

$$k(x_i, x_j) = (\langle x_i, x_j \rangle + 1)^b$$

$$b=2$$

$$\Rightarrow k(x_i, x_j) = (\langle x_i, x_j \rangle + 1)^2$$

$$= \underbrace{\langle x_i, x_j \rangle^2}_{\text{green}} + \underbrace{1}_{\text{yellow}} + \underbrace{2 \langle x_i, x_j \rangle}_{\text{red}}$$

$$\begin{aligned} &= \sum_k \underbrace{\sqrt{2}}_{\text{yellow}} x_i^{(k)} \underbrace{\sqrt{2}}_{\text{yellow}} x_j^{(k)} \\ &= x_i^0 \cdot x_j^0 \\ &= \left(\sum_k x_i^{(k)} x_j^{(k)} \right)^2 \\ &= \sum_{u,v} \begin{pmatrix} x_i^{(u)} & x_i^{(v)} \end{pmatrix} \cdot \begin{pmatrix} x_j^{(u)} & x_j^{(v)} \end{pmatrix} \end{aligned}$$

break down into products between x_i, x_j .

$$f(x) = \left[\underbrace{x^0}_{\text{0th order}}, \underbrace{\sqrt{2}x^{(1)} \dots \sqrt{2}x^{(d)}}_{\text{1st order}}, \underbrace{\sum_{u,v} x^{(u)} x^{(v)}}_{\text{2nd order}} \right]$$