

Transformada de Laplace de $f'(t)$

$$\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0)$$

$$\mathcal{L}[f'(t)] = \int_0^{\infty} f'(t) e^{-st} dt$$

$$u = e^{-st}$$

$$dv = f'(t) dt$$

$$du = -s e^{-st} dt$$

$$v = f(t)$$

$$= e^{-st} f(t) \Big|_0^{\infty} - \int_0^{\infty} f(t) (-s e^{-st}) dt$$

$$= \lim_{t \rightarrow \infty} e^{-st} f(t) - e^{-s(0)} f(0) + s \int_0^{\infty} f(t) e^{-st} dt$$

$$= -f(0) + s \mathcal{L}[f(t)]$$

\therefore se cumple para cualquier valor de $-\infty < s < \infty$