

Lecture 11

Shear flow transition – case study

AE209 Hydrodynamic stability

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- 1. Transition in boundary layer**
- 2. Transition in flow over a circular cylinder**
- 3. Is transition relevant to turbulence?**

General features on instabilities in boundary layer

1. Boundary layer instability (TS wave) is convectively unstable.

⇒ sensitive to noise.

2. Large transient growth is also possible below critical Reynolds number of TS instability.

3. Primary instabilities (either TS wave or transient growth) undergo secondary instability, eventually leading to turbulence.

↳ Turbulence spots

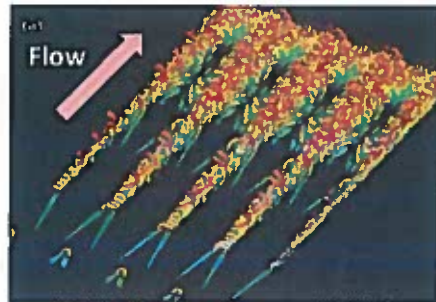
At low noise level

At high noise level

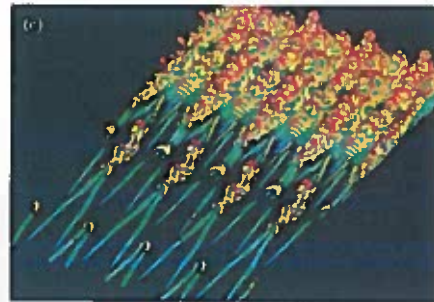
1. Transition in boundary layer

Scenario I : Modal transition via Tollmien-Schlichting wave

Two types of secondary instabilities of TS wave



K-type transition
(Klebanoff 1962)



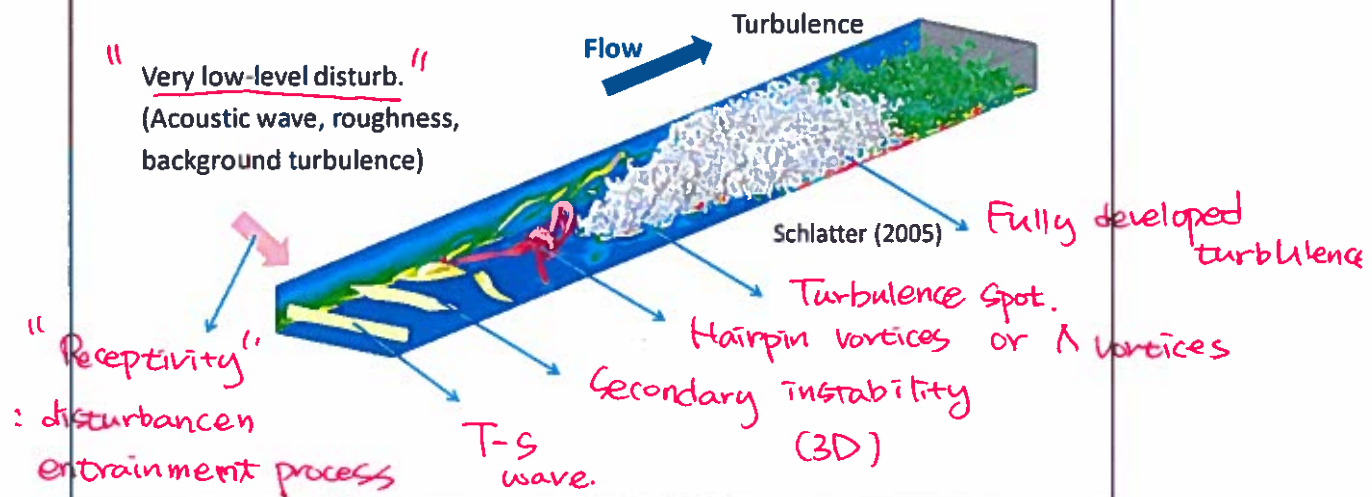
H-type transition
(Herbert 1988)

Theory by
Orszag and Patera
(1982)

DNS by Sayadi et al. (2012)

Scenario I : Modal transition via Tollmien-Schlichting wave

Secondary instabilities of Tollmien-Schlichting wave



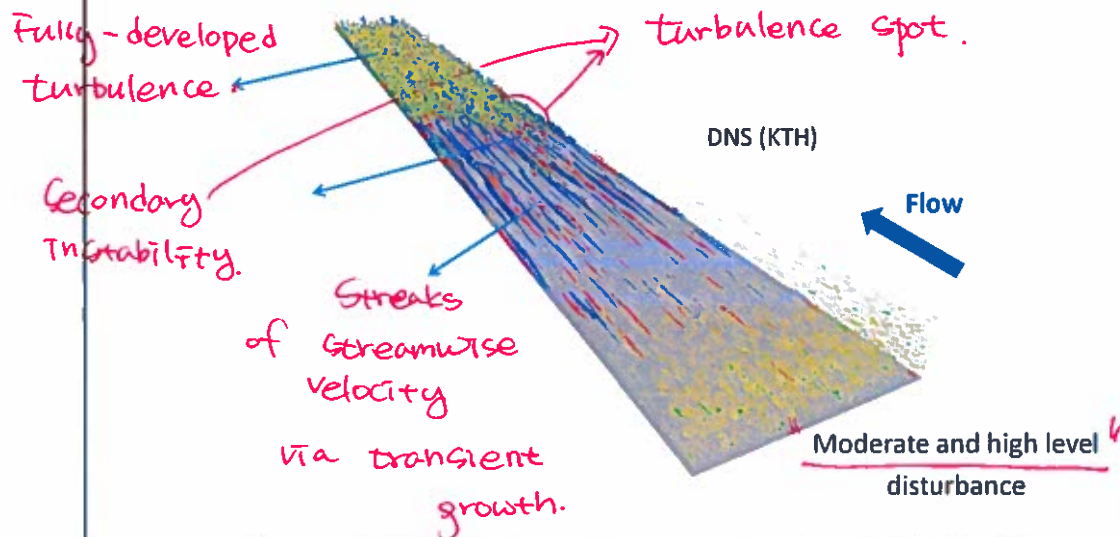
Scenario II : Bypass transition via transient growth

➡ Flow (Top view)

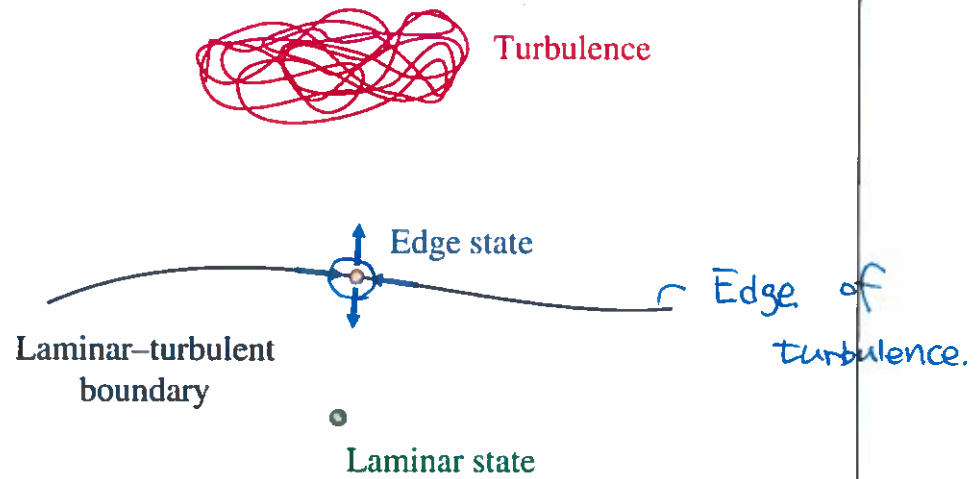


Matsubara & Alfredsson (2001)

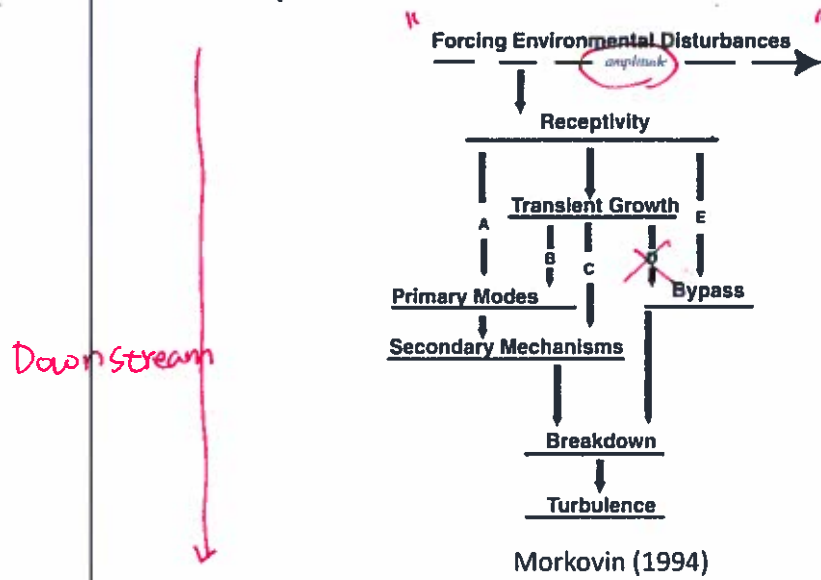
Scenario II : Bypass transition via transient growth



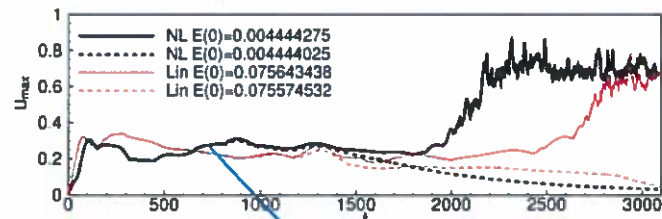
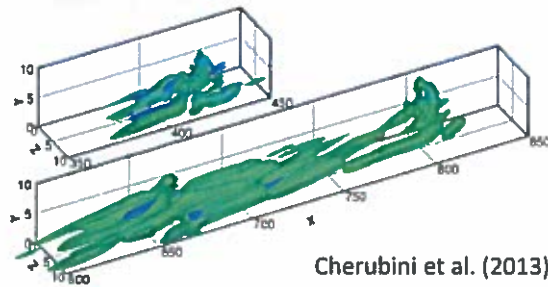
Beyond nonlinearity – Dynamical system approach (since mid 2000)



Summary



Edge state in boundary layer

 $t = 300$ $t = 700$ 

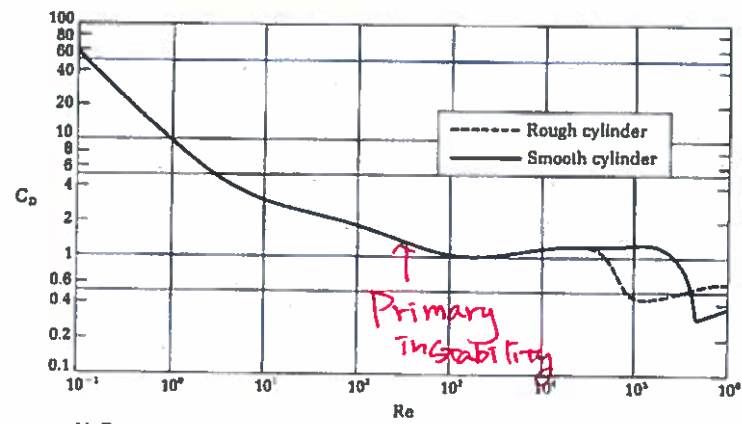
Cherubini et al. (2013)

2. Transition in flow over a circular cylinder

Wake behind a circular cylinder

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Drag coefficient with Re



$$Re = \frac{\rho U_{\infty} D}{\mu}$$

ρ = density of fluid (M/L^3)

U_{∞} = free-stream fluid velocity (L/T)

D = diameter of cylinder (L)

μ = viscosity of fluid ($F \cdot T/L^2$)

Wake behind a circular cylinder

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General features on instabilities in bluff-body wake

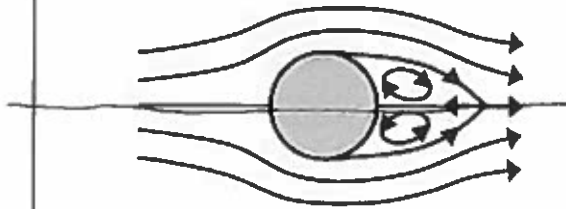
1. Instability (Karman vortex shedding) is **absolutely unstable** and is driven by **inflectional mechanism** (Rayleigh criterion).
2. The role of **transient growth** is **not very important** typically because the critical Reynolds number of instability itself is also quite low.
3. **Spatial development** is **not of main interest**, as the instability process is often dominated by the near-wake region.

→ Insensitive to noise
: For There exists a very well defined critical Re.

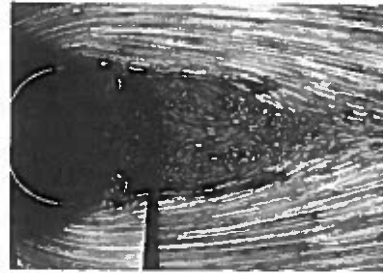
Wake behind a circular cylinder

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Steady symmetric flow ($5 < Re < 47$)

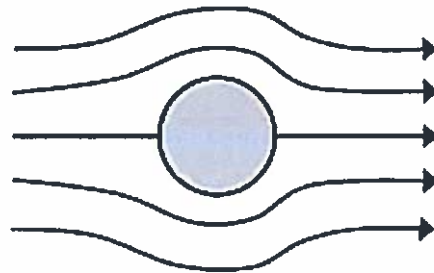


Sketch



Coutanceau & Bouard (1977)

Creeping flow ($0 < Re < 5$): Hele-Shaw flow



Sketch



Schlichting (1979)

Wake behind a circular cylinder

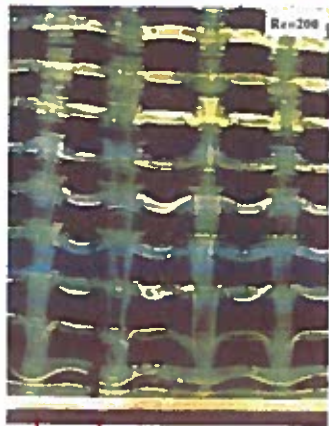
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Onset of three-dimensional Karman vortex shedding ($189 < Re < 300$)

Turbulence appears at far downstream ($300 < Re < 1000$)

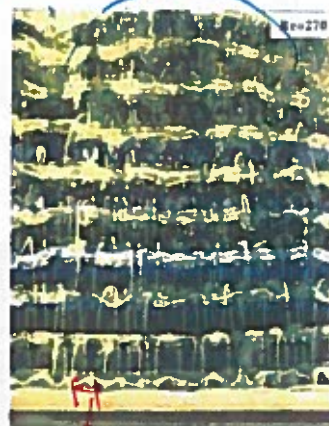
Secondary
Instability
of
vortex
shedding.

Flow ↑



Mode A $Re_D = 200$

4D. (diameter) Williamson (1996)



Mode B $Re_D = 270$

1 ~ 2D.

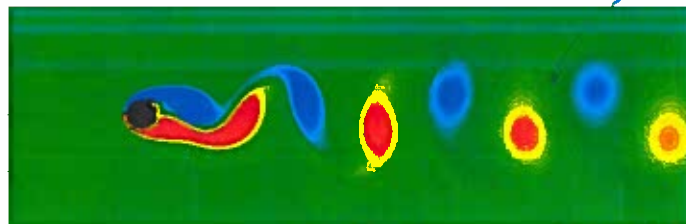
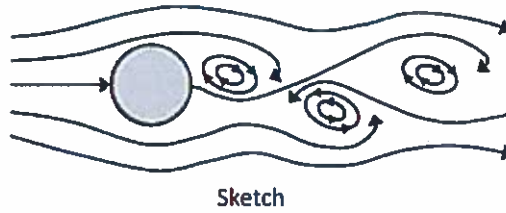
Wake behind a circular cylinder

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Two-dimensional laminar Karman vortex shedding ($47 < Re < 189$)

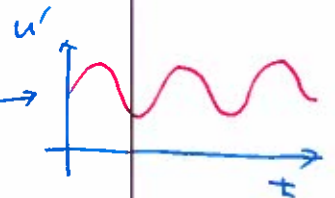
$Re_c \approx 47$

No variation
in spanwise
direction



Hwang (2005)

$Re_D = 100$



"Completely
periodic
in time"

⇒ Supercritical
Hopf bifurcation

Transition in boundary layer ($15000 < Re < 10^6$)

$Re_D = 2.5 \times 10^5$



$Re_D = 5.3 \times 10^5$

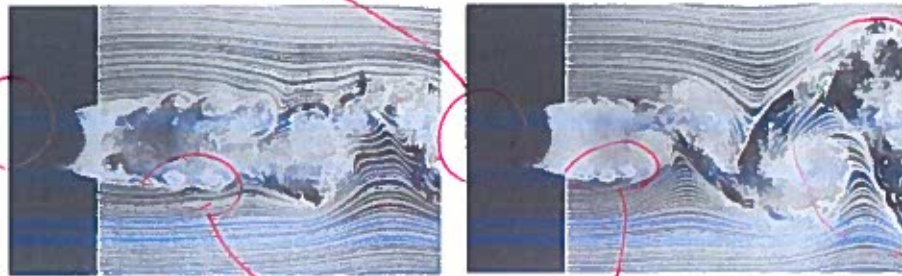


Lehmkuhl et al. (2014)

Wake behind a circular cylinder

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Instability in separating shear layer ($1000 < Re < 15000$)



$Re_D = 7000$

$Re_D = 10000$

Prasad & Williamson (1997)

K-H instability in the separating shear layer.

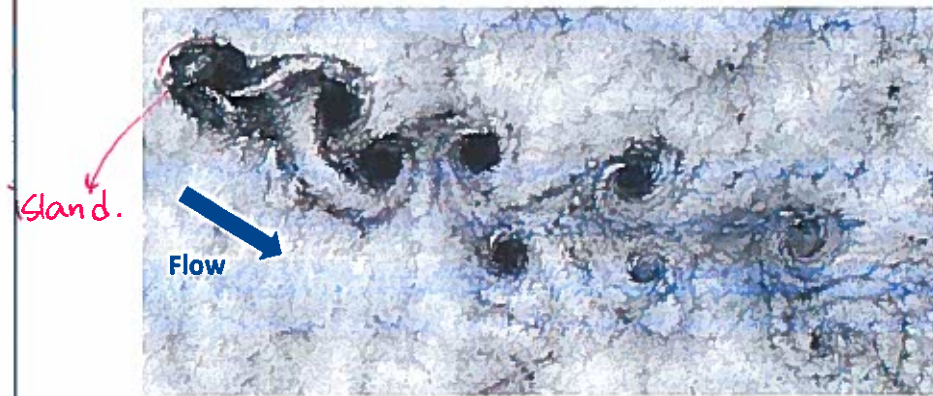
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- 2 Transition in flow over a circular cylinder

3. Is transition relevant to turbulence?

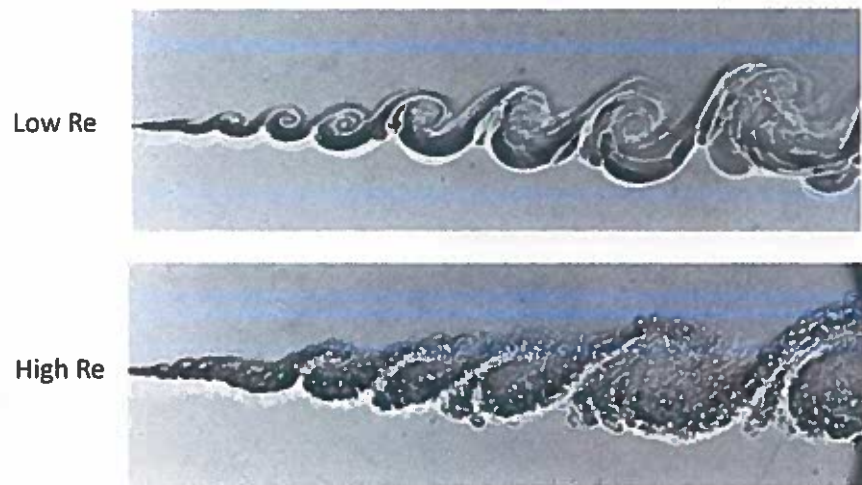
Wake behind a circular cylinder

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Fully turbulent everywhere ($Re > 10^6$)

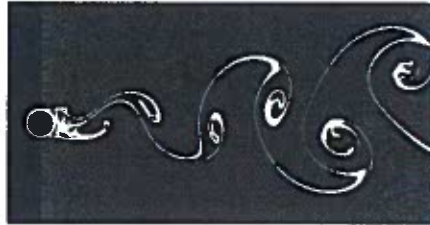


Transitional and turbulent mixing layers



Brown & Roshko (1974)

Laminar and turbulent vortex shedding in bluff body wakes



Low Re

High Re

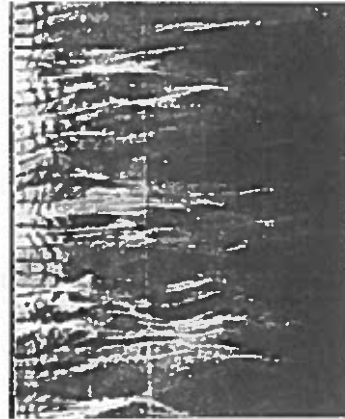


Boundary layers

Streaks in bypass transition



Matsubara & Alfredsson (2001)



Streaks in buffer layer

Kline et al. (1967)

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