$$\hat{y} = \langle \hat{\beta}, f(x) \rangle$$

$$= \langle (F^T F)^{-1} F^T y, f(x) \rangle$$

$$= f (F^T F)^{-1} F^T y$$

$$= \overset{h}{\boxed{\qquad\qquad}} \ \overset{y}{\boxed{\ }}$$

$$Var[\hat{y}] = E[\hat{y}^2] - E^2[\hat{y}]$$

$$= E[(hy)^2] - E^2[hy]$$

$$= E[(hf + h\epsilon)^2] - E^2[h(f+\epsilon)]$$

$$= E[(hf)^2 + (h\epsilon)^2 + 2hfh\epsilon] - [E(hf) + \underbrace{E(h\epsilon)}_{0}]^2$$

$$= E[(hf)^2] + E[(h\epsilon)^2] + E[2hfh\epsilon] - E^2(hf)$$

0

$$E[hf] = hf$$

$$= E[(h\epsilon)^2]$$

$$= E[\epsilon^T h^T h \epsilon]$$

$$= tr[h^T h] \, cov[\epsilon]$$

$$= h h^T \cdot \sigma^2$$

concludes the proof

$$\boxed{\begin{aligned} & h\epsilon \, h\epsilon \\ & = \epsilon^T h^T h \, \epsilon \end{aligned}}$$

$$\boxed{\begin{aligned} & E_x[x^T A x] \\ & = tr[A \cdot \Sigma] \\ & cov[x] = \Sigma \\ & E[x] = 0 \end{aligned}}$$