Lecture 1

Course outline & Introduction

AEM-ADV12 Hydrodynamic stability

Dr Yongyun Hwang

Lecture outline

1. Course outline
2. Hydrodynamic stability
3. Navier-Stokes equation as a dynamical system

Lectures

Lecturer

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Office hour

E-mail me to arrange an appointment

Feel free to come !!!

Lecture composition

11 lectures

1 tutorial (last year exam)

1 exam to test your knowledge (not to test problem solving technique)

Lecture outline

1. Course outline

Further readings (Books)

1. **Introduction to hydrodynamic stability (2002) P.G. Drazin,** Cambridge University Press – an introductory text for classical stability analysis Ch. 3.
2. **Stability of fluid motions (1976) D.D. Joseph**, Springer – focused on nonlinear perspective of stability analysis focused on energy stability method Ch 6.
3. **Hydrodynamic stability (1982) P.G. Drazin and W.R. Reid**, Cambridge University Press – a comprehensive bible for classical stability analysis (1980's research monograph) Ch 4 – Ch 6.
4. **Hydrodynamic instabilities in open flows** (1998) **P. Huerre and M Rossi**, (Chapter 2 in Hydrodynamics and nonlinear instabilities, edited by C. Godreche and P Manneville, Cambridge University Press) – A high level text with focus on spatio-temporal development of instabilities (Absolute/convective and local/global instabilities) Ch 9. – Ch 10
5. **Stability and transition in shear flow (2001) P.J. Schmid & D.S. Henningson**, Springer – a comprehensive research monograph up to 2000 with focus on non-modal stability analysis. Ch 7. – Ch 8.

Objectives of the course

Objectives

: to deliver basic theoretical tools of hydrodynamic stability for advanced study

1. Fundamental of hydrodynamic stability

a) Basic methodologies of hydrodynamic stability theory

b) Main results from 1850 to 1970

c) Will cover only 'linear' stability analysis.

2. Introduction to modern hydrodynamic stability theories

a) Two major breakthroughs made in 1980s and 1990s

b) Case studies of transition in shear flows

: Boundary layer and cylinder wake

c) Case studies of transition control

Course structure

PART II – Introduction to modern hydrodynamic stability

Lecture 7-8. Non-modal stability analysis

- Initial value problem of linearised equation
- Algebraic instability, transient growth and non-normality
- Lift-up effect and Orr mechanism

Lecture 9-10. Spatio-temporal development of instabilities

- Temporal vs spatial stability theories
- Absolute and convective instabilities
- Briggs-Bers criterion
- Physical examples

Lecture 11. Transition in shear flows: case studies

- Case studies (boundary layer and cylinder wake)
- Is hydrodynamic stability relevant to turbulence?

Course structure

PART I – Fundamentals of hydrodynamic stability

Lecture 1. Definition of stability
- Why do we study hydrodynamic stability?
- Mathematical definition of stability

Lecture 2-3. Basic dynamical system theory
- Nonlinear dynamical system
- Phase portrait and linear stability
- Introductory bifurcation theory

Lecture 4-6. Linear stability of parallel shear flows
- Rayleigh equation for inviscid flow
- Squire's transformation
- Rayleigh inflectional point theorem
- Shear layer instabilities (Broken profile analysis)
- Orr-Sommerfeld-Squire equation for viscous flow
- Eigenspectra and neutral stability curves
- Spatial stability analysis and vibrating ribbon problem

Lecture outline

2. **Hydrodynamic stability**

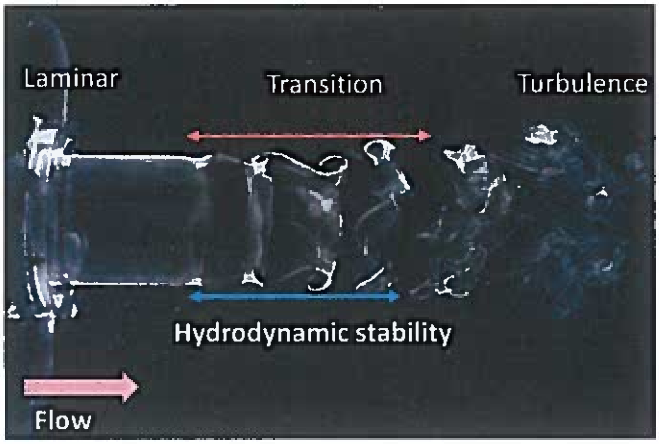
Hydrodynamic stability

What is hydrodynamic stability?

A branch of fluid dynamics investigating transition to turbulence

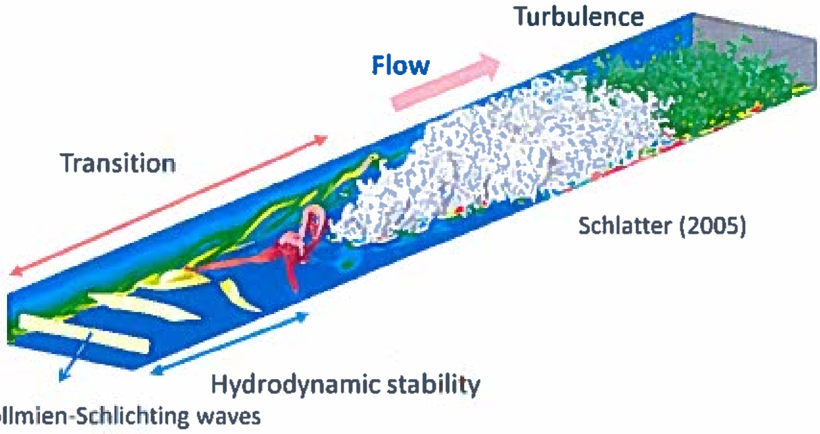
Transition in shear flows

Example 1 – Axisymmetric jet



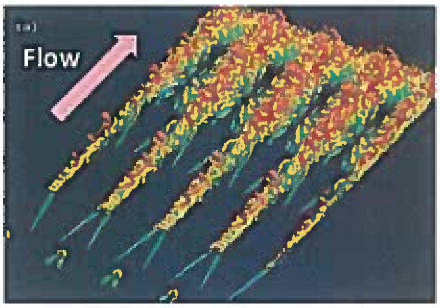
Transition in shear flows

Example 2 – Spatially developing turbulent channel

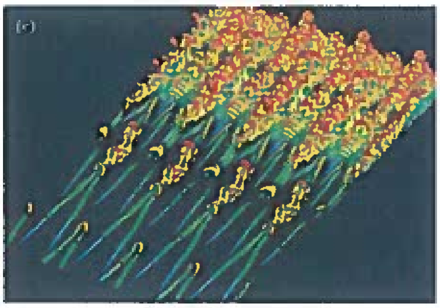


Transition in shear flows

Example 4 – Transition in boundary layer



K-type transition



H-type transition

Sayadi et al. (2012)

Transition is often quite **sensitive to external noise and disturbances,** and can lead to a dramatic change in the flow field.

Transition in shear flows

Example 3 – bifurcating axisymmetric jet

Unforced jet

Jet with a helical forcing at the exit

Reynolds et al. (2001)

Transition is often quite **sensitive to external noise and disturbances**, and can lead to a dramatic change in the flow field.

Hydrodynamic stability

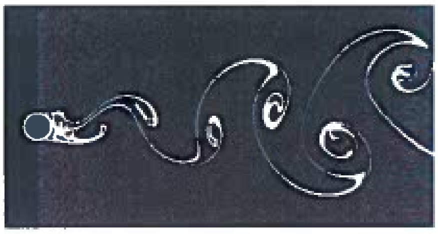
What is hydrodynamic stability?

A branch of fluid dynamics investigating **transition to turbulence**

What you can actually do with hydrodynamic stability are to study:

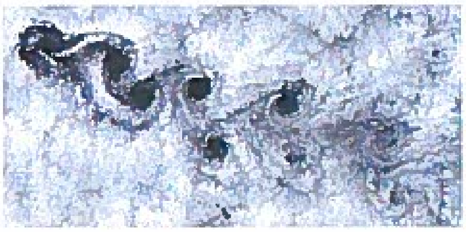
1. Transition to turbulence (the goal of this course)
2. Control of transition and turbulence
3. Coherent structure dynamics in turbulent flows

Example 5 – Karman vortex shedding



Laminar vortex shedding
at low Reynolds number

Turbulent vortex shedding
at high Reynolds number



The **structures** observed **in transition** often **persist** even in **turbulent flows**

3. Navier-Stokes equation as a dynamical system

Dynamical system

Definition: Nonlinear dynamical system

A general nonlinear dynamical system is defined as

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t), \qquad \mathbf{x}(t=0) = \mathbf{x}_0$$

where $\mathbf{x} = [x_1, x_2, x_3, \ldots, x_n]^T$ with

$$\mathbf{f} = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), \ldots, f_n(\mathbf{x})]^T$$

Example 2: Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

$u(x,t)$

$= [\, u(x_1,t) \quad u(x_2,t) \quad \cdots \quad u(x_n,t) \,]$

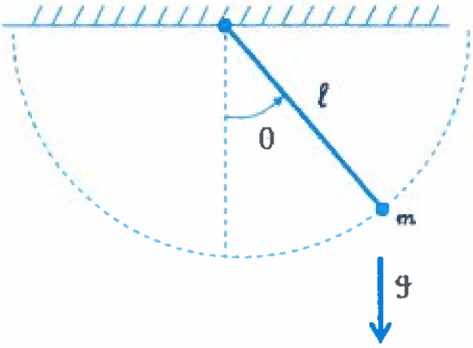
total number of grid points

Let $\mathbf{x} = \begin{bmatrix} \mathbf{u}^T & p \end{bmatrix}^T$, then

$$\frac{\partial}{\partial t}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} 1/\mathrm{Re}\,\nabla^2 & -\nabla \\ \nabla\cdot & 0 \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} + \begin{bmatrix} -(\mathbf{u}\cdot\nabla)\mathbf{u} \\ 0 \end{bmatrix}$$

We discretise the system with e.g. FVM or FEM, then it becomes a finite dimensional dynamical system. In fact, the Navier-Stokes equation is an infinite dimensional dynamical system.

Example: Nonlinear pendulum



$$\ddot{\theta} + \omega \sin \theta = 0$$

where

$$\omega = \sqrt{g/l}$$

Let $\omega = 1$. and

$x = \theta$. $y = \dot{\theta}$

$\Rightarrow \dot{x} = \dot{\theta} = y$

$\dot{y} = \ddot{\theta} = -\sin\theta = -\sin x$

$$\underbrace{\frac{d}{dt}\underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\underline{x} \in \mathbb{R}^2} = \underbrace{\begin{bmatrix} y \\ -\sin x \end{bmatrix}}_{\underline{f}(\underline{x})}}_{\frac{d\underline{x}}{dt} = f(\underline{x})}$$

Summary

1. Course outline
2. Hydrodynamic stability
3. Navier-Stokes equation as a dynamical system