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Primer Examen de Cálculo Aplicado

1) i) $\lim_{x \to 1} \left[\frac{x}{x-1} - \frac{x}{\ln x} \right] = \lim_{x \to 1} \left[\frac{\ln x(x) - x(x-1)}{\ln x(x-1)} \right] = \frac{0-0}{0-0} = \frac{0}{0}$

$= \lim_{x \to 1} \left[\frac{(\ln x(x) - x(x-1))'}{((\ln x)(x-1))'} \right] = \frac{(\ln x(1) + (x)(\frac{1}{x})) - x(1) + (x-1)(1)}{(\ln x(1) + (x-1)\frac{1}{x})}$

$= \frac{(\ln x + 1) - (x + x - 1)}{\ln x + (x-1)\frac{1}{x}} = \frac{2 + \ln x - 2x}{1 - \frac{1}{x} + \ln x} = \frac{2 + \ln x - 2x}{\frac{x - 1 + x\ln x}{x}} = \frac{(2x + \ln x\, x - 2x^2)}{(x - 1 + x\ln(x))} = \frac{0}{0}$

$= \frac{(2x + \ln x(x) - 2x^2)'}{(x - 1 + x\ln x)'} = \frac{2 + (\ln x + 1) - 4x}{2 + \ln x} = \frac{3 + \ln x - 4x}{2 + \ln x}$

$= \lim_{x \to 1} \frac{3 + \ln x - 4x}{2 + \ln x} = \frac{3 + \ln(1) - 4(1)}{2 + \ln(1)} = \frac{3 + 0 - 4}{2 + 0} = -\frac{1}{2}$

ii) $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\frac{\pi}{2} - x} = y = (\cos x)^{\frac{\pi}{2} - x}$

$\ln y = \frac{\pi}{2} - x \ln \cos x \to \lim y = \lim \frac{\pi}{2} - x \ln \cos x$

$= \frac{\pi}{2} \lim -x \ln \cos x \to \frac{(\ln \cos x)'}{(-\frac{1}{x})'} = \frac{\tan x}{+\frac{1}{x^2}} = \frac{\frac{-\operatorname{sen} x}{\cos x}}{\frac{1}{x^2}}$

$= \lim_{x \to \frac{\pi}{2}} \frac{(\operatorname{sen} x^2)'}{(\cos x)'} = \frac{-\cos x\, x^2 - 2x \operatorname{sen} x}{-\operatorname{sen} x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x(x^2) - 2x \operatorname{sen} x}{-\operatorname{sen} x}$

$= \frac{-\cos \frac{\pi}{2} (\frac{\pi}{2})^2 - 2(\frac{\pi}{2}) \operatorname{sen} \frac{\pi}{2}}{-\operatorname{sen} \frac{\pi}{2}} = \frac{0 - 2\frac{\pi}{2}(1)}{-1} = 2\frac{\pi}{2}$

$= e^{\ln_{x \to \frac{\pi}{2}}} = 2\frac{\pi}{2} \longrightarrow e^{\ln y} = e^{\pi}$

2.- $\int_1^\infty \frac{dx}{x^{2\alpha}} = \int_1^\infty x^{-2\alpha}$

$= \frac{1}{2\alpha x^{2\alpha-1} - x^{2\alpha-1}} = \lim_{b\to\infty} -\frac{1}{2\alpha x^{2\alpha-1} - x^{2\alpha-1}}\Big|_1^b$

$= \lim_{b\to\infty} \left[-\frac{1}{2\alpha b^{\alpha-1} - b^{2\alpha-1}} + \frac{1}{2\alpha^{2\alpha-1} - 1^{2\alpha-1}} \right]$

$= \frac{1}{4}\lim_{b\to\infty} \left[\frac{1}{\alpha b^{\alpha-1} - b^{2\alpha-1}} + \frac{1}{\alpha^{2\alpha-1} - 1^{2\alpha-1}} \right] = \infty$

Es divergente en $\alpha \leq 1$

Es convergente en $\alpha > 1$

3.- i) $\int_0^\infty e^{-x} \frac{\operatorname{sen} 5x}{x} dx = \lim_{b\to\infty} \int_0^b e^{-x} \frac{\operatorname{sen}^{5x}}{x} dx$

$= \int_0^\infty e^{-x} \cos 5x \, dx$

$u = \cos 5x \quad dv = e^{-x}$

$du = -5 \operatorname{sen} 5x \quad v = -e^{-x}$

$= \lim_{b\to\infty} \int_0^b e^{-x} \cos 5x \, dx =$

$= \cos 5x - e^{-x} \Big|_0^b + \int -e^{-x} - 5 \operatorname{sen} 5x \, dx$

$= [\cos 5b - e^{-b}] - [1 - 1] + 5 \int e^{-x} \operatorname{sen} 5x \, dx$

$= 5 \int e^{-x} \operatorname{sen} 5x \, dx$

$u = \operatorname{sen} 5x \quad dv = e^{-x}$

$du = \cos \quad v = -e^{-x}$

$= 5 \left[-\operatorname{sen} 5x \, e^{-x} \right]_0^b + \int e^{-x} \, 5 \cos 5x \, dx$

$= 5 - 5 \int e^{-x} \cos 5x \, dx$

$= 0$

ii) $\int_0^\infty \frac{dx}{x^2-x-2} = \int_0^\infty \frac{1}{(x-2)(x+1)}\,dx = \frac{A}{x-2} + \frac{B}{x+1} = (x+1)A + (x-2)B$

$= \lim_{a\to 3} \int_0^a \frac{dx}{x^2-x-2} + \lim_{b\to\infty} \int_a^\infty \frac{dx}{x^2-x-2}$

$= Ax + A + Bx - 2B = (A+B)x \quad A-2B$

$= A+B=0$
$= A-2B=1$ $\Big\}$ $(2(A+B) = 2A + \cancel{2B} = 0$

$A - \cancel{2B} = 1$

$3A = 1$

$A = \frac{1}{3}$

$A+B=0$

$\frac{1}{3} + B = 0$

$B = -\frac{1}{3}$

$= \frac{1}{3}\int \frac{dx}{x-2} - \frac{1}{3}\int \frac{dx}{x+1} = \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + C$

$= \lim_{a\to 3} \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1|\Big|_0^3 + \lim_{b\to\infty} \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x-1|\Big|_3^\infty$

$= \frac{1}{3}\lim_{a\to 3}\left[\ln|a-2| - \ln|a+1|\right] - \left[\ln|-2| - \ln|1|\right] +$

$= \frac{1}{3}\lim_{b\to\infty}\left[\ln|b-2| - \ln|b+1|\right] - \left[\ln|1| - \ln|4|\right]$

$= \frac{1}{3}\lim_{a\to 3}\left[0 - \ln|2|\right] - \left[\ln|+2| - 0\right] + \frac{1}{3}\left[\infty - \infty\right] - \left[0 - \ln|4|\right]$

$= \frac{1}{3}\lim_{a\to 2^-}\left[\ln|a+1| - \ln|a-2| - \ln|2| + \ln|1|\right] \longrightarrow -\infty$

$= \frac{1}{3}\lim_{a\to 2^+}\left[\ln|3+1| - \ln|3-2| - \ln|a+1| + \ln|a-2|\right] \longrightarrow +\infty$

De todo esto, podemos concluir que el área entre 0 y $+\infty$ es infinita