3er examen departamental EC.

2.1 $F(p) = \frac{1}{p}\cos\frac{1}{p}$

Para $F(p) = \frac{1}{p}\cos\frac{1}{p}$, para $p=\infty$

$= \frac{1}{p}\sum_{n=0}^{\infty}\frac{\cos(n)}{p^n} = \sum_{n=0}^{\infty}\cos t\,\frac{t^n}{n!\,p^{n+1}}$

$p(t) = \sum_{n=0}^{\infty}\cos(n)\frac{t^n}{n!\,p^{n+1}} \rightarrow p(t) = \cos(n) I_\alpha \sqrt{t}$

I_α = función de Besel

2.2 $F(p) = \frac{1}{\sqrt{t}}\operatorname{sen}\frac{1}{\sqrt{p}}$

Para $F(p) = \frac{1}{\sqrt{t}}\operatorname{sen}\frac{1}{\sqrt{p}}$, para $p=\infty$

$= \frac{1}{\sqrt{p}}\sum_{n=0}^{\infty}\frac{\operatorname{sen}(n)}{\sqrt{p}} = \sum_{n=0}^{\infty}\frac{\operatorname{sen}(n)}{p} = P(t) = \sum_{n=0}^{\infty}\operatorname{sen}(n)\frac{t^n}{n!\,p}$

$f(t) = \operatorname{sen}(n) I_\alpha(t) \rightarrow I_\alpha$ = función de Besel

2.3 $F(p) = \frac{1}{(p^2+1)^2+(p^2-4)}$

$f(0) = \frac{1}{(0+1)^2(0-4)} = \frac{1}{-4} = -\frac{1}{4} + \frac{\frac{d}{dp}\left(\frac{1}{(p^2+1)(p^2-4)}\right)}{1!}p$

$+ \frac{\frac{d^2}{dp^2}\left(\frac{1}{(p^2+1)^2(p^2-4)}\right)^{(0)}}{2!}p^2 + \dots$

$\frac{d}{dp}\left(\frac{1}{(p^2+1)^2(p^2-4)}\right)^{(0)} = (9p^2-1)(p^2+1)^3(p^2-4)^2 - p$

$= \frac{1}{(p^2+1)^2(p^2-0)}\Big) = - 124\,\frac{(14p^8 - 64p^6 + 309p^4 - 919p^2 + 16p)}{(p+2)^4(p-2)^4(p^2+1)^5}$

$$= \frac{1}{4} + \frac{6}{1!} p + \frac{\frac{9}{8}}{2!} p^2 + \frac{6}{3!} p^3 + \frac{-\frac{127}{8}}{4!} p^4 + \dots$$

$$F(p) = \frac{1}{(p^2+1)^2(p^2-4)} \rightarrow (p^2)^2 + 2p^2(1+1)^2$$

$$= (p^2)^2 + 2(p^2+1) \rightarrow (p^2)^2 = p^4$$

$$= \frac{1}{(p^4+2p^2+1)(p^2-4)}$$

$$= p^4 p^2 - 4p^4 + 2p^2 p^2 - 2(4p^2+1)(p^2-1)$$

$$= p^6 - 2p^4 - p^2 - 4 \rightarrow \frac{1}{p^6 - 2p^4 - p^2 - 4}$$

2.4 $$F(p) = \frac{1}{\sqrt{1+p^2}}$$

$$f(t) = \operatorname{sen} t$$

$$\mathcal{L}(f(t)) = \frac{1}{p^2+1} \rightarrow \frac{p^2+1}{(p^2+1)^2} = \frac{-2p}{(p^2+1)^2}$$

2.5 $$F(p) = \frac{1}{2p} \ln \frac{p+1}{p-1}$$

$$= \frac{1}{2p} \sum_{n=0}^{\infty} e^{\frac{n!\, p^n + 1}{n!\, p^2 1}}$$

$$f(1) = \sum_{n=0}^{\infty} e^{\frac{n!\, t^n + 1}{n!\, t^2 1}} = L_o \frac{(s+1)^n}{(s-1)^n}$$

$$= f(A) \operatorname{res}\left(\frac{\ln pA}{(2p^2+\beta^3} ! \beta i\right) + \operatorname{res}\left(\frac{\ln pA}{(2p^2+\beta^2)^3}\right)$$

$$\Rightarrow \frac{1}{4} \lim_{p \to \beta i} \frac{d^2}{2dp^2} \left(\frac{\ln pA}{(2p+\beta i)^3}\right) = \frac{2A^2 \ln(\beta i A)}{8\beta^3 i} \longrightarrow$$

$$- \frac{3 \ln(\beta i A)}{8\beta^2} + \frac{3 \ln \beta i A}{8\beta^3 A}$$

2.8 $\dfrac{\cos \frac{1}{p}}{p}$

$= \mathcal{L}\left(\frac{1}{\sqrt{p}}\left(\frac{e^{ip}+e^{-ip}}{2}\right)\right)$

$= \frac{1}{2}\mathcal{L}\left(\frac{1}{p}\right) \rightarrow \frac{1}{2}\mathcal{L}\left(\frac{1}{\sqrt{t}}\right)\Big|_{s \to s+i}$

$= \frac{1}{2}\sqrt{\frac{\pi}{s}}\Big|_{s \to s-i} + \frac{1}{2}\sqrt{\frac{\pi}{s}}\Big|_{s \to s+i}$

$= \frac{1}{2}\frac{\sqrt{\pi}\sqrt{s+i}}{\sqrt{s-i}} + \frac{\sqrt{\pi}\sqrt{s-i}}{(s+i)} = \frac{\sqrt{\pi}}{2}\left(\frac{\sqrt{s+i}+\sqrt{s-i}}{\sqrt{s^2+1}}\right.$

$= \frac{\sqrt{2}\sqrt{\pi}}{2}\,\frac{\sqrt{s+\sqrt{s^2+1}}}{\sqrt{s^2+1}} = \sqrt{\frac{\pi}{2}}\sqrt{\frac{s+\sqrt{s^2+1}}{s^2+1}}$

2.9 $F(p) = \frac{p}{p^2+4p+5}$

$\frac{p}{p^2+4p+5} = \frac{p}{(p+4)p+5}$

$f(A) = \sum_{n=0}^{\infty} \frac{n!\,A^n}{n!\,A^n + 4n!\,A^n+5} = I_0 A_n \frac{A^n}{A^n+4A^{n}+5}$

$f(A) = \text{res}\left(\frac{A\,p^A}{((p^2)^{B}p)^3}\, A\, Bi\right) + \text{res}\left(\frac{A n p A}{(p^2+Bi)^3}\right.$

$= \frac{1}{2}\lim_{p \to Bi} \frac{d^2}{dp^2}\left(\frac{p^A}{pA^2+4p^2+5}\right)$

$= \frac{-A^2 \beta A}{16\beta^3 i} - \frac{3ABi}{32B^2} + \frac{3eBiA}{25B^3A}$

2.11 $F(p) = \frac{Q'(p)}{Q(p)}$ donde $Q(p) = (p-p_1)(p-p_2)\ldots$

$$\frac{dQ}{dA} \longrightarrow q = \frac{\frac{dQ}{dA}\left(\frac{dq}{dA}\right)}{Q\left(\frac{dq}{dA}\right)}$$

para $(p-q^n) \longrightarrow \left(\frac{q'}{q} - Q'_n\right) \longrightarrow \boxed{p = \frac{Q'}{q}}$

2.12 $F(p) = \frac{1}{(p^4-1)^2}$

$(p^4-1)^2 \Rightarrow (a^2-b^2)^2 = ((a+b)(a-b))^2$

$a = p^2 \quad b = 1 \Rightarrow ((p^2+1)(p^2-1))^2 \rightarrow (p+1)(p-1)$

$\therefore (p^4-1)^2 = ((p^2+1)(p+1)(p-1))^2 = (p^2+1)^2(p+1)^2(p-1)^2$

$$\frac{1}{(p^2+1)^2(p+1)^2(p-1)^2} = \frac{Ap+B}{(p^2+1)} + \frac{Cp+D}{(p^2+1)^2} + \frac{E}{(p+1)} + \frac{F}{(p+1)^2} + \frac{G}{(p-1)} + \frac{H}{(p-1)^2}$$

$\downarrow$

$1 = (-1^2+1)(-1+1)^2(-1-1)^2(A(-1)+B) + (-1+1)^2(-1-1)^2(C(-1)+D)$
$+ ((-1)^2+1)^2(-1+1)(-1-1)^2(E + (-1)^2(-1-1)^2 F +$
$+ ((-1)^2+1)^2(-1+1)^2(-1-1)(G + ((-1)^2+1)^2(-1+1)(H)$

$1 = (4)(4)F \longrightarrow F = 1/16$

$1 = (1^2+1)(1+1)^2(1-1)^2(A(1)+B) + (1+1)^2(1-1)^2$
$(C(1)+D) + (1+1)^2(1+1)(1-1)^2(1^2+1)^2(1-1)^2(F)$

$1 = (4)(4)H \longrightarrow H = 1/16$

$1 = Ap^7 - Ap^5 - Ap^3 + Bp^6 - Bp^4 - Bp^2 + Ap + B + Cp^5 + Dp^4 - 2Cp^3$
$-2Dp^2 + C + D + Ep^7 - Ep^6 + Ep^5 - Ep^4 - Ep^3 + Ep^2 Ep + E + \frac{p^6}{16}$
$-\frac{p^5}{8} + \frac{3p^4}{16} - \frac{p^3}{4} + \frac{3p^2}{16} - \frac{p}{8}$

$1 = p^7(A+E+6) + p^6(B-E+\frac{1}{8}+6) + p^5(-A+C+E+6) +$
$p^4(-B+D-E+\frac{3}{8}+6) + p^3(-A-2C-E-6) + p^2$
$(-B-2D+E+\frac{3}{8}-6) + p(A+C-E-6+B+D+E-6$
$+\frac{1}{8})$

$A+C-E-6=0$
$-B-2D+E-6+\frac{3}{8}=0$
$-A-2C-E-6=0$
$-B+D-E+6+\frac{3}{8}=0$
$-A+C+E+6=0$
$B-E+6+\frac{1}{8}=0$
$A+E+6=0$
$D=1/4 \longrightarrow A = -\frac{16G+3}{8} \rightarrow E = \frac{3}{16}$
$B = \frac{3}{16} - (-\frac{3}{16}) - \frac{1}{8} = \frac{1}{4}$

$= \frac{1}{4(p^2+1)} + \frac{1}{4(p^2+1)^2} + \frac{3}{16(p+1)} + \frac{1}{16(p+1)^2} - \frac{3}{16(p-1)}$
$+ \frac{1}{16(p-1)^2}$

$\frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{p^2+1}\right) + \frac{1}{4}\left(\frac{1}{(p^2+1)^2}\right) + \frac{3}{16}\mathcal{L}^{-1}\left(\frac{1}{p+1}\right) + \frac{1}{16}\mathcal{L}^{-1}\left(\frac{1}{(p+1)^2}\right.$

$= \frac{3}{8}\operatorname{sen} t - \frac{t\cos t}{8} + \frac{3}{16}(e^{-t} - e^{t}) + \frac{t}{16}(e^{-t} + e^{t})$

Si $\operatorname{senh} t = \frac{e^t - e^{-t}}{2}$ y $\cosh t = \frac{e^t + e^{-t}}{2}$

$= \frac{3}{8}(\operatorname{sen} t - \operatorname{senh} t) + \frac{t}{8}(\cosh t - \cos t)$

2.3 $F(p) = \frac{1}{\sqrt{p}} \operatorname{arctg} \frac{1}{\sqrt{p}}$

$$\frac{\operatorname{arctg}\left(\frac{\sqrt{p}}{p}\right)}{\sqrt{p}} = \frac{\operatorname{arctg}\left(\frac{1}{\sqrt{p}}\right)}{\sqrt{p}}$$

$$\frac{1}{\sqrt{p}} = \frac{\sqrt{p}}{p} \longrightarrow F(p) = \frac{\operatorname{arctg}\left(\frac{\sqrt{p}}{p}\right)}{\sqrt{p}}$$

2.14 $F(p) = \frac{p^2}{(p^4-1)^2}$

$$\frac{p^2}{(p^2+1)^2(p+1)^2(p-1)^2} = \frac{Ap+B}{(p^2+1)} + \frac{Cp+D}{(p^2+1)^2} + \frac{E}{(p+1)} + \frac{F}{(p+1)^2} + \frac{G}{(p-1)} + \frac{H}{(p-1)^2}$$

$$p^2 = p^7(A+E+G) + p^6\left(B-E+G+\frac{1}{8}\right) + p^5(-A+C+E+G) + p^4\left(-B + D - E + G + \frac{3}{8}\right) + p^3(-A-2C-E-G) + p^2\left(-B-2D+E+\frac{3}{8}-G\right)$$

$F = 1/16$, $H = 1/16$

$B+D+E-G+\frac{1}{8}=0$

$A+C-E-G=0$

$-B-2D+E-G+\frac{3}{8}=1$

$-A-2C-E-G=0$

$-B+D-E+G=0$

$B-E+G+\frac{1}{8}=0$

$A+E+G=0$

$E=-\frac{8G+1}{8}$

$-A-2C+\frac{16G+1}{8}=0$

$A+\frac{16G+1}{8}=0$

$B=E-G-1/8$

$(E-G-1/8)+D+E-G+1/8=0$

$A+C-E-G=0$

$-(E-G-1/8)-2D+E-G+3/8=1$

$-A-2C-E-G=0$

$-(E-G-1/8)+D-E+G+3/8=0$

$A+E+G=0$

$A=-\frac{16G+1}{8}$

$-\left(-\frac{16G+1}{8}\right)-2C+\frac{-16G-1}{8}=0$

$\frac{-16G+1}{4}=0 \longrightarrow G=-\frac{1}{16}$

$$A = \frac{-16(-1/16)+1}{4} = 0$$

$$E = \frac{8(-1/16)+1}{8} = 1/16$$

$$B = \frac{1}{16} - \frac{1}{16} - \frac{1}{8} = 0$$

$$= -\frac{1}{4(p^2+1)^2} + \frac{1}{16(p+1)} + \frac{1}{16(p+1)^2} - \frac{1}{16(p-1)}$$

$$-\frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{(p^2+1)^2}\right) + \frac{1}{16}\mathcal{L}^{-1}\left(\frac{1}{(p+1)}\right) + \frac{1}{16}\mathcal{L}^{-1}\left(\frac{1}{(p+1)^2}\right)$$

$$-\frac{1}{16}\mathcal{L}^{-1}\left(\frac{1}{(p-1)}\right) + \frac{1}{16}\mathcal{L}^{-1}\left(\frac{1}{(p-1)^2}\right)$$

$$= -\frac{1}{4}\left(\frac{1}{2}(\text{sen}t - t\cos t)\right) + \frac{e^{-t}}{16} + \frac{te^{-t}}{16} - \frac{e^{t}}{16} + \frac{te^{t}}{16}$$

$$-\frac{\text{sen}t}{8} + \frac{t\cos t}{8} + \frac{1}{16}(e^{-t} - e^{t}) + \frac{t}{16}(e^{-t} + e^{t})$$

$$\text{senh}t = \frac{e^{t} - e^{-t}}{2} \quad y \quad \cosh t = \frac{e^{t} + e^{-t}}{2}$$

$$= -\frac{1}{8}(\text{sen}t + \text{senh}t) + \frac{t}{8}(\cos t + \cosh t)$$

2.15 $F(p) = \frac{p^3}{p^6-1}$

$$\frac{p^3}{p^6-1} = \frac{1}{6(p+1)} + \frac{1}{6(p-1)} - \frac{p^3+p}{3(p^4+p^2+1)} = \frac{1}{6}\left(\frac{1}{p+1}+\frac{1}{p-1}\right)$$

$$+\frac{-p^3+p}{3(p^4+p^2+1)} = \frac{1}{6}\left(\frac{1}{p+1}+\frac{1}{p-1}\right) + \frac{-p^3+p}{(p^2+p+1)(p^2-p+1)}$$

3er elemento por fracciones parciales

$$\frac{-p^3+p}{(p^2+p+1)(p^2-p+1)} = \frac{-p-2}{2(p^2+p+1)} - \frac{p+2}{2(p^2-p+1)}$$

$$= -\frac{1}{2}\frac{p+1/2}{(p+1/2)^2+3/4} - \frac{3}{4}\frac{1}{(p+1/2)^2+3/4} - \frac{1}{2}\frac{p-1/2}{(p-1/2)^2+3/4}$$

$$\frac{p^3}{p^6-1} = \frac{1}{6}\left(\frac{1}{p+1}+\frac{1}{p-1}\right) - \frac{1}{2}\frac{p+1/2}{(p+1/2)^2+3/4} - \frac{3}{4}\frac{1}{(p+1/2)^2+3/4}$$

Con la formula $\mathcal{L}\{e^{\alpha t}\} = \frac{1}{(p-\alpha)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{6}\left(\frac{1}{p+1}+\frac{1}{p-1}\right)\right\} = \frac{1}{6}\left(e^{-t}+e^{t}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{3}{4}\left(\frac{1}{(p-1/2)^2+3/4} - \frac{1}{(p+1/2)^2+3/4}\right)\right\} = \frac{3}{4}\left(\frac{2}{\sqrt{3}}e^{t/2}\operatorname{sen}\frac{\sqrt{3}t}{2}\right.$$

$$\therefore f(t) = \frac{1}{6}(e^{-t}+e^{t}) - \frac{1}{2}\left(e^{-t\pi}\cos\frac{\sqrt{3}t}{2} + e^{t\pi}\cos\frac{\sqrt{3}t}{2} + \frac{3}{2\sqrt{3}}\right.$$

$$\left(e^{t\pi}\operatorname{sen}\frac{\sqrt{3}t}{2} - e^{-t\pi}\operatorname{sen}\frac{\sqrt{3}t}{2}\right)$$

2.17 $F(p) = \frac{1}{p^2-4p+3}$

$p^2-4p+3 = (p-1)(p-3)$

$\frac{1}{p^2-4p+3} = \frac{1}{(p-1)(p-3)} = \frac{a_0}{p-1} + \frac{a_1}{p-3}$

$1 = a_0(p-3) + a_1(p-1) \rightarrow$ Si, $p=1$

$1 = a_0(1-3) + a_1(1-1)$

$1 = -2a_0 \longrightarrow a_0 = -1/2$

Si $p=3$

$1 = a_0(3-3) + a_1(3-1) \Rightarrow \frac{1}{(p-1)(p-3)} = -\frac{1}{2(p-1)} + \frac{1}{2(p-3)}$

$1 = 2a_1 \longrightarrow a_1 = 1/2$

$\mathcal{L}^{-1}\left\{\frac{1}{p^2-4p+3}\right\} = -1/2\,\mathcal{L}^{-1}\left\{\frac{1}{p-1}\right\} + \frac{1}{2}\left\{\frac{1}{p-3}\right\}$

$= -1/2\,e^{t} + 1/2\,e^{3t}$

2.18 $F(p) = \frac{p^2+1}{p^2(p^2-1)^2}$

$p^2(p^2-1)^2 = p^2(p+1)^2(p-1)^2$

$\frac{p^2+1}{p^2(p+1)^2(p-1)^2} = \frac{a_0}{p} + \frac{a_1}{p^2} + \frac{a_2}{p+1} + \frac{a_3}{(p+1)^2} + \frac{a_4}{p-1} + \frac{a_5}{(p-1)^2}$

$p^2+1 = a_0\left(p(p+1)^2(p-1)^2\right) + a_1\left((p+1)^2(p-1)^2\right) +$
$a_2\left(p^2(p+1)(p-1)^2\right) + a_3\left(p^2(p-1)^2\right) + a_4\left(p^2(p+1)^2\right)$
$a_5\left(p^2(p+1)^2\right)$

$\rightarrow$ Si, $p=0$

$\downarrow$

$1 = a_0(0(1)^2(-1)^2) + a_1((0+1)^2(0-1)^2) + a_2(0^2(0+1)(0-1)^2)$
$+ a_3(0^2(0-1)^2) + a_4(0^2(0+1)^2(0-1)) + a_5(0^2(0+1)^2)$

$\to$ si $p = 1$

$(-1)^2+1) = a_0(-1(-1+1)^2(-1-1)^2) + a_1((-1+1)^2(-1-1)^2)$
$+ a_2((-1)^2(-1+1)(-1-1)^2) + a_3((-1)^2(-1-1)^2)$
$+ a_4(-1)^2(-1+1)^2(-1-1)) + a_5((-1)^2(-1+1)^2)$

$4a_5 = 2 \to a_5 = \frac{1}{2}$

$\downarrow$

$p^2+1 = p^5(a_0+a_2+a_4) + p^4(a_4+2-a_2) + p^3(-2a_0-a_2-a_4)$
$+ p^2(a^2-a_4-1) + a_0 p + 1 \to a_0 = 0,\ a_2 = 1$
$a_4 = -1$

Sustituyendo las "a"

$= \frac{0}{p} + \frac{1}{p^2} + \frac{1}{p+1} + \frac{1/2}{(p+1)^2} + \frac{(-1)}{p-1} + \frac{1/2}{(p-1)^2}$

$\mathcal{L}^{-1}\left\{\frac{p^2+1}{p^2(-p^2-1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{p^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{p+1}\right\} + \frac{1}{2}$
$\mathcal{L}^{-1}\left\{\frac{1}{(p-1)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{p-1}\right\} + \frac{1}{2}\mathcal{L}\left\{\frac{1}{(p-1)^2}\right\}$

$\to \mathcal{L}^{-1}\left\{\frac{1}{(p-1)^2}\right\} = a = 1,\ F(p) = \frac{1}{2p^2}$

$= e^t\mathcal{L}^{-1}\left\{\frac{1}{2p^2}\right\} \longrightarrow e^t\left(\frac{t}{2}\right)$

$\therefore = t + e^{-t} + \frac{e^t t}{2} - e^t + \frac{e^t t}{2}$

2.19 $F(p) = \frac{p}{p^4 - 5p^2 + 4}$

$p^4 - 5p^2 + 4 = (p+1)(p-1)(p+2)(p-2)$

$\frac{p}{(p+1)(p-1)(p+2)(p-2)} = \frac{a_0}{p+1} + \frac{a_1}{p+1} + \frac{a_2}{p+2} + \frac{a_3}{p-2}$

$p = a_0((p-1)(p+2)(p-2)) + a_1((p+1)(p+2)(p-2)) + a_2$
$(p+1)(p-1)(p-2) + a_3((p-1)(p+1)(p-2)$

$\rightarrow$ si $p=-1$

$-1 = a_0((-1-1)(-1+2)(-1-2)) + a_1((-1+1)(-1+2)(-1-2)$
$a_2((-1+1)(-1-1)(-1-2)) + a_3((-1-1)(-1+1)(-1+2))$

$-12a_2 = -2 \longrightarrow a_2 = \frac{1}{6}$

$12a_3 = 2 \longrightarrow a_3 = 1/6$

Sustituyendo las a

$\mathcal{L}^{-1}\left\{\frac{p}{p^4 - 5p^2 + 4}\right\} = -\frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{p+1}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{p-1}\right\}$

$+ \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{p-2}\right\} + \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{p+2}\right\}$

$= -\frac{1}{6}e^{-t} - \frac{1}{6}e^{t} + \frac{1}{6}e^{-2t} + \frac{1}{6}e^{2t}$

220 $F(p) = \frac{p^3}{(p^4-1)(p^4+4)}$

$$\mathcal{L}^{-1}\left\{\frac{p^3}{(p^4-1)(p^4+4)}\right\} = \frac{p^3}{(p+1)(p-1)(p^2+1)(p^2+2p+2)(p^2-2p+2)}$$

$$= \frac{Bp+A}{p^2+1} + \frac{A_2}{p+1} + \frac{A_3}{p-1} + \frac{A_5 p + A_4}{p^2-2p+2}$$

$$= \frac{(A_7 p + A_6)\ (p+1)(p-1)(p^2+1)\ (p^2+2p+2)(p^2-2p+2)}{p^2-2p+2}$$

$p^3 = (Bp+A)(p+1)(p^2+2p+2)(p^2-2p+2) + A_2(p^2+1)$
$(p-1)(p^2+2p+2)(p^2-2p+2) + A_3(p^2+1)(p+1)$
$(p^2+2p+2)(p^2-2p+2) + (A_5 p + A_4)(p^2+1)(p+1)$
$(p-1)(p^2-2p+2) + A_7 p + A_6)(p^2+4)(p+1)(p-1)$
$(p^2+2p+2$

$p=-1$

$(-1)^3 = (B(-1)+A)((-1)+1)((-1)-1)((-1)^2+2(-1)+2)((-1)^2$
$-2(-1)+2) + A_2((-1^2+1)((-1)-1)((-1)^2+2(-1)+2)$
$((-1)^2-2(-1)+2) + A_3((-1)^2+1)((-1)+1)((-1)^2+2$
$(-1)+2)((-1)^2-2(-1)+2) + (A_5(-1)+A_4)((-1)^2+1$

$-1 = -20A_2 \longrightarrow a_2 = \frac{1}{20}$

$p=1\ (B(1)+a)(1+1)(1-1)(1^2+2(1+2)(1^2-2(1+2)+$
$A_2(1^2+1)(1-1)(1^2+2(1+2)(1^2-2(1+2)) + A_3(1^2+1)(1+1)$
$(1^2+2(1+2))(1^2-2(1+2) + (A_5(1A_4)(1^2+1)(1+1)$
$(1-1)(1^2-2(1+2)(A_3+(A_6))(1^2+1)(1-1)(1^2+2(1+2))$

$1 = 20A_3$

$A_3 = \frac{1}{20} \longrightarrow a = \frac{-A_4 + A_6}{2}$

$A_5 = A_7$

$\downarrow$

$$\mathcal{L}^{-1}\left(\frac{p}{10(p^2+1)} + \frac{1}{20(p+1)} + \frac{1}{20(p-1)} + \frac{-p-1}{10(p^2+2p+2)} + \frac{-p+1}{10(p^2-2p+2)}\right)$$

$\downarrow$

$$\frac{1}{10}\cos t + \frac{1}{20}e^{-t} + \frac{1}{20}e^{t} + \frac{1}{10}\left(-e^{-t}\cos t\right) + \frac{1}{10}\left(e^{t}\cos t\right)$$

Simplificando $\therefore$

$$\Rightarrow \frac{1}{10}\cos t + \frac{1}{20}e^{-t} + \frac{1}{20}e^{t} - \frac{1}{10}e^{-t}\cos t - \frac{1}{10}e^{t}$$

3.1 $x'' + 9x = \cos 3t$

$$\mathcal{L}(x'') + \mathcal{L}(9x) = \mathcal{L}(\cos 3t)$$

$$s^2\mathcal{L}(x) - s\,x(0) - x'(0) + 9\mathcal{L}(x) = \frac{s}{s^2+9}$$

$$\mathcal{L}(x)(s^2+9) - s\,x(0) - x'(0) = \frac{s}{s^2+9}$$

$$\mathcal{L}(x)(s^2+9) = \frac{s}{s^2+9} + s\,x(0) + x'(0)$$

$$\mathcal{L}(x) = \frac{s - (s^2+9)(s\,x(0)) + s - (s^2+9)(x'(0))}{(s^2+9)(s^2+9)}$$

$$= \frac{s\,x(0) - x'(0) + 2s}{s^2+9} = x = \mathcal{L}^{-1}\left(\frac{s\,x(0)}{s^2+9}\right) - \mathcal{L}^{-1}\left(\frac{x'(0)}{s^2+9}\right) + \mathcal{L}\left(\frac{2s}{s^2+9}\right)$$

$$= \frac{s\,x(0)}{3}(\operatorname{sen}(3t)) - \frac{x'(0)}{3}(\operatorname{sen}(3t)) + 2\cos(3t)$$

3.2 $x'' - 4x' + 4x = e^{2t}$

$\mathcal{L}(x'') - 4\mathcal{L}(x') + 4\mathcal{L}(x) = \mathcal{L}(e^{2t})$

$s^2\mathcal{L}(x) - sx(0) - x'(0) - s\mathcal{L}(x) - x(0) + 4\mathcal{L}(x) = \frac{1}{s-2}$

$\mathcal{L}(x)(s^2 - s + 4) - sx(0) - x'(0) - x(0) = \frac{1}{s-2}$

$\mathcal{L}(x)(s^2 - s + 4) = \frac{1}{s-2} + sx(0) + x'(0) + x(0)$

$= \frac{1 + (s-2)(sx(0)) + (s-2)(x'(0)) + (s-2)(x(0))}{s-2}$

$= (sx(0) + x'(0) + (x(0)) + 3$

$\mathcal{L}(x) = \frac{sx(0) + x'(0) + x(0) + 3}{s^2 - s + 4}$

$\mathcal{L}(x) = \frac{1}{4}sx(0)\,\mathcal{L}^{-1}\left(\frac{1}{8^2 s}\right) + \frac{1}{4}x'(0)\,\mathcal{L}^{-1}\left(\frac{1}{s^2 s}\right)$
$+ \frac{1}{4}x(0)\,\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)$

1. $\frac{A}{s} + \frac{B}{s-1} \longrightarrow \frac{s}{s(s-1)} \longrightarrow -\cancel{1}$

2. $\frac{B}{s-1} = \frac{1}{s(s-1)} - B = \frac{\cancel{s-1}(1)}{\cancel{(s-1)}s} = \frac{1}{s} = \frac{1}{1} = \frac{1}{\cancel{1}}$

$\therefore \mathcal{L}(x) = \operatorname{senh}(t) - 1\left(\frac{6x(0)}{4} + \frac{x'(0)}{4} + \frac{x(0)}{4} + \frac{3}{4}\right)$

$x = \operatorname{senh} t - 1\left(\frac{sx(0) + x'(0) + x(0) + 3}{3}\right)$

3.3 $x'' + 2x' = te^{2t}$

$\mathcal{L}(x'' + 2x' = te^{2t}) \to \mathcal{L}(x'') + 2\mathcal{L}(x') = \mathcal{L}(te^{2t})$

$(p^2 + 2p)\,X(p) - x_0 p - x'_0 - 2x_0 = \frac{1}{(p+2)^2}$

$X(p) = \frac{1}{p(p+2)^3} + \frac{x_0}{p+2} + \frac{x_0'}{p^2+2p} + \frac{2x_0}{p^2+2p}$

$\mathcal{L}^{-1}\left(\frac{1}{p(p+2)^3}\right) = \frac{1}{8}H(t) - \frac{1}{8}e^{-2t} - \frac{1}{4}te^{-2t} - \frac{1}{4}t^2e^{-2t}$

$\mathcal{L}^{-1}\left(\frac{x_0}{p+2}\right) = X_0 e^{-2t}$

$\mathcal{L}^{-1}\left(\frac{2x_0}{p^2+2p}\right) = X_0 H(t) - e^{-2t}X_0$

$\downarrow$

$x(p) = \left(-\frac{1}{8} - \frac{t-t^2}{4} + x_0 - \frac{1}{2}x_0' - x_0\right)e^{-2t} + \frac{1}{8}H(t)$

$+ \frac{1}{2}H(t)\,x_0' + X_0 H(t)$

$C_1 = \frac{1}{8}H(t) + \frac{1}{2}H(t)x_0' + X_0 H(t)$

$x(p) = \left(-\frac{1}{8} - \frac{1}{2}x_0' - \frac{t-t^2}{4}\right)e^{-2t} + C_1$

$C_2 = -\frac{1}{8} - \frac{1}{2}x_0'$

$x(p) = \left(C_2 - \frac{t-t^2}{4}\right)e^{-2t} + C_1$

3.4 $x'' + x' - 2x = e^t$

$\mathcal{L}(x'') + \mathcal{L}(x') - 2\mathcal{L}(x) = \mathcal{L}(e^t)$

$p^2 X(p) - x_0 p - x_0' + p X(p) - x_0 - 2X(p) = \frac{1}{p-1}$

$X(p)(p^2+p-2) - x_0 p - x_0' + x_0' + x_0$

$X(p) = \frac{1}{(p-1)(p+2)(p-1)} + \frac{x_0 p}{(p+2)(p-1)} + \frac{x_0'}{(p+2)(p-1)} + \frac{x_0}{(p+2)(p-1)}$

$\mathcal{L}^{-1}\left(\frac{x_0}{(p+2)(p-1)}\right) = -\frac{1}{3}x_0 e^{-2t} + \frac{1}{3}x_0 e^t$

$x(p) = -\frac{1}{9}e^t + \frac{1}{3}te^t + \frac{1}{9}e^{-2t} + \frac{1}{3}x_0 e^t + \frac{2}{3}x_0 e^{-2t}$

$-\frac{1}{3}x_0' e^{-2t} + \frac{1}{3}x_0' e^t - \frac{1}{3}x_0 e^{-2t} + \frac{1}{3}x_0 e^t$

$x(p) = (-\frac{1}{9} + \frac{1}{3}t + \frac{1}{3}x_0 + \frac{1}{3}x_0' + \frac{1}{3}x_0)e^t +$

$(\frac{1}{9} + \frac{2}{3}x_0 - \frac{1}{3}x_0' - \frac{1}{3}x_0)e^{-2t}$

$C_1 = -\frac{1}{9} + \frac{1}{3}x_0 + \frac{1}{3}x_0' + \frac{1}{3}x_0$

$C_2 = \frac{1}{9} + \frac{2}{3}x_0 - \frac{1}{3}x_0' - \frac{1}{3}x_0$

$x(p) = (C_1 + \frac{1}{3}t)e^t + C_2 e^{-2t}$

3.5 $x'' + x' = e^{t} sen(t)$

$\mathcal{L}(x'') + \mathcal{L}(x') = e^{t} sen\, t$

$\downarrow$

Coef Indeter

$m^2 + m = 0 \longrightarrow m(m+1) = 0 \quad m_1 = 0, m_2 = -1$

$x_c = C_1 e^{t} + C_2$

$x_p = Ae^{-t} cos\, t + Be^{-t} sent$

$\downarrow$

$x_p' = 2Ae^{t} sent - 2Be^{t} cos\, t$

$= 2Ae^{-t} sent - 2Be^{t} cost - Ae^{t} cos\, t - Ae^{-t} sent$
$- Be^{-t} sent + Be^{t} cost \longrightarrow e^{t} sent$

$\downarrow$

$A - B = 1 \quad -A - B = 0$ Luego $-2B = 1 \Rightarrow B = -1/2$
$A = 1/2$

$x_p = \frac{1}{2} e^{-t} cost - \frac{1}{2} e^{-t} sent + C_1 e^{t} + C_2$

$x = C_1 e^{-t} + C_2 + \frac{1}{2} e^{t} cost - \frac{1}{2} e^{t} sent$

3.6 $x''' + x = 0 \ ; \ x(0) = 0 \ , \ x'(0) = -1, \ x''(0) = 2$

$\mathcal{L}\{x'''\} + \mathcal{L}\{x\} = 0$

$s^3 x(s) - s^2 x(0) - s x'(0) - x''(0) + x(s) = 0$

$(s^3 + 1)\, x(s) = -2 - s$

$x(s) = \frac{2 - s}{s^3 + 1} = \frac{2 - s}{(s+1)(s^2 - s + 1)}$

$$\frac{2-s}{(s+1)(s^2-s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2 s+1}$$

$$2-s = A(s^2-s+1) + B(s^2+s) + (s+1)$$

① $B=-A$, ② $-2A+C=-1$, ③ $A+C=2$

$\therefore C=1$, $A=1$, $B=-1$

$$\mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2 s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2-s+1}\right\}$$

$$s^2-s+1 = \left(s-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)$$

$$x(t) = e^{t} - e^{1/2 t}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{3}e^{1/2 t}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$$

3.7 $x'' + 2x' + x = e^{-t}$; $x(0)=1$; $x'(0)=0$

$$\mathcal{L}(x'') + 2\mathcal{L}(x') + \mathcal{L}(x) = \mathcal{L}(e^{-t})$$

$$s^2 X(s) - sX(0) - x'(0) + 2sX(s) - 2x(0) + X(s) = \frac{1}{s+1}$$

$$(s^2+2s+1)X(s) - s - 2 = \frac{1}{s+1}$$

$$X(s) = \frac{s^2+3s+3}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$s^2+3s+3 = A(s^2+2s+1) + B(s+1) + C$$

$A=1$, $2A+B=3 \Rightarrow B=1$; $A+B+C=3 \Rightarrow C=1$

$$\mathcal{L}^{-1}(X(s)) = \mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+1)^3}\right)$$

$$x(t) = e^{-t} + te^{-t} + \frac{t^2 e^{-t}}{2}$$

3.8 $x'' + 3x' = e^{-3t}$, $x(0)=0$; $x'(0)=1$

$$\mathcal{L}(x'') + 3\mathcal{L}(x') = \mathcal{L}(e^{-3t})$$

$$s^2 x(s) - s x(0) - x'(0) + 3 s x(s) - 3x(0) = \frac{1}{s+3}$$

$$(s^2+3s)\, x(s) - (-1) = \frac{1}{s+3}$$

$$s(s+3)\, x(s) = \frac{1-s-3}{s+3}$$

$$x(s) = \frac{-s-2}{s(s+3)^2} = \frac{A}{s} + \frac{B}{(s+3)} + \frac{C}{(s+3)^2}$$

$$-s-2 = A(s^2+6s+9) + B(s^2+3s) + Cs$$

$A+B=0$, $6A+3B+C=-1$, $9A=-2 \Rightarrow A=\frac{-2}{9}$

$B=\frac{2}{9}$

$C=-\frac{1}{3}$

$$\mathcal{L}^{-1}(x(s)) = -\frac{2}{9}\mathcal{L}^{-1}\left(\frac{1}{s}\right) + \frac{2}{9}\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) - \frac{1}{3}\mathcal{L}^{-1}\left(\frac{1}{(s+3)^2}\right)$$

$$x(t) = -\frac{2}{9} + \frac{2}{9} e^{-3t} - \frac{1}{3} t e^{-3t}$$

3.9 $x'' - 2x' + 2x = \operatorname{sen} t$; $x(0) = 0$, $x'(0) = 1$

$$\mathcal{L}(x'' - 2x' + 2x) = \mathcal{L}(\operatorname{sen} t)$$

$$p^2\mathcal{L}(x) - p\,x(0) - x'(0) - 2(p\mathcal{L}(x) - x(0)) + 2\mathcal{L}(x) = \frac{1}{p^2+1}$$

$$p^2\mathcal{L}(x) - 0 - 1 - 2(p\mathcal{L}(x)) + 2\mathcal{L}(x) = \frac{1}{p^2+1}$$

$$p^2\mathcal{L}(x) + 2\mathcal{L}(x) - 2p\mathcal{L}(x) - 1 = \frac{1}{p^2+1}$$

$$(p^2+2-2p)\mathcal{L}(x) = \frac{1}{p^2+1}$$

$$\mathcal{L}(x) = \frac{p^2+2}{(p^2+1)(p^2+2-2p} \longrightarrow x = \mathcal{L}^{-1}\left(\frac{p^2+2}{(p^2+1)(p^2+2-2p)}\right)$$

$$x = \frac{2}{5}\cos t + \frac{1}{5}\operatorname{sen} t - \frac{2}{5}e^t\cos t + \frac{6}{5}e^t \operatorname{sen} t$$

3.10 $x'' + 4x = \operatorname{sen} 2t$; $x(0) = 1$, $x'(0) = -2$

$$\mathcal{L}(x'' + 4x) = \mathcal{L}(\operatorname{sen} 2t)$$

$$p^2\mathcal{L}(x) - p\,x(0) - x'(0) + 4\mathcal{L}(x) = \frac{2}{p^2+4}$$

$$p^2\mathcal{L}(x) - p + 2 + 4\mathcal{L}(x) = \frac{2}{p^2+4}$$

$$(p^2+4)\mathcal{L}(x) - p + 2 = \frac{2}{p^2+4}$$

$$\mathcal{L}(x) = \frac{p^3 - 2p^2 + 4p - 6}{(p^2+4)^2}$$

$$x = \mathcal{L}^{-1}\left(\frac{p^3 - 2p^2 + 4p - 6}{(p^2+4)^2}\right)$$

$$x = \cos 2t - \operatorname{sen} 2t + \frac{1}{8}(\operatorname{sen} 2t - 2t\cos 2t)$$

3.11 $x'' - 9x = \operatorname{senh} t \; ; \; x(0) = -1 \; , \; x'(0) = 3$

$$\mathcal{L}(x'') - 9\mathcal{L}(x) = \mathcal{L}(\operatorname{senh} t)$$

$$p^2 x(p) - p\cancel{x(0)} - \cancel{x'(0)}^{3} - 9x(p) = \frac{1}{p^2-1}$$

$$p^2 x(p) + p - 3 - 9x(p) = \frac{1}{p^2-1}$$

$$x(p)(p^2-9) = \frac{1}{p^2-1} + 3 - p$$

$$x(p) = \frac{1}{(p^2-1)(p^2-9)} + \frac{3}{p^2-9} - \frac{p}{p^2-9} \quad \cancel{\frac{1}{(p^2-1)(p^2-9)}}$$

$$= \frac{Ap+B}{p^2-1} + \frac{Cp+D}{p^2-9} = \frac{((Ap+B)(p^2-9)) + ((Cp+D)(p^2-1))}{\cancel{(p^2-1)(p^2-9)}}$$

$$1 = Ap^3 - 9Ap + Bp^2 - 9B + Cp^3 - Cp + Dp^2 - D$$

$\downarrow$

$$ax^3 + bx^2 + cx + d = (A+C)p^3 + (B+D)p^2 + (-9A-C)p - 9B - D$$

$$A+C=0 \; , \; B+D=0 \; , \; -9A-C=0 \; , \; -9B-D=1$$

$\downarrow$

$$\begin{array}{l} B+D=0 \\ -9B-D=1 \\ \hline -8B=1 \end{array} > B = -\frac{1}{8} \; , \quad -\frac{1}{8} + D = 0 \longrightarrow D = \frac{1}{8}$$

$$A = C = 0$$

$$\frac{1}{(p^2-1)(p^2-9)} = -\frac{1}{8(p^2-1)} + \frac{1}{8(p^2-9)}$$

$$x(p) = -\frac{1}{8(p^2-1)} + \frac{1}{8(p^2-9)} + \frac{3}{p^2-9} - \frac{p}{p^2-9}$$

$$\mathcal{L}^{-1}(x(p)) = -\frac{1}{8}\mathcal{L}^{-1}\left(\frac{1}{p^2-1}\right) + \frac{25}{24}\mathcal{L}^{-1}\left(\frac{3}{p^2-9}\right) - \mathcal{L}\left(\frac{p}{p^2-9}\right.$$

$$x(t) = -\frac{1}{8}\operatorname{senh} t + \frac{25}{24}\operatorname{sen} 3t - \cosh 3t$$

3.12 $x''' - x'' = e^t \quad , \; x(0) = 1 \quad , \; x'(0) = x''(0) = 0$

$$\mathcal{L}(x''') - \mathcal{L}(x'') = \mathcal{L}(e^t)$$

$$p^3 x(p) - p^2 x(0) - p x'(0) - x''(0) - p^2 x(p) + p x(0) + x'(0) = \frac{1}{p-1}$$

$$x(p)(p^3 - p^2) - p^2 + 3 = \frac{1}{p-1}$$

$$x(p) = \frac{1}{(p-1)(p^3-p^2)} + \frac{p^2-p}{p^3-p^2} = \frac{1}{p^2(p-1)^2} + \frac{p^2-p}{p^2(p-1)} +$$

$$\frac{1}{p^2(p-1)^2} + \frac{p^2-p}{p^2(p-1)} = \frac{1+p^3-p^2-p^2+p}{p^2(p-1)^2}$$

$$x(p) = \frac{p^3-2p^2+p+1}{p^2(p-1)} \longrightarrow \frac{(Ap+B)(p-1)^2 + Cp^2(p-1) + Dp^2}{p^2(p-1)^2}$$

$$\hookrightarrow Ap^3 - 2Ap^2 + Ap + Bp^2 - 2Bp + B + Cp^3 - Cp^2 - Dp^2$$

$$A = 3 \quad , \; B = 1 \quad , \; C = -2 \quad , \; D = 1$$

$$\frac{p^3-2p^2+3+1}{p^2(p-1)^2} = \frac{3p+1}{p^2} - \frac{2}{p-1} + \frac{1}{(p-1)^2}$$

$$= \frac{3}{p} + \frac{1}{p^2} - \frac{2}{p-1} + \frac{1}{(p-1)^2}$$

$$\mathcal{L}^{-1}(x(p)) = 3\mathcal{L}^{-1}\left(\frac{1}{p}\right) + \mathcal{L}^{-1}\left(\frac{1}{p^2}\right) - 2\mathcal{L}^{-1}\left(\frac{1}{p-1}\right) + \mathcal{L}^{-1}\left(\frac{1}{(p-1)^2}\right)$$

$$x(t) = 3 + t - 2e^t + te^t$$

3.13 $x^{IV} - x = \operatorname{sh} t$; $x(0) = x'(0) = x''(0) = 0$, $= x'''(0) = 1$

$\mathcal{L}(x^{IV})\ \mathcal{L}(x''') - \mathcal{L}(x'') = \mathcal{L}(e^t)$

$h(x) = \sqrt{\frac{x^{IV} - x}{t}}$, $h(x) = -\sqrt{\frac{x^{IV} - x}{t}}$

$x^{IV} - x = h(x)\, h(x)\, t \longrightarrow h(x)\, h(x)\, t = x^{IV} - x$

$h(x)^2 t = x^{IV} - x \implies \frac{h(x)^2 t}{t} = \frac{x^{IV} - x}{t}$; $t \neq 0$

$h(x)^2 = \frac{x^{IV} - x}{t}$, $t \neq 0$

Para $x^2 = f(a)$

$\downarrow$

$x = \sqrt{f(a)}$, $-\sqrt{f(a)}$

$h(x) = \sqrt{\frac{x^{IV} - x}{t}}$, $h(x) = -\sqrt{\frac{x^{IV} - x}{t}}$; $t \neq 0$

3.14 $x''' + 3x'' + 3x' + x = te^{-t}$; $x(0) = x'(0) = x''(0) = 0$

$x''' + 3x'' + 3x' + x = \frac{t}{e^t} \to x = \frac{C_2 t^2 - C_1 t}{e^t} - \frac{-t^4 - C}{24e^t}$

$a_0 t^{(n)} + a_1 t^{(n-1)} + \dots + a_{n-1} t' + a_n t = f(t)$

$\downarrow$

$\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0 \to (x+1)^3 = 0$

$\downarrow$

$(\lambda+1)^3 \to \lambda_{1,2,3} = -1 \quad k = 3 \quad r = \frac{C_2 t^2 + C_1 t + C}{e^t}$

S.P.

$x_1 = t^s e^{\alpha t}(Rm(t)\cos\beta t + Tm(t)\operatorname{sen}\beta$

Coefctes

$e^{\alpha t}(Rm(t)\cos\beta t + Qm(t)\operatorname{sen}\beta t)$

$x = \frac{t_0^3(At_0 + B)}{e^{t_0}} \to x' = At\int_0^4 (B - 4A) + \int_0^3 -3Bt$

$x'' = A + \int_0^4 + (B - 8A) + \int_0^3 12A - 6B +$

$x''' = At\int_0^4 (B - 12A) + \int_0^3 (36A - 9B + \int_0^2 (18_B - 24_A) \int_0^2 + 6Bt_0$

Se sustituyeen la original $\quad \to$ Coef

$\begin{cases} 24A = 1 \\ 6B = 0 \end{cases} = \begin{cases} A = \frac{1}{24} \\ B = 0 \end{cases}$

$\frac{24At_0 + 6B}{e^{t_0}} = \frac{t_0}{e^{t_0}}$

$\downarrow$

$x = \frac{t\int_0^4}{24e^{t_0}} \qquad x = \frac{C_2 t^2 + C_1 t + C}{e^t}$

3.15 $x' + x = f(t)$, donde $f(t) = \begin{cases} 1 & \text{para } 0 \le t < 2 \\ 0 & \text{para } t \ge 2 \end{cases}$

$x' + x = y \rightarrow$ Sust. Lineal

$u = t - x \quad x = y - u \longrightarrow 1 - u' = u$

$u' = t - x' \quad x' = t u' \qquad -u' = u - 1$

$\longleftarrow -\frac{u_0}{u_1} = u - 1$

$-u_0 = (u-1) u_1$

$\frac{u_0}{u-1} = -d_y \longrightarrow \int \frac{1}{u-1} u_0 = \int -1 u + \ln(u-1) = d_y$

$u = e^{1-y} + 1 \rightarrow u - 1 \; e^{1-y} \rightarrow t - x - 1 = \frac{c}{e^y}$

$\mathcal{L}(x') + \mathcal{L}(x) = \mathcal{L}(t - x - 1) \rightarrow \mathcal{L}(\ln(u-1) - \mathcal{L}(c - y)$

$f(t) = 3 + e^t + m(2)(1 - e^y) - 2e^{\pm 2}$

$f(t) = 1 + e^{-t} + m(t-2(1 - e^{-y}) - 2e^t$

$f(t) = 1 - e^{-1} - m(t-2)(1 - e^{-(-y)})$

3.16 $x'' + x = f(t)$ donde $f(t) = \begin{cases} t & \text{para } 0 \le t < \pi \\ 0 & \text{para } t \ge \pi \end{cases}$

$x'' f(t) = x \longrightarrow y = \sum P_{k-1}(y e^{\alpha y} \operatorname{sen} \beta y + Q_{k-1} e^{\alpha y} \cos \beta y$

con $\lambda = \alpha + \beta \rightarrow x = C_1 \operatorname{sen} y + C_2 \cos y$

$\left. \begin{array}{l} y = A y_0 + B \\ x'' = 0 \end{array} \right\} \begin{array}{l} A = 1 \\ B = 0 \end{array} \rightarrow \begin{array}{l} A = 1 \\ B = 0 \end{array} \rightarrow x = C_1 \operatorname{sen} x + C_2 \cos y + y$

$\mathcal{L}(x'') + \mathcal{L}(x) = x' + 1 = 0 \qquad C_1 = \frac{1}{2}, \; C_2 = \frac{1}{2}$

$x(t) = C_1 \operatorname{sen}(y) + C_2 \cos y + y \rightarrow$

$x(t) = \frac{t}{2} \operatorname{sen} y = \frac{1}{2} m(t+\pi)(t-\pi) \operatorname{sen}(t-\pi)$

$x(t) = \frac{t}{2} \operatorname{sen} y + \frac{1}{2} m(t+\pi)(t-\pi) \operatorname{sen}(t-\pi)$

3.17 $x'' - x' = f(t)$ donde

$$f(t) = \begin{cases} e^{-t} & \text{para } 0 \leq t < 1 \\ 0 & \text{para } t \geq 1 \end{cases} = U(t-a) = U(t-1)$$

$$f(t) = g(t) - g(t)U(t-a) + h(t)U(t-a)$$

$$f(t) = e^{-t} - e^{-t}U(t-1) \rightarrow f(t-\tau) = e^{-p\tau}F(p)$$

$$\mathcal{L}(x'') - \mathcal{L}(x') = \mathcal{L}(e^{-t} - e^{-t}U(t-1))$$

$$p^2 Y(p) - p\,Y(p) = -e^{-t}\frac{1}{p-1}\frac{e^{-p}}{p}$$

$$Y(p)(p^2-p) = -e^{-t}\frac{e^{-p}}{p^2-p} \rightarrow Y(p) = \frac{-e^{-t}}{(p^2-p)^2}e^{-p}$$

$$Y(p) = pt - e^{-1}U(t-1)$$

$$x(t) = \left(\frac{p}{p^2-1}t - 1 - e^{-1}U(t-1)\right)\left(\frac{p}{p^2-1}(t-1) - 1\right)$$

$\downarrow$

$$x(t) = \left(\frac{p}{p^2-1}t - 1 - \frac{1}{e}U(t-1)\right)\left(\frac{p}{p^2-1}(t-1) - 1\right)$$

como $\frac{p}{p^2-\beta^2} = \operatorname{ch}\beta t$

$\therefore$

$$x(t) = \left(\operatorname{ch}t - 1 - \frac{1}{e}U(t-1)\right)\left(\operatorname{ch}(t-1) - 1\right)$$

3.18 $x'' + x = f(t)$; donde

$$f(t) = \begin{cases} 1 \text{ para } & 0 \leq t < 1 = U(t-1) \\ -1 \text{ para } & 1 \leq t < 2 \\ 0 \text{ para } & t \geq 2 \end{cases} > U(t-2)$$

$$f(t) = 1[U(t-1)] - 1[U(t-1) - U(t-2)]$$

$$f(t) = U(t-1) - U(t-1) + U(t-2) \rightarrow f(t-\tau) \doteq e^{-p\tau} F(p)$$

$$\mathcal{L}(x'') + \mathcal{L}(x') = \mathcal{L}(f(t)) \rightarrow$$

$$p^2 Y_{(p)} - p Y_{(p)} = \frac{e^{-p}}{p} - \frac{e^{-p}}{p} + \frac{e^{-2p}}{p}$$

$$Y_{(p)}(p^2 - p) = \frac{e^{-p}}{p} - \frac{e^{-p}}{p} + \frac{e^{-2p}}{p} \Rightarrow \frac{e^{-2p}}{(p^2-p)p}$$

$$Y(p) = 2\left(\frac{2\beta t}{2p+4^2}\right) - 2U(t-1) \frac{2(t-1)}{2p(p^2+4^2)} + U(t-2) \operatorname{sen}^2 \frac{t-2}{2}$$

$$\downarrow$$

$$x(t) = 2\left(\frac{2\ t}{2(p(p^2+4^2)}\right) - 2U(t-1) \frac{2\ (t-1)}{2(p(p^2+4^2)} + U(t-2)$$

$$\text{Como } \operatorname{sen}^2 \beta t = \frac{2\beta}{p(p^2+4\beta^2)}$$

$$\therefore$$

$$x(t) = 2(\operatorname{sen}^2 \frac{t}{2} - 2U(t-1)) \operatorname{sen}^2 \frac{t-1}{2} + U(t-2) \operatorname{sen}^2 \frac{t-2}{2}$$

3.20 $x_1(0) = x_1(t)$ y $\mathcal{L}(p) = p^n + a_1 p^{n-1} + \dots a_{n-1} p + a_n$

$$\mathcal{L}(p) = \frac{1}{p\mathcal{X}_1(p)} \rightarrow \mathcal{L}(p)\, X(p) = f(p)$$

donde $X(p) = x(t)$ y $f(p) = f(t)$

$$X(p) = \frac{f(p)}{\mathcal{L}(p)} = p\mathcal{X}_1(p) f(p)$$

$$\rightarrow x(t) = x_1(0)\, \delta(t) + \int_0^t x_1(\tau) f(t-\tau)\, d\tau = \int_0^t x_1 \tau\, f(t-\tau)\, dz$$

ya que $x_1(0) = 0$

$$x(t) = f(0)\, x_1(t) + \int_0^t f(\tau)\, x_1(t-\tau)\, dz$$

3.21 $x' - x = \frac{1}{e^t + 3}$

$x(0) = x'(0) = 0 \rightarrow \mathcal{L}(x' - x) = \mathcal{L}\left(\frac{1}{e^t+3}\right)$

$\mathcal{L}(x') - \mathcal{L}(x) = \mathcal{L}(1 \cdot (e^t + 3)^{-1})$

$\mathcal{L}(x') - \mathcal{L}(x) = \mathcal{L}(1(e^{-t} + \frac{1}{3}))$

$\mathcal{L}(x') = s\mathcal{L}(x) - x_0, \quad \mathcal{L}(x) = L$

$\mathcal{L}(x') = sL - 0 \rightarrow sL - 0 - L = \frac{1}{3}\mathcal{L}(e^{-t})$

$$sL - L = \frac{1}{3}\left(\frac{1}{s-(-1)}\right) \rightarrow (s-1)L = \frac{1}{3s+3}$$

$$\mathcal{L}(x) = \frac{1}{(3s+3)(s-1)} \rightarrow x = \mathcal{L}^{-1}\left(\frac{1}{(3s+3)(s-1)}\right)$$

$$x = \mathcal{L}^{-1}\left(\frac{1}{3s^2-3}\right) = \frac{1}{3(s^2-1)}$$

$$x = \frac{1}{3}\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right) \longrightarrow \mathcal{L}^{-1}\left(\frac{k}{s^2-k^2}\right) = \operatorname{senh} kt$$

$$\therefore x = \frac{1}{3}\operatorname{senh}(t)$$

3.22 $x'' + x = \frac{1}{1+e^t}$, $x(0)=0$, $x''(0)=0$

$\mathcal{L}(x'') + \mathcal{L}(x) = \mathcal{L}\left(\frac{1}{1+e^t}\right)$

$\mathcal{L}(x'') = s^2\mathcal{L}(x)\, x_0 s - x'_0$

$\mathcal{L}(x) = L$

$\mathcal{L}(x'') = s^2 L(s) - 0$

$s^2 L + L = \mathcal{L}(1(1+e^t)^{-1})$

$s^2 L + L = \mathcal{L}(1+e^{-t})$

$s^2 L + L = \mathcal{L}(1) + \mathcal{L}(e^{-t})$

$\mathcal{L}k = \frac{k}{s}$, $\mathcal{L}(e^{kt}) = \frac{1}{s-k}$

$s^2 L + L = \frac{1}{s} + \frac{1}{s-(-1)} = \frac{1}{s} + \frac{1}{s+1}$

$L = \frac{1}{s(s^2+1)} + \frac{1}{(s+1)(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$

$1 = s(s^2+1)\left(\frac{A}{s} + \frac{Bs+C}{s^2+1}\right) = (s^2+1)A + s(Bs+C)$

$A=1$, $s=0$, $s=1$, $B+C=-1$, $C=0$

$B=-1$

$L = \frac{1}{s} + \frac{-s}{s^2+1} + \frac{1}{s+1} + \frac{1}{s^2+1}$

$x = 1 - \cos t - e^{-t} + \operatorname{sen} t$

3.23 $x'' - x' + \frac{1}{1+e^t}$, $x'(0)=0$, $x''(0)=0$

$\mathcal{L}(x'') - \mathcal{L}(x') + \mathcal{L}(1(1+e^t)^{-1})$

$\mathcal{L}(x'') = s^2 \mathcal{L} - (0)s)$

$\mathcal{L}(x') = s\,\mathcal{L}(x) - x0$

$s^2\mathcal{L} - s\mathcal{L} + \mathcal{L}(1) + \mathcal{L}(e^{-z})$

$\mathcal{L}k = \frac{k}{s}$ $\mathcal{L}(e^{-kt}) = \frac{1}{s-k}$

$s^2\mathcal{L} - s\,\mathcal{L} + \frac{1}{s} + \frac{1}{s+1}$

$x(0) = e^t - 1 - (t + \ln 2)(e^t+1) + (e^t+1)$

$\ln(e^t+1)$

3.24 $x'' + x = \frac{1}{2+\cos t}$, $x''(0)$, $x(0)=0)$

$\mathcal{L}(x'') + \mathcal{L}(x) = \mathcal{L}((2+\cos t)^{-1})$

$\mathcal{L}(x) = \mathcal{L}$

$s^2\mathcal{L} + 1 = \mathcal{L}(2+\cos t)^{-1})$

$s^2\mathcal{L} + \mathcal{L} = \mathcal{L}(2 + 1\cos^{-t})$

$(s^2+1)\mathcal{L} = \frac{2}{s} + \frac{1}{s+1}$

$\mathcal{L} = \frac{2}{s(s^2+1)} + \frac{1}{s+1(s^2+1}$

$x(t) = \text{sen}\,t\,(t - \frac{4}{\sqrt{3}}\,\text{arctg}\left(\frac{tg\frac{t}{2}}{\sqrt{3}}\right) + \cos t$

$\ln\frac{2+\cos t}{3}$

3.25 $x'' + x = e^{-t^2}$

Hallar y_h

$\to x = e^{yt}$

$\downarrow$

$(e^{yt})'' = y^2 e^{yt}$

$y^2 e^{yt} + e^{yt} = 0$

$e^{yt}(y^2+1) = 0$

$e^0 (C_1 \cos(t) + C_2 \operatorname{sen}(t))$

$x = C_1 \cos t + C_2 \operatorname{sen} t$

$\to g(t) = e^{-t^2}$

$x_p = u_1 x_1 + u_2 x_2 \to x_h = C_1 x_1 + C_2 x_2$

$\begin{pmatrix} u_1' x_1 + u_2' x_2 = 0 \\ u_1' x_1' + u_2' x_2' = g(t) \end{pmatrix}$

$\downarrow$

$u_1 = \int -\frac{x_2 g(t)}{W(x_1, x_2)} dt$ y $u_2 = \int \frac{x_1 g(t)}{W(x_1, x_2)} dt$

$W(x_1, x_2) = x_1 x_2' - x_1' x_2$

$x_1 = \cos t \longrightarrow x_1' = -\operatorname{sen}(t)$

$x_2 = \operatorname{sen} t \qquad x_2' = \cos t)$

$W(x_1, x_2 = 1$

$\downarrow$

$x(p) = \cos(t) \int -e^{-t^2} \operatorname{sen}(t) dt + \operatorname{sen} t \int e^{-t^2} \cos t \, dt$

$\downarrow$

$x(t) = C_1 \cos t + C_2 \operatorname{sen} t + \cos t - \int -e^{t^2} \operatorname{sen} t \, dt + \operatorname{sen} t \int e^{-t^2} dt$

3.26 $x'' + y' = t$

$y'' - x' = \emptyset$

$\begin{cases} \mathcal{L}(x'') + \mathcal{L}(y') = t \\ \mathcal{L}(y'') - \mathcal{L}(x') = \emptyset \end{cases} \longrightarrow \begin{cases} D^2(x) + D(y) = t \\ D^2(y) - D(y) = 0 \end{cases}$

$\begin{pmatrix} D^2(x) = t \\ D^2(y) = \emptyset \end{pmatrix} \quad \begin{pmatrix} r^2 = t \to r = \pm\sqrt{t} \\ r^2 = \emptyset \quad r = \emptyset \end{pmatrix}$

$x = C_1 e^{\sqrt{t}t} + C_2 e^{\sqrt{t}t} \qquad y = C_3 e^{t} + C_4 e^{t\emptyset} = C_3 + C_4$

$x = \frac{-9te^{-t}(C_1 e^{2t^{3/2}})}{4} + \frac{3e^{-t^{3/2}}(C_1 e^{2t} + 2)}{\sqrt{t}} + \frac{3C_1\sqrt{t}e^{t^{3/2}}}{2}$

$- \frac{3C_2\sqrt{t}e^{t^{3/2}}}{2} = \emptyset$

$\vec{X} = \begin{pmatrix} C_1 e^{\sqrt{t}} \\ C_3 \end{pmatrix} + \begin{pmatrix} C_2 e^{-\sqrt{t}} \\ C_4 \end{pmatrix}$

3.27 $x''' + y' = sh(t) - sen(t)$

$y''' + x' = ch(t) - cos(t)$

$\begin{cases} \mathcal{L}(x''') + \mathcal{L}(y') = sh(t) - sen(t) \\ \mathcal{L}(y''') + \mathcal{L}(x') = ch(t) - cos(t) \end{cases} \to \begin{cases} D^2(x) + D^2(y) = \dots \\ D^2(y) + D(x) = \dots \end{cases}$

$\to r^2 = sh\,t - sen\,t$

$\to r^2 = ch\,t - cos\,t$

$x' = \frac{e^{-t\sqrt{sh\,t - sen\,t}}(C_1 e^{2t\sqrt{sh\,t - sen\,t}}\ 2sh\,t - 2sen\,t)}{2\sqrt{sh\,t - sen\,t}}$

$$\vec{x} = \begin{pmatrix} C_1 e^{\sqrt{sht - sent}\; t} \\ C_3 e^{\sqrt{sh(t) - cost}\; t} \end{pmatrix} + \begin{pmatrix} C_2 e^{-\sqrt{sht - sent}\; t} \\ C_4 e^{-\sqrt{cht - cost}\; t} \end{pmatrix}$$

3.28 $x' + y = 0$, $x(0) = 1$, $y(0) = -1$
$y = 0$

$x' + y = 0 \; (1)$ | $x' + y = 0$
$y' = 0 \; (2)$ | $y = -x' \; (3)$

$\downarrow$

$x' + y = 0 \; (1)$ | $x'' + 0 = 0$
$x'' + y' = 0$ | $x'' = 0 \mid r^2 = r_1, r^2$

$x = C_1 e^{r_1 t} + C_2 e^{r_2 t}$
$x' = r_1 C_1 e^{r_1 t} + C_2 e^{r_2 t}$ $\quad \therefore$ $\cancel{x(t)} = e^{t}$ con $x(0) = 1$
$y = -\cancel{C r_1} C_1 e^{rt} - C_2 e^{r_2 t}$ $\quad y(t) = e^{t}$ con $y(0) = 1$

3.30 $x'' - y' = 0$ $\quad x(0) = -1$, $x'(0) = y(0) =$
$x - y'' = 2sent$; $\quad y'(0) = 1$
$x(t) = sent - cost$, $y(t) = sent + cost$

3.31 $x'' - y' = 0$ $\quad x(0) = y(0)$ $\quad x'(0) = y(0) = 2$
$x' - y'' = 2cost$
$x(t) = sent + sht$, $y(t) = cost + cht$

3.32 $x'' - y' = e^{t}$ $\quad x(0) = 1$, $y(0) = -1$
$x' + y'' - y = 0$ $\quad x'(0) = y'(0) = 0$

$-y' = x'' + e^t$ $\quad x' = -y'' - y$
$y' = x' - e^t$ $\quad x' = -y'' - t - 1$
$y(t) = t - e^t$ $\quad x' = -y'' \frac{t}{2}$
$x' = 1 + \frac{t}{2}$

3.33 $x'' + y' = 2\text{sen}t$ $\quad x(0) = z(0) = y'(0) = 0$

$y'' + z' = 2\cos t$ $\quad x'(0) = y(0) = -1$

$z'' - x = 0$ $\quad z'(0) = 1$

$x'' = 2\text{sen}t \rightarrow x'' = \frac{2}{s^2+1}$

$y'' + 1 = 2\cos t \rightarrow y'' = -1 + 2\cos t \rightarrow y'' = -\frac{1}{s} + \frac{2s}{s^2+1}$

$z'' - 0 = 0 \rightarrow z'' = 0$

$\therefore y' = -t + 2\text{sen}(t)$ $\quad x' = -2\cos t$

$y(t) = \frac{-t^2}{2} - 2\cos(t)$ $\quad x(t) = -2\text{sen}(t)$

$z'' = x \rightarrow z'' = -2\text{sen}t$

$z = 2\text{sen}t$

3.34 $x'' - y' = f_1(t)$ donde $f_1(t) = \begin{cases} 1 & \text{para } 0 < t \leq n \\ 0 & \text{para } t > n \end{cases}$

$y' + x = f_2(t)$

$\mathcal{L}(x') + \mathcal{L}(y) = \mathcal{L}(0 - u(t-1))$

$x' + y = 0 - u(t-1)$

$x(s) = \frac{1}{s(s+1)} e^{-s} \rightarrow x(t) = \mathcal{L}^{-1}(Y(s))$

$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = A(s+1) + Bs$

$\therefore y(t) = u(t-1) - Se^{-(t-1)} u(t-1$

3.35 $x'' - y = 0$
$y'' - x = f(t)$

donde $\begin{cases} 1 & \text{para } 0 \le t \le \pi \\ -1 & \text{para } \pi \le t \le 2\pi \\ 0 & \text{para } t \ge 2\pi \end{cases}$

$x(t) = \mathcal{L}(x'' + y) = \mathcal{L}(f(t))$

$\left.\begin{array}{l} g(t) = 1 \\ a = \pi \\ b = 2\pi \end{array}\right\} f(t) = 1\left(U(1-\pi) - U(t-2\pi)\right)$

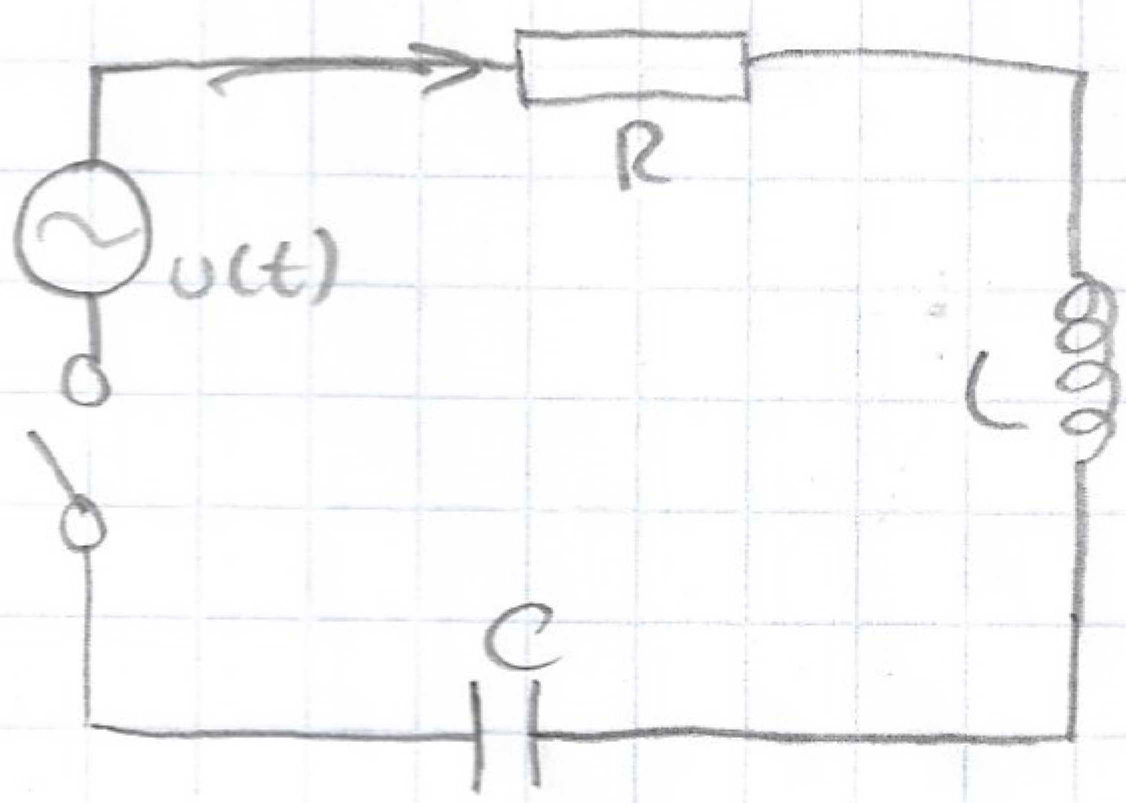
$\mathcal{L}(x'') + \mathcal{L}(y) = \mathcal{L}(1\,U(t-\pi)) - \mathcal{L}(1\,U(t-2\pi))$

$y(s) = e^{-\pi s}\dfrac{1}{s(s^2+1)} \quad e^{-\pi s}\dfrac{1}{s(s^2+1)} + \dfrac{1}{s^2+1}$

$y(t) = \mathcal{L}^{-1}\left(e^{-\pi s} \cdot \dfrac{1}{s(s^2+1)}\right) - \mathcal{L}^{-1}\left(e^{-2\pi s}\dfrac{1}{s(s^2+1)}\right) + \operatorname{sen} t$

$\therefore y(t) = U(t-\pi) - \cos(t-\pi)\,U(t-\pi) - U(t-2\pi) +$
$\cos(t-2\pi)\,U(t-2\pi) + \operatorname{sen} t$

3.36



$\text{si } \dfrac{1}{LC} - \dfrac{R^2}{4L^2} = n^2 > 0$

$\therefore i(t) = \dfrac{E_1}{L n \theta} e^{-Kt} \operatorname{sen} nt$
$+ \dfrac{E_2 - E_1}{L} n(t-T)\, x e^{-k(t-T)}$

$\operatorname{sen} n(t-T);$ 1

$\therefore k = \dfrac{R}{2L}$