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Grupo: 3CM1

Laplace por definición

1.-

Funciones de transformada de laplace

1.- $f(t) = sen^3(t)$

$L\{sen^3(t)\}$ $\qquad sen^3(x) = \frac{1}{4}(-sen(3x) + 3sen(x))$

$= L\{\frac{1}{4}(-sen(3t) + 3sen(t))$

$= L\{-\frac{1}{4}sen(3t) + \frac{3}{4}sen(t)\}$

$= -\frac{1}{4}L\{sen(3t)\} + \frac{3}{4}L\{sen(t)\}$

$= -\frac{1}{4}\int_0^\infty (sen(3t))e^{-st}dt + \frac{3}{4}\int_0^\infty (sen(t))e^{-st}dt$

$= -\frac{1}{4}\left(\frac{3}{s^2+9}\right) + \frac{3}{4}\left(\frac{1}{s^2+1}\right)$

$= -\frac{3}{4(s^2+9)} + \frac{3}{4(s^2+1)} = \frac{-3(s^2+1) + 3(s^2+9)}{4(s^2+9)(s^2+1)}$

$= \frac{\cancel{-3s^2} - 3 + \cancel{3s^2} + 27}{4(s^2+9)(s^2+1)} = \frac{24}{4(s^2+9)(s^2+1)} = \frac{6}{(s^2+9)(s^2+1)}$

2.-

$p = t$

$dp = 1$

2.- $f(t) = te^{\alpha t}\cos\beta t = \int_0^{\infty} te^{\alpha t}\cos\beta t\, e^{-st}\,dt$ $\qquad q = \frac{e^{\alpha t - st}}{\alpha - s}$

$dq = e^{\alpha t - st}$

$= \int te^{\alpha t - st}\cos\beta t\,dt \qquad u = \cos\beta t \qquad V = -$

$du = -\beta\,\text{sen}\,\beta t \qquad dv = te^{\alpha t - st}\,dt$

$= ($

No la pude concluir

3.-

$$3.\ f(t) = \begin{cases} \operatorname{sen} t & \text{para } 0 < t < \pi \\ 0 & \text{para } t \geq \pi \end{cases}$$

$$\mathcal{L}[f(t)] = \int_0^{\pi} \operatorname{sen} t\, e^{-st}\, dt + \cancel{\int_{\pi}^{\infty} (0)\, e^{-st}\, dt}$$

$$= \int_0^{\pi} \operatorname{sen} t\, e^{-st}\, dt \longrightarrow \text{por partes}$$

$u = \operatorname{sen} t$ $\quad v = -\frac{1}{s} e^{-st}$ | $u = \cos t$ $\quad v = -\frac{1}{s} e^{-st}$

$du = \cos t\, dt$ $\quad dv = e^{-st} dt$ | $du = -\operatorname{sen} t$ $\quad dv = e^{-st} dt$

$$= \operatorname{sen} t \left(-\frac{1}{s} e^{-st}\right) - \int_0^{\pi} -\frac{1}{s} e^{-st} \cos t\, dt$$

$$= -\frac{\operatorname{sen} t}{s} e^{-st} + \frac{1}{s} \int_0^{\pi} e^{-st} \cos t\, dt$$

$$= -\frac{\operatorname{sen} t}{s} e^{-st} \Big|_0^{\pi} + \frac{1}{s} \int_0^{\pi} e^{-st} \cos t\, dt$$

$$-\frac{1}{s}\left[\lim_{t \to \pi} \operatorname{sen} t\, e^{-st} - e^{-s(0)} \operatorname{sen}(0)\right] + \frac{1}{s} \int_0^{\pi} e^{-st} \cos t\, dt$$

$$= -\frac{1}{s}\left[\lim_{t \to \pi} \operatorname{sen} t\, e^{-st} - e^{-s(0)} \operatorname{sen}(0)\right] + \frac{1}{s}\left[-\frac{\cos t}{s} e^{-st}\Big|_0^{\pi} - \frac{1}{s}\int_0^{\pi} e^{-st} \operatorname{sen} t\, dt\right]$$

$$= -\frac{1}{s}[0 - 0] + \frac{1}{s}\left[-\frac{1}{s}\left(\lim_{t \to \pi} \cos t\, e^{-st} - e^{-s(0)} \cos 0\right)\right] - \frac{1}{s}\left(\mathcal{L}(\operatorname{sen} t)\right)$$

$$= 0 + \frac{1}{s}\left[-\frac{1}{s}\left(\lim_{t \to \pi} \cos \pi\, e^{-s\pi} - 1\right)\right] - \frac{1}{s}\left(\mathcal{L}(\operatorname{sen} t)\right)$$

$$= 0 + \frac{1}{s}\left[-\frac{1}{s}\left(-\frac{1}{e^{s\pi}} - 1\right)\right] - \frac{1}{s} f(s)$$

$$= \frac{1}{s}\left[\frac{1}{s e^{s\pi}} + \frac{1}{s} - \frac{1}{s} f(s)\right] \rightarrow \frac{1}{s^2 e^{s\pi}} + \frac{1}{s^2} - \frac{1}{s^2} F(s)$$

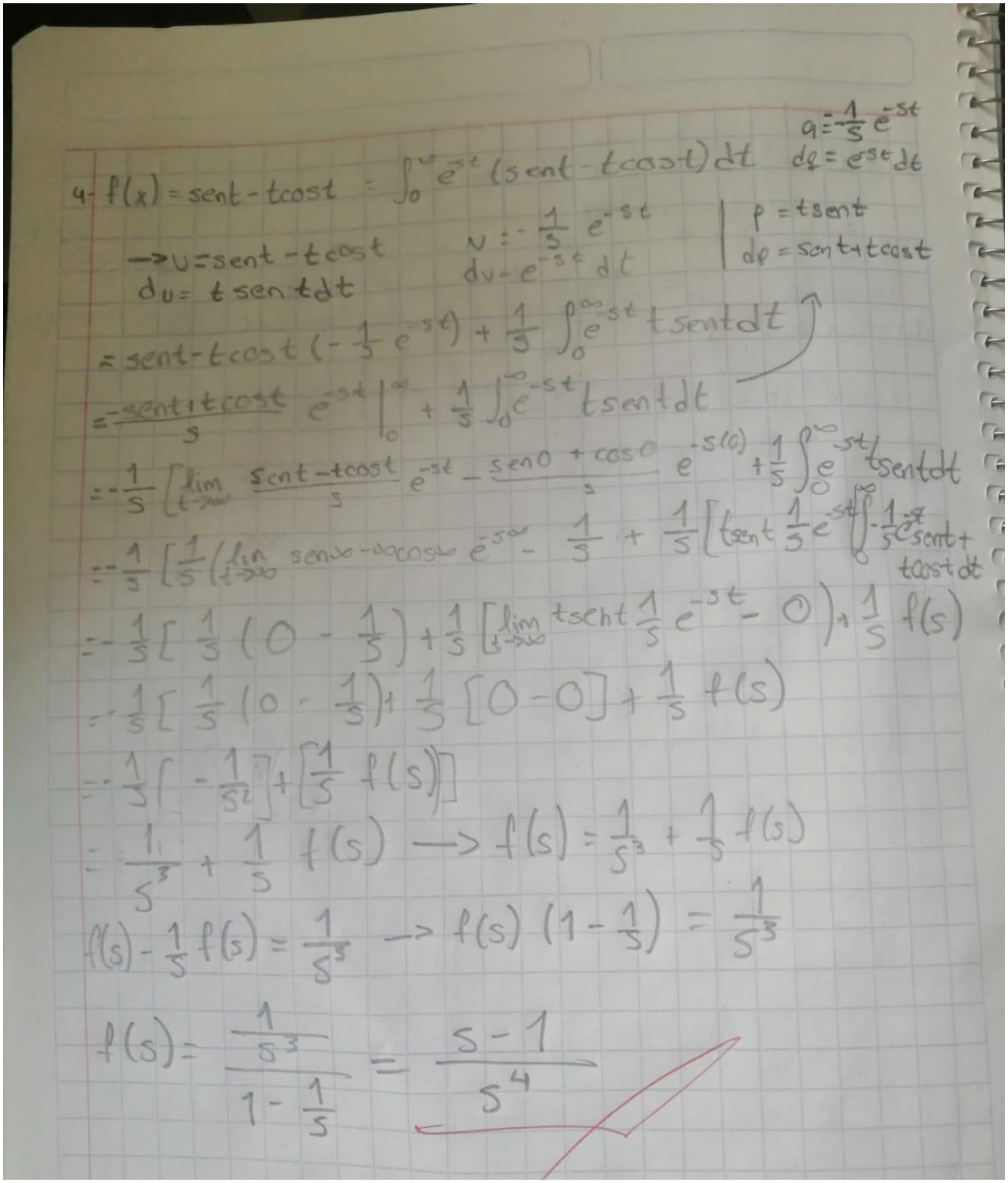
$$= \frac{1 + e^{s\pi}}{s^2 e^{s\pi}} - \frac{1}{s^2} f(s) \rightarrow f(s) = \frac{1 + e^{s\pi}}{s^2 e^{s\pi}} - \frac{1}{s^2} f(s)$$

$$f(s) + \frac{1}{s^2} f(s) = \frac{1+e^{s\pi}}{s^2 e^{s\pi}}$$

$$f(s)\left(1 + \frac{1}{s^2}\right) = \frac{1+e^{s\pi}}{s^2 e^{s\pi}}$$

$$f(s) = \frac{\frac{1+e^{s\pi}}{s^2 e^{s\pi}}}{1+\frac{1}{s^2}} = \frac{s^2+1+s^2e^{s\pi}+e^{s\pi}}{s^4 e^{s\pi}}$$

4.-



4.- $f(x) = \text{sen}t - t\cos t = \int_0^\infty e^{-st}(\text{sen}t - t\cos t)\,dt$ $\quad q = -\frac{1}{s}e^{-st}$, $dq = e^{-st}dt$

$\rightarrow u = \text{sen}t - t\cos t$ $\quad v = -\frac{1}{s}e^{-st}$ $\quad p = t\,\text{sen}t$

$du = t\,\text{sen}t\,dt$ $\quad dv = e^{-st}dt$ $\quad dp = \text{sen}t + t\cos t$

$= \text{sen}t - t\cos t\left(-\frac{1}{s}e^{-st}\right) + \frac{1}{s}\int_0^\infty e^{-st}\, t\,\text{sen}t\,dt$

$= \frac{-\text{sen}t + t\cos t}{s}\, e^{-st}\Big|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st}\, t\,\text{sen}t\,dt$

$= -\frac{1}{s}\left[\lim_{t\to\infty} \frac{\text{sen}t - t\cos t}{s}\, e^{-st} - \frac{\text{sen}0 + \cos 0}{s}\, e^{-s(0)}\right] + \frac{1}{s}\int_0^\infty e^{-st}\, t\,\text{sen}t\,dt$

$= -\frac{1}{s}\left[\frac{1}{s}\left(\lim_{t\to\infty} \text{sen}\infty - \infty\cos\infty\, e^{-s\infty} - \frac{1}{s}\right) + \frac{1}{s}\left[t\,\text{sen}t\,\frac{1}{s}e^{-st}\Big| - \int_0^\infty -\frac{1}{s}e^{-st}\,\text{sen}t + t\cos t\,dt\right]\right]$

$= -\frac{1}{s}\left[\frac{1}{s}\left(0 - \frac{1}{s}\right) + \frac{1}{s}\left[\lim_{t\to\infty} t\,\text{sen}t\,\frac{1}{s}e^{-st} - 0\right) + \frac{1}{s}f(s)\right)$

$= -\frac{1}{s}\left[\frac{1}{s}\left(0 - \frac{1}{s}\right) + \frac{1}{s}[0-0] + \frac{1}{s}f(s)\right)$

$= -\frac{1}{s}\left[-\frac{1}{s^2}\right] + \left[\frac{1}{s}f(s)\right]$

$= \frac{1}{s^3} + \frac{1}{s}f(s) \longrightarrow f(s) = \frac{1}{s^3} + \frac{1}{s}f(s)$

$f(s) - \frac{1}{s}f(s) = \frac{1}{s^3} \longrightarrow f(s)\left(1-\frac{1}{s}\right) = \frac{1}{s^3}$

$f(s) = \frac{\frac{1}{s^3}}{1-\frac{1}{s}} = \frac{s-1}{s^4}$

¿

5.-

5.- $F_{(t)} = e^{-t}\, sen^2(t)$

$\mathcal{L}\left\{e^{-t}\left(\frac{1}{2} - \cos(2t)\,\frac{1}{2}\right)\right\}$

$\mathcal{L}\left\{\frac{1}{2}e^{-t} - \frac{1}{2}e^{-t}\cos(2t)\right\}$

$= \frac{1}{2}\mathcal{L}\{e^{-t}\} - \frac{1}{2}\mathcal{L}\{e^{-t}\cos(2t)\}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ ①

$\mathcal{L}\{e^{-t}\} = \frac{1}{s-1}$

$\mathcal{L}\{e^{-t}\cos(2t)\}$

$\mathcal{L}\{\cos(2t)\}\ (s+1)$

$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$ ②

$\mathcal{L}\{\cos(2t)\} = \frac{s}{s^2+2^2} = \frac{s}{s^2+4} = \frac{s+1}{(s+1)^2+4}$

Juntamos a ① y ②

$= \frac{1}{2} \cdot \frac{1}{s+1} - \frac{1}{2} \cdot \frac{s+1}{(s+1)^2+4}$

Simplificamos

$\frac{2}{(s+1)(s^2+2s+5)}$

R= $\frac{2}{(s+1)(s^2+2s+5)}$