$$\begin{vmatrix} -14 & -8 & 9 \\ -6 & -6 & 6 \\ 4 & 4 & -3 \end{vmatrix}$$

Tarea 5.

38.- $A = \begin{vmatrix} -1 & 1 & 0 \\ 2 & 1 & 4 \\ 1 & 5 & 6 \end{vmatrix}$ $|A| = ?$ a) cofactores fila 2 b) pivotal c) Sarrus

$B_{11} = (-1)^2 \begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix} = 6-20 = -14$

$B_{12} = (-1)^3 \begin{vmatrix} 1 & 0 \\ 5 & 6 \end{vmatrix} = 6-0 = -6$

$B_{13} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} = 4-0 = 4$

$B_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = 10-1 = 9$

$B_{32} = (-1)^5 \begin{vmatrix} -1 & 1 \\ 1 & 5 \end{vmatrix} = -5-1 = 6$

$B_{21} = (-1)^3 \begin{vmatrix} 2 & 4 \\ 1 & 6 \end{vmatrix} = 12-4 = -8$

$B_{22} = (-1)^4 \begin{vmatrix} -1 & 0 \\ 1 & 6 \end{vmatrix} = -6-0 = -6$

$B_{23} = (-1)^5 \begin{vmatrix} -1 & 0 \\ 2 & 4 \end{vmatrix} = -4-0 = 4$

$B_{33} = (-1)^6 \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = -1-2 = -3$

$\det(A) = a_{12}A_{12} + a_{22}A_{22} + a_{23}A_{23}$
$= 2(-6) + 1(-6) + 4(6) = -12-6+24 = 6$ a)

$$\begin{vmatrix} -1 & 1 & 0 \\ 2 & 1 & 4 \\ 1 & 5 & 6 \end{vmatrix} = (-1)\left(\frac{1}{-1}\right)^1 \begin{vmatrix} \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 2 & 4 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 1 & 6 \end{vmatrix} \end{vmatrix}$$

b)

$= (-1) \begin{vmatrix} -3 & -4 \\ -6 & -6 \end{vmatrix}$ $18-24 = -6(-1) = 6$

$$\begin{vmatrix} -1 & 1 & 0 \\ 2 & 1 & 4 \\ 1 & 5 & 6 \end{vmatrix} \begin{matrix} -1 & 1 \\ 2 & 1 \\ 1 & 5 \end{matrix} = -1(1)(6) + 1(4)(1) + 0(2)(5) - 1(2)(6) + 1(4)(5) - 0(1)(1)$$

$= -6 + 4 + 0 - 12 + 20 - 0$

c) $= 6$

39.- Sean A y B matrices $M_{n\times n}$ tales que:

$$A = \begin{vmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_n \end{vmatrix} \quad y \quad B = \begin{vmatrix} b_1 & 0 & \cdots & 0 \\ 0 & b_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & b_n \end{vmatrix}$$

$$|A| = \prod_{i=1}^{n} a_i \qquad |B| = \prod_{i=1}^{n} b_i$$

$$AB = \begin{pmatrix} a_1 b_1 & 0 & \cdots & 0 \\ 0 & a_2 b_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_n b_n \end{pmatrix} \quad |AB| = \prod_{i=1}^{n} a_i b_i$$

$$|A||B| = \left(\prod_{i=1}^{n} a_i\right)\left(\prod_{i=1}^{n} b_i\right) = \prod_{i=1}^{n} a_i b_i = |AB|$$

40.- $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 8$

a) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2(8) = \underline{16}$

b) $\begin{vmatrix} -3a_{11} & -3a_{12} & -3a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 5a_{31} & 5a_{32} & 5a_{33} \end{vmatrix} = \begin{matrix} -3 \\ 2 \\ 5 \end{matrix}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -3(2)(5)(8) = \underline{-240}$

c) $\begin{vmatrix} 2a_{11}-3a_{21} & 2a_{12}-3a_{22} & 2a_{13}-3a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} \qquad 2-3\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{33} & a_{33} \end{vmatrix}$

$2\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} 8 = 2(8) = 16 \qquad 16-24 = -8 \qquad (2-3)(8) = -8$

$-3\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} -3(8) = -24 \qquad = -8-8 = \underline{-16}$

41-a) $C_2 \leftrightarrow C_1$ $\quad$ $R_2 \leftrightarrow R_1$

$$A=\begin{vmatrix}2&-1&0&4&1\\3&1&-1&2&0\\3&2&-2&5&1\\0&0&4&-1&6\\2&2&1&-1&1\end{vmatrix}=(-1)\begin{vmatrix}-1&2&0&4&1\\1&3&-1&2&0\\2&3&-2&5&1\\0&0&4&-1&6\\2&2&1&-1&1\end{vmatrix}\rightarrow(-1)(-1)\begin{vmatrix}1&3&-1&2&0\\-1&2&0&4&1\\2&3&-2&5&1\\0&0&4&-1&6\\2&2&1&-1&1\end{vmatrix}$$

$R_3 \cdot -\frac{5}{3}$

$$\begin{array}{l}R_2+R_1=R_2\\ \\R_3-2R_1=R_3\\ \\R_5-2R_1=R_5\end{array}\begin{vmatrix}1&3&-1&2&0\\0&5&-1&6&1\\0&-3&0&1&1\\0&0&4&-1&6\\0&-4&3&-5&1\end{vmatrix}\xrightarrow{R_2\cdot\frac{1}{5}}\begin{array}{l}\\ \\R_3+3R_2\\ \\R_5+4R_2\end{array}\begin{vmatrix}1&3&-1&2&0\\0&1&-1/5&6/5&1/5\\0&0&-3/5&23/5&8/5\\0&0&4&-1&6\\0&0&11/5&-1/5&9/5\end{vmatrix}\quad(5)$$

$$(5)\left(-\frac{3}{5}\right)\begin{array}{l}\\ \\ \\R_4-4R_3\\R_5-\frac{11}{5}R_3\end{array}\begin{vmatrix}1&3&-1&2&0\\0&1&-1/5&6/5&1/5\\0&0&1&-23/3&8/3\\0&0&0&89/3&50/3\\0&0&0&50/3&-23/3\end{vmatrix}\xrightarrow{R_4\cdot\frac{3}{89}}(5)\left(-\frac{3}{5}\right)\left(\frac{89}{3}\right)\begin{array}{l}\\ \\ \\ \\R_5-\frac{50}{3}R_4\end{array}\begin{vmatrix}1&3&-1&2&0\\0&1&-1/5&6/5&1/5\\0&0&1&-23/3&8/3\\0&0&0&1&50/89\\0&0&0&0&-151/89\end{vmatrix}$$

$$|A|=(5)\left(-\frac{3}{5}\right)\left(\frac{89}{3}\right)\left(-\frac{151}{89}\right)(1)(1)(1)(1)=151$$

b) $C_3 \leftrightarrow C_2$

$$B=\begin{vmatrix}-2&3&1\\4&6&5\\0&2&1\end{vmatrix}\longrightarrow(-1)\begin{vmatrix}1&3&-2\\5&6&4\\1&2&0\end{vmatrix}$$

$$\begin{array}{l}\\R_2-5R_1\\R_3-R_1\end{array}\begin{vmatrix}1&3&-2\\0&-9&14\\0&-1&2\end{vmatrix}(-1)\xrightarrow{R_2\cdot-\frac{1}{9}}(-1)(-9)\begin{array}{l}\\ \\R_3+R_2\end{array}\begin{vmatrix}1&3&-2\\0&1&-14/9\\0&0&4/9\end{vmatrix}$$

$$|B|=(-1)(-9)(1)(1)(4/9)=4$$

c)

$$A=\begin{vmatrix} -2 & 0 & 0 & 7 \\ 1 & 2 & -1 & 4 \\ 3 & 0 & -1 & 5 \\ 4 & 2 & 3 & 0 \end{vmatrix}$$

$R_2 \leftrightarrow R_1$

$$(-1)\begin{array}{r} \\ R_2+2R_1 \\ R_3-3R_1 \\ R_4-4R_1 \end{array}\begin{vmatrix} 1 & 2 & -1 & 4 \\ -2 & 0 & 0 & 7 \\ 3 & 0 & -1 & 5 \\ 4 & 2 & 3 & 0 \end{vmatrix}$$

$$(-1)(4)\begin{vmatrix} 1 & 2 & -1 & 4 \\ 0 & 4 & -2 & 15 \\ 0 & -6 & 2 & -7 \\ 0 & -6 & 7 & -16 \end{vmatrix}$$

$R_2 \cdot \frac{1}{4}$

$$(-1)(4)\begin{array}{r} \\ \\ R_3+6R_2 \\ R_4+6R_2 \end{array}\begin{vmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & -1/2 & 15/4 \\ 0 & 0 & -1 & 31/2 \\ 0 & 0 & 4 & 13/2 \end{vmatrix}$$

$R_3 \cdot (-1)$

$$(-1)(4)(-1)\begin{array}{r} \\ \\ \\ R_4-4R_3 \end{array}\begin{vmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & -1/2 & 15/4 \\ 0 & 0 & 1 & -31/2 \\ 0 & 0 & 0 & 137/2 \end{vmatrix} \rightarrow |A| = (-1)(4)(-1)(1)(1)(1)(137/2) = 274$$

d)

$$A=\begin{vmatrix} 4 & 2 & 3 & -4 \\ 3 & -2 & 1 & 5 \\ -2 & 0 & 1 & -3 \\ 8 & -2 & 6 & 4 \end{vmatrix}$$

$R_1 \cdot 1/4$

$$(\;\;)\begin{array}{r} \\ R_2-3R_1 \\ R_3+2R_1 \\ R_4-8R_1 \end{array}\begin{vmatrix} 1 & 1/2 & 3/4 & -1 \\ 0 & -7/2 & -5/4 & 8 \\ 0 & 1 & 5/2 & -5 \\ 0 & -6 & 0 & 12 \end{vmatrix}$$

$R_2 \cdot -\frac{2}{7}$

$$(4)(-\tfrac{7}{2})\begin{array}{r} \\ \\ R_3-R_2 \\ R_4+6R_2 \end{array}\begin{vmatrix} 1 & 1/2 & 3/4 & -1 \\ 0 & 1 & 5/14 & -16/7 \\ 0 & 0 & 15/7 & -19/7 \\ 0 & 0 & 15/7 & -17/7 \end{vmatrix}$$

$$(4)(-7/2)\begin{array}{r} \\ \\ \\ R_4-R_3 \end{array}\begin{vmatrix} 1 & 1/2 & 3/4 & -1 \\ 0 & 1 & 5/14 & -16/7 \\ 0 & 0 & 15/7 & -19/7 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$|A| = (4)(-7/2)(1)(1)(15/7)(1) = -30$$

A B

42- $\begin{vmatrix} 2 & 1 & 0 \\ 0 & -1 & 3 \\ 0 & 1 & a \end{vmatrix} + \begin{vmatrix} 0 & a & 1 \\ 1 & 3a & 0 \\ -2 & a & 2 \end{vmatrix} = 14$

$R_2 \leftrightarrow R_1$ y $C_2 \leftrightarrow C_3$

$C = \begin{vmatrix} 2 & a+1 & 1 \\ 1 & 3a-1 & 3 \\ -2 & a+1 & a+2 \end{vmatrix} = 14 \rightarrow (-1)(-1)\begin{vmatrix} 1 & 3 & 3a-1 \\ 2 & 1 & a+1 \\ -2 & a+2 & a+1 \end{vmatrix}$

$\begin{array}{l} \\ R_2-2R_1 \\ R_3+2R_1 \end{array} \begin{vmatrix} 1 & 3 & 3a-1 \\ 0 & -5 & -5a+3 \\ 0 & a+8 & 7a-1 \end{vmatrix} R_2(-\tfrac{1}{5})\ (-5) \begin{vmatrix} 1 & 3 & 3a-1 \\ 0 & 1 & \frac{5a-3}{5} \\ 0 & 0 & \frac{-5a^2-2a+19}{5} \end{vmatrix}$ $R_3-(a+8)R_2$

$|C| = (-5)(1)(1)\left(\frac{-5a^2-2a+19}{5}\right) \rightarrow -\frac{14(5)}{5} = -5a^2-2a+19$

$-14 = -5a^2-2a+19$

$-14-19 = -5a^2-2a$

$-33 = (-5a-2)a \rightarrow a = \frac{-33}{-5a-2}$

43- Sean A y B matrices $M_{n \times n}$ tales que $A = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix}$ y

$B = \begin{vmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} \cdots & 0 \\ \vdots & & \vdots \\ 0 & 0 \cdots & b_{nn} \end{vmatrix}$

$|A+B| = \begin{vmatrix} a_{11}+b_{11} & 0 & 0 \\ 0 & a_{22}+b_{22} \cdots & 0 \\ 0 & 0 \cdots & a_{nn}b_{nn} \end{vmatrix} = (a_{11}+b_{11})(a_{22}+b_{22})(a_nb_n)$

$|A| = \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{nn} \end{vmatrix} = (a_{11})(a_{22})(a_{nn})$ y $|B| = \begin{vmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{nn} \end{vmatrix} = (b_{11})(b_{22})(b_{nn})$

$|A|+|B| = (a_{11})(a_{22})(a_{nn}) + (b_{11})(b_{22})(b_{nn})$ $\underline{|A+B| \neq |A|+|B|}$

44.- Sea A una $T_{nxn}$ tal que $\begin{vmatrix} a_{11} & x & y & z \\ 0 & a_{22} & w & u \\ 0 & 0 & a_{33} & v \\ 0 & 0 & 0 & a_{44} \end{vmatrix}$ y $\neq$

$\therefore$

$$|A| = (a_{11})(a_{12})(a_{13})(a_{14}) \neq 0$$

Es decir ningun elemento de su diagonal puede ser 0.

45.- a) $\begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix} = \begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix}$

$C_3 - C_1$ $\begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix} = \begin{vmatrix} -b & 1 & a \\ -c & 1 & b \\ -a & 1 & c \end{vmatrix}$ . $C_1 \cdot -1$ $\begin{vmatrix} b & 1 & a \\ c & 1 & b \\ a & 1 & c \end{vmatrix}$

$C_3 \Leftrightarrow C_1$ $\begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix}$

b) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ $R_1(a) \to R_1$, $R_2(b) \to R_2$, $R_3(c) \to R_3$ $\longrightarrow \frac{1}{a} \cdot \frac{1}{b} \frac{1}{c} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & bca \\ c & c^2 & cab \end{vmatrix}$ $C_1\left(\frac{1}{abc}\right)$

$abc\left(\frac{1}{abc}\right) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$ $C_1 \Leftrightarrow C_3$

$(-1) \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix}$ $C_2 \Leftrightarrow C_3$

$(-1)(-1) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$