Lecture 10

Spatio-temporal evolution of instabilities II

AE209 Hydrodynamic stability
Dr Yongyun Hwang

1. Application to Ginzburg-Landau equation
2. Application to wake
3. Physical implications: oscillator vs amplifier flows

Lecture outline

1. **Application to Ginzburg-Landau equation**
2. Application to wake
3. Physical implications: oscillator vs amplifier flows

Application to Ginzburg-Landau equation

Complex linear Ginzburg-Landau equation

$\frac{\partial}{\partial t} \to -i\omega$, $\frac{\partial}{\partial x} \to ik$.

From dispersion relation, $D(k,\omega)=0$:

$$\omega(k) = Uk - c_d k^2 + i\underbrace{(\mu - k^2)}_{\omega_i}$$

Linear stability: Calculate the maximum growth rate ⇒ Check all real k.

$$\omega(k_{max}) = i\omega_{i,max} = i\mu \text{ with } k_{max} = 0$$

if $\mu > 0$, ⇒ linearly unstable.

Absolute instability: Calculate the absolute growth rate

$$\omega_0 = \omega(k_0) = \underbrace{\frac{c_d U^2}{4(1+c_d^2)}}_{\omega_{0,r}} + i\underbrace{\left[\mu - \frac{U^2}{4(1+c_d^2)}\right]}_{\omega_{0,i}} \text{ with } k_0 = \frac{U}{2(c_d+i)}$$

① Calculate k_0: $\frac{\partial \omega(k)}{\partial k} = U - 2C_d k - 2ik = 0$.

⇒ $k_0 = \frac{U}{2(C_d+i)}$

② Check $\omega_{0,i}$: $\omega_{0,i} = \mu - \frac{U^2}{4(1+C_d^2)}$

---

if $\mu > 0$, ⇒ Linearly unstable.

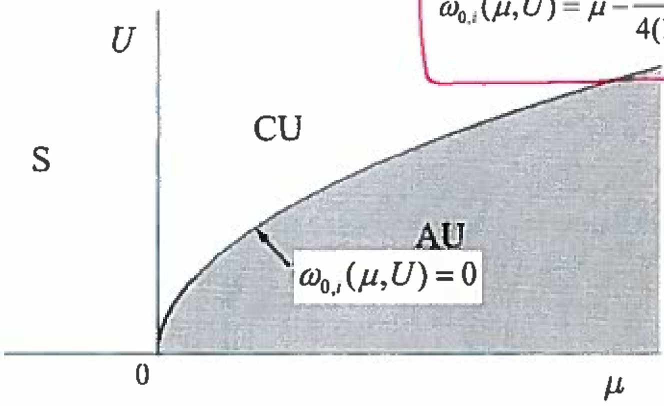
if $\mu > 0$. $\mu < \frac{U^2}{4(1+C_d)^2}$ ⇒ $\omega_{0,i} < 0$. ⇒ Convectively unstable

if $\mu > 0$ $\mu > \frac{U^2}{4(1+C_d)^2}$ ⇒ $\omega_{0,i} > 0$ ⇒ Absolutely unstable.

Application to Ginzburg-Landau equation

Absolute and convective instabilities in parametric space

$$\boxed{\omega_{0,i}(\mu,U)=\mu-\frac{U^2}{4(1+c_d^2)} = 0}$$



AU vs CU

: Competition between advection and instability

Remark

This description is only important for the system with mean advection.

2. Application to bluff-body wake

Application to parallel wake

Family of wake profiles

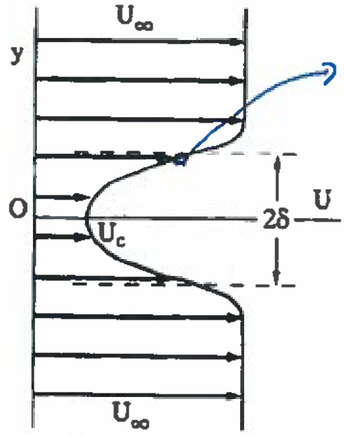
$$U(y) = U_\infty + (U_\infty - U_c)U_1(\frac{y}{\delta}; N)$$

where

$$U_1(\xi; N) = \left[1 + \sinh^{2N}\left\{\xi \sinh^{-1}(1)\right\}\right]^{-1}$$

with

$$\mathrm{Re} = \frac{\overline{U}\delta}{\nu}$$

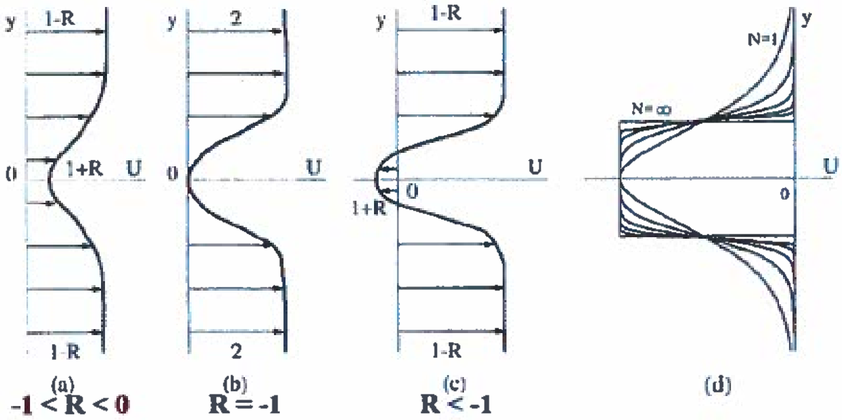


$\overline{U} = \frac{U_\infty + U_c}{2}$

Monkewitz (1988)

Application to parallel wake

Family of wake profiles



Velocity ratio $R = \dfrac{U_c - U_\infty}{U_c + U_\infty}$

N: stiffness

Monkewitz (1988)

Application to parallel wake

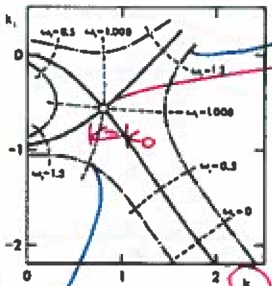
Dispersion relation (obtained by solving the Orr-Sommerfeld equation)

$$\left[(-i\omega+ikU)(D^2-k^2)-ikD^2U-\frac{1}{\mathrm{Re}}(D^2-k^2)^2\right]\tilde{v}=0$$

with the **saddle point** behaviour:

$R=-1$ $\quad N=2$ $\quad \mathrm{Re}=11.3$ contour of ω_i (solid)

k_i



$\dfrac{\partial\omega}{\partial k}=0$

$\omega_0 = 1.008 + 0i$

Monkewitz (1988)

$$\omega-\omega_0 \sim \frac{1}{2}\frac{\partial^2\omega}{\partial k^2}(k_0)(k-k_0)^2$$

$=k_r$

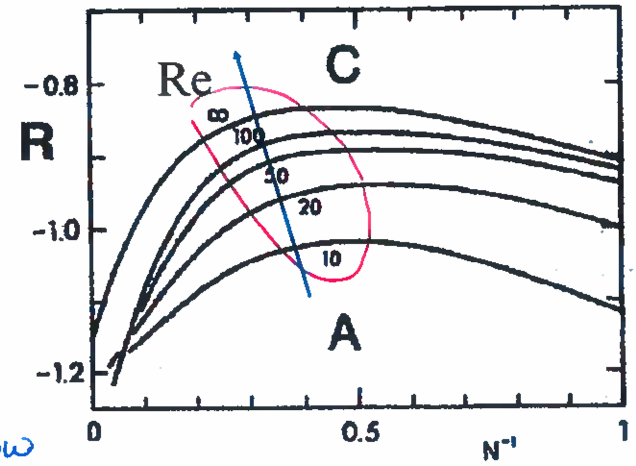
——, curve $\omega_i = 0$; — · — · —, $\omega_i = 0.25$; — ·· — ·· —, $\omega_i = -0.25$

contour of ω_r.

Application to parallel wake

Effect of velocity ratio, stiffness and Reynolds number

Back flow



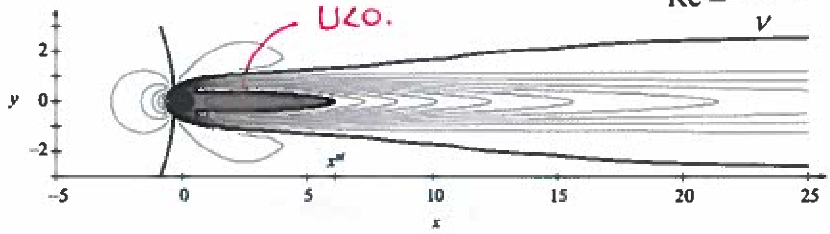
Monkewitz (1988)

AI appears when Re ~ O(10) if you have a backflow.

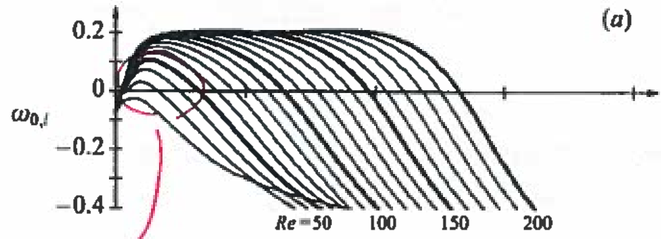
Application to (non-parallel) cylinder wake

Base flow profile at Re=100

$$\mathrm{Re} = \frac{U_\infty d}{\nu}$$



Absolute growth rate of velocity profile at each streamwise location



Pier (2002)

$Re_c \simeq 47$
: Critical Re for vortex Shedding

AU region appears in the near-wake region around $Re \simeq 25$.

⇒ In non-parallel flow, the region of AU is necessary for the onset of Global instability. Vortex Shedding

Emergence of vortex shedding

$5 < \mathrm{Re} < 25$	Some regions are convectively unstable
$25 < \mathrm{Re} < 47$	Some regions are absolutely unstable
$\mathrm{Re} \approx 47$	Strong local absolute instability leads to a **global instability** in the form of $\mathbf{u}'(x,y,t) = \hat{\mathbf{u}}(x,y)e^{-i\omega_G t}$ with $\omega_{G,i} > 0$

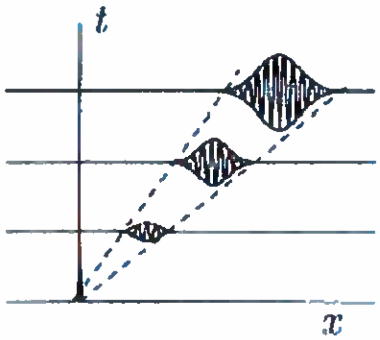
Remark

Local absolute instability is a **necessary condition** for the onset of a **global instability of a fully non-parallel open flow**

Lecture outline

1. Application to Ginzburg-Landau equation
2. Application to bluff-body wake
3. **Physical implications: oscillator vs amplifier flows**

Remarks



Convective instability

Ex) Boundary layer, jet with constant density.

$U_U > 0$

$U_L > 0$

Co-flowing mixing layer.

1. Reference control volume returns to the original state after the impulse moves away downstream.

⇒ Similar to transient growth in the domain of interest.

Convective

non -normalit

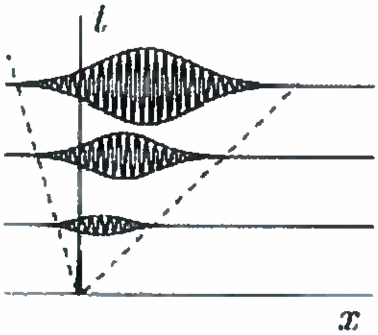
2. **Spatial stability analysis** becomes **meaningful** in this situation.

3. Instability dynamics is **driven by upstream noise**

Noise amplifier flow

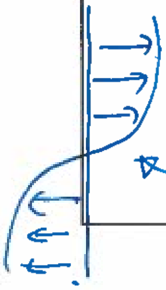
Absolute instability

Remarks



Absolute instability

Ex) Wake, Hot jet.
Counter-flowing mixing layer.

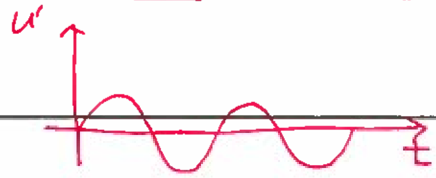


1. Reference control volume never return to the original state.

2. **Spatial stability analysis** becomes **meaningless** in this situation. : less sensitive to noise.

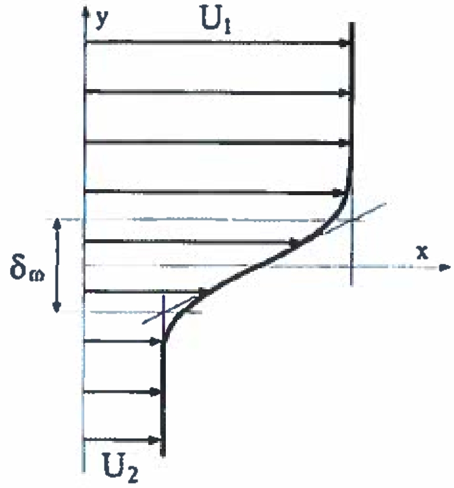
3. Instability dynamics is **intrinsically** driven by the given system and often results in a **nonlinear oscillation** with a **distinct frequency.**

↳ Oscillator flow.



Example: Mixing layer

Hyperbolic tangent mixing layer



Base flow profile

$$U(y) = \overline{U} + \frac{\Delta U}{2} \tanh\left(\frac{2y}{\delta_\omega}\right)$$

Velocity ratio

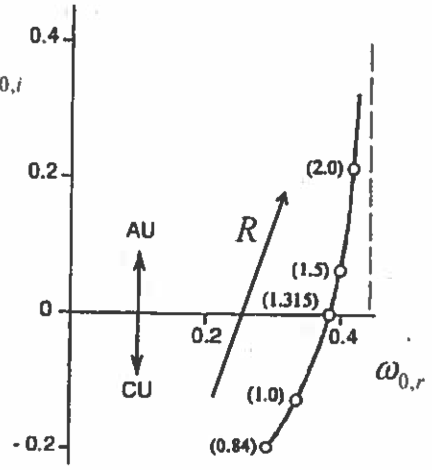
$$\boxed{R = \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2\overline{U}}}$$

$$\delta_\omega = \frac{\Delta U}{(dU/dy)_{\max}}$$

Huerre & Monkewitz (1985)

Example: Mixing layer

Theory: transition from convective to absolute instability with R



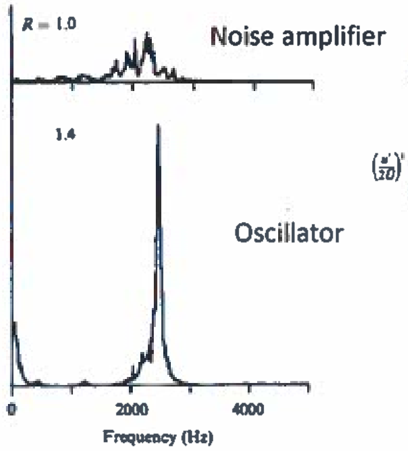
Velocity ratio

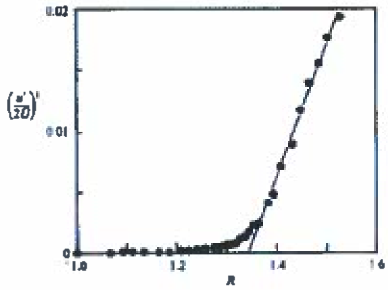
$$R = \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2\overline{U}}$$

Huerre & Monkewitz (1985)

Example: Mixing layer

Experiment: shift from noise amplifier to oscillator with R





Strykowski & Niccum (1991)

Lecture outline

1. Application to Ginzburg-Landau equation
2. Application to wake
3. Physical implications: oscillator vs amplifier flows