Aristotle is a teacher of Plato' is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.
- For every two sentences of L_{simple}, if we put an 'and' or an 'or' between them, then we get sentences of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.
- For every two sentences of L_{simple}, if we put an 'and' or an 'or' between them, then we get sentences of L_{simple}.

'Socrates is a philosopher and Aristotle is not a teacher of Plato' is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.
- For every two sentences of L_{simple}, if we put an 'and' or an 'or' between them, then we get sentences of L_{simple}.

'Socrates is a philosopher and Aristotle is not a teacher of Plato' is a sentence of L_{simple}.

'(Socrates is a philosopher and Aristotle is not a teacher of Plato) or Plato is a teacher of Aristotle' is a sentence of L_{simple}.

Grammar of L_{simple}*:*

- If we put a name before 'is a philosopher' we get a sentence of L_{simple}.
- If we put a name before 'is a teacher of' and another one after it, we get a sentence of L_{simple}.
- If we put 'it is not the case that' ('not') in front of a sentence of L_{simple}, we get a sentence of L_{simple}.
- For every two sentences of L_{simple}, if we put an 'and' or an 'or' between them, then we get sentences of L_{simple}.

‘Socrates is a philosopher and Aristotle is not a teacher of Plato’ is a sentence of L_{simple}.

‘(Socrates is a philosopher and Aristotle is not a teacher of Plato) or Plato is a teacher of Aristotle’ is a sentence of L_{simple}.

‘Socrates is a philosopher and (Aristotle is not a teacher of Plato or Plato is a teacher of Aristotle)’ is a sentence of L_{simple}.

Truth for L_{simple} (first version):

For all sentences x of L_{simple}:

- if x is the result of putting together the name 'Socrates' with the predicate 'is a philosopher', then
  x is true if and only if Socrates is a philosopher;
- if x is the result of putting together the name 'Plato' with the predicate 'is a philosopher', then
  x is true if and only if Plato is a philosopher;
- if x is the result of putting together the name 'Aristotle' with the predicate 'is a philosopher', then
  x is true if and only if Aristotle is a philosopher;
- if x is the result of putting together the name 'Tarski' with the predicate 'is a philosopher', then
  x is true if and only if Tarski is a philosopher;

Truth for $\mathsf{L}_{\mathsf{simple}}$ (first version): [CONTINUED]

- if x is the result of putting together the name 'Socrates' with the predicate 'is a teacher of' and with the name 'Socrates', then
  x is true if and only if Socrates is a teacher of Socrates;
- if x is the result of putting together the name 'Socrates' with the predicate 'is a teacher of' and with the name 'Plato', then
  x is true if and only if Socrates is a teacher of Plato;
  ⋮
  [4 names times 4 names = 16 cases in total for 'is a teacher of']

Truth for L_{simple} (first version): [CONTINUED]

- (case: $x =$ 'not' $+ y$)

  if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol 'it is not the case that' with y, then
  x is true if and only if y is not true;

(Equivalently:
if x is the result of putting together the logical symbol 'it is not the case that' with a sentence y of L_{simple}, then
x is true if and only if y is not true.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, if x is the result of putting together the logical symbol 'it is not the case that' with y, then
x is true if and only if y is not true.)

Truth for L_{simple} (first version): [CONTINUED]

- (case: $x = y +$ 'and' $+ z$)

  if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x is the result of putting together y with the logical symbol 'and' and with z, then x is true if and only if y is true and z is true;

(Equivalently:
if x is the result of putting together a sentence y of L_{simple} with the logical symbol 'and' and with a sentence z of L_{simple}, then
x is true if and only if y is true and z is true.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, for all sentences z of L_{simple}, if x is the result of putting together y with the logical symbol 'and' and with z, then
x is true if and only if y is true and z is true.)

Truth for L_{simple} (first version): [CONTINUED]

- (case: $x = y + \text{`or'} + z$)

  if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x is the result of putting together y with the logical symbol 'or' and with z, then x is true if and only if y is true or z is true;

(Equivalently:
if x is the result of putting together a sentence y of L_{simple} with the logical symbol 'or' and with a sentence z of L_{simple}, then
x is true if and only if y is true or z is true.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, for all sentences z of L_{simple}, if x is the result of putting together y with the logical symbol 'or' and with z, then
x is true if and only if y is true or z is true.)

‘Socrates is a philosopher’ is a sentence of L_{simple}.

‘Socrates is a philosopher’ is a sentence of L_{simple}.

- if x is the result of putting together the name ‘Socrates’ with the predicate ‘is a philosopher’, then
  x is true if and only if Socrates is a philosopher;

‘Socrates is a philosopher’ is a sentence of L_{simple}.

- if x is the result of putting together the name ‘Socrates’ with the predicate ‘is a philosopher’, then
  x is true if and only if Socrates is a philosopher;

‘Socrates is a philosopher’ is true if and only if Socrates is a philosopher.

‘Plato is a teacher of Aristotle’ is true if and only if Plato is a teacher of Aristotle.

‘Aristotle is a teacher of Plato’ is true if and only if Aristotle is a teacher of Plato.

‘Socrates is a teacher of Socrates’ is true if and only if Socrates is a teacher of Socrates.

‘It is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

‘It is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol ‘it is not the case that’ with y, then
  x is true if and only if y is not true;

y: ‘Aristotle is a teacher of Plato’
x: ‘It is not the case that Aristotle is a teacher of Plato’

‘It is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol ‘it is not the case that’ with y, then x is true if and only if y is not true;

‘It is not the case that Aristotle is a teacher of Plato’ is true if and only if it is not the case that ‘Aristotle is a teacher of Plato’ is true.

‘It is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol ‘it is not the case that’ with y, then
  x is true if and only if y is not true;

‘It is not the case that Aristotle is a teacher of Plato’ is true if and only if it is not the case that ‘Aristotle is a teacher of Plato’ is true.

‘It is not the case that Aristotle is a teacher of Plato’ is true if and only if it is not the case that Aristotle is a teacher of Plato.

‘It is not the case that it is not the case that Aristotle is a teacher of Plato’ is true if and only if it is not the case that it is not the case that Aristotle is a teacher of Plato.

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x is the result of putting together y with the logical symbol ‘and’ and with z, then x is true if and only if y is true and z is true;

x: ‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’
y: ‘Socrates is a philosopher’
z: ‘it is not the case that Aristotle is a teacher of Plato’

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x is the result of putting together y with the logical symbol ‘and’ and with z, then x is true if and only if y is true and z is true;

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is true if and only if ‘Socrates is a philosopher’ is true and ‘it is not the case that Aristotle is a teacher of Plato’ is true.

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is a sentence of L_{simple}.

- if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x is the result of putting together y with the logical symbol ‘and’ and with z, then x is true if and only if y is true and z is true;

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is true if and only if ‘Socrates is a philosopher’ is true and ‘it is not the case that Aristotle is a teacher of Plato’ is true.

‘Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato’ is true if and only if Socrates is a philosopher and it is not the case that Aristotle is a teacher of Plato.

‘Socrates is a philosopher’ is true.

‘Aristotle is a teacher of Plato’ is not true.

The definition of truth for L_{simple} is materially adequate.

- if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol ‘it is not the case that’ with y, then
  x is true if and only if y is not true;

- (Multiplying 0 with m yields 0.)
  $mult(0, m) = 0$.
- (Multiplying $1 + n$ with m yields the sum of m and the result of multiplying n with m.)
  $mult(1 + n, m) = m + mult(n, m)$.

- if x is the result of putting together the name ‘Socrates’ with the predicate ‘is a philosopher’, then
  x is true if and only if Socrates is a philosopher;
- if x is the result of putting together the name ‘Aristotle’ with the predicate ‘is a teacher of’ and with the name ‘Plato’, then
  x is true if and only if Aristotle is a teacher of Plato;

Sentencehood for L_{simple} (first version):

For all x:

- if x is the result of putting together the name 'Socrates' with the predicate 'is a philosopher', then x is a sentence of L_{simple};

  ⋮

- if there is a sentence y of L_{simple}, such that x is the result of putting together the logical symbol 'it is not the case that' with y, then x is a sentence of L_{simple};

(Equivalently:
if x is the result of putting together the logical symbol 'it is not the case that' with a sentence y of L_{simple}, then x is a sentence of L_{simple}.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, if x is the result of putting together the logical symbol 'it is not the case that' with y, then x is a sentence of L_{simple}.)

$\vdots$

It is not the case that (Aristotle is a teacher of Tarski, and Tarski is a teacher of Aristotle).

Truth for L_{simple} (second version):

For all sentences x of L_{simple}:

x is true if and only if x is a member of all sets Y of sentences of L_{simple} for which the following holds: for all x',

- if x' is the result of putting together the name 'Socrates' with the predicate 'is a philosopher', then
  x' is a member of Y if and only if Socrates is a philosopher;

  ⋮

⋮

- if $\not{x}$ is the result of putting together the name ‘Socrates’ with the predicate ‘is a teacher of’ and with the name ‘Socrates’, then
  $\not{x}$ is a member of Y if and only if Socrates is a teacher of Socrates;
- if $\not{x}$ is the result of putting together the name ‘Socrates’ with the predicate ‘is a teacher of’ and with the name ‘Plato’, then
  $\not{x}$ is a member of Y if and only if Socrates is a teacher of Plato;

⋮

Truth for L_{simple} (second version): [CONTINUED]

- (case: $x' =$ 'not' $+ y$)

  if there is a sentence y of L_{simple}, such that x' is the result of putting together the logical symbol 'it is not the case that' with y, then
  x' is a member of Y if and only if y is not a member of Y;

(Equivalently:
if x' is the result of putting together the logical symbol 'it is not the case that' with a sentence y of L_{simple}, then
x' is a member of Y if and only if y is not a member of Y.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, if x' is the result of putting together the logical symbol 'it is not the case that' with y, then
x' is a member of Y if and only if y is not a member of Y.)

Truth for L_{simple} (second version): [CONTINUED]

- (case: $x' = y + \text{`and'} + z$)
  if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x' is the result of putting together y with the logical symbol 'and' and with z, then
  x' is a member of Y if and only if y is a member of Y and z is a member of Y;

(Equivalently:
if x' is the result of putting together a sentence y of L_{simple} with the logical symbol 'and' and with a sentence z of L_{simple}, then
x' is a member of Y if and only if y is a member of Y and z is a member of Y.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, for all sentences z of L_{simple}, if x' is the result of putting together y with the logical symbol 'and' and with z, then
x' is a member of Y if and only if y is a member of Y and z is a member of Y.)

Truth for L_{simple} (second version): [CONTINUED]

- (case: $x' = y + \text{`or'} + z$)
  if there is a sentence y of L_{simple} and a sentence z of L_{simple}, such that x' is the result of putting together y with the logical symbol 'or' and with z, then
  x' is a member of Y if and only if y is a member of Y or z is a member of Y.

(Equivalently:
if x' is the result of putting together a sentence y of L_{simple} with the logical symbol 'or' and with a sentence z of L_{simple}, then
x' is a member of Y if and only if y is a member of Y or z is a member of Y.)

(Equivalently, and most precisely:
for all sentences y of L_{simple}, for all sentences z of L_{simple}, if x' is the result of putting together y with the logical symbol 'or' and with z, then
x' is a member of Y if and only if y is a member of Y or z is a member of Y.)

Sentencehood for L_{simple} (second version):

For all x:

x is a sentence of L_{simple} if and only if x is a member of all sets Y for which the following holds: for all x',

- if x' is the result of putting together the name 'Socrates' with the predicate 'is a philosopher', then x' is a member of Y;

  ⋮

- if there is a member y of Y, such that x' is the result of putting together the logical symbol 'it is not the case that' with y, then x' is a member of Y;

(Equivalently:
if x' is the result of putting together the logical symbol 'it is not the case that' with a member y of Y, then x' is a member of Y.)

(Equivalently, and most precisely:
for all members y of Y, if x' is the result of putting together the logical symbol 'it is not the case that' with y, then x' is a member of Y.)

$\vdots$

- 0 has that property P;
- for all natural numbers n: if n has the property P, then also $n + 1$ has that property P.

- 0 has that property P;
- for all natural numbers n: if n has the property P, then also $n + 1$ has that property P.

From this, by *complete induction over natural numbers*, it follows:

- All natural numbers n have property P.

- ‘Socrates is a philosopher’ has property P.

  ⋮

- For all sentences x of L_{simple}: if x has the property P, then also the result of putting together the logical symbol ‘it is not the case that’ with x has property P.
- For all sentences x of L_{simple}, for all sentences y of L_{simple}: if x has the property P and also y has the property P, then the sentence of L_{simple} that results from putting together x with ‘and’ and y has the property P.
- For all sentences x of L_{simple}, for all sentences y of L_{simple}: if x has the property P and also y has the property P, then the sentence of L_{simple} that results from putting together x with ‘or’ and y has the property P.

- ‘Socrates is a philosopher’ has property P.

  ⋮

- For all sentences x of L_{simple}: if x has the property P, then also the result of putting together the logical symbol ‘it is not the case that’ with x has property P.
- For all sentences x of L_{simple}, for all sentences y of L_{simple}: if x has the property P and also y has the property P, then the sentence of L_{simple} that results from putting together x with ‘and’ and y has the property P.
- For all sentences x of L_{simple}, for all sentences y of L_{simple}: if x has the property P and also y has the property P, then the sentence of L_{simple} that results from putting together x with ‘or’ and y has the property P.

From this, by *complete induction over sentences in* L_{simple}, it follows:

- All sentences of L_{simple} have property P.

Theorem:

For all sentences x of L_{simple}:

the result of putting together a quotation mark, the sentence x itself, another quotation mark, 'if and only if', and the sentence x again, follows from the definition of truth for L_{simple}.

E.g., the following sentence follows from the definition of truth for L_{simple}:

'$\underbrace{\text{Socrates is a philosopher}}_{x}$' is true if and only if $\underbrace{\text{Socrates is a philosopher}}_{x}$.

Theorem:

For all sentences x of L_{simple}:

x is true or the result of putting together 'it is not the case that' with x is true.

E.g.:

'$\underbrace{\text{Socrates is a philosopher}}_{x}$' is true or

'it is not the case that $\underbrace{\text{Socrates is a philosopher}}_{x}$' is true.

Proof of second theorem (sketch):

- Property P:

  x is true or the result of putting together ‘it is not the case that’ with x is true.

By logic:

- Socrates is a philosopher or it is not the case that Socrates is a philosopher.

By our definition of truth for L_{simple}:

- 'Socrates is a philosopher' is true if and only if Socrates is a philosopher.
- 'It is not the case that Socrates is a philosopher' is true if and only if it is not the case that Socrates is a philosopher.

From this it follows:

- 'Socrates is a philosopher' is true or 'It is not the case that Socrates is a philosopher' is true.

That is:

- 'Socrates is a philosopher' does have property P. ✓

Next we want to prove:

- For all sentences x of L_{simple}:

  if x has the property P, then also the result of putting together 'it is not the case that' with x has property P.

Next we want to prove:

- For all sentences x of L_{simple}:

  if x has the property P, then also $\underbrace{\neg x}_{\text{negation of } x}$ has property P.

Assume x has property P: (i) x is true or (ii) $\neg x$ is true.

Assume the second to be the case: (ii) $\neg x$ is true.

By logic:

- $\neg x$ is true or $\neg\neg x$ is true.

  $\neg x$ has property P. ✓

Assume x has property P: (i) x is true or (ii) $\neg x$ is true.

Assume the other possible case: (i) x is true.

By logic:

- x is not not true.
- It is not the case that it is not the case that x is true.
- It is not the case that x is not true.

By our definition of truth for L_{simple}:

- x is not true if and only if $\neg x$ is true.

By logic:

- It is not the case that $\neg x$ is true.

[CONTINUED]

We had:

- It is not the case that $\neg x$ is true.

By our definition of truth for L_{simple}:

- It is not the case that x is true if and only if $\neg x$ is true.

By logic:

- $\neg\neg x$ is true.

By logic again:

- $\neg x$ is true or $\neg\neg x$ is true.

  $\neg x$ has property P. ✓

From this, by *complete induction over sentences in* L_{simple}, it follows:

- All sentences of L_{simple} have property P.

That is:

- For all sentences x of L_{simple}: x is true or $\neg x$ is true. ✓

(∗) The sentence in Lecture 2 that is introduced by a star symbol is not true.

(P1) The sentence in Lecture 2 that is introduced by a star symbol is identical to the sentence
‘The sentence in Lecture 2 that is introduced by a star symbol is not true’.

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(P2) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

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the sentence in Lecture 2 that is introduced by a star symbol is not true.

(C) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true
if and only if
'The sentence in Lecture 2 that is introduced by a star symbol is not true' is not true.

(P1) The sentence in Lecture 2 that is introduced by a star symbol is identical to the sentence
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(P2) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

Premise 1 entails:
'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is true.

(P1) The sentence in Lecture 2 that is introduced by a star symbol is identical to the sentence
'The sentence in Lecture 2 that is introduced by a star symbol is not true'.

(P2) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

Premise 1 entails:
'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is true.

Premise 2 says:
'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

(P1) The sentence in Lecture 2 that is introduced by a star symbol is identical to the sentence
'The sentence in Lecture 2 that is introduced by a star symbol is not true'.

(P2) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

The sentence in Lecture 2 that is introduced by a star symbol is true
if and only if
The sentence in Lecture 2 that is introduced by a star symbol is not true.

Which we can rewrite, if we want to:

(C) 'The sentence in Lecture 2 that is introduced by a star symbol is not true' is true
if and only if
'The sentence in Lecture 2 that is introduced by a star symbol is not true' is not true.

(∗) The sentence in Lecture 2 that is introduced by a star symbol is not true.

(+) The sentence in Lecture 2 that is introduced by a plus sign consists of 66 signs.

(P2) ‘The sentence in Lecture 2 that is introduced by a star symbol is not true’ is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

(T) ‘A’ is true if and only if A.

- The object language: the language *for* which one defines truth.
- The metalanguage: the language *in* which one defines truth for the object language.

(P2) ‘The sentence in Lecture 2 that is introduced by a star symbol is not true’ is true if and only if
the sentence in Lecture 2 that is introduced by a star symbol is not true.

‘Plato is a teacher of Aristotle’ is a sentence of L_{simple}.

‘ ‘Plato is a teacher of Aristotle’ is true’ is not a sentence of L_{simple}.

We *can* derive from our definition of truth for L_{simple}:

- ‘Plato is a teacher of Aristotle’ is true if and only if Plato is a teacher of Aristotle.

We *cannot* derive from our definition of truth for L_{simple}:

- ‘ ‘Plato is a teacher of Aristotle’ is true’ is true if and only if ‘Plato is a teacher of Aristotle’ is true.

L_{simple}: object language.

L_{simple}: object language.

L_1: metalanguage of L_{simple} (includes 'true' for L_{simple}).

L_{simple}: object language.

L_1: metalanguage of L_{simple} (includes 'true' for L_{simple}).

L_2: metametalanguage of L_{simple} (includes 'true_1' for L_1).

L_{simple}: object language.

L_1: metalanguage of L_{simple} (includes 'true' for L_{simple}).

L_2: metametalanguage of L_{simple} (includes 'true_1' for L_1).

$\vdots$

- ‘Plato is a teacher of Aristotle’ is true if and only if Plato is a teacher of Aristotle

is derivable from the definition of truth for L_{simple} in the metalanguage L_1.

- ‘‘Plato is a teacher of Aristotle’ is true’ is true_1 if and only if ‘Plato is a teacher of Aristotle’ is true

is derivable from the definition of truth for L_1 in the metametalanguage L_2.

- ‘‘‘Plato is a teacher of Aristotle’ is true’ is true_1’ is true_2 if and only if ‘‘Plato is a teacher of Aristotle’ is true’ is true_1

is derivable from the definition of truth for L_2 in the metametametalanguage L_3.

And so on.