

$$\begin{aligned}
\hat{y} &= \langle \hat{\beta}, f(x) \rangle \\
&= \langle (F^T F)^{-1} F^T y, f(x) \rangle \\
&= f(F^T F)^{-1} F^T y \\
&= \underbrace{h}_{\text{matrix}} \underbrace{y}_{\text{vector}}
\end{aligned}$$

$$\begin{aligned}
\text{Var}[\hat{y}] &= E[\hat{y}^2] - E^2[\hat{y}] \\
&= E[(hy)^2] - E^2[hy] \\
&= E[(hf + h\epsilon)^2] - E^2[h(f + \epsilon)] \\
&= E[(hf)^2 + (h\epsilon)^2 + 2hf h\epsilon] - [E(hf) + \underbrace{E(h\epsilon)}_0]^2 \\
&= E[(hf)^2] + E[(h\epsilon)^2] + E[2hf h\epsilon] - E^2(hf)
\end{aligned}$$

$$\underline{E[hf] = hf}$$

$$\begin{aligned} &= E[(h\epsilon)^2] \\ &= E[\epsilon^T h^T h \epsilon] \quad \downarrow \\ &= \text{tr}[h^T h] \text{cov}[\epsilon] \quad \downarrow \\ &= \underline{hh^T \cdot G^2} \end{aligned}$$

concludes the proof

$$\boxed{\begin{aligned} h\epsilon h\epsilon \\ = \epsilon^T h^T h \epsilon \end{aligned}}$$

$$\boxed{\begin{aligned} E_x[x^T A x] \\ = \text{tr}[A \cdot \Sigma] \\ \text{cov}[x] = \Sigma \\ E[x] = 0 \end{aligned}}$$