MAT 128B: Project I

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1 Introduction

Describe how we are splitting up the project here

2 Filled Julia Sets

2.1

Given the function $\phi(z) = z^2 + c$, where $c \in \mathbb{C}$ is some costant, we consider the problem of finding the fixed points of $\phi(z)$ using an iterative method. The usual process starts with some $z_0 \in \mathbb{C}$ and uses the relation $z_{k+1} = \phi(z_k)$ to generate a sequence $\{z_n\}$ which may, or may not, converge to a fixed point of $\phi(z)$, depending on the choice of z_0.

Julia sets study the set of initial points which generate a converging sequence. That is, we define the filled Julia set by $\{z \in \mathbb{C} : \text{the sequence } z_{k+1} = \phi(z_k), \text{ with initial value } z_0 = z, \text{ converges}\}$, and define the Julia set to be the boundary of the filled Julia set.

We start with the simplest case: $\phi(z) = z^2$, which has two fixed points $u = 1$ and $v = 0$. Clearly, if $|z_0| \leq 1$, the sequence will converge because $|z_1| = |\phi(z_0)| = |(z_0)^2| \leq 1$, which implies $|z_2| \leq 1$ and so forth. Also, it is clear that if $|z_0| > 1$, the sequence will diverge, because the modulus of the following terms will continue to grow. Thus, the filled Julia set for $\phi(z)$ is the unit disc $D^2 = \{z : |z| \leq 1\}$, and the Julia set is the boundary of this disc.

Below is a plot of the set, which was generated with the MATLAB code provided in section 3.

We also graph the Julia set for the function $\phi(z) = z^2 - c$, for $c = 1.25,\ 0.36 + 0.1i,\ -0.123 - 0.745i$ and display their graph's below.

3 MATLAB Code

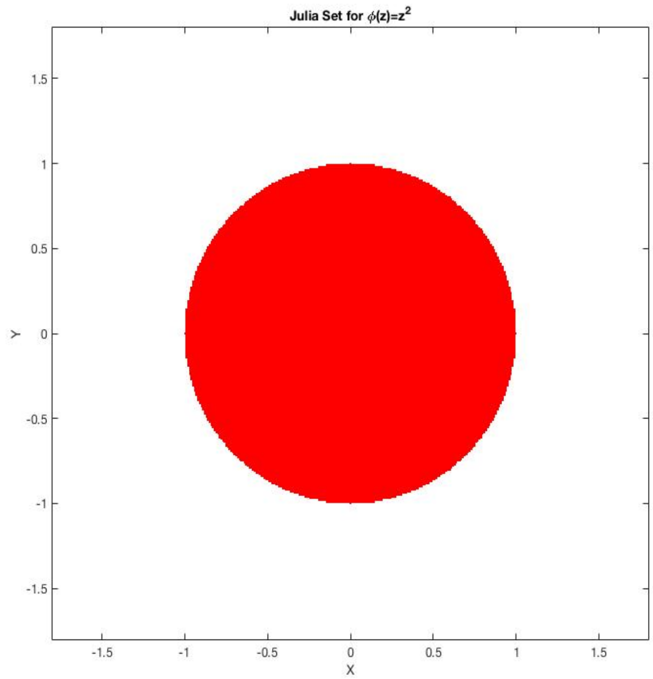

Figure 1:

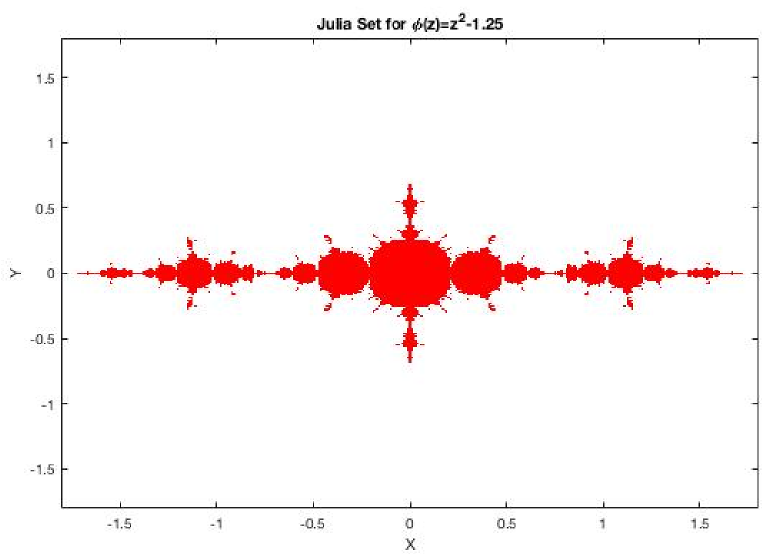

Figure 2: