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Tarea # 1

Lista 1 de ejercicios

Pag 8) Encuentre la suma, diferencia, producto y cociente de cada par de N.C

9 - $3-2i,\ 4+i$

$z_1 = 3-2i \ ; \ z_2 = 4+i$

Suma

$z_1 + z_2 = (x_1, iy_1) + (x_2, iy_2) = (x_1 + x_2, i(y_1 + y_2))$

$(3-2i)+(4+i) = \underline{(7-i)}$

Diferencia

$z_1 - z_2 = (x_1, iy_1) - (x_2, iy_2) = (x_1 - x_2, i(y_1 - y_2))$

$(3-2i)-(4+i) = (3-2i-4-i) = \underline{-1-3i}$

Producto

$z_1 \cdot z_2 = (x_1 x_2 - iy_1 iy_2), (x_1 iy_2 + iy_1 x_2)$

$(3-2i)(4+i) = (12+2)+(3-8)i = \underline{14-5i}$

Cociente

$\frac{z_1}{z_2} = z_1\left(\frac{1}{z_2}\right) = z_1 z_2^{-1} \longrightarrow z_2^{-1} = \frac{1}{z_2 z_2^*} z_2^* \quad ; z_2^* = 4-i$

$z_2^{-1} = \frac{1}{(4+i)(4-i)}(4-i) = \frac{1}{4^2+1^2}(4-i)$

$\frac{z_1}{z_2} = \frac{(3-2i)(4-i)}{4^2+1^2} = \frac{12-2}{4^2+1^2} + \frac{-8i-3i}{4^2+1^2} = \frac{12-2}{4^2+1^2} + \frac{-8-3}{4^2+1^2} i$

$= \underline{\frac{10}{17} + \frac{-11}{17} i}$

11.- $4+5i,\ 1-i$

$z_1 = 4+5i\ ;\ z_2 = 1-i$

Suma

$z_1 + z_2 = (4+5i) + (1-i) = 5+4i$

Diferencia

$z_1 - z_2 = (4+5i) - (1-i) = (4+5i-1+i) = 3+6i$

Producto

$z_1 \cdot z_2 = (4+5i)\cdot(1-i) = (4+5) + (-4+5)i = 9+i$

Cociente

$$\frac{z_1}{z_2} = \frac{(4+5i)(1+i)}{1^2+1^2} = \frac{4-5}{1^2+1^2} + \frac{4i+5i}{1^2+1^2} = \frac{-1}{2} + \frac{9}{2}i$$

Escribir el número dado en la forma $x+iy$

17.- $\frac{2+i}{3-i} - \frac{4+i}{1+2i}$ $\qquad \frac{a+bi}{c+di} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

$$\frac{2+i}{3-i} = \frac{(2(3)+1(-1)) + (1(3)-2(-1))i}{3^2+(-1)^2} = \frac{(6-1)+(3+2)i}{9+1} = \frac{5+5i}{10} = \frac{5(1+i)}{2(5)} = \frac{1}{2}+\frac{1}{2}i$$

$$\frac{4+i}{1+2i} = \frac{(4(1)+1(2)) + (1(1)-4(2))i}{1^2+2^2} = \frac{(4+2)+(1-8)i}{1+4} = \frac{6-7i}{5} = \frac{6}{5}-\frac{7}{5}i$$

$$\left(\frac{1}{2}+\frac{1}{2}i\right) - \left(\frac{6}{5}-\frac{7}{5}i\right) = \left(\frac{1}{2}+\frac{1}{2}i\right) + \left(-\frac{6}{5}+\frac{7}{5}i\right) = \left(\frac{1}{2}-\frac{6}{5},\ \frac{1}{2}i+\frac{7}{5}i\right)$$

$$= \left(\frac{5-12}{10},\ \frac{5+14}{10}i\right) = -\frac{7}{10} + \frac{19}{10}i$$

18.- $\frac{3+2i}{1+i} + \frac{5-2i}{-1+i}$

$$\frac{3+2i}{1+i} = \frac{(3(1)+2(1)) + (2(1)-3(1))i}{1^2+1^2} = \frac{(3+2)+(2-3)i}{2} = \frac{5-i}{2} = \frac{5}{2}-\frac{1}{2}i$$

$$\frac{5-2i}{-1+i} = \frac{(5(-1)-2(1)) + (-2(-1)-5(1))i}{-1^2+1^2} = \frac{(-5-2)+(2-5)i}{2} = \frac{-7-3i}{2} = -\frac{7}{2}-\frac{3}{2}i$$

$$\left(\frac{5}{2}-\frac{1}{2}i\right) + \left(-\frac{7}{2}-\frac{3}{2}i\right) = \left(\frac{5}{2}-\frac{7}{2},\ -\frac{1}{2}i-\frac{3}{2}i\right) = -1-2i$$

26.- Pruebe el teorema binomial para números complejos

$$(z_1+z_2)^n = z_1^n + \binom{n}{1} z_1^{n-1} z_2 + \binom{n}{2} z_1^{n-2} z_2^2 + \dots + z_2^n,$$

donde n es un entero positivo y $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

$n=1$

$$(z_1+z_2)^1 = z_1^{(1)} + \binom{1}{1} z_1^{(1-1)} z_2$$

$$= z_1 + 1(z_1)^0 z_2 \longrightarrow z_1+z_2$$

$$(z_1+z_2)^n = \binom{n}{0} z_1^n + \binom{n}{1} z_1^{n-1} z_2 + \binom{n}{2} z_1^{n-2} z_2^2 + \dots + \binom{n}{k} z_2^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} z_1^k z_2^{n-k}$$

$$= \sum_{k=0}^{n} \binom{n}{k} z_1^{k+1} z_2^{m-k} + \sum \binom{m}{k} z_1^k \cdot z_2^{m-k+1}$$

$$= \sum_{k=0}^{m} \binom{m}{k-1} z_1^{k} z_2^{m-k+1} + \binom{m}{m} z_1^k z_2^0 + \sum_{k=1}^{m} \binom{m}{k} z_1^k z_2^{m-k} + \binom{m}{0} z_1 z_2$$

$$= \sum_{k=0}^{n} \binom{m+1}{k} z_1^k z_2^{m-k+1} + \binom{m+1}{0} z_1^{m+1} z_2^k + \binom{m+1}{n+1} z_1^0 z_2^{m+1}$$

$$= \sum_{k=0}^{m+1} \binom{m+1}{k} z_1^k z_2^{m-k}$$

30.- Muestre que la sustitución $w = z + p/3$ reduce la ecuación cúbica general a una ecuación de la forma:

$$w^3 + aw + b = 0$$

$$z = (w+\alpha) \longleftrightarrow sz^3 + pz^2 + qz + r = 0$$

$$s(w+\alpha)^3 + p(w+\alpha)^2 + q(w+\alpha) + r = 0$$

$$sw^3 + 3\alpha w + 3\alpha^2 w + \alpha^3 + p(w^2 + 2\alpha w + \alpha^2) + q(w+\alpha) + r = 0$$

$$sw^3 + w^2(s3\alpha + p) + w(3\alpha^2 s + 2\alpha p + q) + s\alpha^3 + \alpha^2 p + q\alpha + r = 0$$

$$s3\alpha + p = 0 \longrightarrow s3\alpha = -p \qquad \alpha = \frac{-p}{3s}$$

$$z = w + \left(\frac{-p}{3\alpha}\right)$$

$$z = w - \frac{p}{3\alpha}$$

$$a = \frac{3sq - p}{3s^2}, \quad b = \frac{2p^2 - 9sp + 27s^2 d}{27s^3}$$

$$w^3 + aw + b = 0$$

Pag 18) Encuentra el valor absoluto, el argumento y la representación polar de los números complejos dados.

9.- $5+2i$

$z=5+2i$; $|z|=\sqrt{5^2+2^2}$ $\qquad \theta=\tan^{-1}\left(\frac{Im\,z}{Re\,z}\right)$

$z=|z|[\text{Cos}(\arg z)+i\,\text{Sen}(\arg z)]$ $\qquad \theta=\tan^{-1}\left(\frac{2}{5}\right)=0.38\pi$

$z=\sqrt{29}\left[\text{Cos}\left(\tan^{-1}\left(\frac{2}{5}\right)\right)+i\,\text{Sen}\left(\tan^{-1}\left(\frac{2}{5}\right)\right)\right]$

$z\approx 5.38[\text{Cos}(0.38)+i\,\text{Sen}(0.38)]$

Use el teorema de Moivre para expresar cada número en la forma $x+iy$, donde x y Y son reales

12.- $(-1-i)^{36}$

$z=-1-i$

$z^{36}=\left|\sqrt{-1^2+1^2}\right|^{36}\left[\text{Cos}\,36\left(\tan^{-1}\left(\frac{-1}{-1}\right)\right)+i\,\text{Sen}\,36\left(\tan^{-1}\left(\frac{-1}{-1}\right)\right)\right]$

$z^{36}=(2^{1/2})^{36}\left[\text{Cos}\,36\left(\frac{-\pi}{4}\right)+i\,\text{Sen}\,36\left(\frac{-\pi}{4}\right)\right]$

$z^{36}=2^{18}\left[\text{Cos}\,\frac{36}{4}\pi+i\,\text{Sen}\,\frac{36\pi}{4}\right]$ $\qquad \arg z=\frac{-4\pi}{4}+2k\pi$

$\frac{\frac{36}{4}\pi}{2\pi}=\frac{36}{8}\Rightarrow$ $36=8\cdot4+4$

$\frac{36\pi}{4}=2\pi\cdot4+\frac{4}{4}\pi\Rightarrow k=4$

$8\overline{)36}$ 4

$z=2^{18}\left[\text{Cos}\left(\frac{36}{4}\pi\right)+i\,\text{Sen}\,\frac{36}{4}\pi\right]=2^{18}(-1+i(0))$

$=-2^{18}$

13.- $(2+2i)^{12}$

$z=2+2i$

$z^{12}=\left|\sqrt{2^2+2^2}\right|^{12}\left[\text{Cos}\,12\left(\tan^{-1}\left(\frac{2}{2}\right)\right)+i\,\text{Sen}\,12\left(\tan^{-1}\left(\frac{2}{2}\right)\right)\right]$

$z^{12}=|2\sqrt{2}|^{12}\left[\text{Cos}\,12\left(\frac{\pi}{4}\right)+i\,\text{Sen}\,12\left(\frac{\pi}{4}\right)\right]$

$z^{12}=|2\sqrt{2}|^{12}\left[\text{Cos}\,\frac{12}{4}\pi+i\,\text{Sen}\,\frac{12}{4}\pi\right]$

$=2^{12}[\text{Cos}\,3\pi+i\,\text{sen}(3\pi)]$

$=2^{12}(-1+0)$

$=-2^{12}$

Encuentre todas las soluciones de las ecuaciones señaladas

19.- $-z^2 = \sqrt{3} + i$

$|z| = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$-z^2 = |z|^2 [(\cos 2\theta + i \operatorname{Sen} 2\theta) \longrightarrow -|z|^2 = -4 \longrightarrow k = 0, 1$

$z_0 = -4 (\cos \frac{\pi}{6} + 2(0)\pi) + i \operatorname{Sen}(\frac{\pi}{6} + 2(0)\pi)]$

$z_0 = -4 (\cos \frac{\pi}{6} + i \operatorname{Sen} \frac{\pi}{6}))$

$z_1 = -4 [(\cos(\frac{\pi}{6} + 2\pi) + i \operatorname{Sen}(\frac{\pi}{6} + 2\pi)]$

21.- $z^3 = 1 + \sqrt{3}\, i$

$|z|^3 = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

Teorema de Moivre

$z^3 = |z|^3 (\cos 3\theta + i \operatorname{Sen} 3\theta)$

$z^3 = 2 [\cos(\frac{\pi}{3} + 2k\pi) + i \operatorname{sen}(\frac{\pi}{3} + 2k\pi)$

$|z| = 2^{1/3} \longrightarrow \theta = \frac{\pi}{9} + \frac{2k\pi}{3}, \quad k = 0, 1, 2$

$k = 0 \longrightarrow 0$

$k = 1 \longrightarrow \frac{6}{9}\pi$

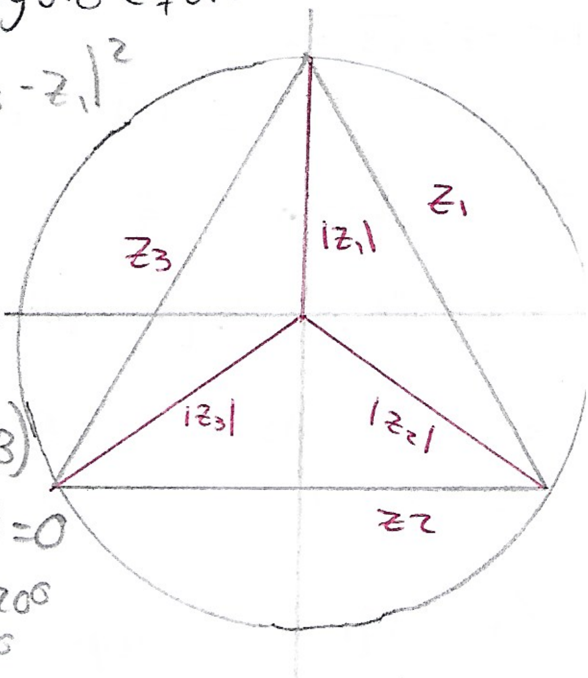
$k = 2 \longrightarrow \frac{12}{9}\pi$

$z_0 = 2^{1/3} [\cos(\frac{\pi}{9}) + i \operatorname{Sen}(\frac{\pi}{9})]$

$z_1 = 2^{1/3} [\cos(\frac{7}{9}\pi) + i \operatorname{Sen}(\frac{7}{9}\pi)]$

$z_2 = 2^{1/3} [\cos(\frac{13}{9}\pi) + i \operatorname{Sen}(\frac{13}{9}\pi)]$

29.- Demuestre que, si $|z_1| = |z_2| = |z_3|$ y $z_1 + z_2 + z_3 = 0$, entonces z_1, z_2, z_3 son los vértices de un triángulo equilátero



Sug. Mueste que $|z_1 - z_2|^2 = |z_2 - z_3|^2 = |z_3 - z_1|^2$

$\downarrow$

$|z_1| = |z_2| = |z_3| = r$

Se supone que el primer N.C, se encuentra en el eje (z_2 y z_3)

$\downarrow$

$z_1 = r$, $z_2 = r(\cos\alpha + i\,\mathrm{sen}\,\alpha)$, $z_3 = r((\cos\beta + i\,\mathrm{sen}\,\beta)$

$\therefore z_1 + z_2 + z_3 = r(1 + \cos\alpha + \cos\beta) + ir(\mathrm{sen}\,\alpha + \mathrm{sen}\,\beta) = 0$

$\cos\alpha + \cos\beta = 1 \ldots$ ① $\longrightarrow \alpha = -\beta \longrightarrow \cos\alpha = -120°$

$\mathrm{sen}\,\alpha + \mathrm{sen}\,\beta = 0 \ldots$ ② $\longrightarrow \beta = 180° - \alpha \longrightarrow \beta = 120°$

$\therefore$ se demuestra que si $|z_1| = |z_2| = |z_3|$ y $z_1 + z_2 + z_3 = 0$ son vértices de un triángulo equilatero

37.- Muestre que si z_0 es una raiz del polinomio $P(z)$ con coeficientes reales, entonces $\overline{z_0}$ es también una raiz de $P(z)$.

Se tiene $z_0 = x + iy \longrightarrow \overline{z_0} = x - iy$

$$P(z) = \left.\begin{aligned} a_1 &= \frac{-b + \sqrt{-\Delta i}}{2a} \\ a_2 &= \frac{-b - \sqrt{-\Delta i}}{2a} \end{aligned}\right\} P(x) = \sum_{i=0}^{n} a_i x^i \in C[x]$$

$\downarrow$

$$\overline{P(x)} = \sum_{i=0}^{n} \overline{a_i}\, x^i$$

Si se aplica

$\downarrow$

$\overline{z+w} = \overline{z} + \overline{w}$, $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$ $\longrightarrow$ Si $P, Q \in C[x]$ son polinomios

$\overline{P+Q} = \overline{P} + \overline{Q}$, $\overline{P \cdot Q} = \overline{P} \cdot \overline{Q}$

$\overline{P}(z) = \overline{P}(\overline{z})$

$\therefore$ Si $\overline{P} = P$ entonces $\overline{P(z)} = P(\overline{z})$

Se demuestra que si $P(z) = 0$, se tiene $P(\overline{z}) = 0$

$= P(z)$ con coef reales, presentan raices conjugadas $[z, \overline{z}]$

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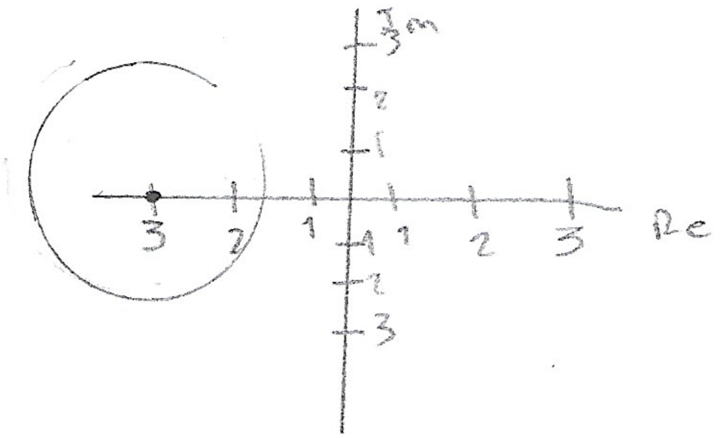
1.- $|z+3|<2$ $\qquad z = x + iy$

$= |x+iy+3|<2 \longrightarrow Q = \sqrt{(x-x_0)^2+(y-y_0)^2}$

$= |(x+3)^2 + i(y+0)|<2$

$= \sqrt{(x+3)^2+(y+0)^2}<2$

$O \longrightarrow (-3,0)$



2.- $|Re\ z|<1$ $\qquad z = x+iy \quad , \quad Re\ z = x$

$= |Re\ z| = \sqrt{x^2} = |x| \Leftarrow$ Módulo de x

$|x|<1 \Rightarrow -1<x<1$

∴ Al solo poseer puntos interiores, es un conj. Abierto