2do examen parcial Algebra Lineal

1.- Si V_1 y V_2 son subespacios de $\mathbb{R}^n$, demuestra que $V_1 \cap V_2$ es un subespacio de $\mathbb{R}^n$

Sean $\underline{u}, \underline{v} \in \mathbb{R}^n$ y $\alpha \in \mathbb{R}$ tales que $\underline{u} = u = V_1$, $\underline{v} = v = V_2$ $= \underline{u} \cap \underline{v}$ $\forall n \geq 0$

① ¿$\underline{u} + \underline{v} = \underline{v} + \underline{u}$?

② ¿$\alpha \underline{u} \in \mathbb{R}^n$?

2.- $S = \{v_1, v_2, v_3\}$, $T = \{w_1, w_2, w_3\}$ $w_1 = (3, 2, 0)$, $w_2 = (2, 1, 0)$, $w_3 = (3, 1,$

$P_{T \to S} \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$ ¿Cuales son los vectores de la base S?

3.- $S=\{(-1,2,1),(0,1,1),(-2,2,1)\}$ y $T=\{(-1,1,0),(0,1,0),(0,1,1)\}$

$[V]_S = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

a) $T_{T\to S} = \left(\begin{array}{ccc|ccc} -1 & 0 & -2 & -1 & 0 & 0 \\ 2 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)^{(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ R_2-2R_1 \\ R_3-R_1 \end{matrix} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 1 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right)$

$\begin{matrix} \\ \\ R_3-R_2 \end{matrix} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right) \begin{matrix} R_1-2R_3 \\ R_2+2R_3 \\ \\ \end{matrix} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right) \therefore T_{T\to S} \begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

$T_{S\to T} = \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & -1 & 0 & -2 \\ 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)^{(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right) R_2-R_1 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$

$R_2-R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right) \therefore T_{S\to T} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$

b) $[V]_T = T_{S\to T}[V]_S = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} =$

$= \begin{pmatrix} 1(1)+0(-2)+2(0) \\ 0(1)+0(-2)+(-1(0)) \\ 1(1)+1(-2)+1(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \underline{-1} \end{pmatrix}$

c) ¿$\underline{v}$?

$\underline{v} = 1(-1,1,0)+0(0,1,0)-1(0,1,1)$

$=(-1,1,0)+(0,0,0)-(0,1,1)$

$\underline{v} = (-1,2,1)$

4.- $x_1 + x_2 + 2x_4 = 0$

$-2x_1 - 2x_2 + x_3 - 5x_4 = 0$

$x_1 + x_2 - x_3 + 3x_4 = 0$

$4x_1 + 4x_2 - x_3 + 9x_4 = 0$

$$A = \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ -2 & -2 & 1 & -5 & 0 \\ 1 & 1 & -1 & 3 & 0 \\ 4 & 4 & -1 & 9 & 0 \end{array}\right)$$

a)

$$\begin{array}{l} \\ R_2+R_1 \\ R_3-R_1 \\ R_4-4R_1 \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} \\ \\ R_3+R_2 \\ R_4+R_2 \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = -x_2 - 2x_4$

$x_3 = x_4$

$x_2 = x_2$

$x_4 = x_4$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -x_2 - 2x_4 \\ x_2 \\ x_4 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ son la base para el espacio solución del sistema

b)

$$\begin{pmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{pmatrix}$$

Con respecto al inciso anterior se concluye que la nulidad $\rightarrow$ $null(A) = 2$

Y su rango por renglones estaría dado por

$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$ y $\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$ $\therefore$ su $rango(A) = 2$

$\therefore$ esto cumple la condición que dice que:

rango + nulidad = no. de columnas

5.- $v_1=(1,-1,1)$, $v_2=(-2,3,-1)$, $v_3=(-3,5,-1)$, $v_4=(1,2,-4)$

$v_1\cdot v_1=3$ $\quad v_2\cdot v_2=14$ $\quad v_3\cdot v_3=35$ $\quad v_4\cdot v_4=19$ $\quad u_3\cdot v_1=-9$

$u_2\cdot v_1=-6$ $\quad u_3\cdot v_2=22$ $\quad u_4\cdot v_1=-5$ $\quad u_4\cdot v_2=8$ $\quad u_4\cdot v_3=11$

$$V_2=U_2-\frac{u_2\cdot v_1}{v_1\cdot v_1}v_1=\begin{pmatrix}-2\\3\\-1\end{pmatrix}-\frac{-6}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}-2\\3\\-1\end{pmatrix}+\begin{pmatrix}2\\-2\\2\end{pmatrix}=\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$V_3=U_3-\frac{U_3\cdot V_1}{V_1\cdot V_1}V_1-\frac{U_3\cdot V_2}{V_2\cdot V_2}V_2=\begin{pmatrix}-3\\5\\-1\end{pmatrix}-\frac{-9}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}-\frac{22}{14}\begin{pmatrix}-2\\3\\-1\end{pmatrix}=\begin{matrix}22/7\\-19/7\\25/7\end{matrix}$$

$$V_4=U_4-\frac{U_4\cdot V_1}{V_1\cdot V_1}V_1-\frac{U_4\cdot V_2}{V_2\cdot V_2}V_2-\frac{U_4\cdot V_3}{V_3\cdot V_3}V_3$$

$$=\begin{pmatrix}1\\2\\-4\end{pmatrix}-\frac{-5}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}-\frac{8}{14}\begin{pmatrix}-2\\3\\-1\end{pmatrix}-\frac{11}{35}\begin{pmatrix}-3\\5\\-1\end{pmatrix}$$

$$=\begin{pmatrix}1\\2\\-4\end{pmatrix}+\begin{matrix}5/3\\-5/3\\5/3\end{matrix}-\begin{pmatrix}-8/7\\12/7\\-4/7\end{pmatrix}-\begin{matrix}33/35\\11/7\\11/35\end{matrix}=\begin{matrix}8/3\\1/3\\-7/3\end{matrix}=\begin{matrix}80/21\\1/21\\23/21\end{matrix}=\begin{matrix}43/15\\34/21\\148/105\end{matrix}$$