Ejercicios 3.2

a)

1- $\frac{dN}{dt} = N(1-0.0005N)$, $N(0)=1$

$N=0$, $N=2000$ $\quad 0<N<2000$

$\frac{dN}{dt} > 0 \quad \lim_{t\to\infty} N(t) = 2000$

b)

$$\frac{dN}{N(1-0.0005N)} = \left(\frac{1}{N} - \frac{1}{N-2000}\right) dN = dt$$

$$\ln N - \ln(N-2000) = t + c$$

$$\to N(t) = \frac{2000\, e^{c_1+t}}{1+e^{c+t}} \to \frac{2000\, e^{c} e^{t}}{(1+e^{c}e^{t})}$$

$$N(0) = 1 \to e^{c} = \frac{1}{1999} \to N(t) = \frac{2000\, e^{t}}{1999+e^{t}}$$

$N(10) = 1833$

6)

$$P(5-P) - \frac{25}{4} = 0$$

$$\to P = \frac{5}{2} \qquad \frac{dP}{dt} < 0 \;,\; P \neq \frac{5}{2}$$

$$\to P(t) = \frac{[4P_0 + (10P_0 - 25)t]}{[4 + (4P_0 - 10)t]}$$

11.- a) $\frac{dh}{dt} = \frac{-8A_n\sqrt{h}}{A_w}$, $h(0) = H$

$\rightarrow \frac{dh}{\sqrt{h}} = -\frac{8A_n}{A_w}dt$, $2\sqrt{h} = -\frac{8A_n}{A_w}t + c$

$h(0) = H \rightarrow c = 2\sqrt{H}$

$\sqrt{h(t)} = \frac{(A_w\sqrt{H} - 4A_n t)}{A_w}$ donde $A_w\sqrt{H} - 4A_n t \geq 0$

$\therefore h(t) = \frac{(A_w\sqrt{H} - 4A_n t)^2}{A_w^2}$ $0 \leq t \leq \frac{A_w\sqrt{H}}{4A_n}$

16.-

$m\frac{dv}{dt} = -mg - kv^2$

$\frac{dv}{mg + kv^2} = -\frac{dt}{m} \rightarrow \frac{1}{\sqrt{mgk}}\arctan\left(\frac{\sqrt{k}v}{\sqrt{mg}}\right) = -\frac{1}{m}t + C$

$\arctan\left(\frac{\sqrt{k}v}{\sqrt{mg}}\right) = -\frac{1}{m}t + C$

$\arctan\left(\frac{\sqrt{k}v}{\sqrt{mg}}\right) = -\sqrt{\frac{gk}{m}}t + C_1$

$v(t) = \sqrt{\frac{mg}{k}}\tan\left(C_1 - \sqrt{\frac{gk}{m}}t\right)$

$v(t) = 230.94\tan(0.91 - 0.13)$

21.- a)

t	P(t)	Q(t)
0	3.9	0.03
10	5.3	0.03
20	7.2	0.03
30	9.6	0.03
40	12.8	0.03
50	17.6	0.03

b) $Q = 0.034 - 0.00016\,P$

c) $a = 0.034$, $b = 0.00016$

$$P(t) = \frac{aP_0}{bP_0 + (a - bP_0)e^{-at}}$$

d) $P_0 = 3.929$

$$P(t) = \frac{0.136}{0.00066 + 0.0341e^{-0.034}}$$

26.- $V = V_s + V_r = (-V_s\cos\theta, -V_s\operatorname{sen}\theta) + (0, V_r) = (-V_s\cos\theta, -V_s\operatorname{sen}\theta + V_r) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

$$\frac{dx}{dt} = -V_s\cos\theta \quad , \quad \frac{dy}{dt} = -V_s\operatorname{sen}\theta + V_r$$

$$\rightarrow \frac{dx}{dt} = -V_s\frac{x}{\sqrt{x^2+y^2}} \qquad \frac{dy}{dt} = -V_s\frac{y}{\sqrt{x^2+y^2}} + V_r$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-V_s y + V_r\sqrt{x^2+y^2}}{-V_s x} = \frac{V_s y - V_r\sqrt{x^2+y^2}}{V_s x}$$

31.- a) $c = 0.6$, $Ah = \pi\left(\frac{1}{32}\cdot\frac{1}{32}\right)^2$, $Aw = \pi \cdot 1^2 = \pi$, $g = 32$

$\frac{dh}{dt} = -0.00003255\sqrt{h}$, $2\sqrt{h} = -0.00003255t + C$

$\rightarrow h = (c_1 - 0.00001628t)^2$ $h(0) = 2 \rightarrow c = \sqrt{2}$
$h(t) = (\sqrt{2} - 0.00001628t)^2$

b)
$h(3600) = (\sqrt{2} - 0.00001628(3600))^2 = 1.838 \text{ ft}$

Ejercicios 3.3

1.- $\frac{dx}{dt} = -\lambda_1 x$

$\frac{dx}{x} = -\lambda_1 dt \rightarrow \ln|x| = -\lambda t + c \rightarrow x = c_1 e^{-\lambda_1 t}$

$x(0) = x_0 \rightarrow c_1 = x_0 \rightarrow x = x_0 e^{-\lambda_1 t}$

$\frac{dy}{dt} + \lambda_2 y = \lambda_1 x_0 e^{-\lambda_1 t}$

$\frac{d}{dt}\left[e^{\lambda_2 t} y\right] = \lambda_1 x_0 e^{(\lambda_2 - \lambda_1)t} + c_2$

$y = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} e^{-\lambda_2 t} + c_2 e^{-\lambda_2 t} = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t}$

$$y = \frac{\lambda_1 x_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

6.- $x_1' = \frac{1}{100} x_2 2 - \frac{1}{100} x_1 6 = \frac{1}{50} x_2 - \frac{3}{50} x_1$

$x_2' = \frac{1}{100} x_1 6 + \frac{1}{100} x_3 - \frac{1}{100} x_2 2 - \frac{1}{100} x_2 5 = \frac{3}{50} x_1 - \frac{7}{100} x_2 + \frac{1}{100} x_3$

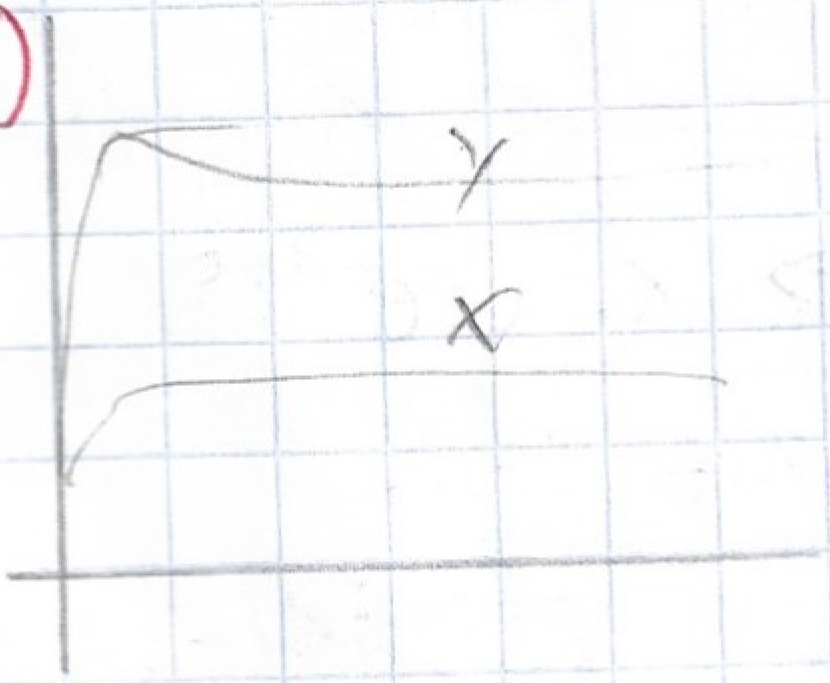
$x_3' = \frac{1}{100} x_2 5 - \frac{1}{100} x_3 - \frac{1}{100} x_3 4 = \frac{1}{20} x_2 - \frac{1}{20} x_3$

11.-

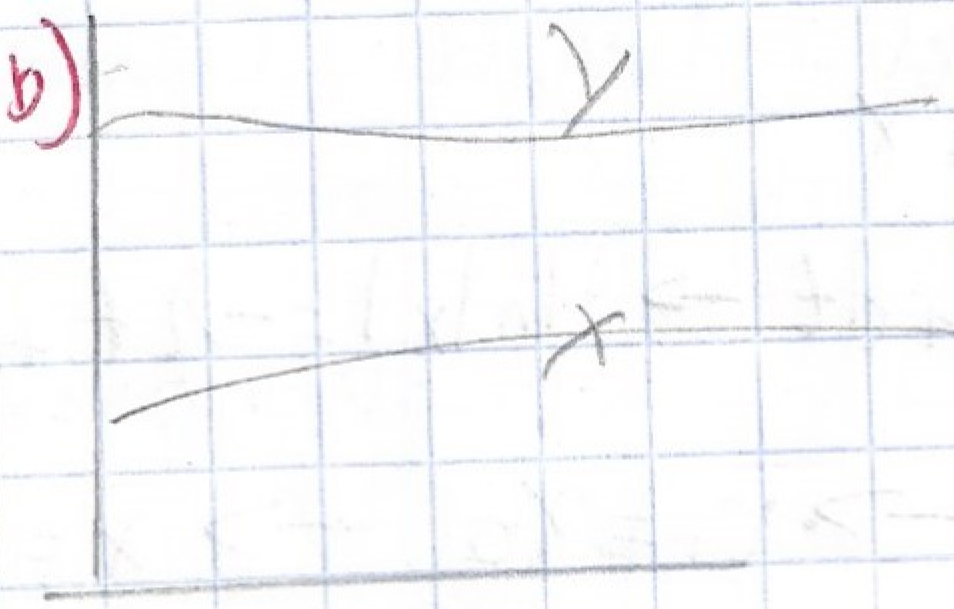
$\frac{dx}{dt} = x(1 - 0.1x - 0.05y)$

$\frac{dy}{dt} = y(1.7 - 0.1y - 0.15x)$

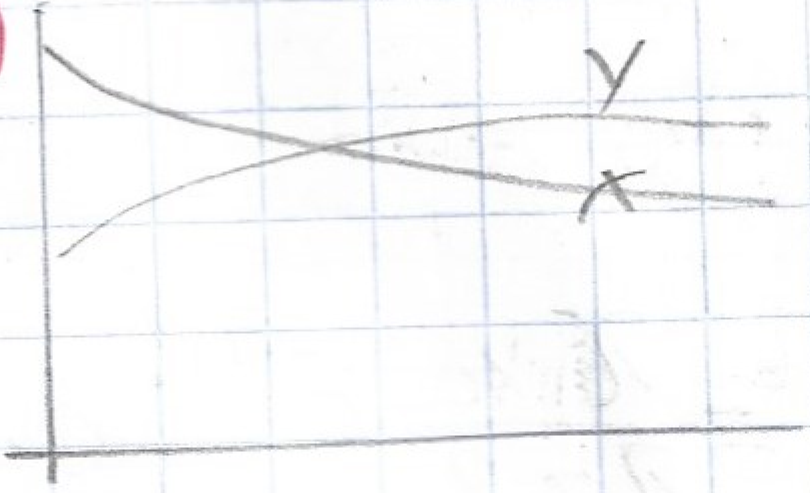
a)



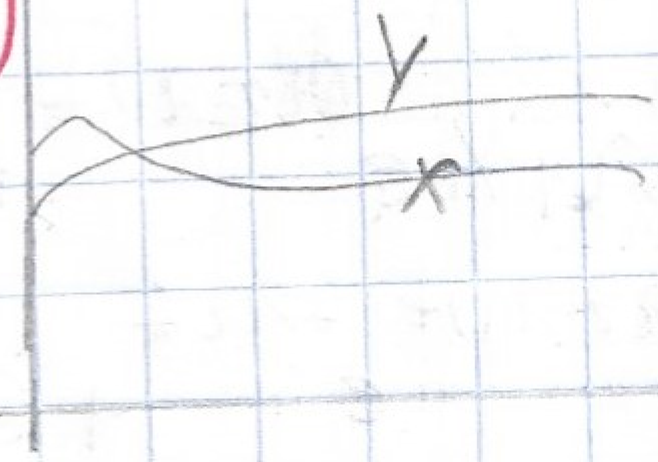
b)



c)

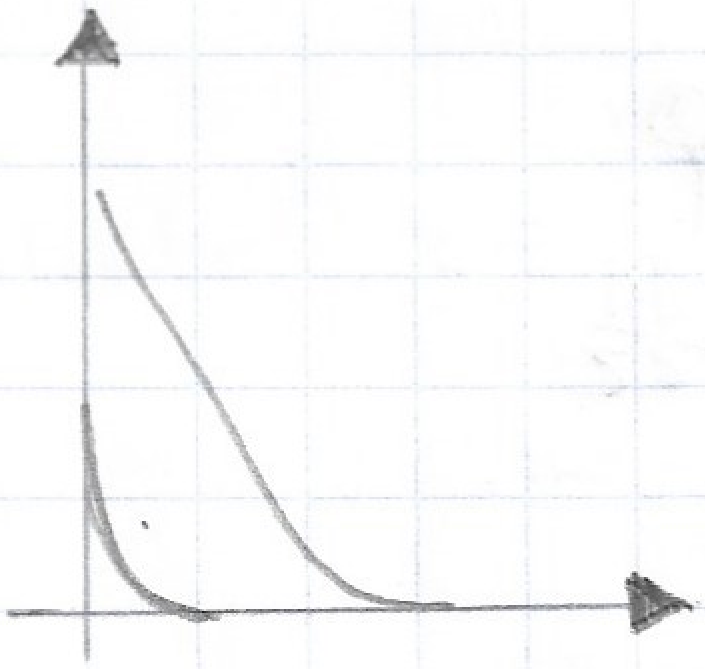


d)



16.- a) $s+i+r=n$

b) $\frac{ds}{dt}=-0.25si$, $\frac{di}{dt}=-0.7i+0.25si$



21.-

a) 20% de etanol

b) $100P''=\frac{1}{50}P-\frac{1}{10}Q-P'$

c) $Q=50P'-30+\frac{P}{2}$

d) $P'(0)=\frac{6}{10}+\frac{7}{50}-\frac{200}{100}=\frac{-63}{50}$

$P(t)=\frac{-604}{19}e^{-\frac{t}{400}}\operatorname{sen}\left(\frac{\sqrt{95}t}{2000}\right)\sqrt{95}-100e^{-\frac{t}{400}}\cos\left(\frac{\sqrt{95}t}{2000}\right)+100$

e) $Q(t)=-\frac{270}{19}e^{-\frac{t}{400}}\cos\left(\frac{\sqrt{95}t}{2000}\right)-\frac{130}{19}e^{-\frac{t}{400}}\operatorname{sen}\left(\frac{\sqrt{95}t}{2000}\right)\sqrt{95}+20+\frac{23}{19}e^{-\frac{t}{20}}$

Ejercicios 4.1

1.- $y = C_1e^x + C_2e^{-x}$, $(-\infty, \infty)$
$y'' - y = 0$, $y(0) = 0$, $y'(0) = 1$

$y' = C_1e^x - C_2e^{-x} \rightarrow y(0) = C_1 + C_2 = 0$

$y'(0) = C_1 - C_2 = 1 \therefore C_1 = \frac{1}{2}$, $C_2 = -\frac{1}{2}$

$\hookrightarrow y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

6.- $y(0) = C_1 = 0$, $y'(0) = 2C_2 \cdot 0 = 0 \rightarrow C_1 = 0$

C_2 es arbitrario

$\therefore y = x^2$, $y = 2x^2$

11.- a) $y(0) = C_1 + C_2 = 0$, $y(1) = C_1e + C_2e^{-1} = 1$

$\rightarrow C_1 = \frac{e}{e^2-1}$, $C_2 = \frac{-e}{e^2-1} \therefore y = \frac{e(e^x - e^{-x})}{e^2-1}$

b) $y(0) = C_3\cosh 0 + C_4\operatorname{senh} 0 = C_3 = 0$
$y(1) = C_3\cosh 1 + C_4\operatorname{senh} 1 = C_4\operatorname{senh} 1 = C_3 = 0$

c) $y = \frac{1}{\operatorname{senh} 1}\operatorname{senh} x = \frac{2}{e^1 - e^{-1}}\,\frac{e^x - e^{-x}}{2} = \frac{e^x - e^{-x}}{e - \frac{1}{e}} = \frac{e}{e^2-1}(e^x - e^{-x})$

16.- $(1)0 + (0)x + (0)e^{x} = 0$ es li

21.- $c_1(1+x) + c_2 x + c_3 x^2 = 0$ Entonces
$c_1 + (c_1+c_2)x + c_3x^2 = 0$ y $c_1 = 0$, $c_1 + c_2 = 0$, $c_3 = 0$

26.-

$$\omega(e^{\frac{x}{2}}, xe^{\frac{x}{2}}) = e^{x} \neq 0$$

$$-\infty < x < \infty \therefore y = C_1 e^{\frac{x}{2}} + C_2 x e^{\frac{x}{2}}$$

31.- $y_1 = e^{2x}$, $y_2 = e^{5x} \rightarrow y_p = 6e^{x}$

36.-

a) $y_{p1} = 5$

b) $y_{p2} = -2x$

c) $y_p = y_{p1} + y_{p2} = 5 - 2x$

d) $y_p = \frac{1}{2} y_{p1} - 2 y_{p2} = \frac{5}{2} + 4x$

41.- $0y_1 + 0y_2 + \dots 0y_n + 1y_{n+1} = 0$ son li

Ejercicios 4.2

1.- $y''-4y'+4y=0$; $y_1=e^{2x}$

$y=u\cdot y_1 \rightarrow y=e^{2x}\cdot u$ $\quad y'=2e^{2x}\cdot u+e^{2x}u'$

$y''=4e^{2x}u+2e^{2x}u'+2e^{2x}\cdot u'+e^{2x}u''$

$y''=4e^{2x}u+4e^{2x}\cdot u'+e^{2x}u''$

$4e^{2x}u+4e^{2x}\cdot u'+e^{2x}u''-4(2e^{2x}\cdot u+e^{2x}u')+4(e^{2x}u)=0$

$\cancel{4e^{2x}u}+\cancel{4e^{2x}u'}+e^{2x}u''-\cancel{8e^{2x}\cdot u}-\cancel{4e^{2x}u'}+\cancel{4e^{2x}u}=0$

$e^{2x}u''=0 \rightarrow u''=0$

$u=c_1x+c_2$, $c_1=1$, $c_2=0$

$y_2=xe^{2x}$

6.- $y''-25y=0$; $y_1=e^{5x}$

$y=uy \rightarrow y=e^{5x}$ $\quad y'=5e^{5x}\cdot u+e^{5x}u'$

$y''=25e^{5x}u+5e^{5x}\cdot u'+5e^{5x}u'+e^{5x}u''$

$y''=25e^{5x}u+10e^{5x}u'+e^{5x}u''$

$25e^{5x}u+10e^{5x}u'+e^{5x}u''-25e^{5x}=0$

$e^{5x}(25u+10u'+u''-25)=0$ $\rightarrow w=u'$

$u''+10u'+25u-25=0$

$u''+10u'=0$

$w=u' \rightarrow w'+10w=0 \rightarrow e^{10\int dx}=e^{10x}$

$\rightarrow e^{10x}w=C \rightarrow w=ce^{10x}=u'$

$\therefore y_2=e^{-10x}e^{5x}=e^{-5x}$

11.- $xy'' + y' = 0$; $y_1 = \ln x$

$P(x) = \frac{1}{x}$

$\therefore y_2 = \ln x \int \frac{e^{-\int \frac{dx}{x}}}{(\ln x)^2} dx = \ln x \int \frac{dx}{x(\ln x)^2} = \ln x \left(-\frac{1}{\ln x}\right) = -1$

$\therefore y_2 = 1$

16.- $(1-x^2)y'' + 2xy' = 0$; $y_1 = 1$

$P(x) = -\frac{2x}{(1-x^2)}$

$\therefore y_2 = \int e^{\int \frac{2x\,dx}{(1-x^2)}} dx = \int \cancel{e}^{\cancel{\ln}(1-x^2)} dx = \int \frac{1}{1-x^2} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$

$\therefore y_2 = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$

21-

$y_1 = e^{m_1x} \rightarrow y_1' = m_1 e^{m_1x} \quad y_1'' = m_1^2 e^{m_1x}$

$ay_1'' + by_1' + cy_1 = am_1^2 e^{m_1x} + bm_1 e^{m_1x} + ce^{m_1x}$

$= e^{m_1x}(am_1^2 + bm_1 + c) = 0$

$y = e^{m_1x} \longrightarrow am_1^2 + bm_1 + c = 0$

$\therefore y'' + \frac{b}{a} y' + \frac{c}{a} y = 0$

Ejercicios 4.3

1- $4y'' + y' = 0$

$4D^2 + D = 0$

$D = 0 \quad D = \frac{1}{4} \longrightarrow \therefore y = C_1 + C_2 e^{-\frac{x}{4}}$

6.- $y'' - 10y' + 25y = 0$

$D^2 - 10D + 25 = 0$

$D = 5$

$y = C_1 e^{-5x} + C_2 x e^{5x}$

11.- $y'' - 4y' + 5y = 0$

$D^2 - 4D + 5 = 0$

$D = 2$ $\longrightarrow$ $y = C_1 \cos x\, e^{2x} + C_2 \operatorname{sen} x\, e^{2x}$

$\downarrow$

$y = e^{2x}(C_1 \cos x + C_2 \operatorname{sen} x)$

16.- $y''' - y = 0$

$D^3 - D = 0$

$$\begin{array}{cc|c} 1 & -1 & D=1 \\ & 1 & \\ \hline 1 & 0 & \end{array}$$

$D = 1, \quad D = -\frac{1}{2} \pm \sqrt{\frac{3i}{2}}$

$y_1 = C_1 e^{x} + e^{-\frac{x}{2}} \left(\left(C_2 \cos \frac{\sqrt{3x}}{2} \right) + \left(C_3 \operatorname{sen} \frac{\sqrt{3x}}{2} \right) \right)$

21.- $y''' + 3y'' + 3y' + y = 0$

$D^3 + 3D^2 + 3D + 1 = 0$

$D = -1$

$y = C_1 e^{-x} + C_2 x e^{-x} + C_3 x^2 e^{-x}$

26.- $\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} - 18y = 0$

$D^4 - 7D^2 - 18 = 0$

$D = 3\ ,\ D = -3 \quad D = \pm\sqrt{2}$

$\downarrow$

$\therefore\ y = C_1e^{3x} + C_2e^{-3x} + C_3\cos\sqrt{2}x + C_4\,\text{sen}\,\sqrt{2}x$

31.- $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = 0\ ,\ y(1) = 0\ ,\ y'(1) = 2$

$D^2 - 4D - 5 = 0\ ,\ y(1) = 0\ ,\ y'(1) = 2$

$D = 5\ ,\ D = -1$

$\therefore\ y = c_1e^{-t} + c_2e^{5t}$

$y(1) = 0\ ,\ y'(1) = 2$

$\downarrow$

$0 = C_1e^{-1} + C_2e^{5}\ ,\ 2 = -c_1e^{-1} + C_2e^{5}$

$\therefore\ c_1 = -\frac{e}{3}\ ,\ C_2 = \frac{e^{-5}}{3}$

$y = -\frac{1}{3}e^{1-t} + \frac{1}{3}e^{5t-5}$

36.- $y''' + 2y'' - 5y' - 6y = 0$, $y(0) = y'(0) = 0$, $y''(0) = 1$

$D^3 + 2D^2 - 5D - 6 = 0$

$D_1 = -3$, $D_2 = -1$, $D_3 = 2$

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

$c_1 + c_2 + c_3 = 0$, $-c_1 + 2c_2 - 3c_3 = 0$, $c_1 + 4c_2 + 9c_3 = 1$

$c_1 = -\frac{1}{6}$, $c_2 = \frac{1}{15}$, $c_3 = \frac{1}{10}$

$$y = -\frac{1}{6} e^{-x} + \frac{1}{15} e^{2x} + \frac{1}{10} e^{-3x}$$

41.- $y'' - 3y = 0$, $y(0) = 1$, $y'(0) = 5$

$D^2 - 3 = 0$

$D = \pm\sqrt{3} \longrightarrow y = c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}$

$$y = \cosh\sqrt{3}x + \frac{5}{3}\sqrt{3}\,\operatorname{senh}\sqrt{3}x$$

Ejercicios 4.4

1.- $y''+3y'+2y=6$

$D^2+3D+2=0$

$D_1=-1$, $D_2=-2$ $\rightarrow$ $y_c=c_1e^{-x}+c_2e^{-2x}$

$y_p=A$

$\hookrightarrow 2A=6 \rightarrow A=3$, $y_p=3$

$$y=c_1e^{-x}+c_2e^{-2x}+3$$

6.- $y''-8y'+20y=100x^2-26xe^x$

$y''-8y'+20y=0$

$D^2-8D+20=0$

$D=4+2i$, $D=4-2i$

$y_c=e^{4x}(c_1\cos 2x+c_2\operatorname{sen} 2x)$

$y_p=Ax^2+Bx+C+(Dx+E)e^x$

$\hookrightarrow$ $2A-8B+20C=0$

$-6D+13E=0$

$-16A+20B=0$

$13D=-26$

$20A=100$

$\therefore A=5$, $B=4$, $C=\frac{11}{10}$, $D=-2$, $E=-\frac{12}{13}$, $y_p=5x^2+4x+\frac{11}{10}+\left(-2x-\frac{12}{13}\right)e^x$

$$y=e^{4x}(c_1\cos 2x+c_2\operatorname{sen} 2x)+5x^2+4x+\frac{11}{10}+\left(-2x-\frac{12}{13}\right)e^x$$

11.- $y'' - y' + \frac{1}{4}y = 3 + e^{\frac{x}{2}}$

$D^2 - D + \frac{1}{4} = 0$

$D = \frac{1}{2} \longrightarrow y_c = C_1 e^{\frac{x}{2}} + C_2 x e^{\frac{x}{2}}$

$y_p = A + Bx^2 e^{\frac{x}{2}} \longrightarrow \frac{1}{4}A = 3 \quad 2B = 1$

$A = 12, \quad B = \frac{1}{2}, \quad y_p = 12 + \frac{1}{2}x^2 e^{\frac{x}{2}}$

$$y = C_1 e^{\frac{x}{2}} + C_2 x e^{\frac{x}{2}} + 12 + \frac{1}{2}x^2 e^{\frac{x}{2}}$$

16.- $y'' - 5y' = 2x^3 - 4x^2 - x + 6$

$D^2 - 5D = 0$

$D_1 = 5, \quad D_2 = 0 \longrightarrow y_c = C_1 e^{5x} + C_2$

$y_p = Ax^4 + Bx^3 + Cx^2 + Dx$

$\hookrightarrow -20A = 12A - 15B = -4, \; 6B - 10C = -1, \quad 2C - 5D = 6$

$A = -\frac{1}{10}, \quad B = \frac{14}{75}, \quad C = \frac{53}{250}, \quad D = -\frac{697}{625}$

$y_p = -\frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x,$

$$y = C_1 e^{5x} + C_2 - \frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x$$

21.- $y''' - 6y'' = 3 - \cos x$

$D^3 - 6D^2 = 0$

$D_1 = 0 = D_2$, $D_3 = 6 \rightarrow y_c = C_1 + C_2 x + C_3 e^{6x}$

$y_p = Ax^2 + B\cos x + C\operatorname{sen} x$, $-12A = 3, 6B - C = -1$

$B + 6C = 0 \rightarrow A = -\frac{1}{4}, B = -\frac{6}{37}, C = \frac{1}{37}$

$y_p = -\frac{1}{4}x^2 - \frac{6}{37}\cos x + \frac{1}{37}\operatorname{sen} x$

$$y = C_1 + C_2 x + C_3 e^{6x} - \frac{1}{4}x^2 - \frac{6}{37}\cos x + \frac{1}{37}\operatorname{sen} x$$

26.- $y^{(4)} - y'' = 4x + 2xe^{-x}$

$D^4 - D^2 = 0$

$D_1 = D_2 = 0$, $D_3 = 1$, $D_4 = -1$

$y_c = C_1 + C_2 x + C_3 e^x + C_4$

$y_p = Ax^3 + Bx^2 + (Cx^2 + Dx)e^{-x}$

$\hookrightarrow -6A = 4$, $-2B = 0$, $10C - 2D = 0$, $-4C = 2$

$\therefore A = -\frac{2}{3}$, $B = 0$, $C = -\frac{2}{3}$, $D = -\frac{5}{2}$

$y_p = -\frac{2}{3}x^3 - \left(\frac{1}{2}x^2 + \frac{5}{2}x\right)e^{-x}$

$$y = C_1 + C_2 x + C_3 e^x + C_4 e^{-x} - \frac{2}{3}x^3 - \left(\frac{1}{2}x^2 + \frac{5}{2}x\right)e^{-x}$$

31.- $y'' + 4y' + 5y = 35e^{-4x}$, $y(0) = -3$, $y'(0) = 1$

$y_c = e^{-2x}(c_1 \cos x + c_2 \operatorname{sen} x)$

$y_p = Ae^{-4x}$, $A = 7$ $\therefore$ $y = e^{-2x}(c_1 \cos x + c_2 \operatorname{sen} x) + 7e^{-4x}$

$c_1 = -10$, $c_2 = 9$

$$y = e^{-2x}(-10\cos x + 9\operatorname{sen} x) + 7e^{-4x}$$

36.- $y''' + 8y = 2x - 5 + 8e^{-2x}$, $y(0) = -5$, $y'(0) = 3$, $y''(0) = -4$

$y_c = c_1 e^{-2x} + e^{x}(c_2 \cos\sqrt{3}x + c_3 \operatorname{sen}\sqrt{3}x)$

$y_p = Ax + B + Cxe^{-2x}$, $A = \frac{1}{4}$, $B = -\frac{5}{8}$, $C = \frac{2}{3}$

$$y = c_1 e^{-2x} + e^{x}(c_2 \cos\sqrt{3}x + c_3 \operatorname{sen}\sqrt{3}x) + \frac{1}{4}x - \frac{5}{8} + \frac{2}{3}xe^{-2x}$$

41.- $y'' + 4y = g(x)$, $y(0) = 1$, $y'(0) = 2$, donde

$$g(x) = \begin{cases} \operatorname{sen} x, & 0 \le x \le \frac{\pi}{2} \\ 0, & x > \frac{\pi}{2} \end{cases}$$

$y_c = C_1 \cos 2x + C_2 \operatorname{sen} 2x$

$y_p = A\cos x + B \operatorname{sen} x \rightarrow (0, \frac{\pi}{2})$

$A = 0$, $B = \frac{1}{3}$ $\therefore$ $y = C_1 \cos 2x + C_2 \operatorname{sen} 2x + \frac{1}{3} \operatorname{sen} x$

$y = C_3 \cos 2x + C_4 \operatorname{sen} 2x$

$C_1 = 1$, $\frac{1}{3} + 2C_2 = 2$

$$y(x) = \begin{cases} \cos 2x + \frac{5}{6} \operatorname{sen} 2x + \frac{1}{3} \operatorname{sen} x, & 0 \le x \le \frac{\pi}{2} \\ \frac{2}{3} \cos 2x + \frac{5}{6} \operatorname{sen} 2x, & x > \frac{\pi}{2} \end{cases}$$

46.- $y'' - 4y' + 8y = (2x^2 - 3x) e^{2x} \cos 2x + (10x^2 - x - 1) e^{2x} \operatorname{sen} 2x$

$y_c = e^{2x} (C_1 \cos 2x + C_2 \operatorname{sen} 2x$

$y_p = (Ax^3 + Bx^2 + Cx) e^{2x} \cos 2x + (Dx^3 + Ex^2 + F) e^{2x} \operatorname{sen} 2x$

$(12Dx^2 + (6A + 8E)x + (2B + 4F)) e^{2x} + (-12Ax^2 + (-8B + 6D) x + (-4C + 2E)) e^{2x} \operatorname{sen} 2x$

$= (2x^2 - 3x) e^{2x} \cos 2x + (10x^2 - x - 1) e^{2x} \operatorname{sen} 2x$

$y_p = (-\frac{5}{6}x^3 + \frac{1}{4}x^2 + \frac{3}{8}x) e^{2x} \cos 2x + (\frac{1}{6}x^3 + \frac{1}{4}x^2 - \frac{1}{8}x) e^{2x}$ $\operatorname{sen} 2x$

Ejercicios 4.5

1.- $9y'' - 4y = \text{sen } x$

$9D^2 - 4y = (3D-2)(3D+2)y = \text{sen} x$

6.- $y''' + 4y' = e^x \cos 2x$

$(D^3 + 4D)y = D(D^2+4)y = e^x \cos 2x$

11.- D^4; $y = 10x^3 - 2x$

$D^4 y = D^4 (10x^3 - 2x) = D^3(30x^2 - 2) = D^2(60x) = D(60) = 0$

16.- $x^3(1-5x)$

D^4 debido al x^3

21.- $13x + 9x^2 - \text{sen} 4x$

$D^3(D^2+16)$ debido al x^2 y al $\text{sen} 4x$

26.- $e^{-x} \text{sen} x - e^{2x} \cos x$

$(D^2+2D+2)(D^2-4D+5)$ debido a $e^{-x} \text{sen} x$ y $e^{2x} \cos x$

31.- $D^2 + 5$

$\cos \sqrt{5}x$ y $\text{sen} \sqrt{5}x$

36.- $2y'' - 7y' + 5y = -29$

$$D(2D^2 - 7D + 5)y = 0$$

$\therefore \quad y = C_1 e^{\frac{5x}{2}} + C_2 e^{x} + C_3 \longrightarrow y_p = A$

$5A = -29 \quad ó \quad A = -\frac{29}{5}$

$$y = C_1 e^{\frac{5x}{2}} + C_2 e^{x} - \frac{29}{5}$$

41.- $y''' + y'' = 8x^2$

$$D^3(D^3 + D^2)y = D^5(D+1)y = 0$$

$y = C_1 + C_2 x + C_3 e^{-x} + C_4 x^4 + C_5 x^3 + C_6 x^2$

$y_p = Ax^4 + Bx^3 + Cx^2$

$12Ax^2 + (24A + 6B)x + (6B + 2C) = 8x^2$

$12A = 8, \quad 24A + 6B = 0, \quad 6B + 2C = 0$

$$y = C_1 + C_2 + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$$

46.- $y'' + 6y' + 8y = 3e^{-2x} + 2x$

$D^2(D+2)(D^2 + 6D + 8)y = D^2(D+2)^2(D+4)y = 0$

$y = C_1 e^{-2x} + C_2 e^{-4x} + C_3 x e^{-2x} + C_4 x + C_5$

$2Ae^{-2x} + 8Bx + (6B + 8C) = 3e^{-2x} + 2x$

$$y = C_1 e^{-2x} + C_2 e^{-4x} + \frac{3}{2}xe^{-2x} + \frac{1}{4}x - \frac{3}{16}$$

51.- $y'' - y = x^2 e^{x} + 5$

$D(D-1)^3 (D^2-1)y = D(D-1)^4(D+1)y = 0$

$y = c_1 e^x + c_2 e^{-x} + c_3 x^3 e^x + c_4 x^2 e^x + c_5 x e^x + c_6$

$y_p = Ax^3e^x + Bx^2e^x + Cxe^x + E$

$6Ax^2e^x + (6A+4B)xe^x + (2B+2C)e^x - E = x^2e^x + 5$

$y = c_1 e^x + c_2 e^x + c_2 e^{-x} + \frac{1}{6}x^3e^x - \frac{1}{4}x^2e^x + \frac{1}{4}xe^x - 5$

56.- $y'' + y = 4\cos x - \operatorname{sen} x$

$(D^2+1)(D^2+1) = (D^2+1)^2 = 0$

$y = c_1 \cos x + c_2 \operatorname{sen} x + c_3 x \cos x + c_4 x \cos x$

$y = c_1 \cos x + c_2 \operatorname{sen} x + \frac{1}{2} x \cos x - 2x \operatorname{sen} x$

61.- $y''' - 3y'' + 3y' - y = e^x - x + 16$

$D^2(D-1)(D^3 - 3D^2 + 3D - 1) = D^2(D-1)^4 = 0$

$y = c_1 e^x + c_2 xe^x + c_3 x^2 e^x + c_4 + c_5 x + c_6 x^3 e^x$

$y = c_1 e^x + c_2 xe^x + c_3 x^2 e^x - 13 + x + \frac{1}{6}x^3 e^x$

66.- $y'' + y' = x, \quad y(0) = 1, \quad y'(0) = 2$

$y_c = c_1 + c_2 e^{-x} \rightarrow D^2 \rightarrow y_p = Ax - 2B$

$(A + 2B) + 2Bx = x \quad \therefore \quad y = c_1 + c_2 e^{-x} - x + \frac{1}{2}x^2$

$y' = -c_2 e^{-x} - 1 + x, \quad c_1 + c_2 = 1, \quad -c_2 = 1$

$y = 2 - e^{-x} - x + \frac{1}{2}x^2$

Ejercicios 4.6

1.- $y'' + y = \sec x$

$D^2 + 1 = 0 \rightarrow y_c = c_1 \cos x + c_2 \operatorname{sen} x$

$W = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = 1 \qquad f(x) = \sec x$

$u_1' = -\frac{\operatorname{sen} x \sec x}{1} = -\tan x \quad , \quad u_1 = \ln|\cos x|$

$u_2' = \frac{\cos x \sec x}{1} = 1 \quad , \quad u_2 = x$

$y = c_1 \cos x + c_2 \operatorname{sen} x + \cos x \ln|\cos x| + x \operatorname{sen} x$

6.- $y'' + y = \sec^2 x$

$D^2 + 1 = 0 \rightarrow y_c = c_1 \cos x + c_2 \operatorname{sen} x$

$W = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = 1 \quad f(x) = \sec^2 x$

$u_1' = \frac{-\operatorname{sen} x}{\cos^2 x} \quad , \quad u_2' = \sec x$

$u_1 = -\frac{1}{\cos x} = -\sec x \quad , \quad u_2 = \ln|\sec x + \tan x|$

$y = c_1 \cos x + c_2 \operatorname{sen} x - \cos x \sec x + \operatorname{sen} x \ln|\sec x + \tan x|$

$y = c_1 \cos x + c_2 \operatorname{sen} x - 1 + \operatorname{sen} x \ln|\sec x + \tan x|$

11.- $y'' + 3y' + 2y = \frac{1}{1+e^x}$

$D^2 + 3D + 2 = 0$

$= (m+1)(m+2) = 0 \longrightarrow y_c = c_1 e^{-x} + c_2 e^{-2x}$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x} \quad f(x) = \frac{1}{1+e^x}$

$u_1' = \frac{e^x}{1+e^x}, \quad u_2' = -\frac{e^{2x}}{1+e^x} = \frac{e^x}{1+e^x} - e^x$

$y = c_3 e^{-x} + c_2 e^{-2x} + (1+e^{-x}) e^{-x} \ln(1+e^x)$

16.- $2y'' + 2y' + y = 4\sqrt{x}$

$2D^2 + 2D + 1 = 0 \longrightarrow y_c = e^{-\frac{x}{2}}(c_1 \cos\frac{x}{2} + c_2 \operatorname{sen}\frac{x}{2})$

$W \begin{vmatrix} e^{-\frac{x}{2}} \cos\frac{x}{2} & e^{-\frac{x}{2}} \operatorname{sen}\frac{x}{2} \\ -\frac{1}{2}e^{-\frac{x}{2}}\cos\frac{x}{2} - \frac{1}{2}e^{-\frac{x}{2}}\operatorname{sen}\frac{x}{2} & \frac{1}{2}e^{-\frac{x}{2}}\cos\frac{x}{2} - \frac{1}{2}e^{-\frac{x}{2}}\operatorname{sen}\frac{x}{2} \end{vmatrix} = \frac{1}{2}e^{-x}$

$u_1' = -\frac{e^{-\frac{x}{2}} \operatorname{sen}\frac{x}{2}\, 2\sqrt{x}}{e^{-\frac{x}{2}}} = -4e^{\frac{x}{2}} \sqrt{x} \operatorname{sen}\frac{x}{2}$

$u_2' = -\frac{e^{-\frac{x}{2}} \cos\frac{x}{2}\, 2\sqrt{x}}{e^{-\frac{x}{2}}} = 4e^{\frac{x}{2}} \sqrt{x} \cos\frac{x}{2}$

$y = e^{-\frac{x}{2}}(c_1 \cos\frac{x}{2} + c_2 \operatorname{sen}\frac{x}{2}) - 4e^{-\frac{x}{2}} \cos\frac{x}{2} \int_{x_0}^{x} e^{\frac{t}{2}} \sqrt{t} \operatorname{sen}\frac{t}{2}\, dt$

21.- $y'' + 2y' - 8y = 2e^{-2x} - e^{-x}$

$D^2 + 2D - 8 = (m-2)(m+4) = 0 \rightarrow y_c = c_1 e^{2x} + c_2 e^{-4x}$

$$W = \begin{vmatrix} e^{2x} & e^{-4x} \\ 2e^{2x} & -4e^{-4x} \end{vmatrix} = -6e^{-2x}$$

$$u_1 = -\frac{1}{12} e^{-4x} + \frac{1}{18} e^{-3x}$$

$$u_2 = \frac{1}{18} e^{3x} - \frac{1}{6} e^{2x}$$

$$y = \frac{25}{36} e^{2x} + \frac{4}{9} e^{-4x} - \frac{1}{4} e^{-2x} + \frac{1}{9} e^{-x}$$

26.- $y''' + 4y' = \sec 2x$

$D^3 + 4D = D(D^2+4) = 0 \rightarrow y_c = c_1 + c_2 \cos 2x + c_3 \operatorname{sen} 2x$

$$W = \begin{vmatrix} 1 & \cos 2x & \operatorname{sen} 2x \\ 0 & -2\operatorname{sen} 2x & 2\cos 2x \\ 0 & -4\cos 2x & -4\operatorname{sen} 2x \end{vmatrix} = 8 \qquad f(x) = \sec 2x$$

$$u_1 = \frac{1}{8} \ln |\sec 2x + \tan 2x|$$

$$u_2 = -\frac{1}{4} x$$

$$u_3 = \frac{1}{8} \ln |\cos 2x|$$

$$y = c_1 + c_2 \cos 2x + c_3 \operatorname{sen} 2x + \frac{1}{8} \ln |\sec 2x + \tan 2x| - \frac{1}{4} x \cos 2x + \frac{1}{8} \operatorname{sen} 2x \ln \cos 2x$$

31.- $y_p(x) = u_1(x)\, y_1(x) + u_2(x)\, y_2(x)$

$\to x \quad , x_0$

$$y_p(x) = y_1(x) \int_{x_0}^{x} \frac{-y_2(t) f(t)}{W(t)}\, dt + y_2(x) \int_{x_0}^{x} \frac{y_1(t) f(t)}{W(t)}\, dt$$

$$= \int_{x_0}^{x} \frac{-y_1(x)\, y_2(t) f(t)}{W(t)}\, dt + \int_{x_0}^{x} \frac{y_1(t)\, y_2(x)\, f(t)}{W(t)}\, dt$$

$$= \int_{x_0}^{x} \frac{y_1(t)\, y_2(x) - y_1(x)\, y_2(t)}{W(t)} f(t)\, dt$$

$$= \int_{x_0}^{x} G(x,t) f(t)\, dt$$

Ejercios 4.7

1.- $x^2 y'' - 2y = 0$

$D^2 - D - 2 = (D+1)(D-2) = 0 \longrightarrow y = c_1 x^{-1} + c_2 x^2$

6.- $x^2 y'' + 5xy' + 3y = 0$

$D^2 + 4D + 3 = (m+1)(m+3) = 0 \longrightarrow y = c_1 x^{-1} + c_2 x^{-3}$

11.- $x^2 y'' + 5xy' + 4y = 0$

$D^2 + 4D + 4 = (D+2)^2 = 0 \longrightarrow y = c_1 x^{-2} + c_2 x^{-2} \ln x$

16- $x^3 y''' + xy' - y = 0$

$y = x^D \quad D(D-1)(D-2) - 6 = D^3 - 3D^2 + 2D - 6$
$= (D-3)(D^2+2) = 0$

$y = c_1 x^3 + c_2 \cos(\sqrt{2}\ln x) + c_3 \operatorname{sen}(\sqrt{2}\ln x)$

21- $x^2 y'' - xy' + y = 2x$
$D^2 - 2D + 1 = (D-1)^2 = 0 \rightarrow y_c = c_1 x + c_2 x \ln x$

$W(x, x\ln x) = \begin{vmatrix} x & x\ln x \\ 1 & 1+\ln x \end{vmatrix} = x$

$y = c_1 x + c_2 x \ln x + x(\ln x)^2 \quad , x > 0$

26- $x^2 y'' - 5xy' + 8y = 0$

$D^2 - 6D + 8 = (D-2)(D-4) = 0$
$y = c_1 x^2 + c_2 x^4 \quad , y' = 2c_1 x + 4c_2 x^3$

$4c_1 + 16c_2 = 32, \quad 4c_1 + 32c_2 = 0$

31- $x^2 y'' + 9xy' - 20y = 0$

$x = e^t \longrightarrow \frac{d^2y}{dt^2} + 8\frac{dy}{dt} - 20y = 0$

$y = c_1 e^{-10t} + c_2 e^{2t} = c_1 x^{-10} + c_2 x^2$

36.- $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$

$$\frac{d^2y}{dx^2} = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$\frac{d^3y}{dx^3} = \frac{1}{x^2}\frac{d}{dx}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - \frac{2}{x^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$= \frac{1}{x^3}\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right)$$

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} - 6y = 3 + 3t$$

$$y = c_1e^t + c_2e^{2t} + c_3e^{3t} - \frac{17}{12} - \frac{1}{2}t = c_1x + c_2x^2 + c_3x^3 - \frac{17}{12} - \frac{1}{2}\ln x$$

41.-

$x^2y'' = 0 \rightarrow D(D-1) = 0 \rightarrow y = c_1 + c_2x$

$c_1 = y_0,\ c_2 = y_1,\ y = y_0 + y_1x$

$x^2y'' - 2xy' + 2y = 0 \rightarrow D^2 - 3D + 2 = (D-1)(D-2) = 0$

$y = c_1x + c_2x^2$

$x^2y'' - 4xy' + 6y = 0 \rightarrow D^2 - 5D + 6 = (D-2)(D-3) = 0$

$y = c_1x^2 + c_2x^3$

Ejercicios 4.8

1.- $\frac{dx}{dt} = 2x - y$

$\frac{dy}{dt} = x$

$Dx = 2x - y$ $\longrightarrow y = 2x - Dx,\ Dy = 2Dx - D^2x$

$Dy = x$ $\quad (D^2 - 2D + 1)x = 0$

$x = c_1e^t + c_2te^t$

$y = (c_1 - c_2)e^t + c_2te^t$

6.- $(D+1)x + (D-1)y = 2$

$3x + (D+2)y = -1$

$y = -\frac{1}{3} - \frac{1}{3}(D-2D) \longrightarrow Dx = \frac{1}{3}(D^2+2D)y$

$(D^2+5)y = -7$

$y = c_1\cos\sqrt{5}t + c_2\,\text{sen}\sqrt{5}t - \frac{7}{5}$

$x = \left(-\frac{2}{3}c_1 - \frac{\sqrt{5}}{3}c_2\right)\cos\sqrt{5}t + \left(\frac{\sqrt{5}}{3}c_1 - \frac{2}{3}c_2\right)\text{sen}\sqrt{5}t + \frac{3}{5}$

11.- $(D^2-1)x - y = 0$
$(D-1)x + Dy = 0$

$y = (D^2-1)x \,,\; Dy = (D^3-D)x$

$(D-1)(D^2+D+1)x = 0$

$x = c_1 e^{t} + e^{-\frac{t}{2}}\left(c_2 \cos\frac{\sqrt{3}}{2}t + c_3 \operatorname{sen}\frac{\sqrt{3}}{2}t\right)$

$y = \left(-\frac{3}{2}c_2 - \frac{\sqrt{3}}{2}c_3\right)e^{-\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t + \left(\frac{\sqrt{3}}{2}c_2 - \frac{3}{2}c_3\right)$
$e^{-\frac{t}{2}} \operatorname{sen}\frac{\sqrt{3}}{2}t$

16.- $D^2x - 2(D^2+D)y = \operatorname{sen} t$
$x + Dy = 0$

$y = c_1 + c_2 e^{-t}\cos t + c_3 e^{-t}\operatorname{sen} t + \frac{1}{5}\cos t + \frac{2}{5}\operatorname{sen} t$

$x = (c_2+c_3)e^{-t}\operatorname{sen} t + (c_2-c_2)e^{-t}\cos t + \frac{1}{5}\operatorname{sen} t - \frac{7}{5}\cos t$

21.- $\frac{dx}{dt} = -5x - y$

$\frac{dy}{dt} = 4x - y$

$x(1) = 0, \ y(1) = 1$

$x = c_1 e^{-3t} + c_2 t e^{-3t}$

$y = -(2c_1 + c_2) e^{-3t} - 2c_2 t e^{-3t}$

$c_1 e^{-3} + c_2 e^{-3} = 0, \ -(2c_1 + c_2) e^{-3} - 2c_2 e^{-3} = 1$

$x = e^{-3t+3} - t e^{-3t+3}$

$y = -e^{-3t+3} + 2t e^{-3t+3}$

26.-

$x_1(t) = x_2(t)$

$\downarrow$

$t = 13.73$ min $\rightarrow$ B contiene más al que A

para $t > 13.73$ min

Ejercicios 4.9

1.- $(y'')^2 = y^2$; $y_1 = e^x$, $y_2 = \cos x$

$(y'')^2 = (e^x)^2 = e^{2x} = y_1^2$

$y'_2 = -\operatorname{sen} x$, $y''_2 = -\cos x$

$(y''_2)^2 = (-\cos x)^2 = \cos^2 x = y_2^2$

$y = c_1 y_1 + c_2 y_2 \rightarrow (y'')^2 = (c_1 e^x - c_2 \cos x)^2$

$y^2 = (c_1 e^x + c_2 \cos x)^2$

6.- $(y+1)y'' = (y')^2$

$\frac{du}{u} = \frac{dy}{y+1} \Longrightarrow \ln|u| = \ln|y+1| + \ln c_1$

$\longrightarrow u = c_1(y+1)$

$\longrightarrow \frac{dy}{dx} = c_1(y+1) \longrightarrow \frac{dy}{y+1} = c_1 dx$

$\downarrow$

$\ln|y+1| = c_1 x - c_2 \longrightarrow y+1 = c_3 e^{cx}$

11.- $xy'' = y' + (y')^3$

$$\frac{d}{dx}[x^2 w] = -2x = x^2 w = -x^2 + c_1 \rightarrow w = -1 + \frac{c_1}{x^2}$$

$$\rightarrow u^{-2} = -1 + \frac{c_1}{x^2} \rightarrow u = \frac{x}{\sqrt{c_1 - x^2}}$$

$$\rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{c_1 - x^2}} \rightarrow y = -\sqrt{c_1 - x^2} + c_2$$

$$\rightarrow c_1 - x^2 = (c_2 - y)^2 \rightarrow \underline{x^2 + (c_2 - y)^2 = c_1}$$

16.- $y'' = e^y$, $y(0) = 0$, $y'(0) = -1$

$$y(x) = y(0) + y'(0)x + \frac{1}{2}y''(0)x^2 + \frac{1}{3}y'''(0)x^3 + \frac{1}{4}y^{(4)}x^4$$

$$y'''(x) = e^y y'$$

$$y^{(4)}(x) = e^y (y')^2 + e^y y''$$

$$y^{(5)}(x) = e^y (y')^3 + 3e^y y' y'' + e^y y'''$$

$$y^{(6)}(x) = e^y (y')^4 + 6e^y (y')^2 y'' + 3e^y (y'')^2 + 4e^y y' y''' + e^y y^{(4)}$$

$$y(x) = -x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{24}x^5 + \frac{1}{45}x^6$$

21.- $v = \frac{dx}{dt}$ $\quad \frac{d^2x}{dt^2} = v\frac{dv}{dx} \rightarrow v\frac{dv}{dx} = -\frac{k^2}{x^2}$

$v\,dv = -\frac{k^2}{x^2}dx \rightarrow \frac{1}{2}v^2 = \frac{k^2}{x} + C \rightarrow \frac{1}{2}v^2 = \frac{k^2}{x} + C$

$t=0$, $x = x_0$ y $v = 0 \rightarrow 0 = \frac{k^2}{x_0} + C \therefore C = -\frac{k^2}{x_0}$

$\rightarrow \frac{1}{2}v^2 = k^2\left(\frac{1}{x} - \frac{1}{x_0}\right) \rightarrow \frac{dx}{dt} = -k\sqrt{2}\sqrt{\frac{x_0 - x}{x x_0}}$

$t = \frac{1}{k}\sqrt{\frac{x_0}{2}}\left(\sqrt{x(x_0-x)} + \frac{x_0}{2}\arctan\frac{x_0 - 2x}{2\sqrt{x(x_0-x)}}\right)$

Ejercicios 5.1

1.- $\frac{1}{8}x'' + 16x = 0$

$\downarrow$

$x = c_1\cos 8\sqrt{2}t + c_2\operatorname{sen} 8\sqrt{2}t$

$\therefore$ periodo de movimiento es $\frac{2\pi}{8\sqrt{2}} = \frac{\sqrt{2}\pi}{8}$ seg

6.- $20x'' + 20x = 0$, $x(0) = 0$, $x'(0) = -10 \rightarrow x = -10\operatorname{sen}t$

$x' = -10\cos t$

a) La masa de 20 kg tiene amplitud mayor

b) 20 kg $= x'\left(\frac{\pi}{4}\right) = -5\sqrt{2}$ m/s, $x'\left(\frac{\pi}{2}\right) = 0$ m/s

c) $-5\operatorname{sen}2t = -10\operatorname{sen}t \rightarrow \operatorname{sen}t(\cos t - 1) = 0$

$\rightarrow t = n\pi$ para $n = 0, 1, 2, \ldots$

11.- $2x'' + 200x = 0, \; x(0) = -\frac{2}{3} \quad x'(0) = 5$

a) $x = -\frac{2}{3}\cos 10t + \frac{1}{2}\operatorname{sen} 10t = \frac{5}{6}\operatorname{sen}(10t - 0$
b) amplitud = $\frac{5}{6}$ ft , periodo = $\frac{2\pi}{10} = \frac{\pi}{5}$
c) $3\pi = \frac{\pi k}{5} \rightarrow k = 15$ ciclos
d) $x = 0 \rightarrow 10t - 0.927 = t = 0.721\,s$
e) $x' = \frac{25}{3}\cos(10t - 0.927) = 0 \rightarrow 10t -$
$t = \frac{(2n+1)\pi}{20}$

f) $x(3) = -0.597$ ft
g) $x'(3) = -5.814$ ft
h) $x''(3) = 59.702$ ft^2
i) $x = 0 \rightarrow t = \frac{1}{10}(0.927 + n\pi) \quad n = 0, 1$

j) $x = \frac{5}{12} \rightarrow t = \frac{1}{10}(0.927 + n\pi)$

k) $x = \frac{5}{12}, \; x' < 0 \rightarrow t = \frac{1}{10}(\frac{5\pi}{6}$

16.- Cuando t crece, la masa const

∴ las oscilaciones se vuelven p
esta mas oscila rápidamen

16.- y''

Cua
Cua

y'

λ

21.-

$x=$

∴ $($ 7.97867

26.- $T(x$ 1586,

$m($ 867)x

m :

$Pw^2 >$

21.- $\frac{1}{8}x'' + x' + 2x = 0$, $x(0) = -1$, $x'(0) = 8$

$x = 4te^{-4t} - e^{-4t} \rightarrow = 8e^{-4t} - 16te^{-4t}$

$x = 0 \rightarrow t = \frac{1}{4}s$, $x' = 0 \rightarrow t = \frac{1}{2}s$

$\therefore \quad x = e^{-2} ft$

26.-

a) $\frac{1}{4}x'' + x' + 5x = 0$, $x(0) = \frac{1}{2}$, $x'(0) = 1$

$x = e^{-2t}\left(\frac{1}{2}\cos 4t + \frac{1}{2}\operatorname{sen} 4\right.$

b) $x = \frac{1}{\sqrt{2}} e^{-2t} \operatorname{sen}\left(4t + \frac{\pi}{4}\right)$

c) $x = 0 \rightarrow 4t + \frac{\pi}{4} = \pi, 2\pi, 3\pi \rightarrow t = (7 + 8n)$

para $n = 0, 1, 2 \ldots$

31.- $x(0) = 0$, $x'(0) = -2 \rightarrow c_1 = 0$, $c_2 = -\frac{3}{\sqrt{\beta^2 18t}}$

$\frac{1}{2}x'' + \frac{1}{2}x' + 6x = 10\cos 3t$, $x(0) = -2$, $x'(0) = 0$

$x_c = e^{-\frac{t}{2}}\left(c_1 \cos \frac{\sqrt{47}}{2}t + c_2 \operatorname{sen} \frac{\sqrt{47}}{2}t\right)$

36.-

a) $100x'' + 1600x = 1600\,\text{sen}\,8t,\ x(0) = 0,\ x'(0) = 0$

$\rightarrow x_c = c_1 \cos 4t + c_2,\ x_p = -\frac{1}{3}\text{sen}\,8t$

$x = \frac{2}{3}\text{sen}\,4t - \frac{1}{3}\text{sen}\,8t = \frac{2}{3}\text{sen}\,4t - \frac{2}{3}\text{sen}\,4t\cos 4t$

b) $x = \frac{1}{3}\text{sen}\,4t\,(2 - 2\cos 4t) = 0 \rightarrow t = \frac{n\pi}{4}$ para $n = 0, 1, 2, \ldots$

c) $x' = \frac{8}{3}\cos 4t - \frac{8}{3}\cos 8t = \frac{8}{3}(1 - 4\cos t)(1 + 2\cos 4t)$

41.-

a) $\cos(u-v) = \cos u\cos v + \text{sen}\,u\,\text{sen}\,v,\ \cos(u+v) = \cos u\cos v - \text{sen}\,u\,\text{sen}\,v$

$\rightarrow \text{sen}\,u\,\text{sen}\,v = \frac{1}{2}(\cos(u-v) - \cos(u+v)) \rightarrow u = \frac{1}{2}(\gamma - \omega)t$

$v = \frac{1}{2}(\gamma + \omega)t$

b) $\epsilon = \frac{1}{2}(\gamma + \omega) \rightarrow \gamma = \omega \rightarrow x = \left(\frac{F_0}{2\epsilon\gamma}\right)\text{sen}\,\epsilon t\ \text{sen}\,\gamma t$

46.- $\frac{1}{4}q'' + 10q' + 30q = 300 \rightarrow q(t) = e^{-3t}(c_1\cos 3t + c_2\,\text{sen}\,3t) + 10$

$q(0) = q'(0) = 0 \rightarrow c_1 = c_2 = -10$

$\therefore q(t) = 10 - 10e^{-3t}(\cos 3t + \text{sen}\,3t)$

$i(t) = 60e^{-3t}\,\text{sen}\,3t$

51- $\frac{1}{2}q'' + 20q' + 1000q = 100\,\text{sen}\,60t$

$x = L\gamma - \frac{1}{C\gamma} = \frac{1}{2}(60) - \frac{1}{0.001(60)} = 13.3$

$Z = \sqrt{x^2 + R^2} = \sqrt{x^2 + 400} = 24.03$

$\frac{E_0}{Z} = \frac{100}{Z} = 4.16$

52- $0.1q'' + 10q = 100\,\text{sen}\,\gamma t$

$\rightarrow q(t) = c_1 \cos 10t + c_2 \text{sen}\,10t + q_p(t)$

$q_p(t) = A\,\text{sen}\,\gamma t + B\cos\gamma t$

$(100 - \gamma^2)A\,\text{sen}\,\gamma t + (100 - \gamma^2)B\cos\gamma t = 100\,\text{sen}\,\gamma t$

$A = \frac{100}{(100 - \gamma^2)}$, $B = 0$

$\therefore q_p(t) = \frac{100}{100 - \gamma^2}\text{sen}\,\gamma t$

$\downarrow$

$\therefore i(t) = \frac{100\gamma}{100 - \gamma^2}(\cos\gamma t - \cos 10t)$

Ejercicios 5.2

1.- $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \frac{w_0}{24EI} x^4$

$y(0) = 0,\ y'(0) = 0,\ y''(L) = 0,\ y'''(L) = 0$

$2c_3 + 6c_4 L + \frac{w_0}{2EI} L^2 = 0$

$6c_4 + \frac{w_0}{EI} L = 0$

6.-

a) $y_{max} = y(L) = w_0 \frac{L^4}{8EI}$

b) $L = \frac{L}{2}$ $x =$ $y(x) \longrightarrow \left(\frac{w_0}{24EI}\right)\frac{L^4}{16} = \frac{w_0 L^4}{384EI}$

c) $y_{max} = y\left(\frac{L}{2}\right) = \frac{5 w_0 L^4}{384EI}$

11.- $y'' + \lambda y = 0,\ y(0) = 0,\ y(L) = 0$

Cuando $\lambda \leq 0 \longrightarrow y = 0$

Cuando $\lambda = \alpha^2 > 0 \longrightarrow y = c_1 \cos\alpha x + c_2 \operatorname{sen}\alpha x$

$\therefore y'(x) = -c_1 \alpha \operatorname{sen}\alpha x + c_2 \alpha \cos\alpha x$

$y'(0) = 0 \longrightarrow c_2 = 0$

$y(L) = c_1 \alpha L = 0$

$\alpha L = \frac{(2n-1)\pi}{2}$ ó $\lambda = \alpha^2 = \frac{(2n-1)^2 \pi^2}{4L^2}$

$n = 1, 2, 3 \ldots$

16.- $y'' + (\lambda + 1)y = 0$, $y'(0) = 0$, $y'(1) = 0$

Cuando $\lambda < -1 \rightarrow y = 0$
Cuando $\lambda = -1 \rightarrow y = c_1 x + c_2$

$y' = c_1$ $y'(0) = 0$ $c_1 = 0$ $\therefore y = c_2$, $y'(1) = 0$

$\lambda > -1$ ó $\lambda + 1 = \alpha^2 > 0$

$y = c_1 \cos \alpha x + c_2 \operatorname{sen} \alpha x$
$y' = -c_1 \alpha \operatorname{sen} \alpha x + c_2 \alpha \cos \alpha x$
$y'(1) = -c_1 \alpha \operatorname{sen} \alpha = 0$

21.- $x = \frac{L}{4}$, $x = \frac{L}{2}$, $x = \frac{3L}{4}$

$\therefore$ la carga crítica será P_4

26.- $T(x) = x^2 \rightarrow x^2 y'' + 2xy' + \rho w^2 y = 0$

$m(m-1) + 2m + \rho w^2 = m^2 + m + \rho w^2 = 0$

$m_1 = -\frac{1}{2} - \frac{1}{2}\sqrt{4\rho w^2 - 1}\, i$, $m_2 = -\frac{1}{2} + \frac{1}{2}\sqrt{4\rho w^2 - 1}\, i$

$\rho w^2 > 0.25$ $\therefore y = \frac{c_1 x^{-\frac{1}{2}}}{\cos(\lambda \ln x)} + \frac{c_2 x^{-1/2}}{(\operatorname{sen}(\lambda \ln x))}$

31.- $y'' + 16y = 0, \quad y(0) = y_0, \quad y\left(\frac{\pi}{2}\right) = y_1$

a) $y = c_1\cos 4x + c_2 \operatorname{sen} 4x \rightarrow y_0 = y(0)$

$\rightarrow y = y_0\cos 4x + c_2\operatorname{sen} 4x, \; y_1 = y\left(\frac{\pi}{2}\right) = y_0$

$y_0 = y_1, \; y = y_0 \cos 4x + c_2\operatorname{sen} 4x$

b) $y_0 = y_1$ tiene inf soluciones

c) $y_0 \neq y_1$ no hay soluciones

36.- Si se usa un SAC $\rightarrow \tan x = -x$, 4.91318, 7.97867 y 11.0855

Los correspondientes valores propios son 4.11586, 16,5991 y 122.889, con funciones propias

$\operatorname{sen}(2.02876x)$, $\operatorname{sen}(4.91318x)$, $\operatorname{sen}(7.97867x)$

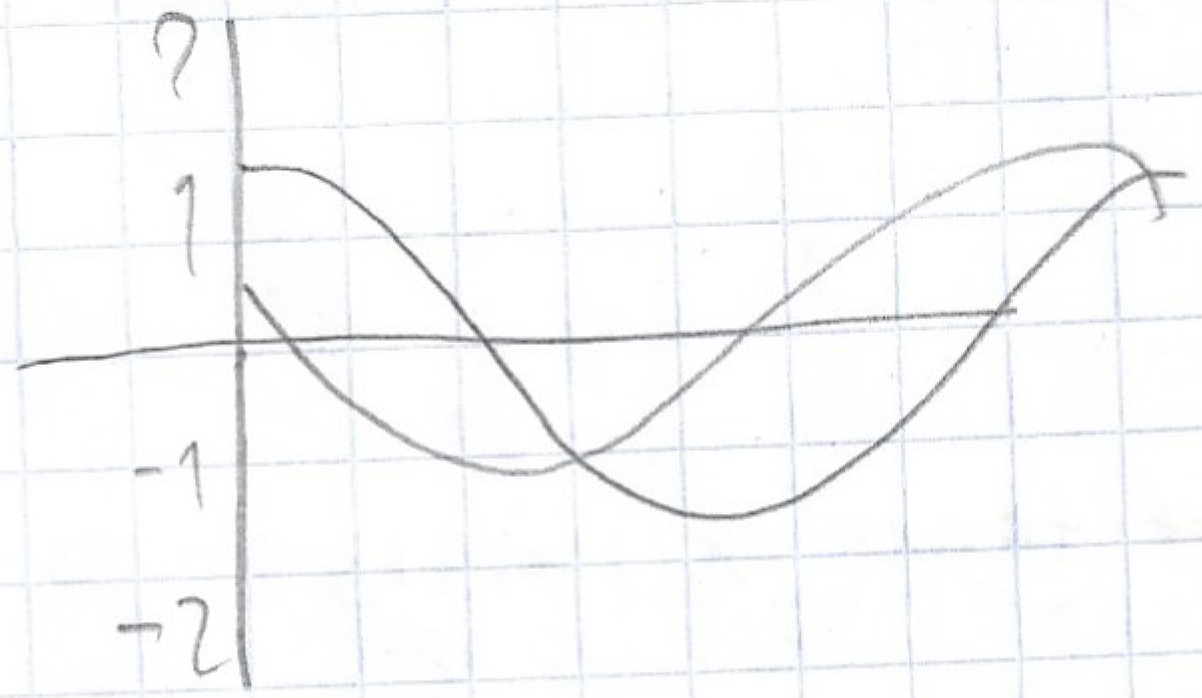
Ejercicios 5.3

1.- $\frac{d^2x}{dt^2} + x^3 = 0$

$x(0)=1$, $x'(0)=1$; $x(0)=\frac{1}{2}$, $x'(0)=-1$

$x(0)=1$, $x'(0)=1$ es aprox 5.6

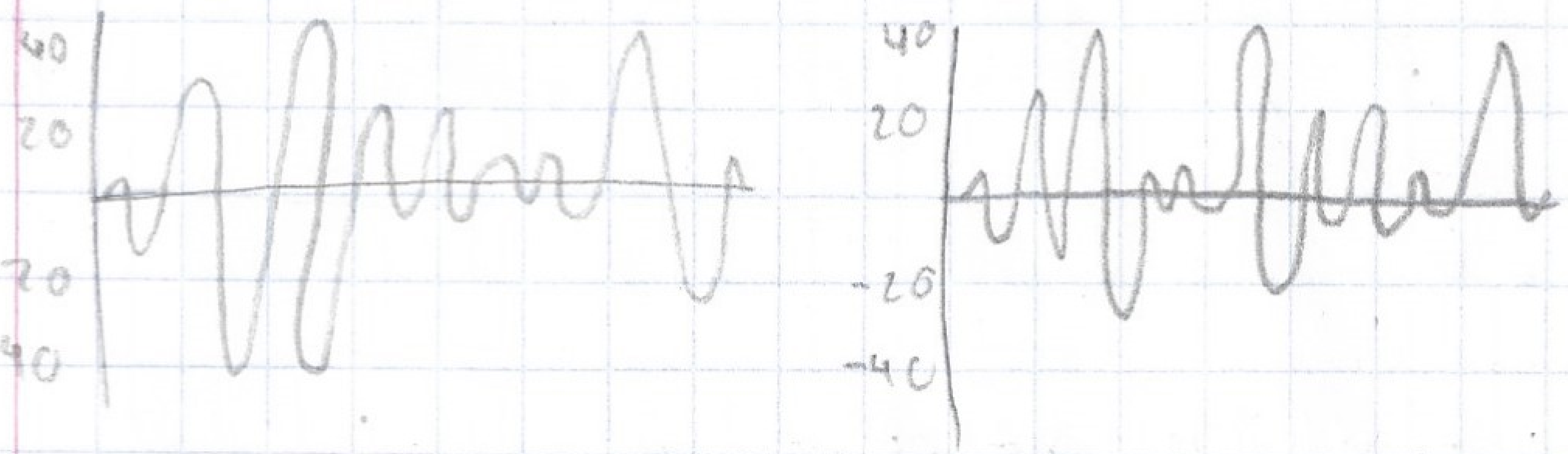
$x(0)=\frac{1}{2}$, $x'(0)=-1$ es aprox 6.2



6.- Para $x(0)=1$, $x'(0)=1$, las oscilaciones son simétricas respecto a $x=0$

Con amplitud >1

11.- Cuando k_1 es muy pequeño, tiene efecto no lineal



16.- $\frac{dx}{dt} = v \rightarrow (L-x)\frac{dv}{dt} - v^2 = Lg$

$\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = \frac{v\,dv}{dx}$, $(L-x)\frac{v\,dv}{dx} - v^2 = Lg$

$\frac{v}{v^2+Lg}dv = \frac{1}{L-x}$, $\frac{1}{2}\ln(v^2+Lg) = -\ln(L-x) + \ln C$

$\therefore \sqrt{v^2+Lg} = \frac{c}{L-x}$, $x=0$, $v=0$, $c = L\sqrt{Lg}$

$\frac{dx}{dt} = v(x) = \frac{\sqrt{Lg(2Lx - x^2)}}{L-x}$

21.- $\frac{d^2\theta}{dt^2} + \omega^2 \operatorname{sen}\theta = 0$

a) $\omega^2 = \frac{g}{l}$, $l=3$ $\theta(0)=1$, $\theta'(0)=2$

$g_{tierra} = 32$, $g_{luna} = 5.5$

b) La amplitud es más grande en la luna que en la tierra

c) $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \rightarrow \omega^2 = \frac{g}{l}$, $g=32$, $l=3$, $\theta(0)=1$,

$\theta' = 2$, $\theta(t) = \cos 3.26t + 0.612 \operatorname{sen} 3.26t$