Use la definición $\epsilon - \delta$ de límite para verificar los ejercicios 1-10

1.- $\lim_{z \to 1} 2z = 2$

$\forall \epsilon > 0$ existe $\delta > 0$
talque $|f(z) - \Lambda| < \epsilon$ siempreque $|z - z_0| < \delta$

$|2z - 2| < \epsilon$

$|2(z-1)| < \epsilon$

$2|z-1| < \epsilon$

$|z-1| < \frac{\epsilon}{2}$

→ Se toma $\delta = \frac{\epsilon}{2}$

$\therefore |z - \underbrace{1}_{z_0}| < \delta$

$z_0 = 1$

3.- $\lim_{z \to -i} z + i = 0$

$\forall \epsilon > 0$ existe $\delta > 0$
talque $|f(z) - \Lambda| < \epsilon$ siempre que $|z - z_0| < \delta$

$|z + i - 0| < \epsilon$

$|z + i| < \epsilon$ → Se toma $\delta = \epsilon$

$|z + i| = \delta$

$|z - \underbrace{(-i)}_{z_0}| = \delta$ $\therefore z_0 = -i$

5.- $\lim_{z \to 1+i} 2z - 3 = -1 + 2i$

$\forall \epsilon > 0$ existe $\delta > 0$
talque $|f(z) - \Lambda| < \epsilon$ siempreque $|z - z_0| < \delta$

$|(2z-3) - (-1+2i)| < \epsilon$

$|2z - 3 + 1 + 2i| < \epsilon$

$|2z - 2 + 2i) < \epsilon$

$|2(z - 1 + i)| < \epsilon$

$2|z - 1 + i| < \epsilon$

$|z - 1 + i| < \frac{\epsilon}{2}$ → Se toma $\delta = \frac{\epsilon}{2}$

$\therefore |z - \underbrace{1 + i}_{z_0}| < \delta$ $z_0 = 1 + i$

7.- $\lim_{z \to 2} \frac{z^2-4}{z-2} = 4$

$\forall\, \epsilon > 0$ existe $\delta > 0$ tal que $|f(z)-1| < \epsilon$ siempre que $|z-z_0| < \delta$

$\left|\frac{z^2-4}{z-2} - 4\right| < \epsilon$

Se toma $\delta = \epsilon$

$\left|\frac{\cancel{z-2}(z+2)}{\cancel{z-2}} - 4\right| < \epsilon$

$\therefore \left|z - \frac{2}{z_0}\right| < \delta \longrightarrow \underline{z_0 = 2}$

$|z+2-4| < \epsilon$

$|z-2| < \epsilon$

9.- $\lim_{z \to 1} \frac{z^3-1}{z-1} = 3$

$\forall\, \epsilon > 0$ existe $\delta > 0$ tal que $|f(z)-1| < \epsilon$ siempre que $|z-z_0| < \delta$

$\left|\frac{z^3-1}{z-1} - 3\right| < \epsilon$

binal cubo $= a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\left|\frac{\cancel{(z-1)}(z^2+z+1)}{\cancel{z-1}} - 3\right| < \epsilon$

$(z-2)|z-1| < \epsilon$

$|z-1| < \frac{\epsilon}{z-2}$

Se toma $\delta = \frac{\epsilon}{z-2}$

$|z^2+z+1-3| < \epsilon$

$|z^2+z-2| < \epsilon$

$|(z+2)(z-1)| < \epsilon$

$\therefore \left|z - \frac{1}{z_0}\right| < \delta \longrightarrow \underline{z_0 = 1}$

Pruebe que las funciones son continuas en $\mathbb{C}$:

11.- $w = \operatorname{Re} z$

$\lim_{\operatorname{Re} z \to a} \operatorname{Re} z = a$

$|\operatorname{Re} z - a| < \epsilon$

si $\operatorname{Re} z = \frac{z+\bar{z}}{2}$

$\left|\frac{z+\bar{z}}{2} - a\right| < \epsilon$

si $\frac{z+\bar{z}}{2} = 2x$

$|2x - a| < \epsilon$

Se toma

$\delta = \epsilon$

$|2x-a| < \delta$

$\left|\operatorname{Re} z - \frac{a}{z_0}\right| < \delta \longrightarrow \underline{z_0 = a}$

13- $w = \overline{z}$

$\lim_{z \to a} \overline{z} = \overline{a}$

$|\overline{z} - \overline{a}| < \epsilon$

$|z - a| < \epsilon$

$\rightarrow |\overline{z} - \overline{a}| = |z - a|$

Se toma $\delta = \epsilon$

$\therefore |z - \underbrace{a}_{z_0}| < \delta \rightarrow z_0 = a$