Lecture 6

Linear stability of parallel shear flows III

AE209 Hydrodynamic stability
Dr Yongyun Hwang

Lecture outline

1. **Eigenspectra and eigenfunctions**
2. **Neutral stability curve**
3. **Spatial stability analysis and vibrating ribbon problem**

1. **Eigenspectra and eigenfunctions**
2. Neutral stability curve
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Review of Orr-Sommerfeld and Squire equations

Orr-Sommerfeld equation (for wall-normal velocity):

$$\left[(-i\omega + i\alpha U)(D^2 - k^2) - i\alpha D^2 U - \frac{1}{\mathrm{Re}}(D^2 - k^2)^2\right]\tilde{v} = 0$$

Squire equation (for wall-normal vorticity):

$$\left[(-i\omega + i\alpha U) - \frac{1}{\mathrm{Re}}(D^2 - k^2)\right]\tilde{\eta} = -i\beta DU\,\tilde{v}$$

where $k^2 = \alpha^2 + \beta^2$ with boundary conditions:

Infinitely many sets of ω_n with the corresponding $\tilde{v}_n$ and $\tilde{\eta}_n$

Eigenspectra and eigenfunctions

Eigenspectra of plane Poiseulle flow

$$U(y) = 1 - y^2$$

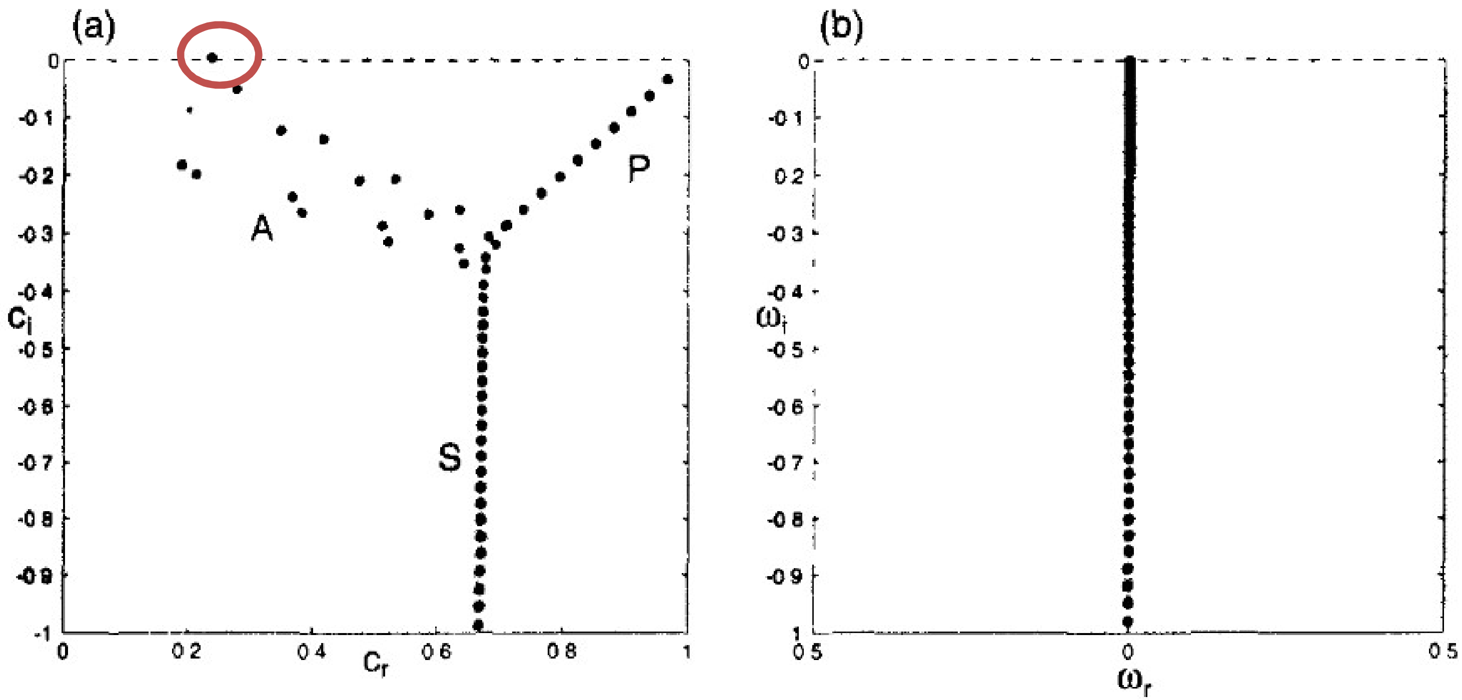


FIGURE 3 1 Orr-Sommerfeld spectrum of plane Poiseuille flow for Re = 10000 (a) wave numbers $\alpha = 1, \beta = 0$. (b) wave numbers $\alpha = 0, \beta = 1$

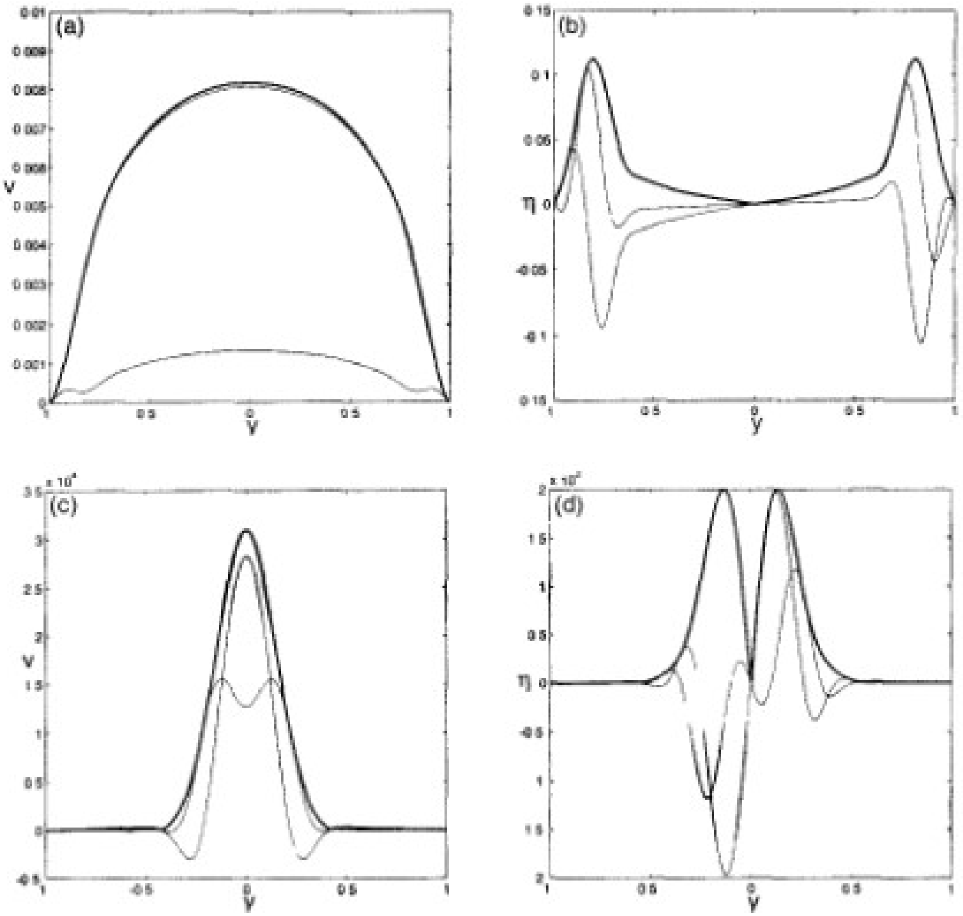
Schmid & Henningson (2001)

Eigenspectra and eigenfunctions

Eigenfunctions of plane Poiseulle flow $(\alpha = 1, \beta = 1, \text{and } \mathrm{Re} = 5000)$

A branch

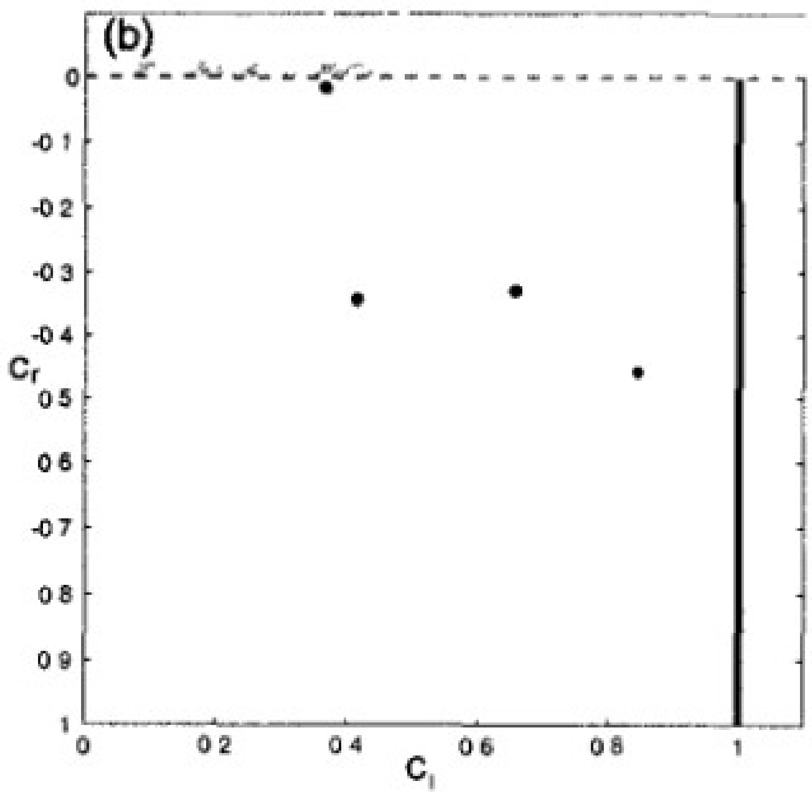
P branch



Schmid & Henningson (2001)

Eigenspectra and eigenfunctions

Eigenspectra of Blasius boundary layer $(\alpha = 0.2, \beta = 0, \text{and } \mathrm{Re} = 500)$



Schmid & Henningson (2001)

Eigenspectra and eigenfunctions

Eigenfunction of Blasius boundary layer $(\alpha = 0.2, \beta = 0, \text{and}\ \mathrm{Re} = 500)$

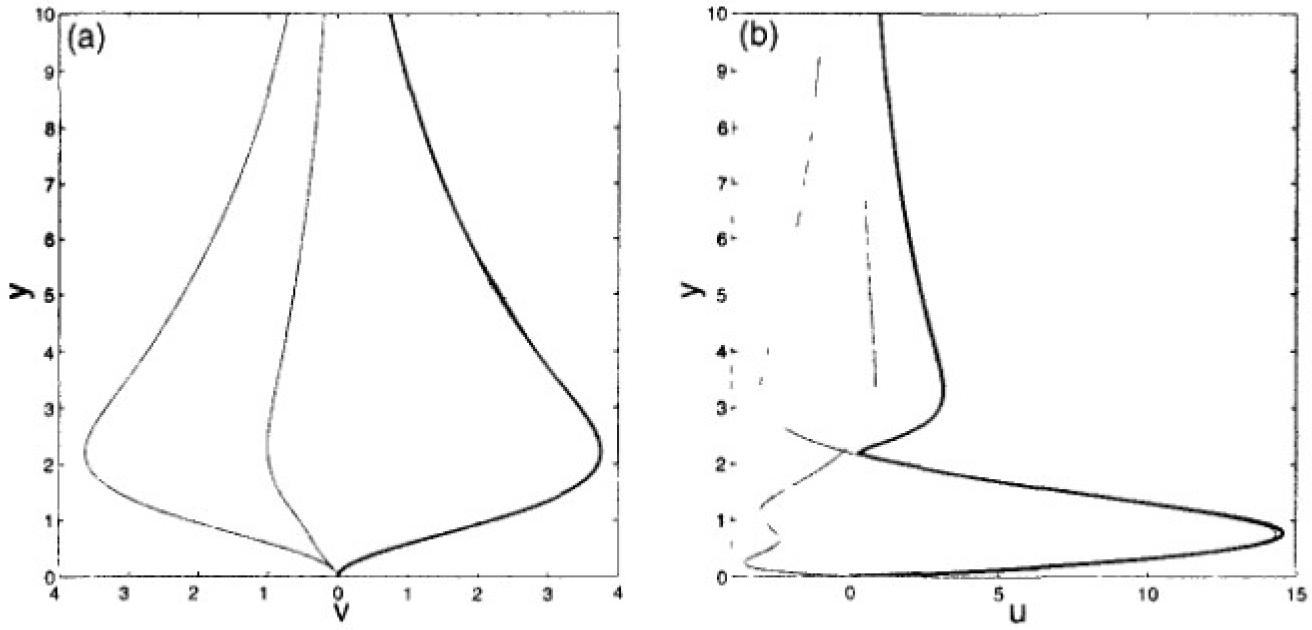


FIGURE 3 5 Eigenfunctions for Blasius boundary layer flow. (a,b) Eigenfunction of the discrete spectrum, vertical (a) and streamwise (b) velocity component for $\alpha = 0\ 2, \mathrm{Re} = 500$ The thick line represents the absolute value of v or u. the thin lines represent the real and imaginary part

Lecture outline

1. Eigenspectra and eigenfunctions
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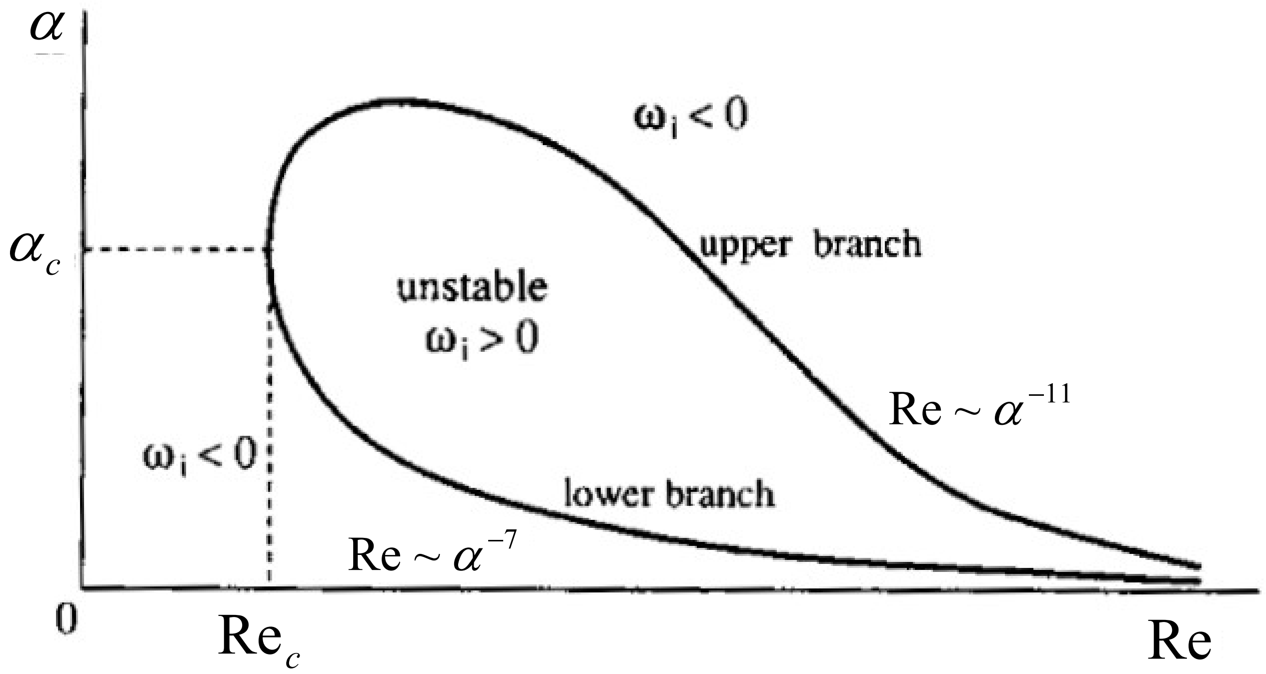
Neutral stability curve

Schematic structure of neutral stability curve for Poiseulle flow

$$\omega_i(\alpha, \beta = 0, \mathrm{Re}) = 0$$



Neutral stability curve of Poiseulle flow

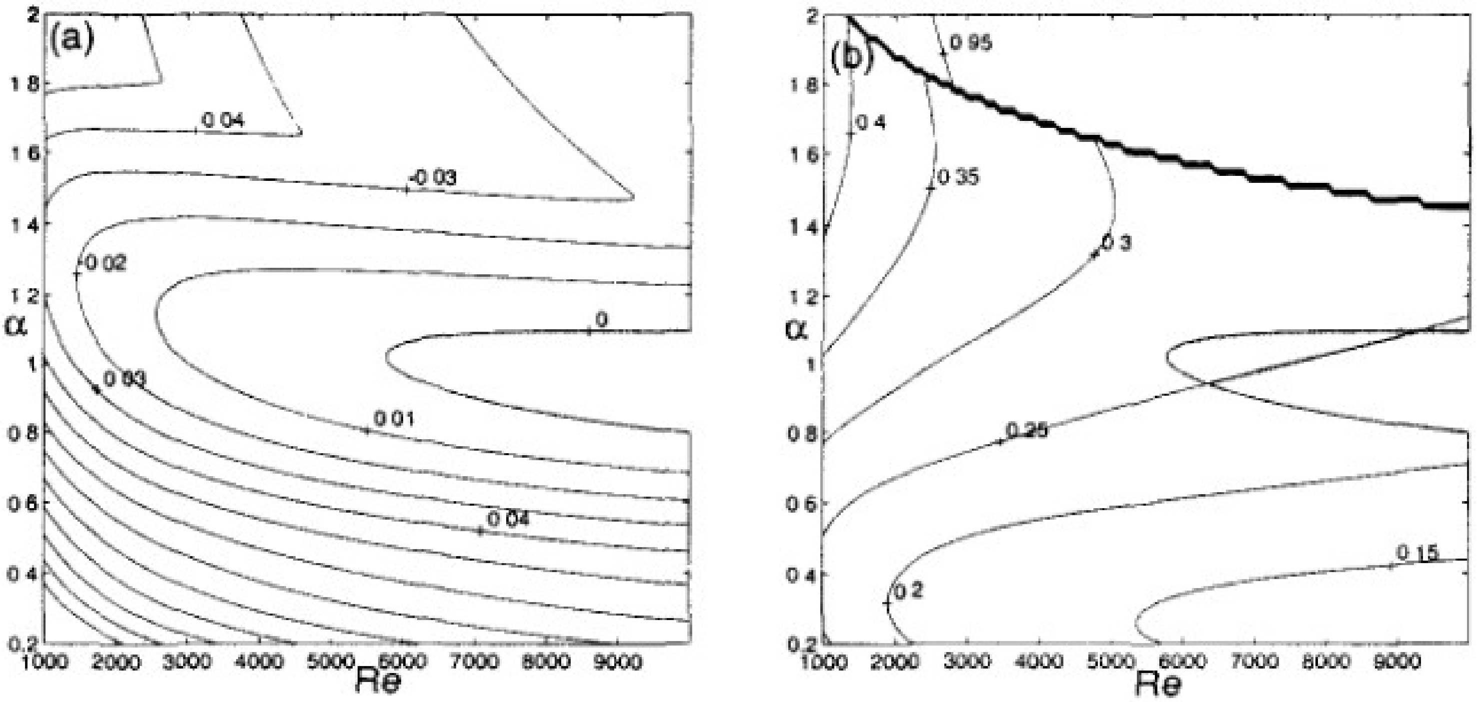


FIGURE 3 8 Neutral curve for plane Poiseuille flow (a) contouis of constant growth rate c_i, (b) contours of constant phase velocity c_r The shaded area represents the region of parameter space where unstable solutions exist

Neutral stability curve of Blasius boundary layer

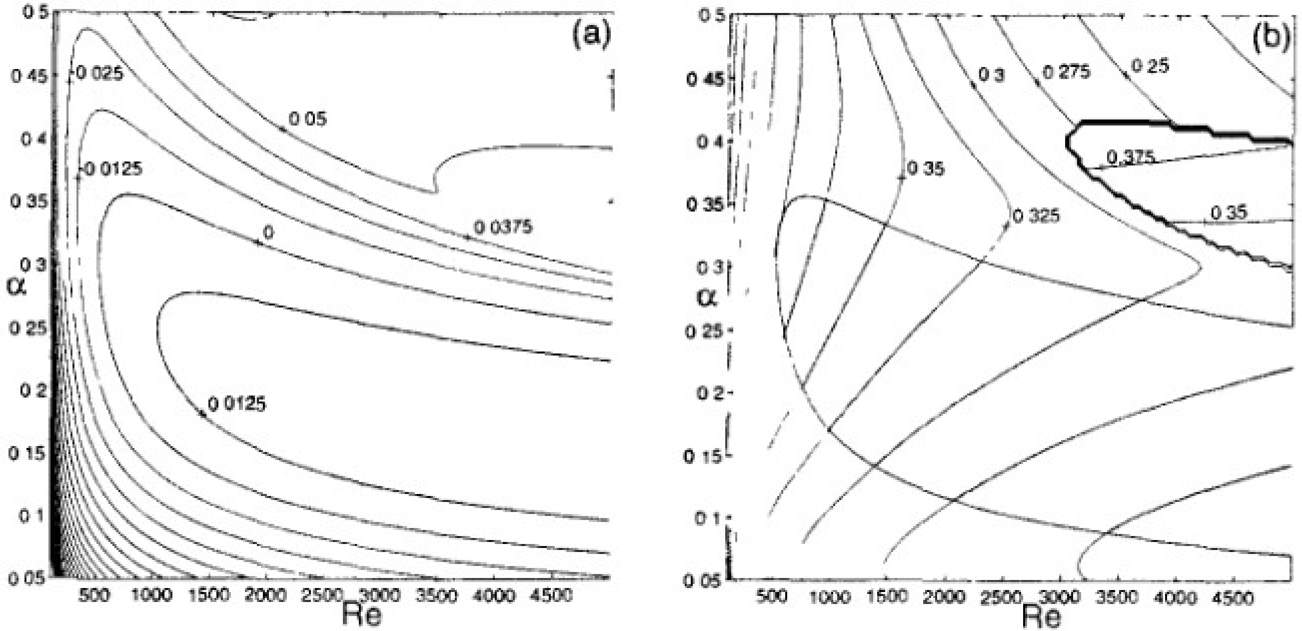


FIGURE 3.9 Neutral curve for Blasius boundary layer flow (a) contours of constant growth rate c_i, (b) contours of constant phase velocity c_r. The shaded area represents the region in parameter space where unstable solutions exist

Neutral stability curve

Critical Reynolds numbers and streamwise wavenumbers

Linear stability analysis

Flow configurations	Critical Re (Linear stability)	Transition Re	Critical wavenumber	Critical phase speed
Couette flow	∞	350-400	-	-
Poiseulle flow	5772.2	1000-2000	1.02	0.2639
Pipe flow	∞	2000-2500	-	-
Boundary layer	519.4	Depends on dist. env.	0.303	0.3935

Remark

Linear stability analysis does **not provide a full explanation** for the onset of **transition**.

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Normal mode solution (2D case) revisited

$$v'(x, y, t) = \tilde{v}(y) e^{i\alpha x - i\omega t} + c.c$$

So far, $\alpha \in R$ is given and $\omega \in C$ unknown

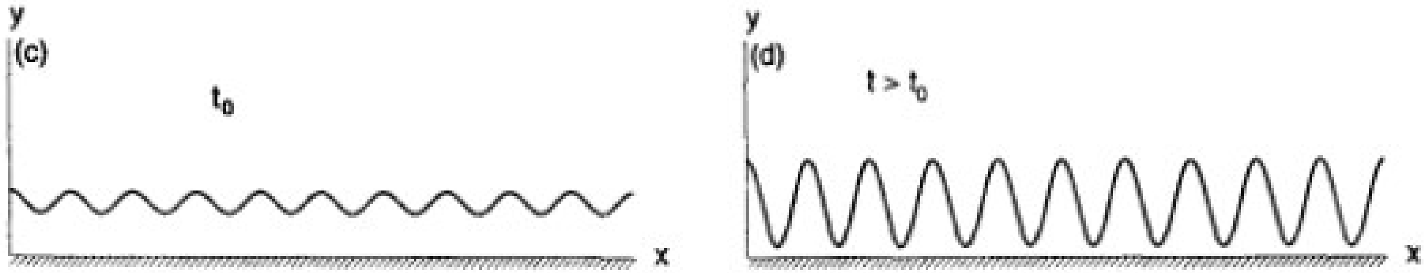
$\omega_i > 0$ Linearly unstable $\omega_i < 0$ Linearly stable

Now, consider $\omega \in R$ is given and $\alpha \in C$ unknown

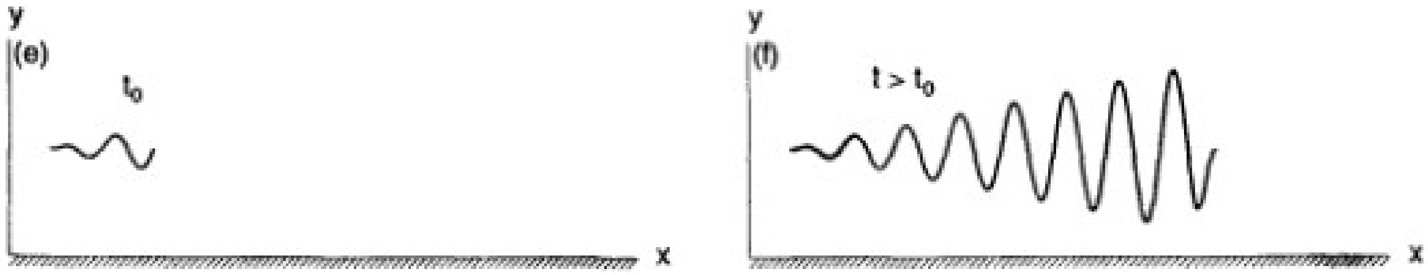
$\alpha_i < 0$ Linearly unstable $\alpha_i > 0$ Linearly stable

Temporal vs spatial stability analysis

Temporal stability analysis

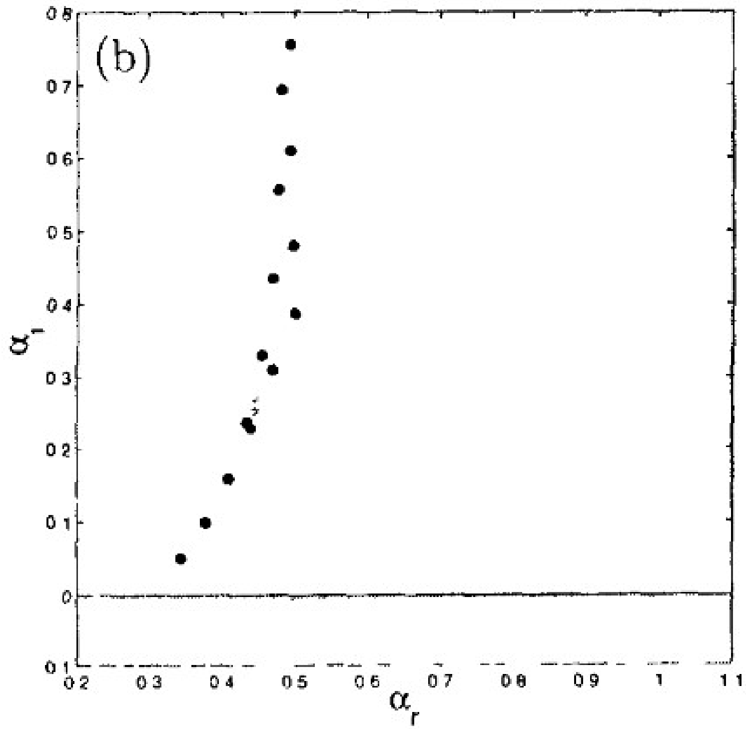


Spatial stability analysis



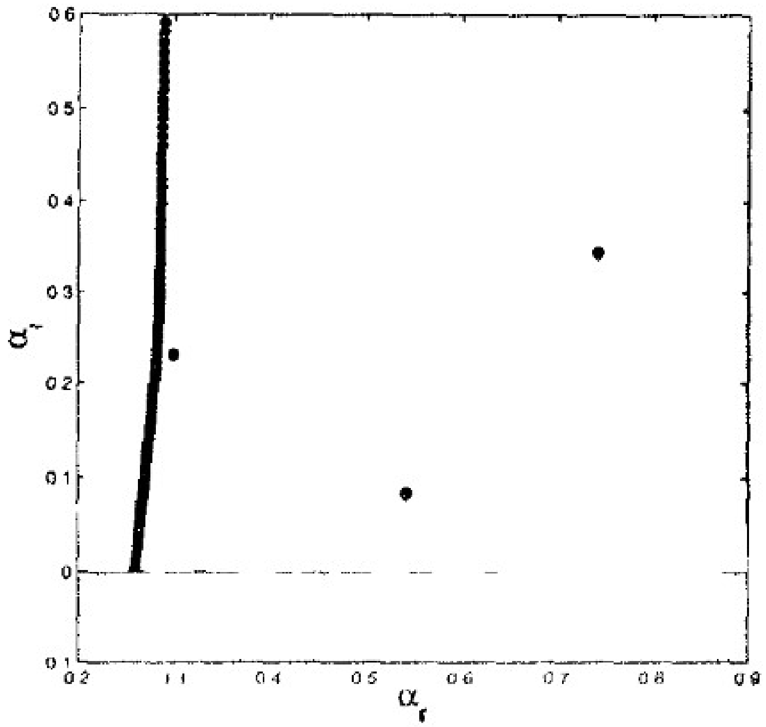
Eigenspectra and neutral stability curve

Eigenspectra of Poiseulle flow and Blasius boundary layer



Poiseulle flow

$(\omega = 0.3$ and $\mathrm{Re} = 1000)$



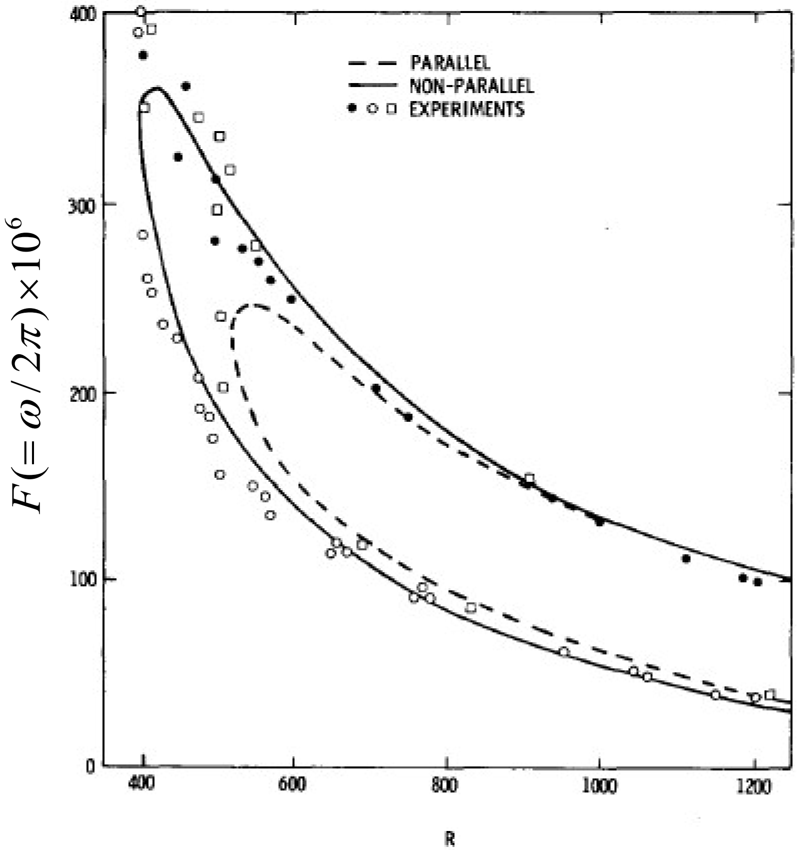
Boundary layer

$(\omega = 0.26$ and $\mathrm{Re} = 1000)$

Schmid & Henningson (2001)

Neutral stability curve

Neutra stability curve of Blasius boundary layer



$$\mathrm{Re}\left(=\frac{U_\infty \delta^*}{\nu}\right)$$

FIG. 1. Comparison between the neutral stability curves based on parallel and nonparallel stability theories and experimental data—□, data of Schubauer and Skramstad, ○, ●, data of Ross *et al*.

Saric & Nayfeh (1975)

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