4CM1 3er examen departamental Colín Ramiro Joel

1.- $\int_{-\infty}^{\infty} e^{kx^2}\cos(ax)dx = \sqrt{\frac{\pi}{k}}e^{-\frac{a^2}{4k}}$, $k>0$, a real

$\downarrow$

$$0=\int_{-a}^{a} e^{-kx^2}dx+\int_{0}^{b} e^{-k(a+iy)^2} i\,dy+\int_{a}^{-a} e^{-k(x+ib)^2}dx+\int_{b}^{0} e^{-k(-a+iy)^2} i\,dy$$

$\downarrow$

$$\int_{-a}^{a} e^{-kx^2}dx - e^{b^2}\int_{-a}^{a} e^{-kx^2}(\cos 2bx - i\,\text{sen}\,2bx)dx - i e^{-a}\int_{0}^{b} e^{ky^2}(e^{2ay}-e^{-2ay})dy$$

$$=\int_{-a}^{a} e^{-kx^2}dx - e^{b^2}\int_{-a}^{a} e^{-kx^2}\cos 2bx\,dx + 2e^{-a^2}\int_{0}^{b} e^{ky^2}\text{sen}\,2ay\,dy$$

$\downarrow$

si se anulan las integrales:

$$\int_{-\infty}^{\infty} e^{-kx^2}dx = \int_{0}^{2\pi}\int_{0}^{\infty} e^{-kr^2} r\,dr\,d\theta = \frac{\pi}{k}$$

$\downarrow$

Se regresa a la integral:

$$\int_{-\infty}^{\infty} e^{-kx^2}\cos ax\,dx = e^{b^2}\int_{-\infty}^{\infty} e^{kx^2}dx = \sqrt{\frac{\pi}{k}}\,e^{-\frac{a^2}{4k}}$$

, donde $k=1$, $a=2b$ para el ejercicio original.

2.- Sea $z(t) = 2e^{it} + 1, \; 0 \leq t \leq 2\pi$ evalue: $\int_c \frac{\text{sen}\, z}{z^2 - z} dz$

$$\int \frac{\text{Sen}\, z}{z(z-1)} dz \rightarrow \oint \frac{f(z)\, dz}{(z-z_0)^{n!}} = \frac{2\pi i}{n!} f(z_0)$$

$$\downarrow$$

$$\frac{A}{z} + \frac{B}{z-1} = \text{sen}\, z \rightarrow \text{sen}\, z \; (A(z-1)) + B(z), \text{ si } B=0$$

$A = -\text{Sen}\, z$

si $A = 1$

$B = \frac{\text{sen}\, z}{z}$

$$\rightarrow -\text{Sen}\, z \int \frac{\text{Sen}\, z}{z} + \frac{\text{sen}\, z}{z} \int \frac{\text{sen}\, z}{z-1}$$

$$\rightarrow \frac{2\pi i}{n!} \left[f^{n}(z) \right]_{z \to 1}$$

$$= \frac{2\pi i}{1!} \left[f'(\cos z) \right]_{z \to 1}$$

$$\downarrow$$

$$\therefore = 2\pi i\, \text{sen}(1)$$

3.- Desarrolle en serie de Taylor alrededor de $z_0 = i$ la siguiente función:

$$f(z) = \frac{1}{1-z}$$

Serie de Taylor

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$= \frac{1}{2} + i\frac{1}{2} + \frac{\frac{d}{dz}\left(\frac{1}{1-z}\right)i}{1!}(z-i) + \frac{\frac{d^2}{dz^2}\left(\frac{1}{1-z}\right)i}{2!}(z-i)^2 + \frac{\frac{d^3}{z^3}\left(\frac{1}{1-z}\right)i}{3!}(z-$$

Evaluando las derivadas

$$\frac{1}{2} + i\frac{1}{2} + \frac{\frac{i}{2}}{1!}(z-i) + \frac{-\frac{1}{2}+i\frac{1}{2}}{2!}(z-i)^2 + \frac{-\frac{3}{2}}{3!}(z-i)^3 + \frac{\frac{24}{(1-i)^5}}{4!}(z-i)^4 +$$

$\downarrow$

$$\frac{1}{2} + i\frac{1}{2} + \frac{i}{2}(z-i) + \left(-\frac{1}{4} + i\frac{1}{4}\right)(z-i)^2 - \frac{1}{4}(z-i)^3 + \frac{1}{(1-i)^5}(z-i)^4 + \dots$$

4.- Desarrolle en serie de Fourier la siguiente función:

$$f(x)=\begin{cases} 4 & , \quad -\pi<x<0 \\ \pi-x & , \quad 0\leq x<\pi \end{cases}$$

$L=\pi \; ; -L=-\pi$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\,\text{sen}\left(\frac{n\pi x}{L}\right)$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx \quad , \quad a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\,\text{sen}\left(\frac{n\pi x}{L}\right)dx$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{0} 4\,dx+\frac{1}{\pi}\int_{0}^{\pi} \pi-x\,dx = 4+\frac{\pi}{2}=\frac{4\pi}{2}=2\pi$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{0} 4\cos\left(\frac{n\pi x}{\pi}\right)dx+\frac{1}{\pi}\int_{0}^{\pi} \pi-x\cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$=\frac{4\,\text{sen}(\pi n)}{\pi n}+\frac{\pi^2 n^2-\pi n\,\text{sen}(\pi n)-\cos(\pi n)+1}{\pi n^2}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{0} 4\,\text{sen}\left(\frac{n\pi x}{\pi}\right)dx+\frac{1}{\pi}\int_{0}^{\pi} \pi-x\,\text{sen}\left(\frac{n\pi x}{\pi}\right)dx$$

$$=\frac{-4+4\cos(\pi n)}{\pi n}+\frac{\pi^2 n^2+\pi n\cos(\pi n)-\text{sen}(\pi n)}{\pi n^2}$$

$$f(x)=\frac{2\pi}{2}+\sum_{n=1}^{\infty}\left[\frac{4n\,\text{sen}(\pi n)+\pi^2 n^2-\pi n\,\text{sen}(\pi n)-\cos(\pi n)+1}{\pi n^2}\left(\cos\frac{n\pi x}{\pi}\right)+\right.$$

$$\frac{-4n+4n\cos(\pi n)+\pi^2 n^2-\pi n\cos(\pi n)-\text{sen}(\pi n)}{\pi n^2}\left(\text{sen}\frac{n\pi x}{\pi}\right)$$