Ejercicios 7.1

1.- $f(t) = \begin{cases} -1 & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$

$\mathcal{L}(f(t)) = \int_0^1 -e^{-st}\,dt + \int_1^\infty e^{-st}\,dt = \frac{1}{s}e^{-st}\Big|_0^1 - \frac{1}{s}e^{-st}\Big|_1^\infty$

$= \frac{1}{s}e^{-s} - \frac{1}{s} - (0 - \frac{1}{s}e^{-s}) = \frac{2}{s}e^{-s} - \frac{1}{s}$ para $s > 0$

6.- $f(t) = \begin{cases} 0 & 0 \le t < \frac{\pi}{2} \\ \cos t & t \ge \frac{\pi}{2} \end{cases}$

$\mathcal{L}(f(t)) = \int_{\frac{\pi}{2}}^\infty (\cos t)\, e^{-st}\,dt = \left(-\frac{s}{s^2+1}e^{-st}\cos t + \frac{1}{s^2+1}e^{-st}\operatorname{sen} t\right)\Big|_{\frac{\pi}{2}}^\infty$

$= 0 - \left(0 + \frac{1}{s^2+1}e^{-\frac{\pi s}{2}}\right) = -\frac{1}{s^2+1}e^{-\frac{\pi s}{2}}$ para $s > 0$

11.- $f(t) = e^{t+7}$

$\mathcal{L}(f(t)) = \int_0^\infty e^{t+7}e^{-st}\,dt = e^7\int_0^\infty e^{(1-s)t}\,dt$

$= -\frac{e^7}{1-s}e^{(1-s)t}\Big|_0^\infty = 0 - \frac{e^7}{1-s} = \frac{e^7}{s-1}$ para $s > 1$

16.- $f(t) = e^t \cos t$

$\mathcal{L}(f(t)) = \int_0^\infty e^t \cos t\, e^{-st}\,dt = \int_0^\infty (\cos t)\, e^{(1-s)t}\,dt$

$= \left(\frac{1-s}{(1-s)^2+1}e^{(1-s)t}\cos t + \frac{1}{(1-s)^2+1}e^{(1-s)t}\operatorname{sen} t\right)\Big|_0^\infty$

$= -\frac{1-s}{(1-s)^2+1} = \frac{s-1}{s^2-2s+2}$ para $s > 1$

21.- $f(t) = 4t - 10$

$\mathcal{L}(f(t)) = \mathcal{L}(4t-10)$

$= \frac{4}{s^2} - \frac{10}{s}$

26.- $f(t) = (2t-1)^3$

$\mathcal{L}(f(t)) = \mathcal{L}((2t-1)^3)$

$= \mathcal{L}(8t^3 - 12t^2 + 6t - 1) = 8\frac{3!}{s^4} - 12\frac{2!}{s^3} + \frac{6}{s^2} - 1$

31.- $f(t) = 4t^2 - 5\operatorname{sen} 3t$

$\mathcal{L}(f(t)) = \mathcal{L}(4t^2 - 5\operatorname{sen}3t)$

$= \mathcal{L}(4t^2 - 5\operatorname{sen}3t) = 4\frac{2}{s^3} - 5\frac{3}{s^2+9}$

36.- $f(t) = e^{-t}\cosh t$

$\mathcal{L}(f(t)) = \mathcal{L}(e^{-t}\cosh t)$

$= \mathcal{L}\left(e^{-t}\frac{e^{t}+e^{-t}}{2}\right) = \mathcal{L}\left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right) = \frac{1}{2s} + \frac{1}{2(s+2)}$

41.- a) $\tau(\alpha+1)=\int_0^\infty t^\alpha e^{-t}dt=-t^\alpha e^{-t}\Big|_0^\infty+\alpha\int_0^\infty t^{\alpha-1}e^{-t}dt$

$=\alpha\tau(\alpha)$

b)

$\mathcal{L}(t^\alpha)=\int_0^\infty e^{-st}t^\alpha dt=\int_0^\infty e^{-u}\left(\frac{u}{s}\right)^\alpha\frac{1}{s}du=\frac{1}{s^{\alpha+1}}\tau(\alpha+1)$

para $\alpha>-1$

46.-

$\mathcal{L}(e^{(a+ib)t})=\frac{1}{s-(a+ib)}=\frac{1}{(s-a)-ib}\,\frac{(s-a)+ib}{(s-a)+ib}$

$=\frac{s-a+ib}{(s-a)^2+b^2}$, $e^{i\theta}+i\,sen\theta$

$\mathcal{L}(e^{(a+ib)t})=\mathcal{L}(e^{at}e^{ibt}=\mathcal{L}(e^{at}(\cos bt+i\,sen\,bt)$

$=\mathcal{L}(e^{at}\cos bt)+i\mathcal{L}(e^{at}sen\,bt$

$=\frac{s-a}{(s-a)^2+b^2}+i\frac{b}{(s-a)^2+b^2}$

$\therefore\ \mathcal{L}(e^{at}\cos bt)=\frac{s-a}{(s-a)^2+b^2}$, $\mathcal{L}(e^{at}sen\,bt)=\frac{b}{(s-a)^2+b^2}$

Ejercicios 7.2

1.- $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = \frac{1}{2}t^2$

6.- $\mathcal{L}^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3}\right\} = 1 + 4t + 2t^2$

11.- $\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{5}{7}\ \frac{7}{s^2+49}\right\} = \frac{5}{7}\operatorname{sen}7t$

16.- $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+2} + \frac{1}{\sqrt{2}}\ \frac{\sqrt{2}}{s^2+2}\right\} = \cos\sqrt{2}\,t + \frac{\sqrt{2}}{2}\operatorname{sen}\sqrt{2}\,t$

21.- $\mathcal{L}^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{0.3}{s-0.1} + \frac{0.6}{s+0.2}\right\} = 0.3e^{0.1t} + 0.6e^{-0.2t}$

26.- $\mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{4(s^2+4)} + \frac{2}{4(s^2+4} - \frac{1}{4(s+2)}\right\} = \frac{1}{4}\cos 2t + \frac{1}{4}\operatorname{sen}2t - \frac{1}{4}$

31.- $\frac{dy}{dt} - y = 1 \quad , \quad y(0) = 0$

$s\mathcal{L}(y) - y(0) - \mathcal{L}(y) = \frac{1}{s}$

$= \mathcal{L}(y) = -\frac{1}{s} + \frac{1}{s-1} \quad \therefore \; y = -1 + e^{t}$

36.- $y'' - 4y' = 6e^{3t} - 3e^{-t} \; , \; y(0) = 1 \quad , \quad y'(0) = -1$

$s^2\mathcal{L}(y) - sy(0) - y'(0) - 4(s\mathcal{L}(y) - y(0)) = \frac{6}{s-3} - \frac{3}{s+1}$

$\mathcal{L}(y) = \frac{6}{(s-3)(s^2-4s)} - \frac{3}{(s+1)(s^2-4s)} + \frac{s-5}{s^2-4s}$

$= \frac{5}{2s} - \frac{2}{s-3} - \frac{3}{5s+5} + \frac{11}{10s-40}$

$\therefore \quad y = \frac{5}{2} - 2e^{3t} - \frac{3}{5}e^{-t} + \frac{11}{10}e^{4t}$

41.- $y' + y = e^{-3t}\cos 2t \; , \; y(0) = 0$

$s\mathcal{L}(y) + \mathcal{L}(y) = \frac{s+3}{s^2+6s+13}$

$\mathcal{L}(y) = \frac{s+3}{(s+1)(s^2+6s+13)} = \frac{1}{4}\frac{1}{s+1} - \frac{1}{4}\frac{s+1}{s^2+6s+13}$

$= \frac{1}{4}\frac{1}{s+1} - \frac{1}{4}\left(\frac{s+3}{(s+3)^2+4} - \frac{2}{(s+3)^2+4}\right)$

$\therefore \; y = \frac{1}{4}e^{-t} - \frac{1}{4}e^{-3t}\cos 2t + \frac{1}{4}e^{-3t}\operatorname{sen} 2t$

Ejercicios 7.3

1.- $\mathcal{L}(te^{10t})$

$= \dfrac{1}{(s-10)^2}$

6.- $\mathcal{L}(e^{2t}(t-1)^2)$

$= \mathcal{L}(t^2e^{2t} - 2te^{2t} + e^{2t}) = \dfrac{2}{(s-2)^3} - \dfrac{2}{(s-2)^2} + \dfrac{1}{s-2}$

11.- $\mathcal{L}^{-1}\left(\dfrac{1}{(s+2)^3}\right)$

$= \mathcal{L}^{-1}\left(\dfrac{1}{2} \dfrac{2}{(s+2)^3}\right) = \dfrac{1}{2}t^2e^{-2t}$

16.- $\mathcal{L}^{-1}\left(\dfrac{2s+5}{s^2+6s+34}\right)$

$= \mathcal{L}^{-1}\left(2\dfrac{(s+3)}{(s+3)^2+5^2} - \dfrac{1}{5} \dfrac{5}{(s+3)^2+5^2}\right)$

$= 2e^{-3t}\cos 5t - \dfrac{1}{5}e^{-3t}\operatorname{sen} 5t$

21.- $y' + 4y = e^{-4t}, \; y(0) = 2$

$s\mathcal{L}(y) - y(0) + 4\mathcal{L}(y) = \dfrac{1}{s+4}$

$\mathcal{L}(y) = \dfrac{1}{(s+4)^2} + \dfrac{2}{s+4}$

$\therefore \; y = te^{-4t} + 2e^{-4t}$

26.- $y'' - 4y' + 4y = t^3$, $y(0) = 1$, $y'(0) = 0$

$$s^2\mathcal{L} - sy(0) - y'(0) - 4(s\mathcal{L}(y) - y(0)) + 4\mathcal{L}(y) = \frac{6}{s^4}$$

$$\mathcal{L}(y) = \frac{s^5 - 4s^4 + 6}{s^4(s-2)^2} = \frac{3}{4}\frac{1}{s} + \frac{9}{8}\frac{1}{s^2} + \frac{3}{4}\frac{2}{s^3} + \frac{1}{4}\frac{3!}{s^4}$$

$$+ \frac{1}{4}\frac{1}{s-2} - \frac{13}{8}\frac{1}{(s-2)^2}$$

$$\therefore\ y = \frac{3}{4} + \frac{9}{8}t + \frac{3}{4}t^2 + \frac{1}{4}t^3 + \frac{1}{4}e^{2t} - \frac{13}{8}te^{2t}$$

31.- $y'' + 2y' + y = 0$, $y'(0) = 2$, $y(1) = 2$

$$\mathcal{L}(y'') + \mathcal{L}(2y') + \mathcal{L}(y) = 0$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + 2s\mathcal{L}(y) - 2y(0) + \mathcal{L}(y) = 0$$

$$s^2\mathcal{L}(y) - cs - 2 + 2s\mathcal{L}(y) - 2c + \mathcal{L}(y) = 0$$

$$(s^2 + 2s + 1)\mathcal{L}(y) = cs + 2c + 2$$

$$\mathcal{L}(y) = \frac{cs}{(s+1)^2} + \frac{2c-2}{(s+1)^2}$$

$$= c\frac{s+1-1}{(s+1)^2} + \frac{2c+2}{(s+1)^2}$$

$$= \frac{c}{s+1} + \frac{c+2}{(s+1)^2}$$

36.- $R\frac{dq}{dt} + \frac{1}{C}q = E_0 e^{-kt}$, $q(0) = 0$

$$Rs\mathcal{L}(q) + \frac{1}{C}\mathcal{L}(q) = E_0\frac{1}{s+k}$$

$$\mathcal{L}(q) = \frac{E_0 C}{(s+k)(RCs+1)} = \frac{\frac{E_0}{R}}{(s+k)\left(s + \frac{1}{RC}\right)}$$

$$\therefore\ q(t) = \frac{E_0 C}{1-kRC}\left(e^{-kt} - e^{-\frac{t}{RC}}\right) \longrightarrow q(t) = \frac{E_0}{R}te^{-kt} = \frac{E_0}{R}te^{-\frac{t}{RC}}$$

41.- $\mathcal{L}(\cos 2t\, \mathcal{U}(t-2))$

$= \mathcal{L}(\cos 2(t-\pi)\, \mathcal{U}(t-\pi)) = \dfrac{se^{-\pi s}}{s^2+4}$

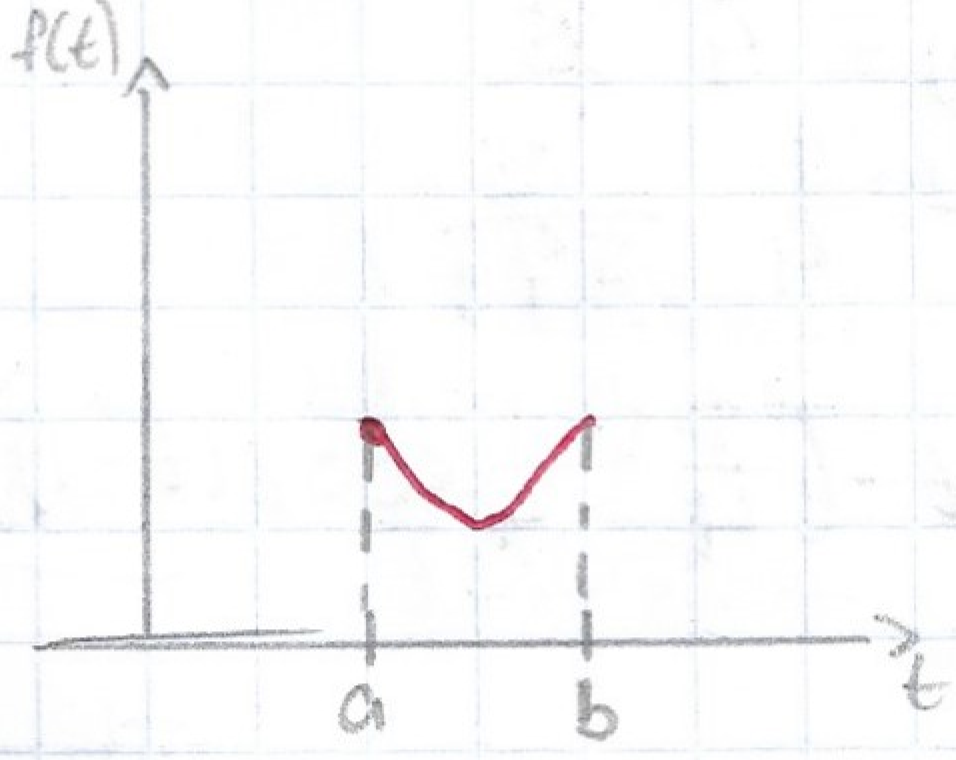
$\mathcal{L}(\cos 2t\, \mathcal{U}(t-\pi)) = e^{-\pi s}\mathcal{L}(\cos 2(t+\pi)) = e^{-\pi s}\mathcal{L}(\cos 2t)$

$= e^{-\pi s}\dfrac{s}{s^2+4}$

46.- $\mathcal{L}^{-1}\left(\dfrac{se^{-\frac{\pi s}{2}}}{s^2+4}\right)$

$= \cos 2(t-\frac{\pi}{2})\, \mathcal{U}(t-\frac{\pi}{2}) = -\cos 2t\, \mathcal{U}(t-\frac{\pi}{2})$

51.-



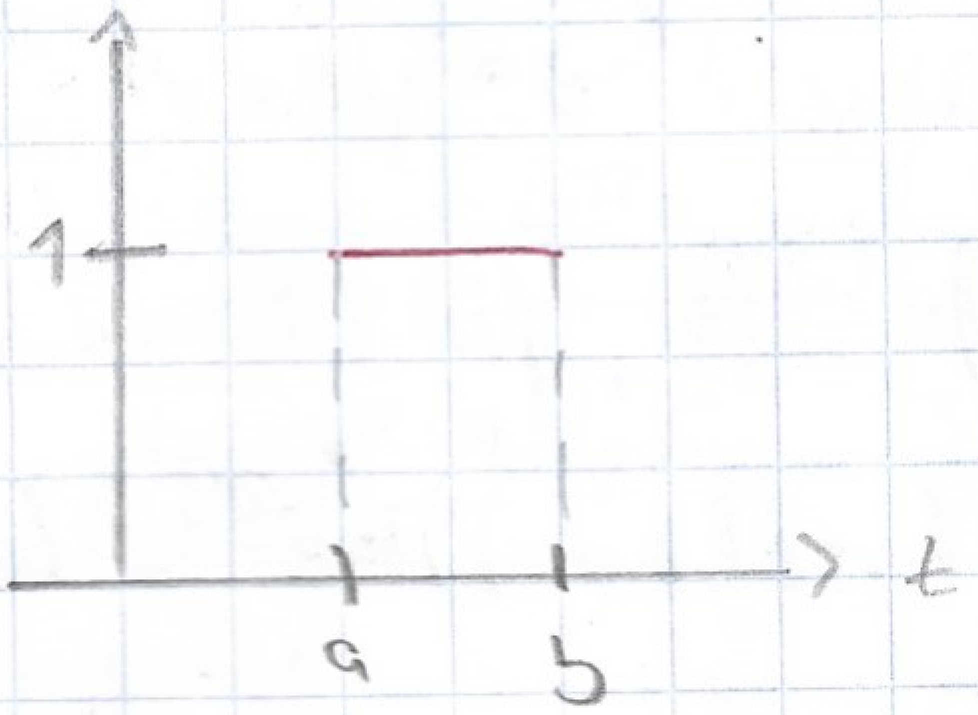
f) $f(t-a)\,\mathcal{U}(t-a) - f(t-a)\,\mathcal{U}(t-b)$

56.-

$$f(t) = \begin{cases} 1, & 0 \le t < 4 \\ 0, & 4 \le t < 5 \\ 1, & t \ge 5 \end{cases}$$

$\mathcal{L}(1 - \mathcal{U}(t-4) + \mathcal{U}(t-5)) = \dfrac{1}{s} - \dfrac{e^{-4s}}{s} + \dfrac{e^{-5s}}{s}$

61.-



$$= \mathcal{L}\left(U(t-a) - U(t-b)\right)$$
$$= \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$$

60.- $y'' + 4y = f(t)$, $y(0) = 0$, $y'(0) = -1$, donde

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + 4\mathcal{L}(y) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$\mathcal{L}(y) = \frac{1-s}{s(s^2+4)} - e^{-s}\frac{1}{s(s^2+4)} = \frac{1}{4}\frac{1}{s}$$
$$-\frac{1}{4}\frac{s}{s^2+4} - e^{-s}\left(\frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4}\right)$$

$$\therefore \quad y = \frac{1}{4} - \frac{1}{4}\cos 2t - \frac{1}{2}\operatorname{sen} 2t - \left(\frac{1}{4} - \frac{1}{4}\cos 2(t-1)\right)U(t-1)$$

71.-

$$f(t) = \begin{cases} 20t, & 0 \leq t < 5 \\ 0, & t \geq 5 \end{cases}$$

$$f(t) = 20t - 20t\,U(t-5) = 20t - 20(t-5)\,U(t-5) - 100\,U(t-5)$$

$$s^2\mathcal{L}(x) + 16\mathcal{L}(x) = \frac{20}{s^2} - \frac{20}{s^2}e^{-5s} - \frac{100}{s}e^{-5s}$$

$$\mathcal{L}(x) = \frac{20}{s^2(s^2+16)} - \frac{20}{s^2(s^2+16)}e^{-5s} - \frac{100}{s(s^2+16)}e^{-5s}$$

$$= \left(\frac{5}{4}\frac{1}{s^2} - \frac{5}{16}\frac{4}{s^2+16}\right)(1-e^{-5s}) - \left(\frac{25}{4}\frac{1}{s} - \frac{25}{4}\frac{s}{s^2+16}\right)e^{-5s}$$

$$\therefore x(t)=\frac{5}{4}t-\frac{5}{16}\operatorname{sen}4t-\frac{5}{4}t\,U(t-5)+\frac{5}{16}\operatorname{sen}4(t-5)\,U(t-5)$$

$$+\frac{25}{4}\cos 4(t-5)\,U(t-5)$$

76.-

$$50\frac{dq}{dt}+\frac{1}{0.01}q=E_0\left(U(t-1)-U(t-3)\right),\quad q(0)=0$$

$$50s\mathcal{L}(q)+100\,\mathcal{L}(q)=E_0\left(\frac{1}{s}e^{-s}-\frac{1}{s}e^{-3s}\right)$$

$$\therefore q(t)=\frac{E_0}{100}\left(\left(1-e^{-2(t-1)}\right)U(t-1)-\left(1-e^{-2(t-3)}\right)U(t-3)\right)$$

81.-

a) $$\frac{dT}{dt}=k(T-T_m)\quad\rightarrow T_m=70+\frac{300-70}{4-0}t=70+57.5t$$

$$T_m=\begin{cases}70+57.5t, & 0\le t<4\\ 300 & t\ge 4\end{cases}$$

$$\therefore\frac{dT}{dt}=k\left(T-70-57.5t-(230-57.5t)-U(t-4)\right),\quad T(0)=70$$

b) $$s t(s)-70=k\left(t(s)-\frac{70}{s}-\frac{57.5}{s^2}+\frac{57.5}{s^2}e^{-4s}\right)$$

$$T(t)=70+57.5\left(\frac{1}{k}+t-\frac{1}{k}e^{kt}\right)-57.5\left(\frac{1}{k}+t-4-\frac{1}{k}e^{k}\right)$$

$$\therefore 300=70+57.5\left(\frac{1}{k}+20-\frac{1}{k}e^{20k}\right)-57.5\left(\frac{1}{k}+16\right)$$

Ejercicios 7.4

1.- $\mathcal{L}(te^{-10t})$

$$= -\frac{d}{ds}\left(\frac{1}{s+10}\right) = \frac{1}{(s+10)^2}$$

6.- $\mathcal{L}(t^2\cos t)$

$$= \frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right) = \frac{d}{ds}\left(\frac{1-s^2}{(s^2+1)^2}\right) = \frac{2s(s^2-3)}{(s^2+1)^3}$$

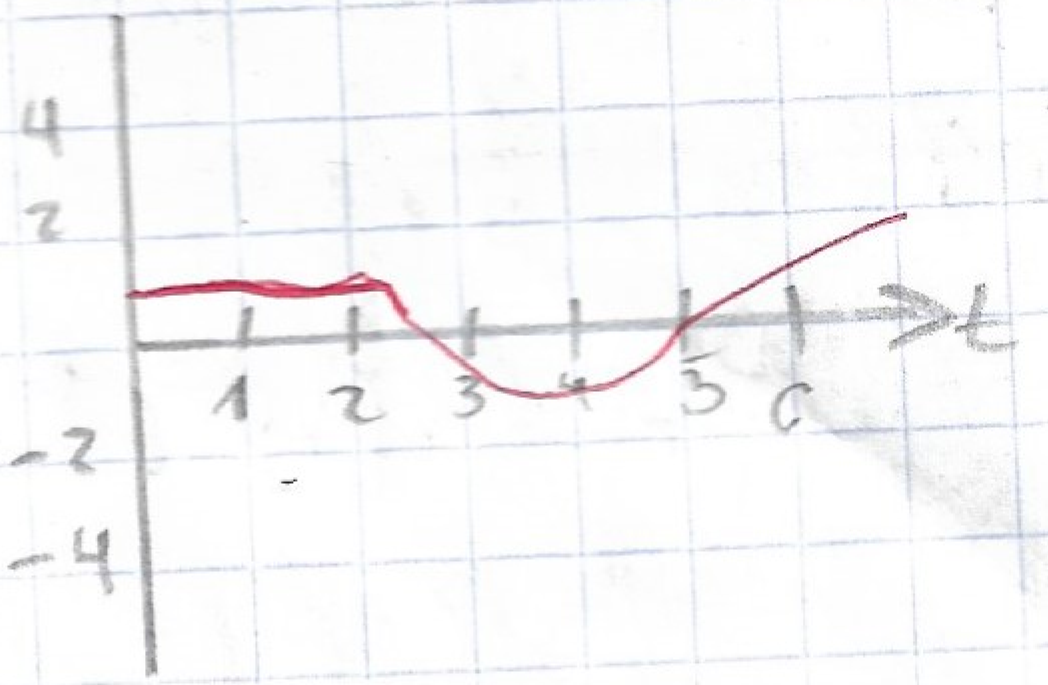
11.- $y''+9y=\cos 3t, \quad y(0)=2, \quad y'(0)=5$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + 9\mathcal{L}(y) = \frac{s}{s^2+9}$$

$$\mathcal{L}(y) = \frac{2s^2+5s^2+19s+45}{(s^2+9)^2} = \frac{2s}{s^2+9} + \frac{5}{s^2+9} + \frac{s}{(s^2+9)^2}$$

$$\therefore\ y = 2\cos 3t + \frac{5}{3}\operatorname{sen}3t + \frac{1}{6}t\operatorname{sen}3t$$

16.-



21.- $\mathcal{L}(e^{-t}\ e^{t}\cos t)$

$$= \frac{s-1}{(s+1)((s-1)^2+1)}$$

26.- $\mathcal{L}\left(\int_0^t \tau \operatorname{sen}\tau\, d\tau\right)$

$$= \frac{1}{s}\mathcal{L}(t\operatorname{sen}t) = \frac{1}{s}\left(-\frac{d}{ds}\frac{1}{s^2+1}\right) = -\frac{1}{s}\,\frac{-2s}{(s^2+1)^2}$$

$$= \frac{2}{(s^2+1)^2}$$

31.- $\mathcal{L}^{-1}\left(\frac{1}{s(s-1)}\right)$

$$= \mathcal{L}^{-1}\left(\frac{\frac{1}{s-1}}{s}\right) = \int_0^t e^{\tau}\, d\tau = e^t - 1$$

36.-

$$s^2\mathcal{L}(y) + \mathcal{L}(y) = \frac{1}{s^2+1} + \frac{2s}{(s^2+1)^2}$$

$$\therefore\ \mathcal{L}(y) = \frac{1}{(s^2+1)^2} + \frac{2s}{(s^2+1)^3}$$

$$\therefore\ y = \frac{1}{2}(\operatorname{sen}t - t\cos t) + \frac{1}{4}(t\operatorname{sen}t - t^2\cos t)$$

41- $f(t)+\int_0^t f(\tau)\,d\tau = 1$

$$\mathcal{L}(f)+\mathcal{L}(1)\mathcal{L}(f)=\mathcal{L}(1)$$

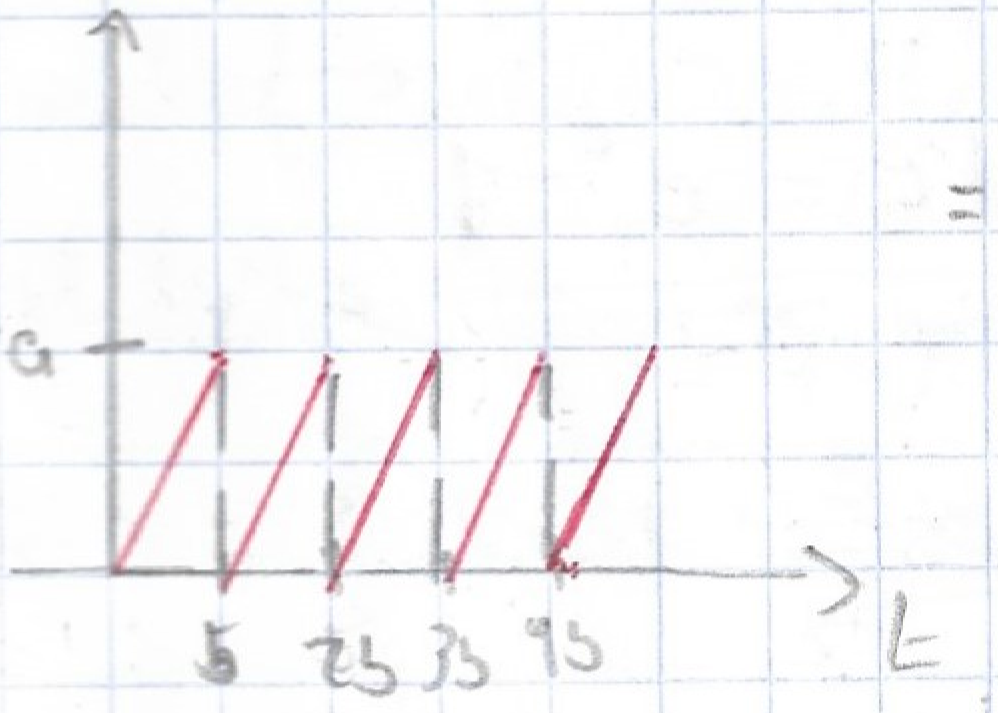
$$\therefore \mathcal{L}(f)=\frac{1}{s+1} \longrightarrow f(t)=e^{-t}$$

46- $\frac{dy}{dt}+6y(t)+9\int_0^t y(\tau)\,d\tau=1,\quad y(0)=0$

$$s\mathcal{L}(y)-y(0)+6\mathcal{L}(y)+9\mathcal{L}(1)\mathcal{L}(y)=\mathcal{L}(1)$$

$$\therefore \mathcal{L}(y)=\frac{1}{(s+3)^2}\longrightarrow y=te^{-3t}$$

51-



$$=\mathcal{L}(f(t))=\frac{1}{1-e^{-bs}}\int_0^b \frac{a}{b}\,te^{-st}\,dt$$

$$=\frac{a}{s}\left(\frac{1}{bs}-\frac{1}{e^{bs}-1}\right)$$

56-

$$Ls\mathcal{L}(i)+R\mathcal{L}(i)=\mathcal{L}(E(t))$$

$$\mathcal{L}(E(t))=\frac{1}{s}\left(\frac{1}{s}-\frac{1}{e^{s}-1}\right)=\frac{1}{s^2}-\frac{1}{s}\frac{1}{e^{s}-1}$$

$$(Ls+R)\mathcal{L}(i)=\frac{1}{s^2}-\frac{1}{s}\frac{1}{e^{s}-1}$$

$$i(t)=\frac{1}{R}\left(t-\frac{L}{R}+\frac{L}{R}e^{-\frac{Rt}{L}}\right)-\frac{1}{R}\left(1-e^{-\frac{R(t-1)}{L}}\right)U(t-1)-$$

$$-\frac{1}{R}\left(1-e^{-\frac{R(t-2)}{L}}\right)U(t-2)-\frac{1}{R}\left(1-e^{-\frac{R(t-3)}{L}}\right)U(t-3)-\cdots$$

$$=\frac{1}{R}\left(t-\frac{L}{R}+\frac{L}{R}e^{-\frac{Rt}{L}}\right)-\frac{1}{R}\sum_{n=1}^{\infty}\left(1-e^{-\frac{R(t-n)}{L}}\right)U(t-n)$$

Ejercicios 7.5

1.- $y' - 3y = \delta(t-2)$, $y(0)=0$

$$\mathcal{L}(y) = \frac{1}{s-3} e^{-2s}$$

$$\hookrightarrow y = e^{3(t-2)} \mathcal{U}(t-2)$$

6.- $y'' + y = \delta(t-2\pi) + \delta(t-4\pi)$, $y(0) = 1$, $y'(0) = 0$

$$\mathcal{L}(y) = \frac{s}{s^2+1} + \frac{1}{s^2+1}\left(e^{-2\pi s} + e^{-4\pi s}\right)$$

$$\therefore y = \cos t + \operatorname{sen} t \left(\mathcal{U}(t-2\pi) + \mathcal{U}(t-4\pi)\right)$$

11.- $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi)$, $y(0) = 1$, $y'(0) = 0$

$$\mathcal{L}(y) = \frac{4+s}{s^2+4s+13} + \frac{e^{-\pi s} + e^{-3\pi s}}{s^2+4s+13}$$

$$= \frac{2}{3}\frac{3}{(s+2)^2+3^2} + \frac{s+2}{(s+2)^2+3^2} + \frac{1}{3}\frac{3}{(s+2)^2+3^2}\left(e^{-\pi s} + e^{-3\pi s}\right)$$

$$\therefore y = \frac{2}{3} e^{-2t} \operatorname{sen} 3t + e^{-2t}\cos 3t + \frac{1}{3} e^{-2(t-\pi)} \operatorname{sen} 3(t-\pi) *$$

$$\mathcal{U}(t-\pi) + \frac{1}{3} e^{-2(t-3\pi)} \operatorname{sen} 3(t-3\pi)\, \mathcal{U}(t-3\pi)$$

Ejercicios 7.6

1.- $\frac{dx}{dt} = -x + y$

$\frac{dy}{dt} = 2x$, $x(0)=0$, $y(0)=1$

$s\mathcal{L}(x) = -\mathcal{L}(x) + \mathcal{L}(y)$

$s\mathcal{L}(y) - 1 = 2\mathcal{L}(x)$

$\mathcal{L}(x) = \frac{1}{(s-1)(s+2)} = \frac{1}{3}\frac{1}{s-1} - \frac{1}{3}\frac{1}{s+2}$

$\mathcal{L}(y) = \frac{1}{s} + \frac{2}{s(s-1)(s+2)} = \frac{2}{3}\frac{1}{s-1} + \frac{1}{3}\frac{1}{s+2}$

$\therefore x = \frac{1}{3}e^{t} - \frac{1}{3}e^{-2t}$, $y = \frac{2}{3}e^{t} + \frac{1}{3}e^{-2t}$

6.- $\frac{dx}{dt} + x - \frac{dy}{dt} + y = 0$

$\frac{dx}{dt} + \frac{dy}{dt} + 2y = 0$, $x(0)=0$, $y(0)=1$

$(s+1)\mathcal{L}(x) - (s-1)\mathcal{L}(y) = -1$

$s\mathcal{L}(x) + (s+2)\mathcal{L}(y) = 1$

$\mathcal{L}(y) = \frac{s+1/2}{s^2+s+1} = \frac{s+1/2}{(s+1/2)^2+(\sqrt{3}/2)^2}$

$\therefore x = -\sqrt{3}\, e^{-t/2} \operatorname{sen} \frac{\sqrt{3}}{2}t$, $y = e^{-t/2} \cos \frac{\sqrt{3}}{2}t$

11.- $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 3y = 0$

$\frac{d^2x}{dt^2} + 3y = te^{-t}$, $x(0) = 0$, $x'(0) = 2$, $y(0) = 0$

$s^2\mathcal{L}(x) + 3(s+1)\mathcal{L}(y) = 2$

$s^2\mathcal{L}(x) + 3\mathcal{L}(y) = \frac{1}{(s+1)^2}$

$\mathcal{L}(x) = -\frac{2s+1}{s^3(s+1)} = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{2}\frac{2}{s^3} - \frac{1}{s+1}$

$\therefore x = 1 + t + \frac{1}{2}t^2 - e^{-t}$, $y = \frac{1}{3}te^{-t} - \frac{1}{3}x'' = \frac{1}{3}te^{-t} + \frac{1}{3}e^{-t} - \frac{1}{3}$

16.-

$i_2' + i_3' + 10i_2 = 120 - 120\,U(t-2)$ $-10i_2' + 5i_3' + 5i_3 = 0$

$(s+10)\mathcal{L}(i_2) + s\mathcal{L}(i_3) = \frac{120}{s}(1-e^{-2s}) - 10s\mathcal{L}(i_2)$

$+ 5(s+1)\mathcal{L}(i_3) = 0$

$\downarrow$

$\mathcal{L}(i_2) = \frac{120(s+1)}{(3s^2+11s+10)s}(1-e^{-2s}) = \left(\frac{48}{s+\frac{5}{3}} - \frac{60}{s+2} + \frac{12}{s}\right)(1-e^{-2s})$

$\mathcal{L}(i_3) = \frac{240}{3s^2+11s+10}(1-e^{-2s}) = \left(\frac{240}{s+\frac{5}{3}} - \frac{240}{s+2}\right)(1-e^{-2s})$

$\therefore i_2 = 12 + 48e^{\frac{-5t}{3}} - 60e^{-2t} - \left(12 + 48e^{\frac{-5(t-2)}{3}} - 60e^{-2(t-2)}\right)U(t-2)$

$i_3 = 240e^{\frac{-5t}{3}} - 240e^{-2t} - \left(240e^{\frac{-5(t-2)}{3}} - 240e^{-2(t-2)}\right)U(t-2)$

21- a) $4\theta_1'' + \theta_2'' + 8\theta_1 = 0$

$\theta_1'' + \theta_2'' + 2\theta_2 = 0$

$4(s^2+2)\,\mathcal{L}(\theta_1) + s^2\mathcal{L}(\theta_2) = 3s$

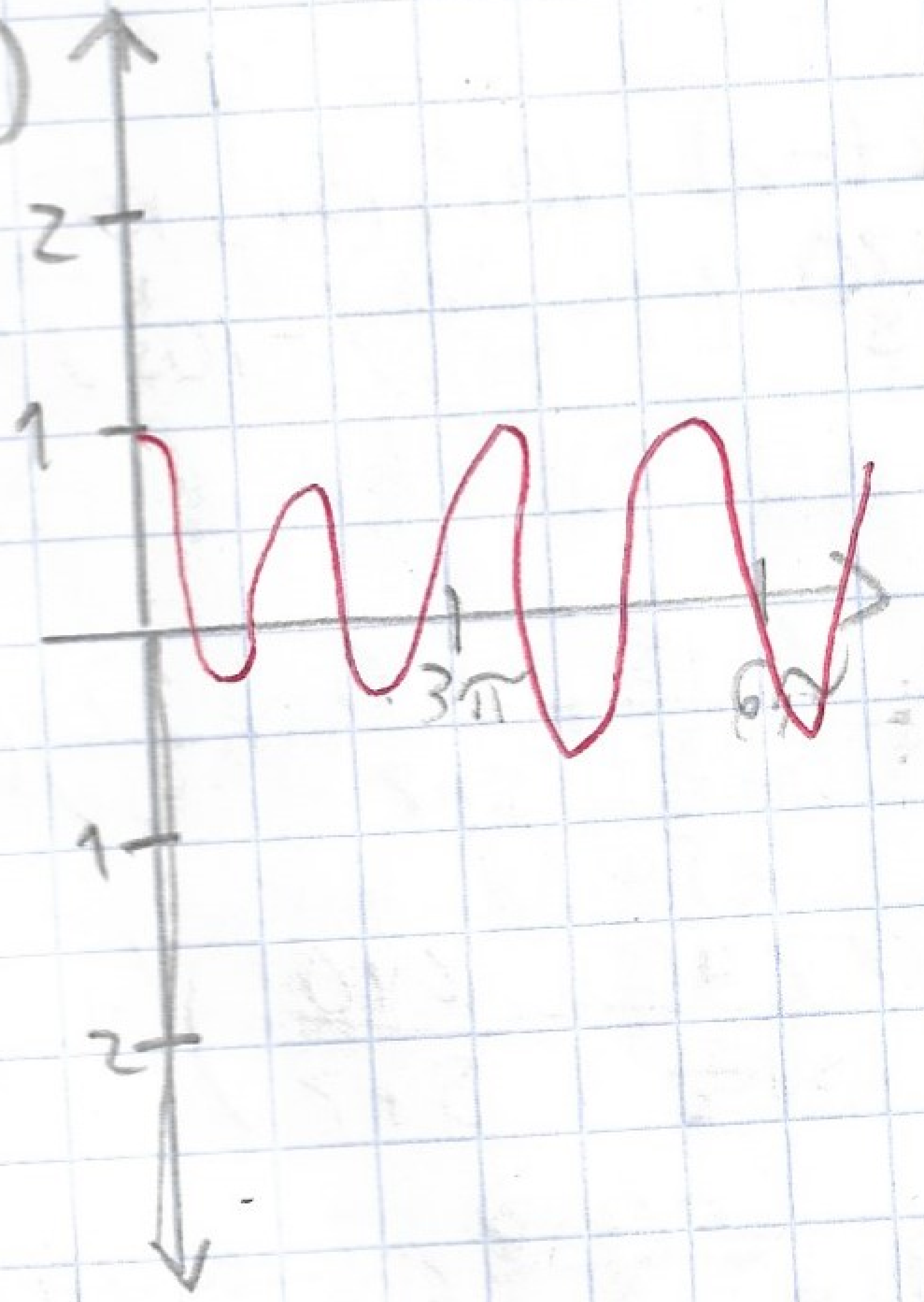
$s^2\mathcal{L}(\theta_1) + (s^2+2)\,\mathcal{L}(\theta_2) = 0$

$(3s^2+4)(s^2+4)\,\mathcal{L}(\theta_2) = -3s^3$

$\mathcal{L}(\theta_2) = \frac{1}{2}\frac{s}{s^2+4/3} - \frac{3}{2}\frac{s}{s^2+4}$

$\therefore \theta_1 = \frac{1}{4}\cos\frac{2}{\sqrt{3}}t + \frac{3}{4}\cos 2t$

b)



c) $0 \leq t \leq 30$