Lecture 3

Basic dynamical system theory II

AEM-ADV12 Hydrodynamic stability

Dr Yongyun Hwang

Lecture outline

1. Transcritical bifurcation
2. Saddle-node bifurcation
3. Pitchfork bifurcation
4. Hopf bifurcation

Lecture outline

1. **Transcritical bifurcation**
2. Saddle-node bifurcation
3. Pitchfork bifurcation
4. Hopf bifurcation

Bifurcation

Definition: Bifurcation

Bifurcation refers to a **sudden topological change** of given nonlinear dynamical system taking place **when a control parameter changes smoothly**.

Example: Transcritical bifurcation

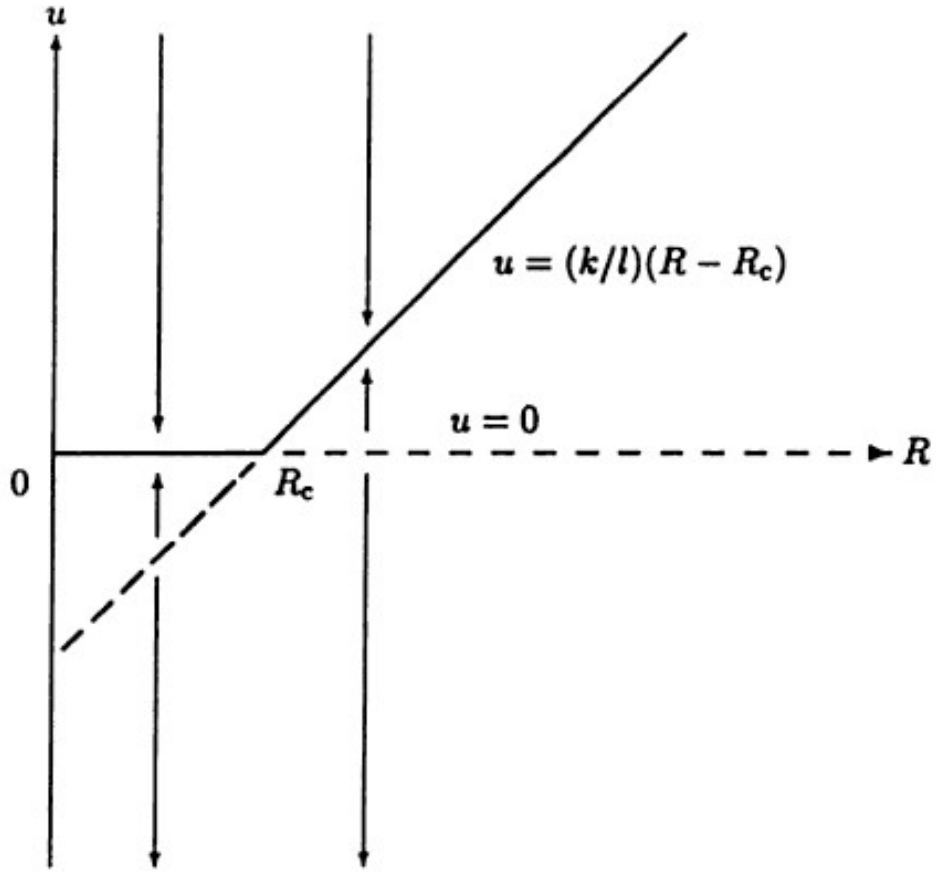
Find the bifurcation diagram of a model given by

$$\frac{du}{dt} = k(R - R_c)u - lu^2$$

where k, l are constants and R is the control parameter.

Step 1) Find equilibrium points

Step 2) Examine linear stability of the equlibrium points



Bifurcation diagram of transcritical bifurcation

Lecture outline

1. Transcritical bifurcation
2. **Saddle-node bifurcation**
3. Pitchfork bifurcation
4. Hopf bifurcation

Example: Saddle-node bifurcation

Find the bifurcation diagram of a model of given by

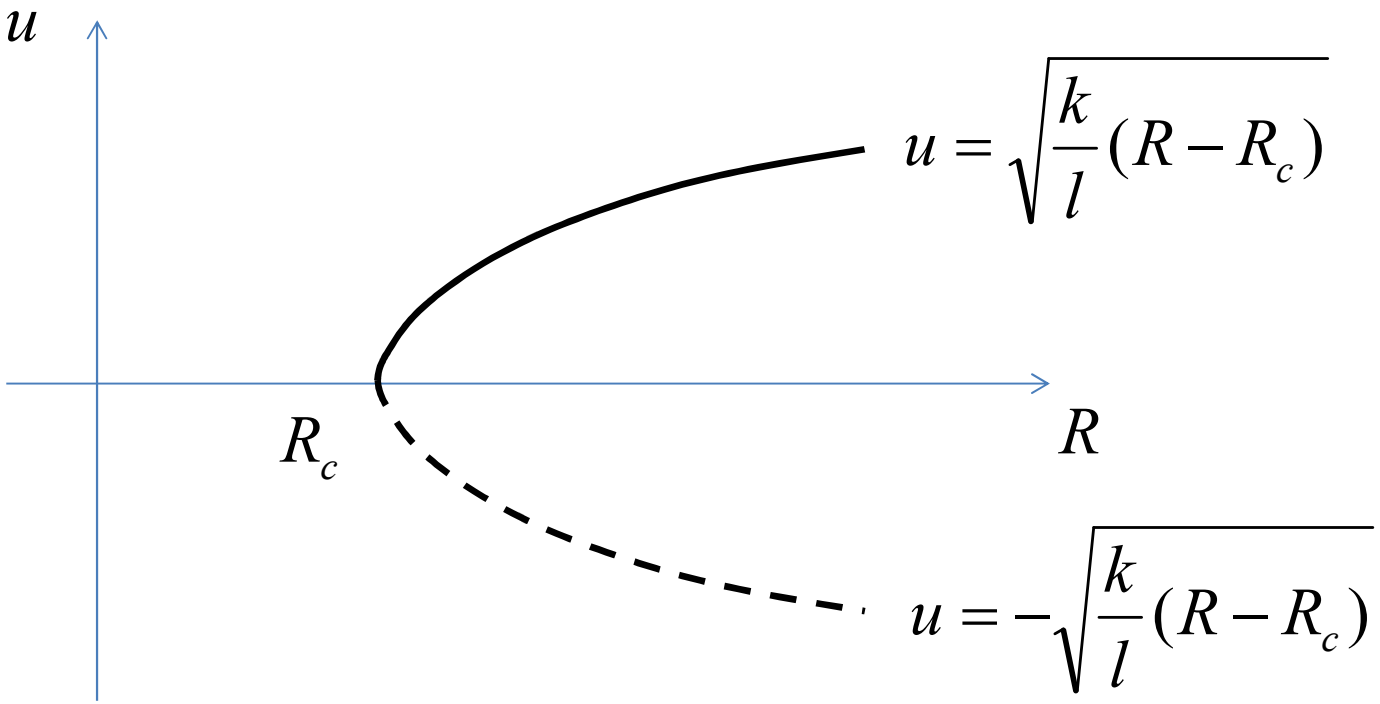
$$\frac{du}{dt} = k(R - R_c) - lu^2$$

where k, l are real constants and R is the control parameter.

Step 1) Find equilibrium points

Step 2) Examine linear stability of the equlibrium points

Example: Saddle-node bifurcation



Bifurcation diagram of Saddle-node bifurcation

Lecture outline

1. Transcritical bifurcation
2. Saddle-node bifurcation
3. **Pitchfork bifurcation**
4. Hopf bifurcation

Example: Pitchfork bifurcation

Find the bifurcation diagram of a model of given by

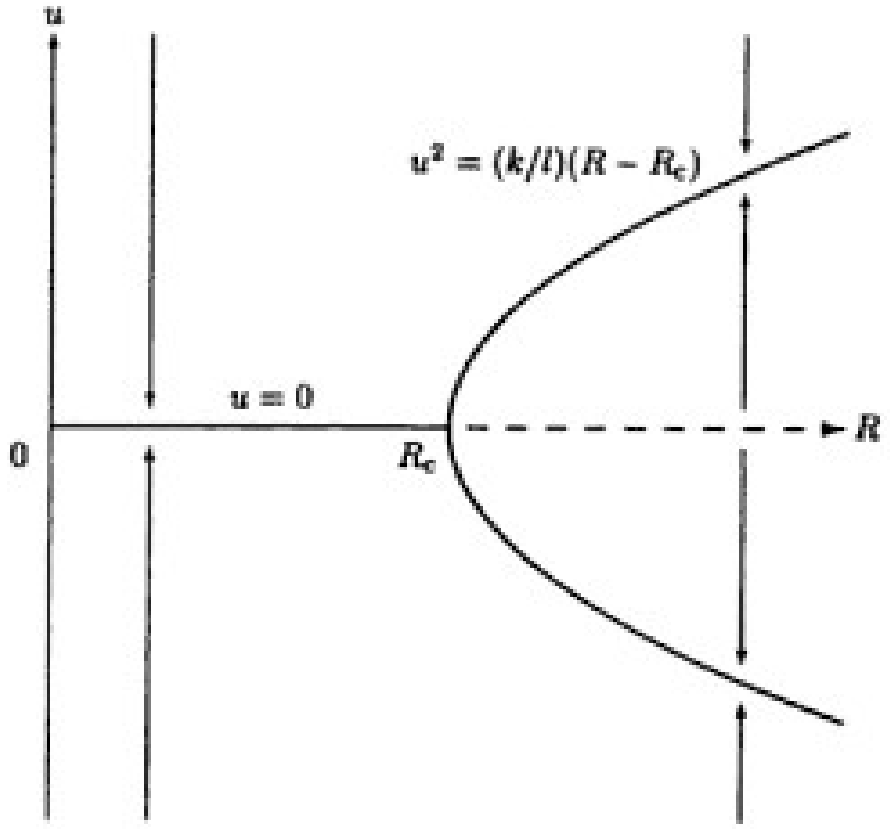
$$\frac{du}{dt} = k(R - R_c)u - lu^3$$

where k, l are real constants and R is the control parameter.

Step 1) Find equilibrium points

Step 2) Examine linear stability of the equlibrium points

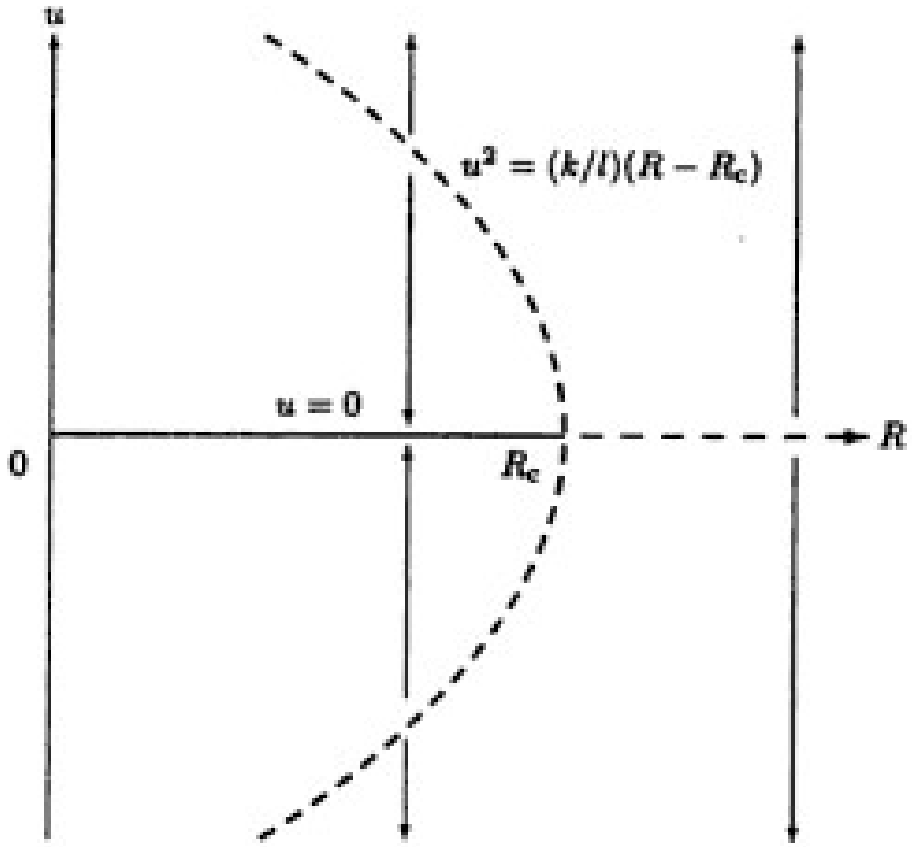
Pitchfork bifurcation



Bifurcation diagram of Pitchfork bifurcation

(supercritical case, i.e. $l > 0$)

Pitchfork bifurcation



Bifurcation diagram of Pitchfork bifurcation

(subcritical case, i.e. $l < 0$)

Pitchfork bifurcation

Flow example: Wake behind a sphere

$$\mathrm{Re}_D = \frac{U_\infty D}{\nu}$$

$$\mathrm{Re}_{D,critical} \approx 210$$

$\mathrm{Re}_D = 100$ → $\mathrm{Re}_D = 250$

Steady axisymmetric **Steady planar symmetric**

Kim & Choi (2001)

Lecture outline

1. Transcritical bifurcation
2. Saddle-node bifurcation
3. Pitchfork bifurcation
4. **Hopf bifurcation**

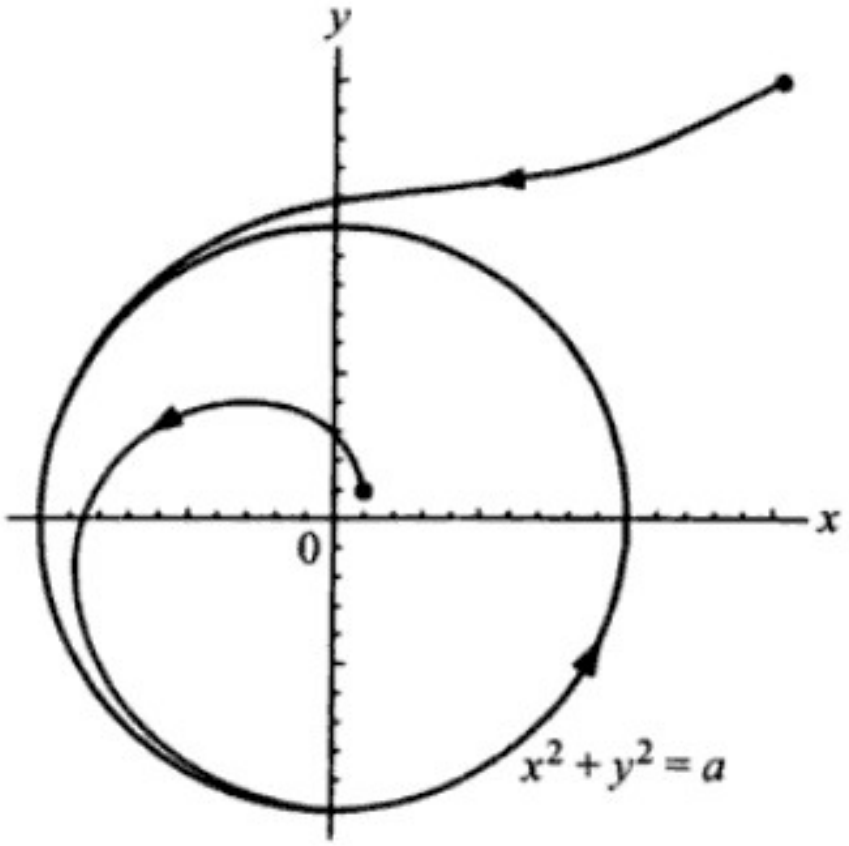
Example 1: Hopf bifurcation

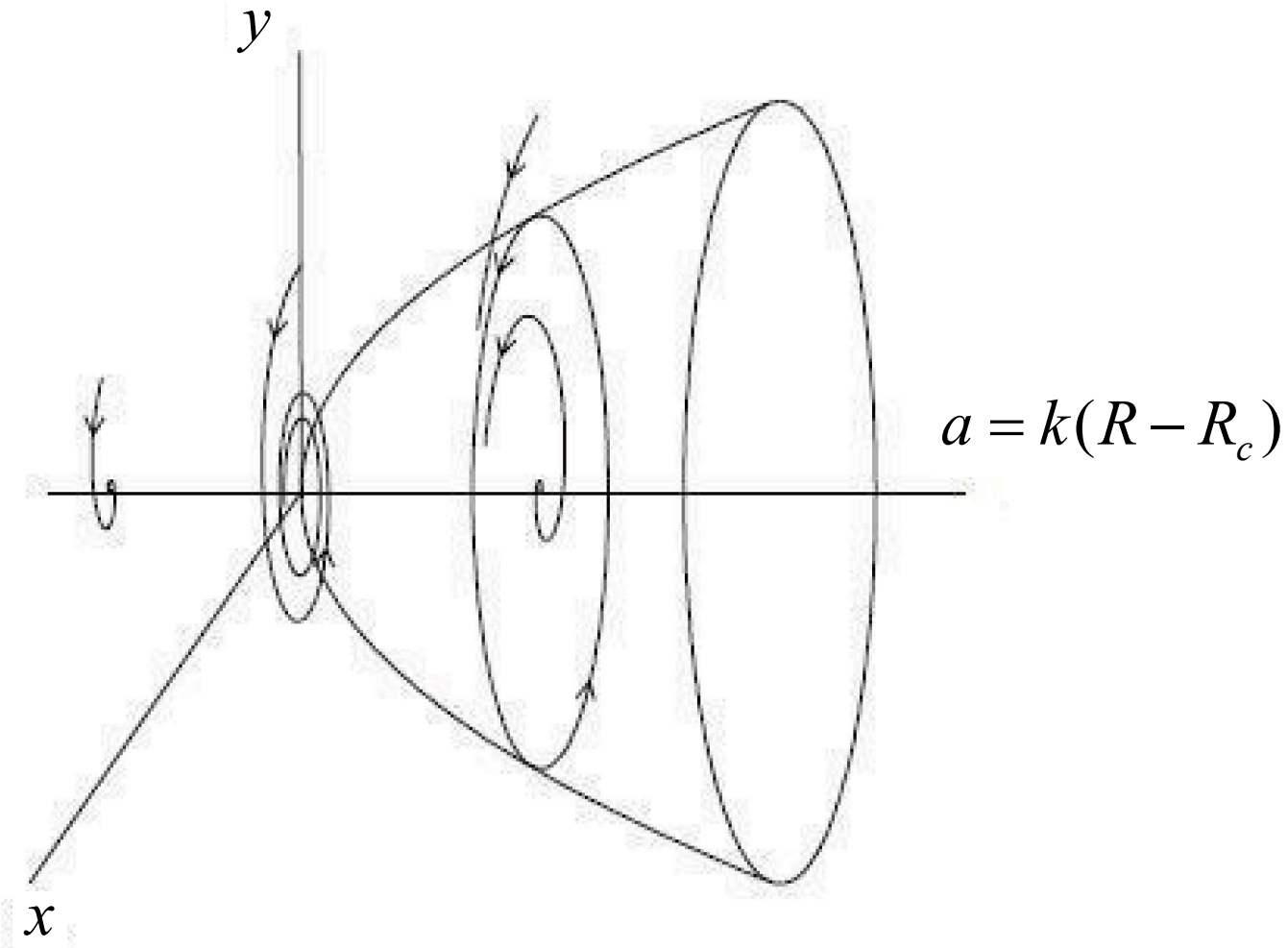
Find the bifurcation diagram a model given by

$$\frac{dx}{dt} = -y + (a - x^2 - y^2)x, \quad \frac{dy}{dt} = x + (a - x^2 - y^2)y,$$

where $a = k(R - R_c)$ and $k > 0$.

Phase portrait for $a > 0$





Bifurcation diagram of
supercritical Hopf bifurcation

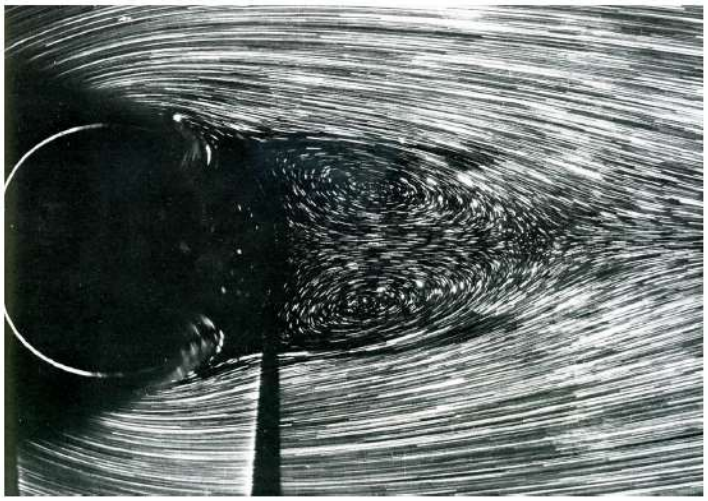
Hopf bifurcation

Flow example: Wake behind a circular cylinder

$$\mathrm{Re}_D = \frac{U_\infty D}{\nu}$$

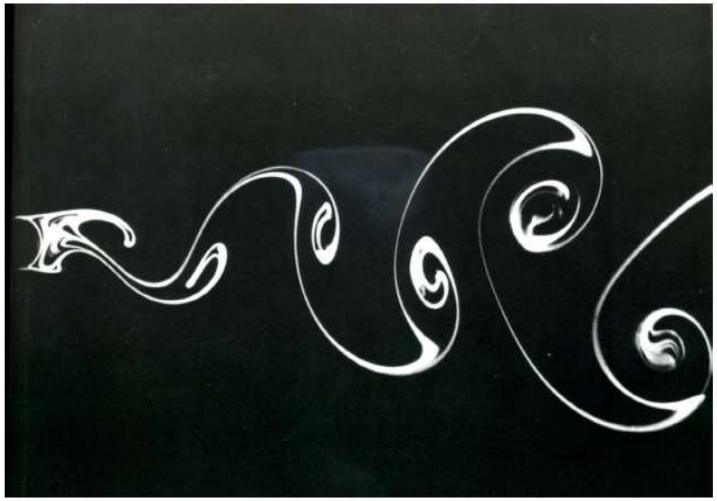
$$\mathrm{Re}_{D,critical} \approx 47$$

$$\mathrm{Re}_D = 27 \quad \rightarrow \quad \mathrm{Re}_D = 140$$



Steady symmetric

Coutanceau & Bouard (1977)



Unsteady time periodic

Taneda (1982)

Summary

1. Transcitical bifurcation
2. Saddle-node bifurcation
3. Pitchfork bifurcation
4. Hopf bifurcation