Lecture 10

Spatio-temporal evolution of instabilities II

AE209 Hydrodynamic stability

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Lecture outline

1. **Application to Ginzburg-Landau equation**
2. **Application to wake**
3. **Physical implications: oscillator vs amplifier flows**

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Application to Ginzburg-Landau equation

Complex linear Ginzburg-Landau equation

From dispersion relation, $D(k,\omega)=0$:

$$\omega(k) = Uk - c_d k^2 + i(\mu - k^2)$$

Linear stability: Calculate the maximum growth rate

$$\omega(k_{\max}) = i\omega_{i,\max} = i\mu \quad \text{with} \quad k_{\max} = 0$$

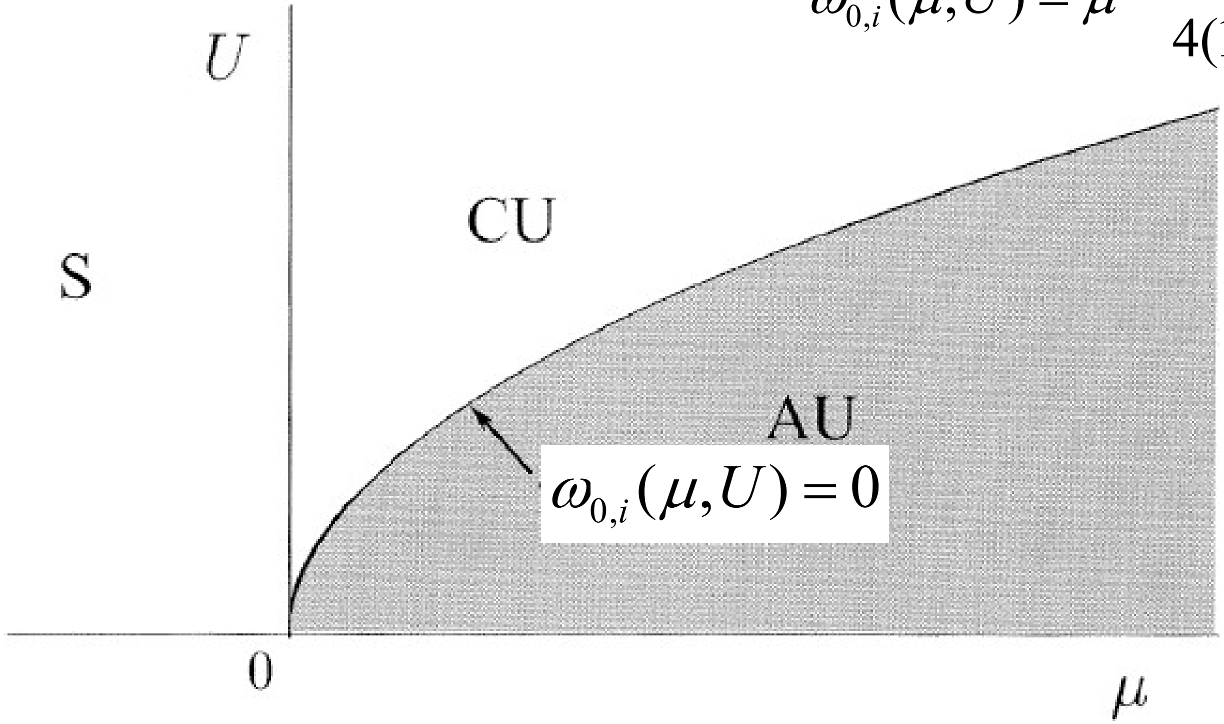
Absolute instability: Calculate the absolute growth rate

$$\omega_0 = \omega(k_0) = \frac{c_d U^2}{4(1+c_d^2)} + i\left[\mu - \frac{U^2}{4(1+c_d^2)}\right] \text{ with } k_0 = \frac{U}{2\left(c_d + i\right)}$$

Application to Ginzburg-Landau equation

Absolute and convective instabilities in parametric space

$$\omega_{0,i}(\mu,U)=\mu-\frac{U^2}{4(1+c_d^2)}$$



Remark

This description is only important for the system with mean advection.

1. Application to Ginzburg-Landau equation
2. **Application to bluff-body wake**
3. Physical implications: oscillator vs amplifier flows

Application to parallel wake

Family of wake profiles

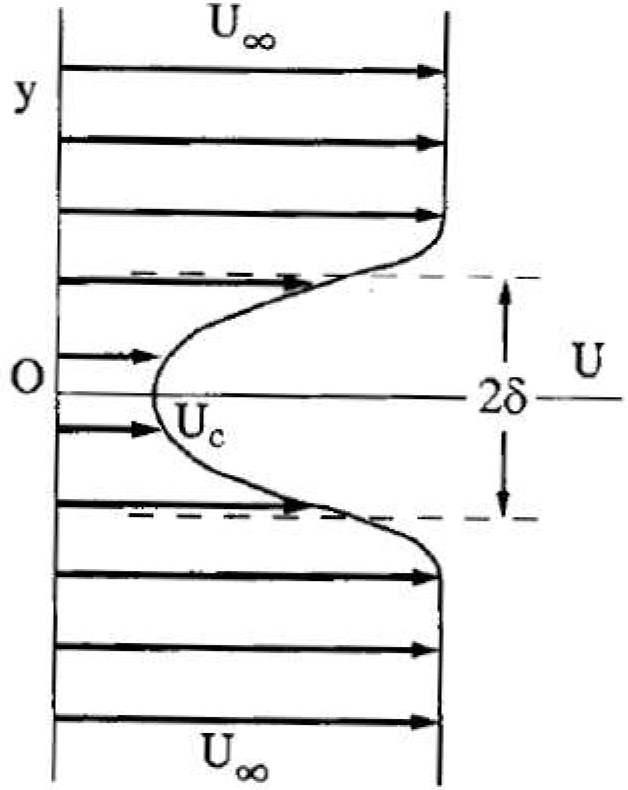
$$U(y) = U_\infty + (U_\infty - U_c)U_1(\frac{y}{\delta}; N)$$

where

$$U_1(\xi; N) = \left[1 + \sinh^{2N}\left\{\xi \sinh^{-1}(1)\right\}\right]^{-1}$$

with

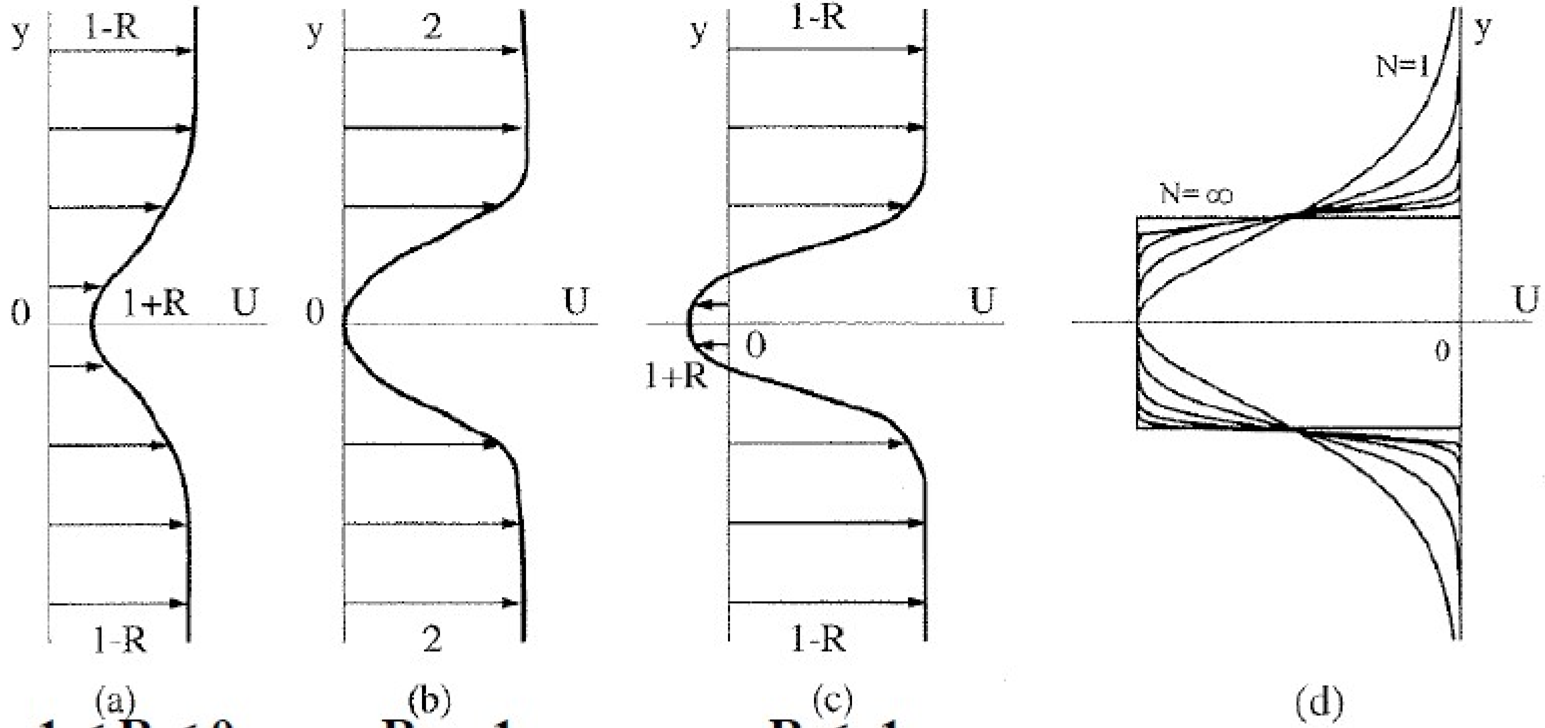
$$\mathrm{Re} = \frac{\overline{U}\delta}{\nu}$$



Monkewitz (1988)

Application to parallel wake

Family of wake profiles



-1 < R < 0 **R = -1** **R < -1**

Velocity ratio $R = \dfrac{U_c - U_\infty}{U_c + U_\infty}$

N: stiffness

Monkewitz (1988)

Application to parallel wake

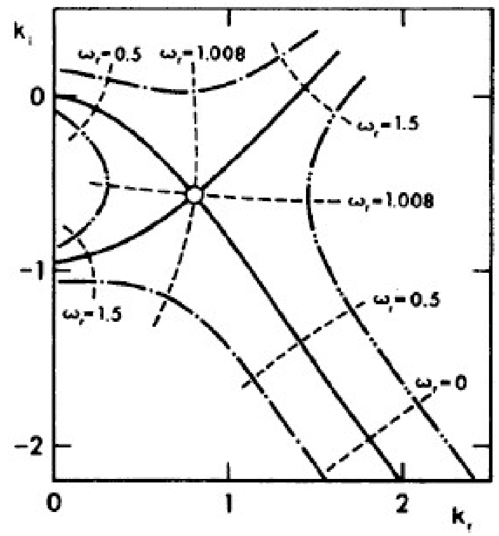
Dispersion relation (obtained by solving the Orr-Sommerfeld equation)

$$\left[\left(-i\omega + ikU\right)\left(D^2 - k^2\right) - ikD^2U - \frac{1}{\mathrm{Re}}\left(D^2 - k^2\right)^2\right]\tilde{v} = 0$$

with the **saddle point** behaviour:

$R = -1$ $\quad N = 2$ $\quad \mathrm{Re} = 11.3$

Monkewitz (1988)

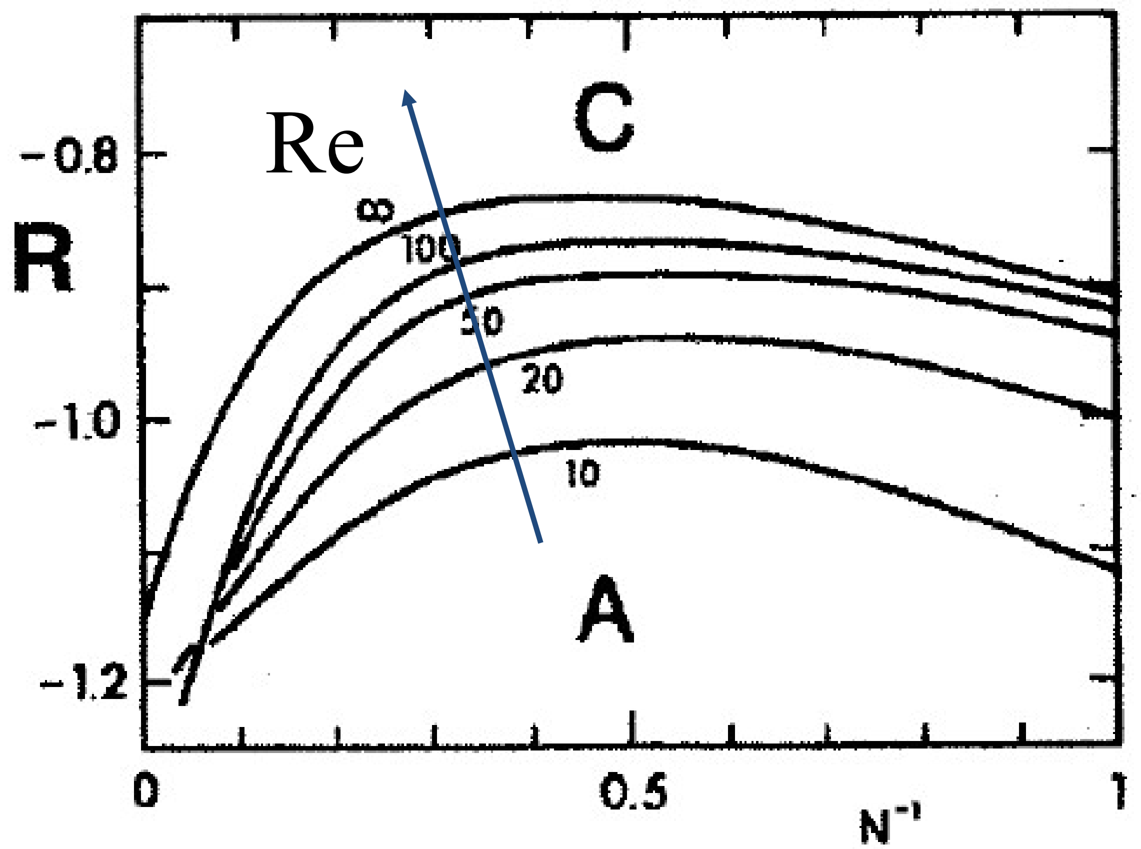


$$\omega_0 = 1.008 + 0i$$

$$\omega - \omega_0 \sim \frac{1}{2}\frac{\partial^2 \omega}{\partial k^2}(k_0)(k - k_0)^2$$

——, curve $\omega_i = 0$; $-\cdot-\cdot-$, $\omega_i = 0.25$; $-\cdot\cdot-\cdot\cdot-$, $\omega_i = -0.25$

Effect of velocity ratio, stiffness and Reynolds number

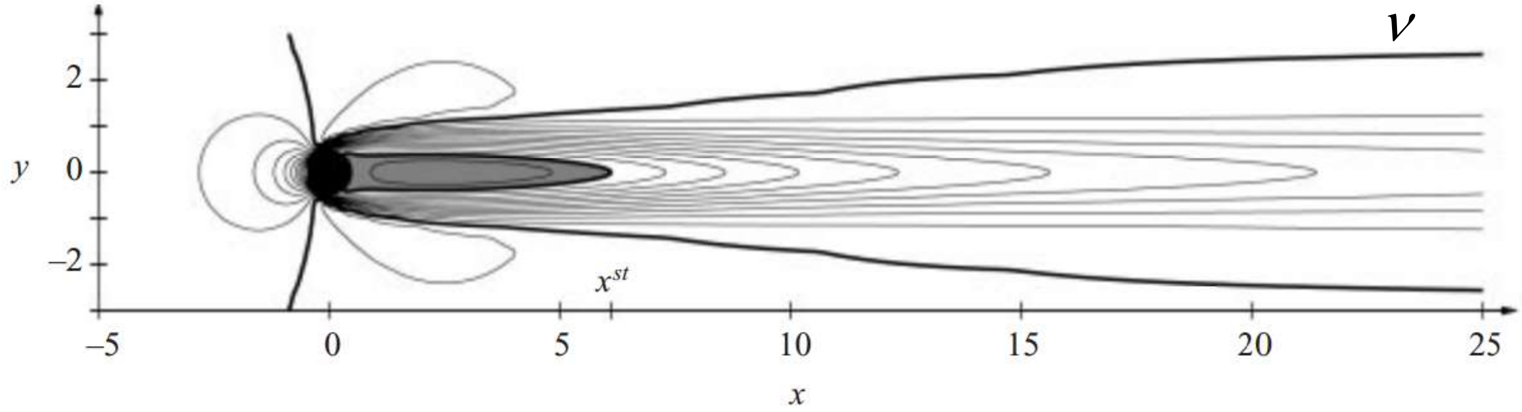


Monkewitz (1988)

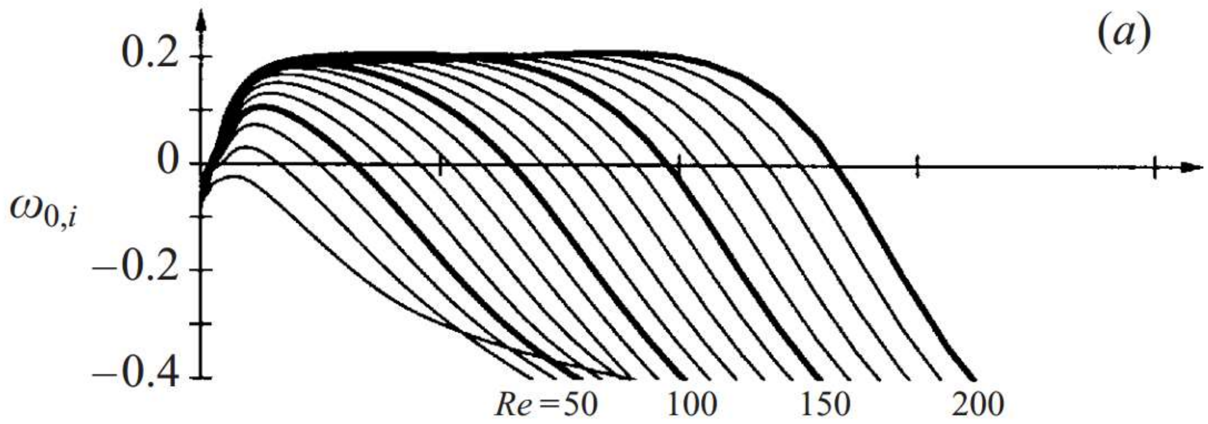
Application to (non-parallel) cylinder wake

Base flow profile at Re=100

$$\mathrm{Re} = \frac{U_\infty d}{\nu}$$



Absolute growth rate of velocity profile at each streamwise location



Pier (2002)

Application to cylinder wake

Emergence of vortex shedding

$5 < \mathrm{Re} < 25$ Some regions are convectively unstable

$25 < \mathrm{Re} < 47$ Some regions are absolutely unstable

$\mathrm{Re} \approx 47$ Strong local absolute instability leads to a **global instability** in the form of

$$\mathbf{u}'(x, y, t) = \hat{\mathbf{u}}(x, y) e^{-i\omega_G t} \quad \text{with} \quad \omega_{G,i} > 0$$

Remark

Local absolute instability is a **necessary condition** for the onset of a **global instability of a fully non-parallel open flow**

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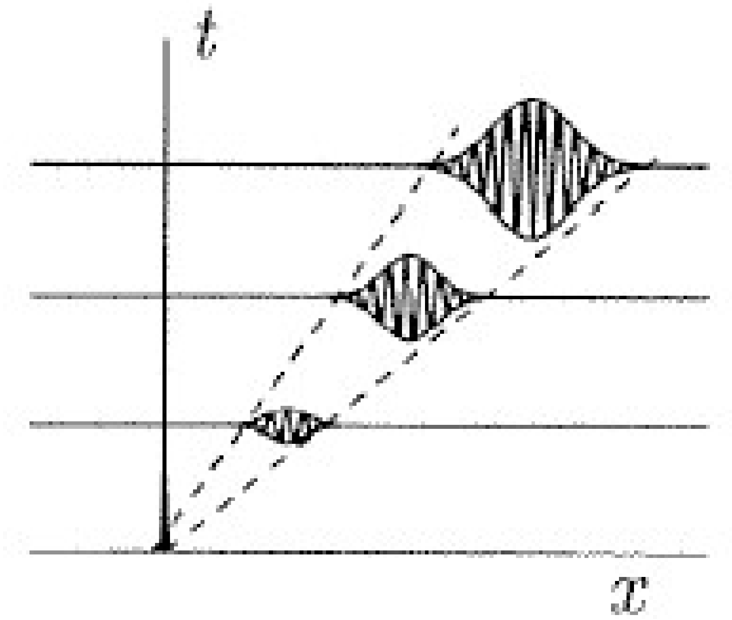
Remarks



Convective instability

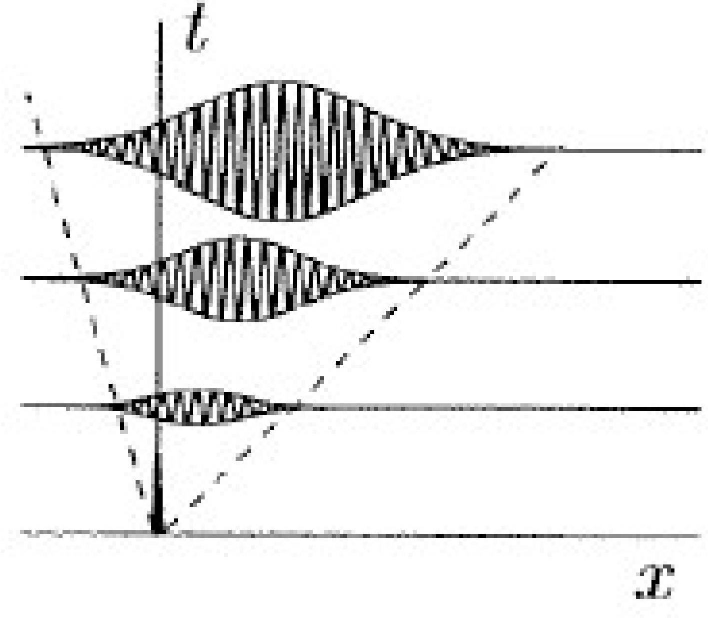
Ex)

1. Reference control volume returns to the original state after the impulse moves away downstream.

2. **Spatial stability analysis** becomes **meaningful** in this situation.

3. Instability dynamics is **driven by upstream noise**

Remarks



Absolute instability

Ex)

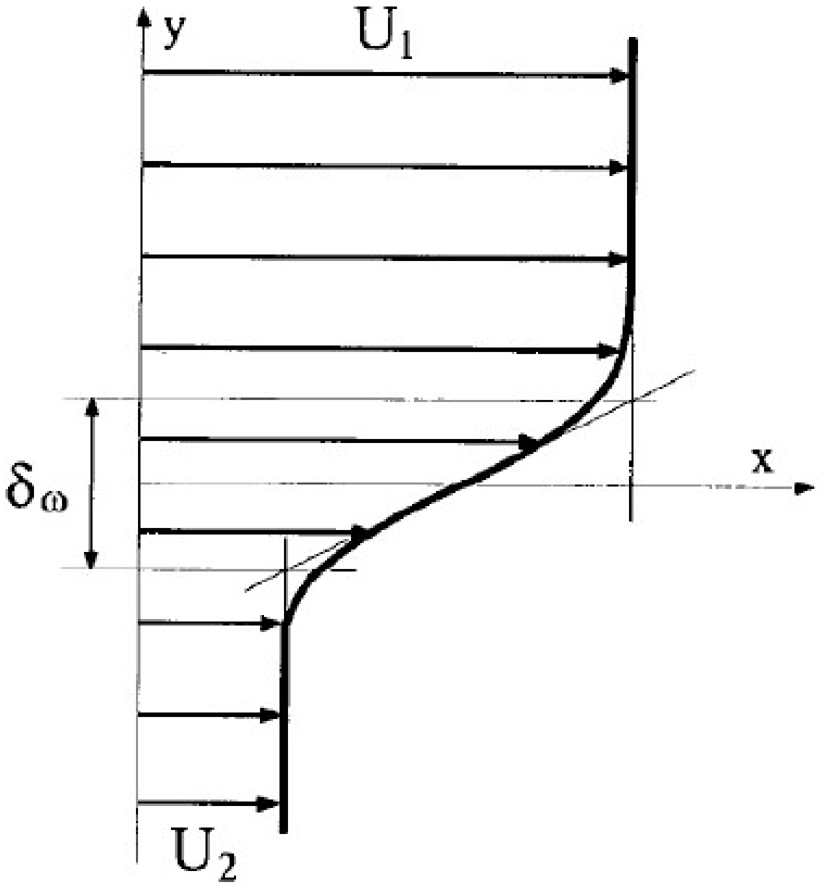
1. Reference control volume never return to the original state.

2. **Spatial stability analysis** becomes **meaningless** in this situation.

3. Instability dynamics is **intrinsically** driven by the given system and often results in a **nonlinear oscillation** with a **distinct frequency.**

Example: Mixing layer

Hyperbolic tangent mixing layer



Base flow profile

$$U(y) = \overline{U} + \frac{\Delta U}{2}\tanh\left(\frac{2y}{\delta_\omega}\right)$$

Velocity ratio

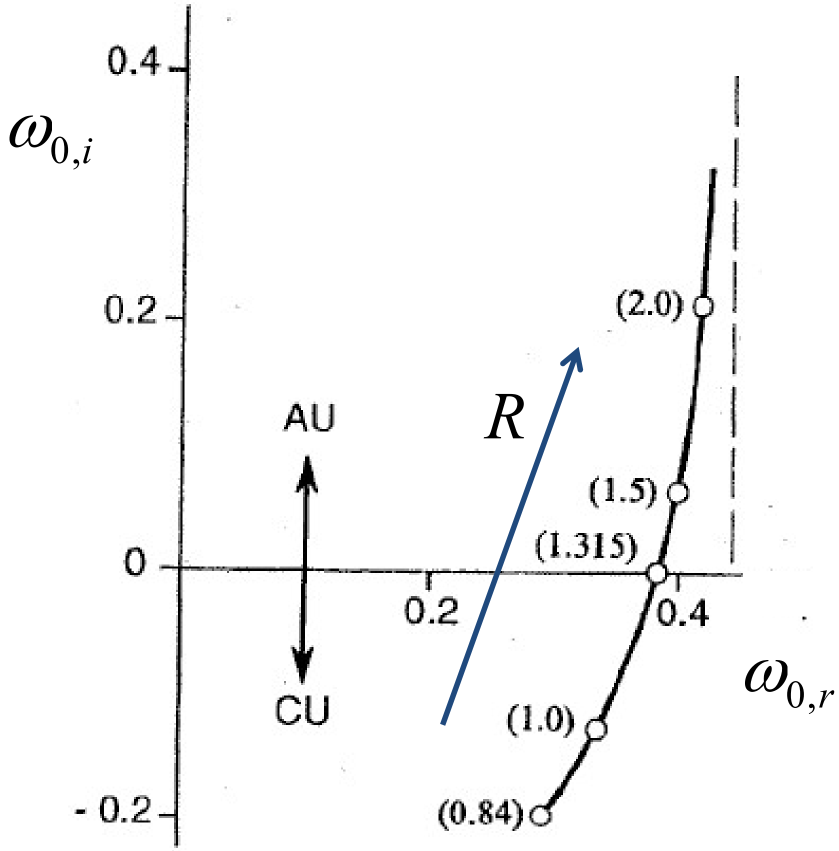
$$R = \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2\overline{U}}$$

$$\delta_\omega = \frac{\Delta U}{(dU/dy)_{\max}}$$

Huerre & Monkewitz (1985)

Example: Mixing layer

Theory: transition from convective to absolute instability with R



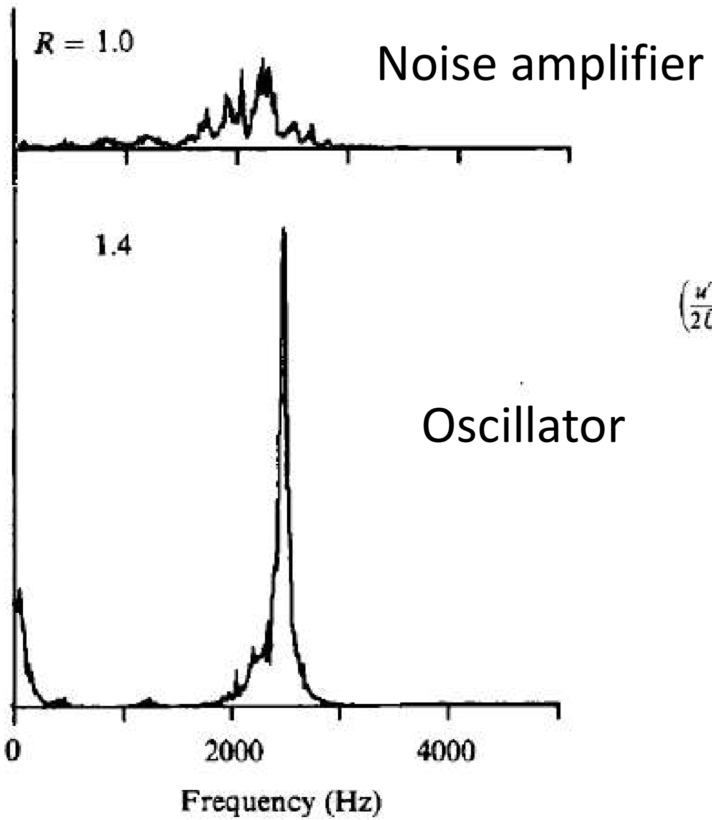
Velocity ratio

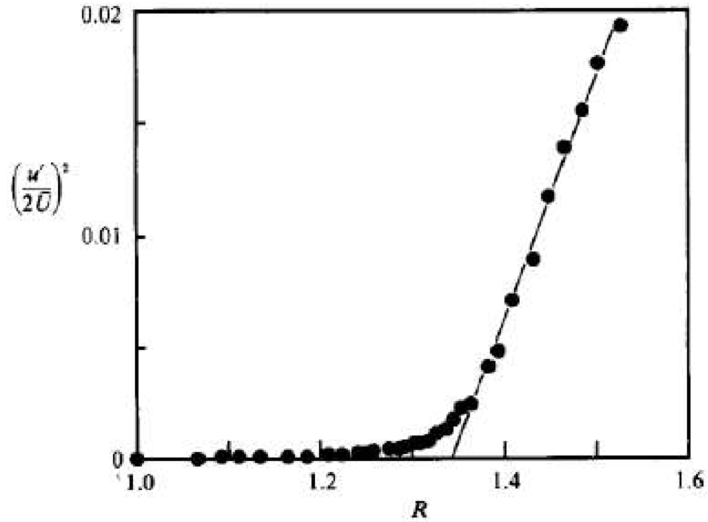
$$R = \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2\overline{U}}$$

Huerre & Monkewitz (1985)

Example: Mixing layer

Experiment: shift from noise amplifier to oscillator with R





Strykowski & Niccum (1991)

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