$y = uv$

$y' = u'v + uv'$

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5.- $e^{2x}\cos x$, $e^{2x}\operatorname{sen} x$, $e^{2x}\operatorname{sen} x$

$$W = \begin{vmatrix} e^{2x}\cos x & e^{2x}\operatorname{sen} x \\ 2e^{2x}\cos x - e^{2x}\operatorname{sen} x & 2e^{2x}\operatorname{sen} x + e^{2x}\cos x \end{vmatrix}$$

$$= e^{2x}\cos x(2e^{2x}\operatorname{sen} x + e^{2x}\cos x) - e^{2x}\operatorname{sen} x(2e^{2x}\cos x - e^{2x}\operatorname{sen} x)$$

$$= xe^{4x}\cos 2x + xe^{4x}\operatorname{sen} 2x$$

$$= xe^{4x}(\cos 2x + \operatorname{sen} 2x)$$

$$= xe^{4x}$$

$$y_p = C_1e^{2x}\cos x + C_2e^{2x}\operatorname{sen} x$$

$$y' = 2C_1e^{2x}\cos x - C_1e^{2x}\operatorname{sen} x + 2C_2e^{2x}\operatorname{sen} x + C_2e^{2x}\cos x$$

$$y'' = 3C_1e^{2x}\cos x - 4C_1e^{2x}\operatorname{sen} x + 4C_2e^{2x}\operatorname{sen} x + 2C_2e^{2x}\cos x$$

$$y + y'' = C_1e^{2x}\cos x$$

$$y + y' = 4C_2e^{2x}\operatorname{sen} x$$

$$y'' + 4xy' = 0$$

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8.- $y''-2y'+y=\frac{e^x}{\sqrt{4-x^2}}$ $\qquad f(x)=\frac{e^x}{\sqrt{4-x^2}}$

$y''-2y'+y=0 \to y=e^{rx}$ $\qquad y_1=e^x$

$y_2=xe^x$

$Ec.c = D^2-2D+1=0$

$(D-1)^2=0 \to D=1,\ D=1 \therefore y_h=C_1e^x+C_2xe^x$

$y_p=u_1e^x+u_2xe^x \quad ; u_1=-\int\frac{y_2 f(x)}{W}dx \quad , u_2=\int\frac{y_1 f(x)}{W}dx$

$W=\begin{vmatrix} e^x & xe^x \\ e^x & e^x+xe^x \end{vmatrix} = e^{2x}\cancel{+xe^{2x}}-\cancel{xe^{2x}}=e^{2x}$

Entonces

$u_1=-\int\frac{xe^x\left(\frac{e^x}{\sqrt{4-x^2}}\right)}{e^{2x}}dx=\int\frac{\frac{x\cancel{e^{2x}}}{\sqrt{4-x^2}}}{\cancel{e^{2x}}}dx=\int\frac{x}{\sqrt{4-x^2}}dx=-\sqrt{x^2-4}+C=u_1$

$u_2=\int\frac{e^x\left(\frac{e^x}{\sqrt{4-x^2}}\right)}{e^{2x}}dx=\int\frac{\frac{\cancel{e^{2x}}}{\sqrt{4-x^2}}}{\cancel{e^{2x}}}dx=\int\frac{1}{\sqrt{4-x^2}}dx=\int\frac{1}{\sqrt{2^2-x^2}}dx=\operatorname{sen}^{-1}\left(\frac{x}{2}\right)+C=u_2$

$\therefore y_p=-\sqrt{x^2-4}\,e^x+\operatorname{sen}^{-1}\left(\frac{x}{2}\right)xe^x$

$y=y_h+y_p$

$\therefore y=C_1e^x+C_2xe^x-\sqrt{x^2-4}\,e^x+\operatorname{sen}^{-1}\left(\frac{x}{2}\right)xe^x$

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$y=e^x\left(C_1+C_2x-\sqrt{x^2-4}+x\operatorname{sen}^{-1}\left(\frac{x}{2}\right)\right)$

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11.- $x^2 y''' - 3xy'' + 3y' = 0$

$u = y'$, $u' = y''$, $u'' = y'''$ $\rightarrow x^2 u'' - 3xu' + 3u = 0$

$u = x^r \rightarrow x > 0$ $\quad u' = rx^{r-1}, \quad u'' = r(r-1)x^{r-2}$

$x^2 r(r-1)x^{r-2} - 3x\, r x^{r-1} + 3x^r = 0$

$(r^2 - r)x^r - 3rx^r + 3x^r = 0$

$x^r(r^2 - r - 3r + 3) = 0$ $\quad$ Como $x > 0 \therefore$

$r^2 - 4r + 3 = 0$ $\quad u_1 = x^1$

$(r-1)(r-3) = 0 \longrightarrow r_1 = 1, \; r_2 = 3$ $\quad u_2 = x^3$

$u = C_1 x + C_2 x^3$

$u = y' \rightarrow y' = C_1 x + C_2 x^3$

$\frac{dy}{dx} = C_1 x + C_2 x^3 \rightarrow dy = (C_1 x + C_2 x^3)\, dx$

$\int dy = \int C_1 x + C_2 x^3 \, dx$

$y = \frac{C_1 x^2}{2} + \frac{C_2 x^4}{4} + C_3$

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$y = C_1 x^2 + C_2 x^4 + C_3$

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$$\left(y^{(k-1)}\right)^n y^{(k)} = \frac{\left(y^{(k-1)}\right)^{n+1}}{n+1}$$

2.- $xyy'' + xy'^2 = 2yy'$

$$xyy'' + xy'^2 - 2yy' = 0$$

$$\int d\left(xyy' - \frac{3y^2}{2}\right)dx = \int 0\, dx \to \left(xyy' - \frac{3y^2}{2} = C\right)2$$

$$2xyy' - 3y^2 = 2C$$

$$2xyy' = 3y^2 + 2C \to yy' = \frac{3y^2+2C}{2x}$$

$$y\frac{dy}{dx} = \frac{3y^2+2C}{2x} = \int y\,dy = \int\left(\frac{3y^2+2C}{2x}\right)dx$$

$$\int \frac{y\,dy}{3y^2+2c} = \int \frac{dx}{2x}$$

$$= \frac{1}{6}\ln|3y^2+2c| = \frac{1}{2}\ln|x| \to \frac{\ln|3y^2+2c|}{6} = \frac{\ln|x|}{2} + C_1$$

$$e^{\frac{\ln|3y^2+2c|}{6}} = e^{\frac{\ln|x|}{2}}$$

$$= (3y^2+2c)^{1/6} = e^{C_1} x^{1/2}$$

$$y^2 = C_1x^3 + C_2, \quad y_1 = \frac{\sqrt{2}\sqrt{-C_1}}{\sqrt{3}}, \quad y_2 = -\frac{\sqrt{2}\sqrt{-C_2}}{\sqrt{3}}$$