$$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_p^{\infty} F(t)\,dp$$

$$\mathcal{L}(\operatorname{sen}\beta t) = \frac{\beta}{p^2+\beta t}$$

1.- $f(t) = \int_0^t \frac{\operatorname{sen}\tau}{\tau}\, d\tau$

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau = \frac{F(p)}{p}, \quad p > \sigma_0\right.$$

$$= \frac{1}{p}\mathcal{L}\left(\frac{\operatorname{sen}\tau}{\tau}\right) = \frac{1}{p}\int_p^{\infty} \operatorname{sen}\tau\, dp$$

$$= \frac{1}{p}\int_p^{\infty} \frac{1}{p^2+1}\, dp \rightarrow \frac{1}{p}\left(\arctan p \Big|_p^{\infty}\right)$$

$$= \frac{1}{p}\left(\frac{\pi}{2} - \arctan p\right) = \underline{\frac{\pi}{2p} - \frac{1}{p}\arctan p} \quad \text{para } p > 0$$

$2.- F(p) = \dfrac{p}{p^4+4}$

$\hookrightarrow \dfrac{p \cdot p}{p(p^4+4)} = \dfrac{p^2}{p(p^4+4)} \longrightarrow p^2 = s \;,\; p = \sqrt{s}$

$\dfrac{s}{\sqrt{s}(s^2+4)} = \dfrac{1}{\sqrt{s}}\left(\dfrac{s}{s^2+4}\right) = \mathcal{L}^{-1}\left(\dfrac{s}{s^2+4}\right)$

$\sqrt{s} = u \longrightarrow \mathcal{L}^{-1}\left(\dfrac{1}{u}\right)\cos 2t = \dfrac{p}{p^4+4}$

3.- $f(t) = \begin{cases} \text{sen } t & \text{para } 0 \leq t \leq \pi \\ 0 & \text{para } t > \pi \end{cases}$

$* \mathcal{L}(u_a(t)f(t)) = e^{-as}\mathcal{L}(f(t+a))$

$* \mathcal{L}(\text{sen}\beta t) = \frac{\beta}{p^2+\beta^2}$

$* \text{sen}(t+\pi) = -\text{sen}(t)$

$f(t) = (1 - u_\pi)\,\text{sen}\,t$

$f(t) = \text{sen}t - u_\pi \text{sen}t$

$\mathcal{L}(f(t)) = \mathcal{L}\,\text{sen}t - \mathcal{L}(u_\pi(t)\text{sen}t)$

$= \frac{1}{p^2+1} - e^{-\pi p}\mathcal{L}(\text{sen}(t+\pi))$

$= \frac{1}{p^2+1} + e^{-\pi p}\mathcal{L}(\text{sen}\,t) = \frac{1}{p^2+1} + \frac{e^{-\pi p}}{p^2+1} = \frac{1+e^{-\pi p}}{p^2+1}$

4.- $F(p) = \frac{p}{p^2+4} - \frac{e^{-p}}{p^2} - \frac{e^{-2p}}{p^2-1}$

$= \mathcal{L}^{-1}\left(\frac{p}{p^2+4}\right) - \mathcal{L}^{-1}\left(\frac{e^{-p}}{p^2}\right) - \mathcal{L}^{-1}\left(\frac{e^{-2p}}{p^2-1}\right)$

* $\mathcal{L}(\cos\beta t) = \frac{p}{p^2+\beta^2} = \mathcal{L}^{-1}\left(\frac{p}{p^2+4}\right) = \underline{\cos 2t}$

* $\mathcal{L}(U_a(t) f(t-a)) = e^{-ap} F(p)$
$= \mathcal{L}^{-1}(e^{-ap}F(p))$

* $\mathcal{L}^{-1}\left(\frac{1}{p^{n+1}}\right) = \frac{t^n}{n!}$

$\hookrightarrow \mathcal{L}^{-1}(e^{-ap}F(p)) = U_a(t)\,\mathcal{L}^{-1}(F(p)$

$= \mathcal{L}^{-1}\left(\frac{e^{-p}}{p^2}\right) = U_a(t)\,\mathcal{L}^{-1}\left(\frac{1}{p^2}\right)$

$= U_a(t)\frac{t^2}{2} \rightarrow \underline{\frac{1}{2}(t-a)^2 U_a(t)}$

* $\mathcal{L}^{-1}\left(\frac{1}{p}e^{-ap}\right) = U_a(t)$

* $p-a \rightarrow p \quad e^{at}$
$p^2-1 \rightarrow p \quad e^{t}$

$= e^{2t}\mathcal{L}^{-1}\left(\frac{e^{-2(p+1)}}{p}\right) = e^{2t}\mathcal{L}^{-1}\left(\frac{e^{-2p-2}}{p}\right)$

$= e^{2t}\mathcal{L}^{-1}\left(\frac{e^{-2p}e^{-2}}{p}\right) = e^{2t}e^{-2}\mathcal{L}^{-1}\left(\frac{1}{p}e^{-2p}\right) = \underline{e^{2t-2}U_2(t)}$

$\therefore \underline{\cos 2t - \frac{1}{2}(t-a)^2 U_a(t) - e^{2t-2}U_2(t)}$