



**Indian Institute of Technology Palakkad**  
**भारतीय प्रौद्योगिकी संस्थान पालक्काड**  
Nurturing Minds For a Better World

**Modelling a controller for a suspension system of a rail vehicle**

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## I ABSTRACT

Trains are one of the most widely used means of transport especially in a huge country such as India. They provide cheap means of transport and facilitate long-distance travel. With increased demands to reduce the duration of travel, trains now travel at a very high speed. The various vibrations and shocks from the tracks can cause discomfort, fatigue, and irritation among passengers. As far as the railways are concerned, these vibrations decrease the durability of the train bogeys, which increases maintenance costs and reduces its performance. To alleviate this problem, we make use of an active suspension system with an electro-hydraulic actuator which applies a specified force to control the motion of the train body. The control strategy used to control the actuators is a Linear Quadratic Regulator (LQR) feedback controller. The LQR controller designed is active and enables the suspension system to dynamically change based on operating conditions. This enables travel without compromising on the rail handling ability and ride comfort. MATLAB will be used to develop and analyze the state-space model for both the passive and active systems and a Simulink will be used to create block diagrams and simulate the systems. Besides the development of the suspension system, we would also investigate the parameters of the suspension system and evaluate both the passive and active suspension system through tests and analyze the results.

## II OBJECTIVES

- Develop a suspension system for a rail vehicle, that can reduce or nullify the vibrations which would in turn improve the passenger experience and vehicle handling for the loco pilot.
- Evaluate the performance of passive and active suspension systems in Simulink and analyze the results obtained
- Modify the parameters of the rail vehicle to ensure minimum vibration and maximum controllability.

## III MOTIVATION/ORIGIN

Travelling by trains and buses can be tedious especially if they are crowded and one can't find a seat. The fact that these vehicles need not travel on smooth surfaces further adds to the inconvenience as one is

forced to stand and grab onto handles for support, especially on turns or on encountering irregularities. This hampers the travel experience and makes it tedious rather than an enjoyable one.

As far as the operators (drivers or loco pilots) are concerned, encountering sudden irregularities or turns proves to be a challenge. At times, control over the vehicle may be lost. In order to have good handling and proper control, stiff suspensions are required in systems to control the vehicle. The passive suspension system, which is the traditional suspension system, always involves a trade-off between the railway handling of the vehicle and the ride quality of the passengers.

The active suspension system aims to solve this issue by utilizing an actuator that provides a controlled force that stabilizes the body and reduces vibrations. Implementing it in a train would be much easier due to the fact that the trains have a fixed specified path (railway tracks). Hence, as a first stage, the suspension system has been designed for a train.

## IV STATE-OF-THE-ART

LQR Controller is a commonly used technique in control systems to design feedback controllers. LQR stands for Linear Quadratic Regulator and employs a cost function that is tuned to ensure maximum stability.

LQR controller defines a cost function which is a function of the steady space variable and inputs and is quadratic in nature. LQR Controllers aim to minimize the cost function.

LQR minimizes the control function by solving the Riccati equation and obtaining a matrix which can then in turn be used to obtain gain.

Among other references went through, the Model Predictive Controller (MPC) was also implemented in certain cases. MPC is a dynamic control algorithm while LQR is a steady-state control algorithm that is utilized mostly for linear, time-invariant purposes.

## V METHODOLOGY/APPROACH

### V.1 THEORY/IMPORTANT CONCEPTS

- LQR Law - The LQR control law is used in minimizing the cost function. It states that

$$u(t) = -Kx(t) \quad (1)$$

Here,  $u(t)$  is the control input, and  $x(t)$ , the state of the system.

The minimum value of gain obtained is

$$K = R^{-1}B'P \quad (2)$$

where  $R$  is the control weighting matrix,  $B'$  is the transpose of matrix  $B$  and  $P$  is the solution to the Riccati Equation.

- Riccati Equation - Riccati equation is a first-order ordinary differential equation of the form

$$y'(x) = a(x)y(x)^2 + b(x)y(x) + c(x) \quad (3)$$

where  $a(x)$ ,  $b(x)$ , and  $c(x)$  are continuous functions of  $x$

- Sprung Mass - Mass of the railway vehicle body. These are supported by the suspension system and involve body, passenger, or cargo.
- Unsprung Mass - Mass of railway vehicle bogey. These are not supported by the suspension system and involve brake gears, wheels, and axles.

## V.2 WORKING

The differential equations for the rail vehicle are found. This is obtained from the force balance equations after obtaining the free-body diagrams.

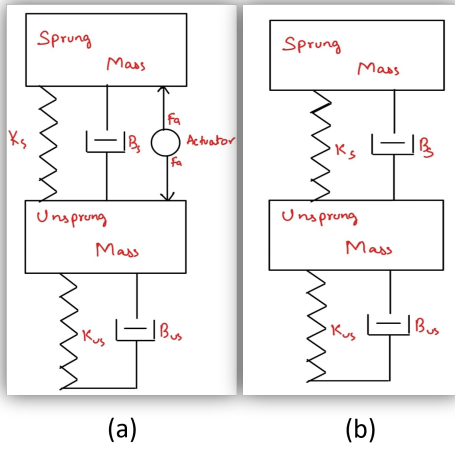


Figure 1: (a) Free body diagram of Active Suspension System, (b) Free body diagram of Passive Suspension System

Here,  $K_S$  and  $K_{US}$  are spring constants,  $B_S$  and  $B_{US}$  are the damping coefficients. The differential equation is obtained and is represented by the steady space matrix. Following that, we obtain the controllability matrix which provides a relationship between the state variables and inputs. The rank of the controllability matrix is compared with the number of state variables.

(Rank should be equal to the number of state variables)

The state weighting matrix and control weighting matrices are obtained.

These are then trained into a Simulink document.

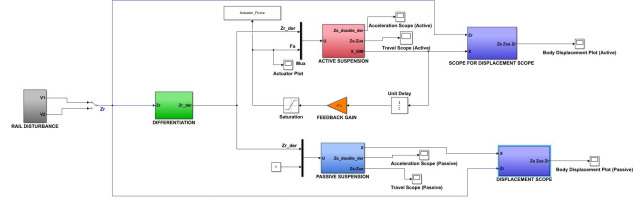


Figure 2: Simulation of the control system

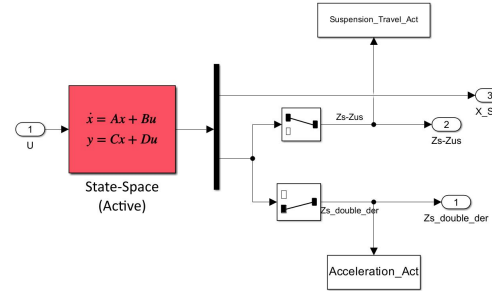


Figure 3: Active Suspension

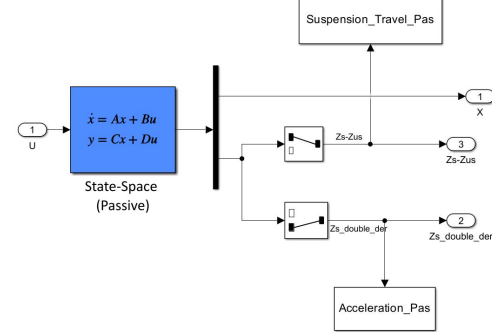


Figure 4: Passive Suspension

For active suspension system after steady space representation, Rail Disturbances were simulated. Vibration 1 was a step input. Vibration 2 is a square wave of duty cycle 50%.

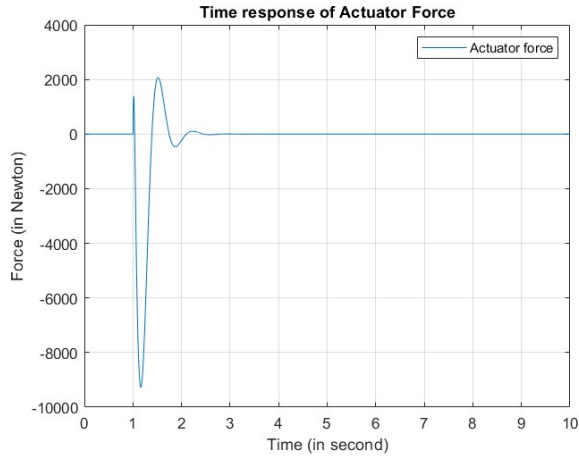
[Link to MATLAB Code](#)

## VI RESULT AND DISCUSSION

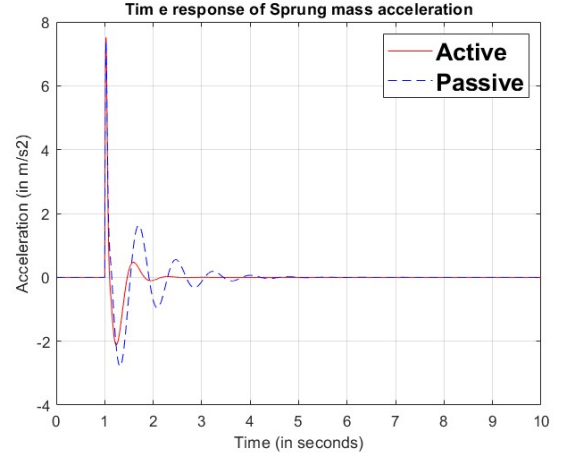
Analysis of the results of the passive suspension system of the quarter car model and then analysis of the response of the active suspension system with the state space controller. The impact of the LQR controller will also be assessed to see the significance

of its impact on the system. When discussing the performance of the two suspension systems the, ride quality and the railcar handling will be monitored, with the parameters of focus being the railcar body acceleration, the travel of the suspension system, and the wheel deflection as well as the body displacements of the quarter car model.

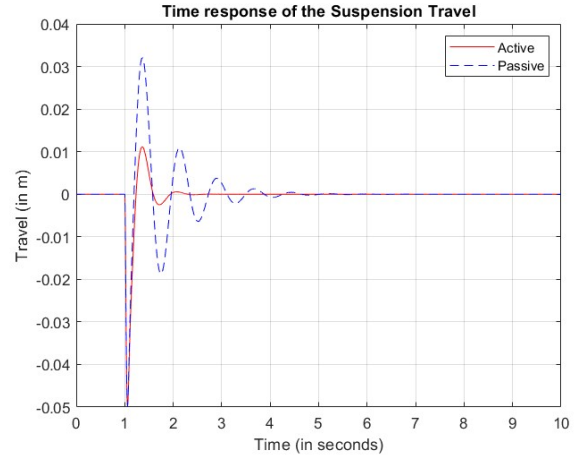
The criteria that will be used to compare the two systems include Settling Time, Rise Time, Overshoot, and Steady-State Error. Settling Time refers to the time taken by the system to stabilize and match the desired value. Rise Time measures how quickly the system responds and reaches a certain percentage of the desired value. Overshoot quantifies how much the system deviates from the desired response initially. Steady-state error measures the final error between the system response and the desired response.



The response of the actuator force in the active suspension controller to a step input disturbance on the railcar is observed to be in the opposite direction of the sprung mass or railcar body, resulting in an overshoot. This behavior is due to the tendency of the sprung mass to move upwards. However, the figure indicates that the actuator response is stable and provides an acceptable response to the step input disturbance of 0.06m.



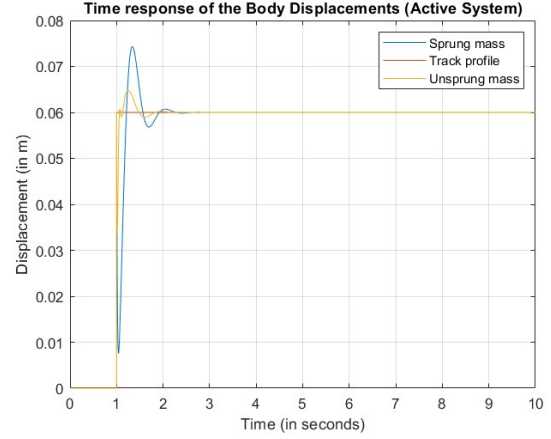
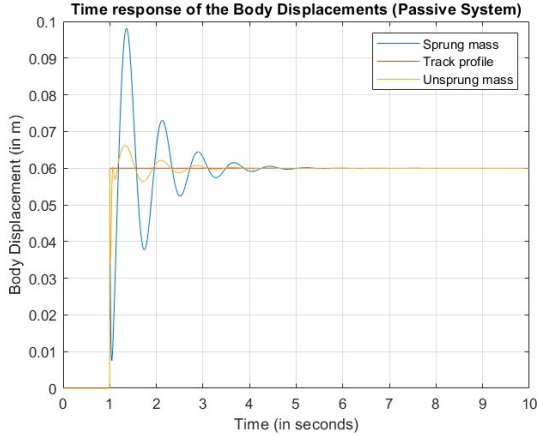
The implementation of an LQR (Linear Quadratic Regulator) controller in the rail car body results in noticeable effects on acceleration, as shown in the graph. Similar to the suspension travel response of the active suspension system compared to the passive suspension, there is an overshoot of  $7.5\text{m/s}^2$ . However, the system also shows a significant 47% improvement in settling time, reduced to 1.75 seconds. This improvement enhances the ride comfort and road handling of the rail car.



The inclusion of a feedback Linear Quadratic Regulator (LQR) controller in the system, as shown in the figure, results in a similar overshoot of 0.05 m compared to the passive suspension. However, when compared to the passive system, the active suspension exhibits a significant reduction in settling time, which is reduced to 1.86 seconds, equivalent to a 50% decrease in response rate compared to the passive suspension. This faster response time improves ride comfort, reduces vibrations for passengers, and enhances overall system performance.



In comparison to the passive suspension, the active suspension shows a similar overshoot in wheel deflection, measuring 0.0255m. However, the active suspension demonstrates a noticeable improvement in settling time, which is reduced to 1.74 seconds, equivalent to a 42% improvement in response rate. Although the initial deflection value at the start of the rail disturbance is slightly higher in the active suspension, the wheel deflection vibration is significantly reduced over the next 1.74 seconds, and system stability is achieved faster. This optimization of both amplitude and settling time with the implementation of the LQR controller helps minimize vibrations and prevent irregular movements of the railcar on the railway, in contrast to the passive suspension which has a longer settling time resulting in larger deflections and poorer rail handling quality.



The active suspension system, when compared to the passive suspension, shows a similar trend in the displacement of both the sprung mass and unsprung mass in response to the rail disturbance. However, with the application of the LQR control method, the overshoot for the sprung mass is slightly lower, measuring 0.0143. The unsprung mass does not change significantly compared to the passive suspension, but the amplitude and settling time are considerably reduced to 1.85 seconds and 1.42 seconds respectively, indicating an improvement of 51% and 43% respectively. The active suspension system with LQR control demonstrates significant improvement in reducing the body displacement of the railway vehicle compared to the passive suspension, resulting in improved ride comfort for passengers.

## VII CONCLUSION

An LQR controller-based system has been developed aimed at reducing the impact low-frequency vibrations have thereby improving passenger safety and comfort and enabling easier rides for loco pilots. The addition of an active suspension system has immensely aided in stabilizing the system. As seen in the plots, the settling time has decreased massively. The overshoot also was found to be less which gives an impression of a more stable system.

## VIII FUTURE PERSPECTIVE

Although the LQR Controller works fine, an implementation that can be done in the future, is setting up LQR controllers with train location and speed. Obtaining real-time speed, acceleration, and position can help LQR Controllers to be better equipped to deal with irregularities

Using machine learning and data to train models that use LQR controllers can make the system much more effective and robust.

Utilising the advancements in communication technology, to build a system that enables coordination of train motion can be done. This can help reduce congestion and facilitate transport.

## IX REFERENCES

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