Tarea 4.

22- a) $A=(r,1,-2)$ $B=(1,3,-1)$ $AB^T=0$

$$B^T=\begin{pmatrix}1\\3\\-1\end{pmatrix}$$

$$AB^T=(r,1,-2)\begin{pmatrix}1\\3\\-1\end{pmatrix}$$

$$AB^T=r+3+2=0$$

$$r+5=0$$

$$r=-5$$

b) $A=(1,r,1)$ $B=(-2,2,5)$ $AB^T=0$

$$B^T=\begin{pmatrix}-2\\2\\5\end{pmatrix} \quad AB^T=(1,r,1)\begin{pmatrix}-2\\2\\5\end{pmatrix}$$

$$AB^T=(-2+2r+5)$$

$$AB^T=2r+3=0$$

$$2r=-3$$

$$r=-3/2$$

$$A = 0_{m\times n} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

24- $AA^T = 0_{m\times n} \therefore A = 0_{m\times n}$

Sea $\underset{A_{m\times n}}{\underline{A_{3\times 2}}}$ tal que $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$ y $A^T = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{pmatrix}$

Entonces

$$AA^T = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{pmatrix} = \begin{pmatrix} a_{11}^2 + a_{12}^2 & a_{11}a_{21} + a_{12}a_{22} & a_{11}a_{31} + a_{12}a_{32} \\ a_{21}a_{11} + a_{22}a_{12} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{22}a_{32} \\ a_{31}a_{11} + a_{32}a_{12} & a_{31}a_{21} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0a_{21} + 0a_{22} & 0a_{31} + 0a_{32} \\ 0a_{11} + 0a_{12} & 0 & 0a_{31} + 0a_{32} \\ 0a_{11} + 0a_{12} & 0a_{21} + 0a_{22} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \underline{0_{m\times n}}$$

26- i) $\underset{2\times 2}{\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}}\underset{2\times 2}{\begin{pmatrix} 4 & 1 \\ 0 & 6 \end{pmatrix}} = \begin{pmatrix} 2(4)+3(0) & 2(1)+3(6) \\ -1(4)+2(0) & -1(1)+2(6) \end{pmatrix}$

$$= \begin{pmatrix} 8+0 & 2+18 \\ -4+0 & -1+12 \end{pmatrix} = \underline{\begin{pmatrix} 8 & 20 \\ -4 & 11 \end{pmatrix}}$$

ii) $\begin{pmatrix} 1 & 6 \\ 0 & 4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 7 & 1 & 4 \\ 2 & -3 & 5 \end{pmatrix} =$

$$\begin{pmatrix} 1(7)+6(2) & 1(1)+6(-3) & 1(4)+6(5) \\ 0(7)+4(7) & 0(1)+4(-3) & 0(4)+4(5) \\ -2(7)+3(2) & -2(1)+3(-3) & -2(4)+3(5) \end{pmatrix} = \begin{pmatrix} 7+12 & 1-18 & 4+30 \\ 0+8 & 0-12 & 0+20 \\ -14+6 & -2-9 & -8+15 \end{pmatrix}$$

$$= \underline{\begin{pmatrix} 19 & -17 & 34 \\ 8 & -12 & 20 \\ -8 & -11 & 7 \end{pmatrix}}$$

$3\times3 \qquad 3\times3$

iii) $\begin{pmatrix} 1 & 4 & 6 \\ -2 & 3 & 5 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & -3 & 5 \\ 1 & 0 & 6 \\ 2 & 3 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} 1(2)+4(1)+6(2) & 1(-3)+4(0)+6(3) & 1(5)+4(6)+6(1) \\ -2(2)+3(1)+5(2) & -2(-3)+3(0)+5(3) & -2(5)+3(6)+5(1) \\ 1(2)+0(1)+4(2) & 1(-3)+0(0)+4(3) & 1(5)+0(6)+4(1) \end{pmatrix}$$

$$= \begin{pmatrix} 2+4+12 & -3+0+18 & 5+24+6 \\ -4+3+10 & 6+0+15 & -10+18+5 \\ 2+0+8 & -3+0+12 & 5+0+4 \end{pmatrix} = \begin{pmatrix} 18 & 15 & 35 \\ 9 & 21 & 13 \\ 10 & 9 & 9 \end{pmatrix}$$

25: $A\begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} b & 2a+3b \\ d & 2c+3d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$b = 1$

$2a+3b = 0 \longrightarrow 2a+3(1)=0 \qquad 2c+3(0)=1$

$d = 0 \qquad 2a=-3 \qquad 2c+0=1$

$2c+3d=1 \qquad a=-3/2 \qquad 2c=1$

$c=1/2$

$\therefore$

$A = \begin{pmatrix} -3/2 & 1 \\ 1/2 & 0 \end{pmatrix}$

Scribe

$u \cdot v = 0$

30.- a) $(2,-3)$, $(3,2)$

$(2(3)+(-3(2)) = (6-6) = 0$

Si es ortogonal

b) $(1,4,-7)$, $(2,3,2)$

$(1(2)+4(3)+(-7(2))) = (2+12-14) = 0$

Si es ortogonal

c) $(1,0,1,0)$, $(0,1,0,1)$

$(1(0)+0(1)+1(0)+0(1)) = 0+0+0+0 = 0$

Si es ortogonal

d) $(a,0,b,0,c)$, $(0,d,0,e,0)$

$(a(0)+0(d)+b(0)+0(e)+c(0))$

$= 0+0+0+0+0 = 0$

Si es ortogonal

32.- $A = \begin{pmatrix} 1 & -3 & 0 \\ 4 & 5 & 1 \\ 3 & 8 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 & -2 \\ 3 & 0 & 4 \\ -1 & 3 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 0 & -2 \\ 4 & 7 & -5 \\ 1 & 0 & -1 \end{pmatrix}$

$D = 2(AB) + C^2$ a) d_{11} b) d_{21} c) d_{32}

a) $AB_{11} = 1(1) + (-3)(3) + (0)(-1) = -8$

$2AB_{11} = -8(2) = -16$

$C^2_{11} = C \cdot C = 2(2) + 0(4) + (-2)(1) = 2$ $D_{11} = 2(AB) + C^2$

$D_{11} = -16 + 2 = \underline{-14}$

b) $AB_{21} = 4(1) + 5(3) + 1(-1) = 18$

$2AB_{21} = 18(2) = 36$

$C^2_{21} = C \cdot C = 4(2) + 7(4) + (-5)(1) = 31$ $D_{21} = 2(AB) + C^2$

$D_{21} = 36 + 31 = \underline{67}$

c) $AB_{32} = 3(1) + 8(0) + 0(3) = 3$

$2AB_{32} = 3(2) = 6$

$C^2_{32} = C\,C = 1(0) + 0(7) - 1(0) = 0$ $D_{32} = 2(AB) + C^2$

$D_{32} = 6 + 0 = \underline{6}$

34.- Sean A, B Matrices tales que $A=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ y $B=\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}$

y se sabe que $AB = BA$

$(AB)^n$

$\downarrow$

$$= \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}^n = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^n \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^n$$

$$\therefore (AB)^n = A^n B^n$$

36.- Sean $A, B \in M_{n \times n}$ tales que: $A=\begin{pmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & a_{nn} \end{pmatrix}$ y

$B=\begin{pmatrix} b_{11} & 0 & 0 & \cdots & 0 \\ 0 & b_{22} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & b_{nn} \end{pmatrix}$

$$AB=\begin{pmatrix} a_{11}b_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22}b_{22} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & a_{nn}b_{nn} \end{pmatrix} = \begin{pmatrix} b_{11}a_{11} & 0 & 0 & \cdots & 0 \\ 0 & b_{22}a_{22} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & b_{nn}a_{nn} \end{pmatrix} = BA$$

$$\therefore$$

$$AB = BA$$