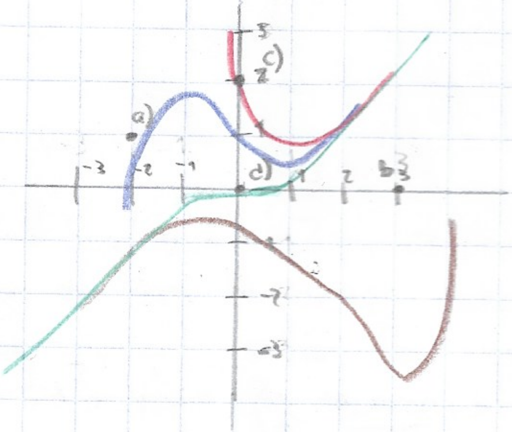
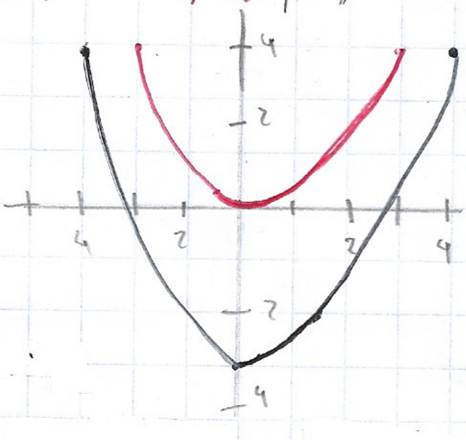
Ejercicios 2.1

1.- $\frac{dy}{dx} = x^2 - y^2$

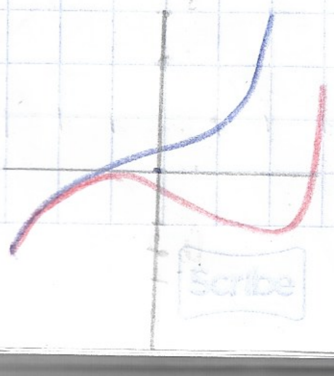
a) $y(-2)=1$, b) $y(3)=0$, c) $y(0)=2$, d) $y(0)=0$



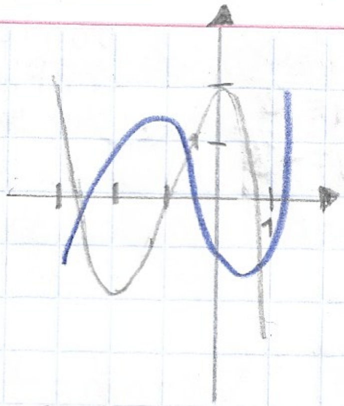
5) $y' = x$ a) $y(0)=0$; b) $y(0)=-3$



9.- $\frac{dy}{dx} = 0.2x^2 + y$ a) $y(0)=\frac{1}{2}$; b) $y(2)=-1$



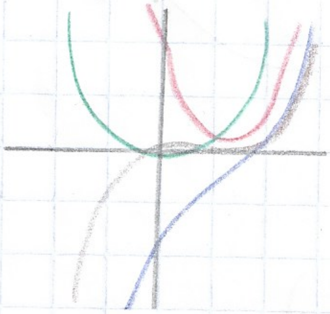
13.-



17.-

Cuando $y < \frac{1}{2}x^2$, $y' = x^2 - 2y$ es positiva y las curvas solución fuera de las ceroclinas están creciendo.

Cuando $y > \frac{1}{2}x^2$, $y' = x^2 - 2y$ es negativa y las curvas dentro de las ceroclinas estan decreciendo



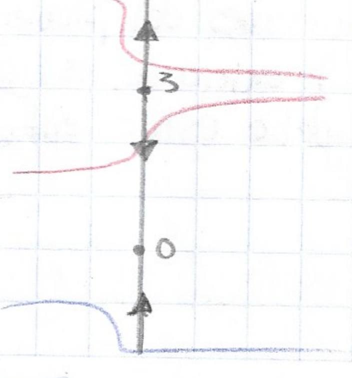
21.- $\frac{dy}{dx} = y^2 - 3y$

$y' = f(y) \rightarrow y = k \Rightarrow y' = 0 \therefore 0 = f(k)$

$y' = y^2 - 3y \rightarrow y^2 - 3y = 0$

$y(y-3) = 0 \quad \begin{cases} y = 0 \\ y = 3 \end{cases}$

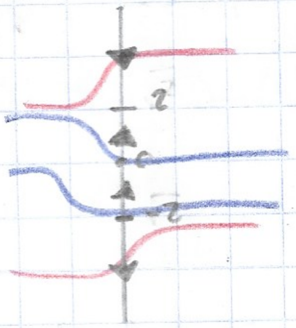
Los puntos críticos



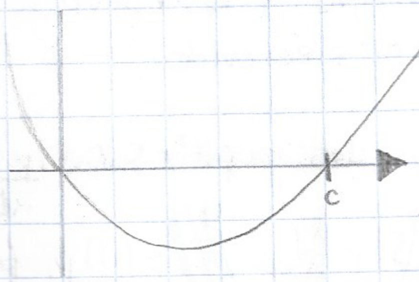
$4-y^2=0$
$y^2=4$
$y=\pm\sqrt{4}$

25.- $\frac{dy}{dx}=y^2(4-y^2)$

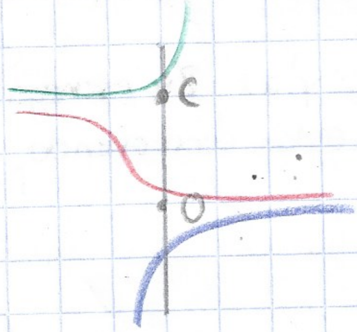
$y'=0 \longrightarrow y^2(4-y^2)=0 \begin{cases} y=0 \\ y=2 \\ y=-2 \end{cases}$ puntos críticos



29.-



Los puntos críticos $\begin{cases} y=0 \\ y=c \end{cases}$ $c>0$



33.-

$\frac{dy}{dx}=f(y)$, se asume que f y f' son funciones continuas en y y en el intervalo I.
Ahora si suponemos que el gráfico de una solución NO constante cruza la línea $y=c$.
Si el punto de incersión se toma como la condición inicial, se tendrá 2 soluciones del problema de valor inicial. Esto contradice a la unicidad, así que el gráfico de una solución no constante debe estar enteramente equilibrada.

37.- Si la ecuación (1) no tiene puntos críticos, entonces no tiene una solución constante. Las soluciones no tienen límite ni superior ni inferior, por lo tanto se dice que cada solución asume todos los valores reales.

41.-

$$m\frac{dv}{dt} = mg - kv^2$$

$$\Downarrow$$

$$\frac{dv}{dt} = \frac{k}{m}\left(\frac{mg}{k} - v^2\right) = \frac{k}{m}\left(\sqrt{\frac{mg}{k}} - v\right)\left(\sqrt{\frac{mg}{k}} + v\right)$$

Podemos observar que físicamente el punto crítico es $\sqrt{\frac{mg}{k}}$

Ejercicios 2.2

1.- $\frac{dy}{dx} = \text{sen}\,5x \rightarrow dy = \text{sen}\,5x\,dx$ $u = 5x$

$du = 5 \rightarrow du = 5dx$

$\int dy = \int \text{sen}\,5x\,dx \rightarrow y = \int \text{sen}\,u\,dx$

$y = \frac{1}{5} - \cos 5x + C \rightarrow y = -\frac{\cos 5x}{5} + C$

4.- $dy - (y-1)^2 dx = 0 \rightarrow dy = (y-1)^2 dx \rightarrow \frac{dy}{(y-1)^2} = dx$

$\int \frac{dy}{(y-1)^2} = \int dx \rightarrow u = y-1 \;;\; du = dy$

$\int \frac{du}{u^2} = x + C \rightarrow -\frac{1}{u} = x \rightarrow -\frac{1}{y-1} = x \rightarrow y - 1 = -\frac{1}{x}$

$y = -\frac{1}{x} - 1 + C \rightarrow y = -\frac{1}{x} + \frac{x}{x} + C$

9.- $y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2 \rightarrow y \ln x\, dx = \frac{(y+1)^2}{x^2} dy$

$\int x^2 \ln x\, dx = \int \frac{(y+1)^2}{y} dy$

$u = \ln x$ $du = \frac{1}{x}dx$

$dv = x^2 dx$ $v = \frac{x^3}{3}$

$= \ln x \frac{x^3}{3} - \int \frac{x^{\cancel{3}}}{3} \frac{1}{\cancel{x}} dx = \ln x \frac{x^3}{3} - \frac{1}{3}\int x^2 dx$

$\ln x \frac{x^3}{3} - \frac{1}{3}\frac{x^3}{3} + C = \ln x \frac{x^3}{3} - \frac{x^3}{9} = \int \frac{(y+1)^2}{y} dy \rightarrow \ln x \frac{x^3}{3} - \frac{x^3}{9} = \frac{y^2}{2} - 2y + \ln y + C$

$\left(\ln x \frac{x^3}{3} - \frac{x^3}{9}\right) 2 = y^2 + \ln y + C \rightarrow y^2 + \ln y = \ln x \frac{x^3}{3} - \frac{x^3}{9} + C$

13.- $(e^y+1)^2 e^{-y}\,dx + (e^x+1)^3 e^{-x}\,dy = 0 \rightarrow (e^x+1)^3 e^{-x}\,dy = -(e^y+1)^2 e^{-y}\,dx$

$$\int \frac{dy}{-(e^y+1)^2 e^{-y}} = \int \frac{dx}{(e^x+1)^3 e^{-x}}$$

$$\int (dy + e^y + 1)^3 e^{-y} =$$

$$\int (e^y+1)^2 e^{+y} = \int (e^y+1)^3 e^{-x}$$

$$\int dx + e^x + 1)^2 e^{-y}$$

$$y = \ln \frac{-2(e^x+1)}{-1+2(e^x+1)^2}$$

$$y = \ln\left(-\frac{2(e^x+1)^2}{1+2c(e^x+1)^2} - \frac{2c(e^x+1)^2}{1+2c(e^x+1)^2}\right)$$

17.- $\frac{dP}{dt} = P - P^2$

$$\int \frac{dP}{P-P^2} = \int dt \longrightarrow \int \frac{dP}{P(1-P)} = \int dt \longrightarrow \int \frac{dP}{P} - \int \frac{dP}{(1-P)} = t + C$$

$u = 1-P$

$du = -dP$

$$\ln P - \int -\frac{du}{u} = t \longrightarrow \ln P + \ln u = t \longrightarrow \ln|P| + \ln|1-P| = t + C$$

21.- $\frac{dy}{dx} = x\sqrt{1-y^2}$

$$dy = x\sqrt{1-y^2}\,dx \longrightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int x\,dx$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{arcsen} \frac{x}{a}$$

$$\operatorname{arcsen} \frac{y}{1} = \frac{x^2}{2} + C$$

$$\operatorname{sen}^{-1} y = \frac{x^2}{2} + C$$

$$y = \operatorname{sen} \frac{x^2}{2} + C$$

25.- $x^2 \frac{dy}{dx} = y - xy$, $y(1) = -1$

$x^2 dy = y - xy \; dx$

$\frac{-\cancel{y}\,dy}{\cancel{y}} = \frac{\cancel{x}\,dx}{x^{\cancel{2}}} = \int -dy = \int \frac{dx}{x}$

$-y = \ln|x| + C$ $\longrightarrow$ $-\ln|1| + 1 = C$

$1 = \ln|1| + C$

$y = -\ln|x| - \ln|1| + 1$ $\longrightarrow$ $y = -\ln|x| + 1$

29.- $\frac{dy}{dx} = ye^{-x^2}$, $y(4) = 1$

$\int_4^x \frac{1}{y} \frac{dy}{dt} dt = \int_4^x e^{-t^2} dt \longrightarrow \ln y(t) \Big|_4^x = \int_4^x e^{-t^2} dt$

$\ln y(x) - \ln y(4) = \int_4^x e^{-t^2} dt$

$\ln y(x) = \ln y(4) + \int_4^x e^{-t^2} dt = \ln 1 + \int_4^x e^{-t^2} dt = \int_4^x e^{-t^2} dt$

$y(x) = e^{\int_4^x e^{-t^2} dt}$

33.-

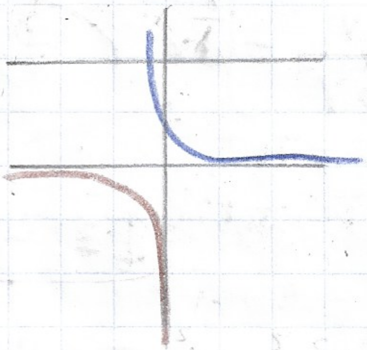
$\frac{dy}{dx} = x\sqrt{1-y^2} \longrightarrow y = -1 \; ; \; y = 1$

$(e^x + e^{-x}) \frac{dy}{dx} = y^2 \longrightarrow y = 0$

37.- $\frac{dy}{dx} = (y-1)^2 + 0.01 \quad , \quad y(0) = 1$

$$\int \frac{dy}{(y-1)^2 + 0.01} = \int dx \longrightarrow -\frac{1}{y-1} = x + C \longrightarrow y = \frac{x-1+C}{x+C}$$

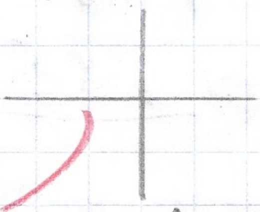
$$1 = \frac{-1+C}{C} \longrightarrow y = \frac{x-101}{x-100}$$



41.- $\frac{dy}{dx} = \frac{2x+1}{2y} \quad , \quad y(-2) = -1$

a) $\int 2y\,dy = \int (2x+1)\,dx \longrightarrow y^2 = x^2 + x + C \longrightarrow C = -1 = y_1 = x^2 + x - 1$

$y = -\sqrt{x^2+x-1}$



b) $(-\infty, -1.65)$

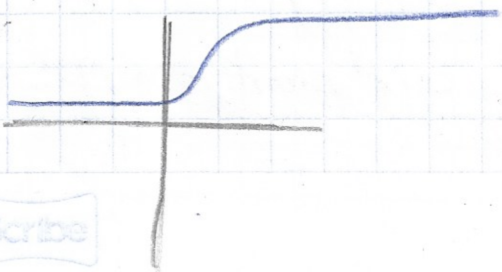
c) $x^2 + x - 1 = 0 \qquad x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5} \longrightarrow \left(-\infty, -\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)$

45.- $\frac{dy}{dx} = \sqrt{1+y^2}\,\text{sen}^2 y \qquad y(0) = \frac{1}{2}$

$$\int \frac{dy}{\sqrt{1+y^2}\,\text{sen}^2 y} = \int dx \longrightarrow \text{No se puede integrar}$$

$\frac{dy}{dx} \geq 0$ para todos los valores de x, y

y que $\frac{dy}{dx} = 0$



Ejercicios 2.3

No hay terminos transitorios

1.- $\frac{dy}{dx} = 5y$

$\int \frac{dy}{5y} = \int dx \rightarrow \frac{1}{5}\ln|y| = x + C$

$\ln y = 5x + 5C$

$y = Ce^{5x}$

$I = -\infty < x < \infty$

5.- $y' + 3x^2y = x^2 \rightarrow \frac{dy}{dx} = x^2 - 3x^2y$

$\frac{dy}{y} = x^2 - 3x^2 dx \rightarrow \int \frac{dy}{y} = \int -2x^2 dx$

$\Rightarrow \ln y = \frac{-2x^3}{3} + C \rightarrow y = Ce^{-\frac{2x^3}{3}}$

$I = -\infty < x < \infty$

ce^{-x^3} es transitorio

9.- $x\frac{dy}{dx} - y = x^2 \operatorname{sen} x$

$x\,dy - y = x^2 \operatorname{sen} x\, dx$

$\int dy + y = \int \frac{x^2 \operatorname{sen} x\, dx}{x} \longrightarrow y - \frac{y^2}{2} = -x\cos x + \operatorname{sen} x + C$

$y = cx - x\cos x$ para $0 < x < \infty$

No hay terminos transitorios

13.- $x^2 y' + x(x+2)y = e^x \longrightarrow x^2 \frac{dy}{dx} + x(x+2)y = e^x$

$x^2 dy + x(x+2)y = e^x dx \longrightarrow \int dy + y = \int \frac{e^x\, dx}{x^2 + x(x+2)}$

$y' + \left(1 + \frac{2}{x}\right) y = \frac{e^x}{x^2}$

$e^{\int \frac{x}{1+x} dx} = x^4 \longrightarrow y = \frac{1}{7}x^3 - \frac{1}{5}x + cx^{-4}$ para $0 < x < \infty$

$\frac{ce^{-x}}{x^2}$ es transitorio

17.- $\cos x \frac{dy}{dx} + (\operatorname{sen} x)y = 1$ $\quad \cos x\, dy + y = \frac{dx}{\operatorname{sen} x} \longrightarrow dy + y = \frac{dx}{\operatorname{sen} x \cos x}$

$y' + (\tan x)y = \sec x \longrightarrow e^{\int \tan x\, dx} = \sec x$

$y = \operatorname{sen} x + \cos x$ para $-\frac{\pi}{2} < x < \frac{\pi}{2}$

No hay terminos transitorios

21.- $\frac{dr}{d\theta} + r\sec\theta = \cos\theta \rightarrow dr = \frac{\cos\theta}{r\sec\theta} d\theta$

$\int r\,dr = \int \frac{\cos\theta}{\sec\theta} d\theta$

$e^{\int \sec\theta\, d\theta} = e^{\ln|\sec r + \tan r|} = \sec\theta + \tan\theta$

asi que $\frac{d}{d\theta}(\sec\theta + \tan\theta)r = 1 + \operatorname{sen}\theta$ y $(\sec\theta + \tan\theta)r = \theta - \cos\theta + c$ para $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

25.- $xy' + y = e^x \quad y(1) = 2$

$y' + \frac{1}{x}y = \frac{1}{x}e^x \rightarrow e^{\int \frac{1}{x} dx} = x$

$\frac{d}{dx}[xy] = e^x$ y $y = \frac{1}{x}e^x + \frac{c}{x}$ para $0 < x < \infty$

Si $y(1) = 2$ entonces $c = 2 - e$ y $y = \frac{1}{x}e^x + \frac{2-e}{x}$

29.- $(x+1)\frac{dy}{dx} + y = \ln x, \quad y(1) = 10$

$y' + \frac{1}{x+1}y = \frac{\ln x}{x+1} \rightarrow e^{\int \frac{1}{x+1} dx} = x+1$

$\frac{d}{dx}(x+1)y = \ln x \quad y = \frac{x}{x+1}\ln x - \frac{x}{x+1} + \frac{c}{x+1}$ para $0 < x < \infty$

Si $y(1) = 10$ entonces $c = 21$

$\therefore y = \frac{x}{x+1}\ln x - \frac{x}{x+1} + \frac{21}{x+1}$

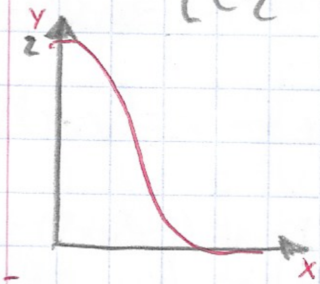
33.- $\frac{dy}{dx} + 2xy = f(x)$, $y(0) = 2$, donde

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$$

$y' + 2xy = f(x) \longrightarrow e^{x^2}$

$$ye^{x^2} = \begin{cases} \frac{1}{2}e^{x^2} + c_1 & 0 \le x \le 1 \\ c_2 & x > 1 \end{cases}$$

Si $y(0)=2$ entonces $c_1 = \frac{3}{2}$ y para que sea continua:



$$y = \begin{cases} \frac{1}{2} + \frac{3}{2}e^{-x^2}, & 0 \le x \le 1 \\ \left(\frac{1}{2}e + \frac{3}{2}\right)e^{-x^2} & x > 1 \end{cases}$$

37.-

$y' - 2xy = 1 \longrightarrow e^{-x^2} \therefore \frac{d}{dx}\left(e^{-x^2}y\right) = e^{-x^2}$

$e^{-x^2}y = \int_0^x e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) + c$

$y = \frac{\sqrt{\pi}}{2}e^{x^2}\operatorname{erf}(x) + ce^{x^2}$

$y(1) = \frac{\sqrt{\pi}}{2}\operatorname{erf}(1) + ce = 1$ se tiene

$c = e^{-1} - \frac{\sqrt{\pi}}{2}\operatorname{erf}(1) = y = e^{x^2-1} + \frac{\sqrt{\pi}}{2}e^{x^2}\left(\operatorname{erf}(x) - \operatorname{erf}(1)\right)$

41.-

$y = 3x - 5 + ce^{-x}$

$\downarrow$

$y' = 3 - ce^{-x} = 3 - (y - 3x + 5) = -y + 3x - 2$

$\therefore$ $y' + y = 3x - 2$ tiene soluciones asintóticas en la línea $y = 3x - 5$

45.- Podemos concluir que $y=0$ es una solución.

48.- $y' + \frac{2}{x} y = \frac{10 \operatorname{sen} x}{x^3} \longrightarrow x^2$

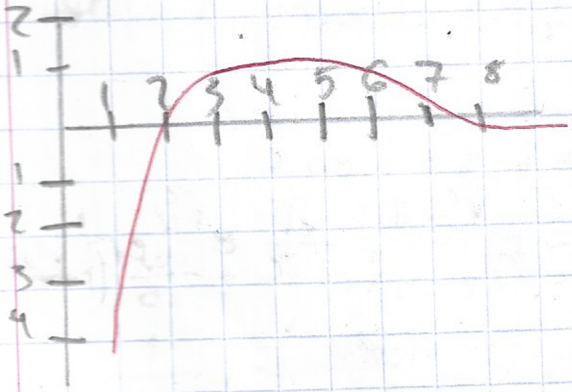
$\therefore \frac{d}{dx}(x^2 y) = 10 \frac{\operatorname{sen} x}{x}$

$x^2 y = 10 \int_0^x \frac{\operatorname{sen} t}{t} \, dt + c$

$y = 10x^{-2} \operatorname{Si}(x) + cx^{-2}$

Desde $y(1)=0$ tenemos $c = -10 \operatorname{Sen}(1)$

$\therefore y = 10x^{-2} \operatorname{Sen} x - 10x^{-2} \operatorname{sen}(1) = 10x^{-2}(\operatorname{Sen} x - \operatorname{Sen} 1)$



Ejercicios 2.4 $P(x,y)dx + Q(x,y)dy = 0$

1.- $(2x-1)dx + (3y+7)dy = 0$

$P(x,y) = (2x-1) = \frac{dP}{dy} = \frac{d}{dy} = 2x-1 = 0$ ∴ si es una E.D exacta

$Q(x,y) = (3y-7) = \frac{dQ}{dx} = \frac{d}{dx} = 3y+7 = 0$

$\frac{dV}{dx} = P(x,y) = \int 2x-1dx \longrightarrow x^2-x+C$

$\frac{dV}{dy} = Q(x,y) = \int 3y+7dy \longrightarrow \frac{3y^2}{2} + 7y + C$

$V(x,y) = x^2 - x + \frac{3y^2}{2} + 7y + C \rightarrow x^2 - x + \frac{3y^2}{2} + 7y = C$

5.- $(2xy^2-3)dx + (2x^2y+4)dy = 0$ $P(x,y) = (2xy^2-3)$; $Q(x,y) = (2x^2y+4)$

$\frac{dP}{dy} = \frac{d}{dy}(2xy^2-3) = 4xy$

∴ si es una E.D. exacta

$\frac{dQ}{dx} = \frac{d}{dx}(2x^2y+4) = 4xy$

$\frac{dV}{dx} = P(x,y) = \int 2xy^2-3dx \rightarrow y^2x^2-3x+C$

$\frac{dV}{dy} = Q(x,y) = \int 2x^2y+4dy \rightarrow x^2y^2+4y+C$

∴ $V(x,y) = x^2y^2 - 3x + x^2y^2 + 4y + C \rightarrow x^2y^2 - 3x + 4y = C$

9.- $(x-y^3+y^2\operatorname{sen}x)dx = (3xy^2+2y\cos x)dy$ $P(x,y) = (x-y^3+y^2\operatorname{sen}x)$

$\frac{d}{dy}(x-y^3+y^2\operatorname{sen}x) = -3y^2+2y\operatorname{sen}x$ $Q(x,y) = (3xy^2+2y\cos x)dy$

$\frac{d}{dx}(3xy^2+2y\cos x) = 3y^2-2y\operatorname{sen}x$

$\frac{dP}{dy} \neq \frac{dQ}{dx}$ ∴ No es una E.D exacta

13.- $x\frac{dy}{dx} = 2xe^{x} - y + 6x^{2} \rightarrow x\,dy = (2xe^{x} - y + 6x^{2})dx$

$\frac{d}{dy} = (2xe^{x} - y + 6x^{2}) = -1$

$\frac{d}{dx} = x = 1$

$\frac{dP}{dy} \neq \frac{dQ}{dx}$ ∴ no es una E.D

17.- $(\tan x - \operatorname{sen}x \operatorname{sen}y)dx + \cos x \cos y\, dy = 0$

$\frac{d}{dy}(\cos x \cos y) = -\cos x \operatorname{sen} y$

$\frac{d}{dx}(\tan x - \operatorname{sen}x \operatorname{sen}y) = \frac{1 - \cos^{3}x \operatorname{sen}y}{\cos^{2}x}$

$\frac{dP}{dy} \neq \frac{dQ}{dx}$ ∴ no es una E.D exacta

21.- $(x+y)^{2}dx + (2xy + x^{2} - 1)dy = 0$, $y(1) = 1$

$\frac{d}{dy}((x+y)^{2}) = 2x + 2y$

$\frac{d}{dx}(2xy + x^{2} - 1) = 2y + 2x$

$\frac{dV}{dx} = P(x,y) = \int (x+y)^{2}dx \rightarrow \frac{x^{3}}{3} + yx^{2} + y^{2}x + C$

$\frac{dV}{dy} = Q(x,y) = \int 2xy + x^{2} - 1\, dy \rightarrow xy^{2} + x^{2}y - y + C$

$V(x,y) = \frac{x^{3}}{3} + x^{2}y + xy^{2} + xy^{2} + x^{2}y - y + C \rightarrow \frac{x^{3}}{3} + x^{2}y + xy^{2} - y = C$

$y(1) = 1 \rightarrow \frac{x^{3}}{3} + x^{2}y + xy^{2} - y = \frac{4}{3}$

25.- $(y^2\cos x - 3x^2y - 2x)\,dx + (2y\,\mathrm{sen}\,x - x^3 + \ln y)\,dy = 0$; $y(0) = e$

$\frac{d}{dy}(y^2\cos x - 3x^2y - 2x) = 2y\cos x - 3x^2$

$\frac{d}{dx}(2y\,\mathrm{sen}x - x^3 + \ln y) = 2y\cos x - 3x^2$

$\frac{dv}{dx} = \int y^2\cos x - 3x^2y - 2x\,dx = y^2\mathrm{sen}x - yx^3 - x^2 + C$

$\frac{dv}{dy} = \int 2y\,\mathrm{sen}x - x^3 + \ln y\,dy = \mathrm{sen}x\,y^2 - x^3y + \ln y\,y - y + C$

$V(x,y) = y^2\mathrm{sen}x - yx^3 - x^2 + \mathrm{sen}x\,y^2 - x^3y + \ln y + C \rightarrow y^2\mathrm{sen}x - x^3y - x^2 + y\ln y = C$

$y(0) = e \rightarrow \quad y^2\mathrm{sen}x - x^3y - x^2 + y\ln y - y = 0$

29.- $(-xy\,\mathrm{sen}x + 2y\cos x)\,dx + 2x\cos x\,dy = 0$; $\mu(x,y) = xy$

$\frac{d}{dx}(-xy\,\mathrm{sen}x + 2y\cos x) = -x\,\mathrm{sen}x + 2\cos x$

$\frac{d}{dy}(2x\cos x) = 2\cos x - 2x\,\mathrm{sen}x \quad \frac{dP}{dy} \neq \frac{dQ}{dx}$

$\therefore$ No es una E.D. exacta

33.- $6xy\,dx + (4y + 9x^2)\,dy = 0$

$\frac{d}{dx}\,6xy = 6y$

$\frac{d}{dy}\,4y + 9x^2 = 4 \qquad \frac{dP}{dy} \neq \frac{dQ}{dx}$

$\therefore$ No son exactos

37.- $x\,dx + (x^2y + 4y)\,dy = 0 \qquad y(4) = 0$

$\frac{d}{dx}\, x = 1$

$\frac{d}{dy}(x^2y + 4y) = x^2 + 4 \qquad \frac{dP}{dy} \neq \frac{dQ}{dx}$

$\therefore$ No son E.D exactas

41.-

$y = \sqrt{\frac{3+\cos^2 x}{1-x^2}} \longrightarrow 3 + \cos^2 x > 0$

Para todo x se tiene que $1 - x^2 > 0$

ó

$-1 < x < 1$ $\therefore$ el intervalo I es $(-1, 1)$

45.-

a) $(v^2 - 32x)\,dx + xv\,dv = 0$ no es exacta

$\mu(x) = x \longrightarrow (xv^2 - 32x^2)\,dx + x^2v\,dv = 0$

$U = \frac{1}{2}x^2v^2 - \frac{32}{3}x^3$ asi que su solución implicita

$\frac{1}{2}x^2v^2 - \frac{32}{3}x^3 = C \longrightarrow x = 3 \quad v = 0 \quad c = -288$

$v = 8\sqrt{\frac{x}{3} - \frac{9}{x^2}}$

b) $v(8) = 8\sqrt{\frac{8}{3} - \frac{9}{64}} = 12\ ft/s$

Ejercicios 2.5

$y = ux \rightarrow u = \frac{y}{x}$

$dy = u\,dx + x\,du$

1.- $(x-y)dx + x\,dy = 0$

$(x-ux)dx + x(u\,dx + x\,du) = 0 \rightarrow x\,dx - ux\,dx + xu\,dx + x^2du = 0$

$x\,dx(1 - \cancel{u} + \cancel{u}) + x^2du \rightarrow x\,dx + x^2du = 0$

$x\,dx = -x^2du \rightarrow \frac{x\,dx}{x^2} = -du$

$\int \frac{x\,dx}{x^2} = -\int du \rightarrow \ln x = -u + C \rightarrow \ln x = -\frac{y}{x} + C$

$y = -x\ln x + Cx$

5.- $(y^2 + yx)dx - x^2dy = 0$ $\quad y = ux \rightarrow u = \frac{y}{x} \rightarrow dy = u\,dx + x\,du$

$((ux)^2 + (ux^2))dx - x^2(u\,dx + x\,du) = 0 \rightarrow u^2x^2dx + u^2x^2dx - x^2u\,dx - x^3du = 0$

$x^2dx(u^2 + u^2 - u) - x^3du = 0$

$u^2dx - x\,du = 0 \rightarrow \frac{dx}{x} - \frac{du}{u^2} = 0 \rightarrow \int\frac{dx}{x} = \int\frac{du}{u^2}$

$\ln x + \frac{1}{u} = C \rightarrow \ln x + \frac{x}{y} = C$

$y\ln x + x = C \rightarrow y = \frac{x}{-\ln x + C}$

9.- $-y\,dx + (x + \sqrt{xy})\,dy = 0$ $\quad x = uy \rightarrow dx = u\,dy + y\,du \rightarrow u = \frac{x}{y}$

$-y(u\,dy + y\,du) + (uy + \sqrt{uy^2})\,dy = 0$

$-uy\,dy - y^2du + uy\,dy + \sqrt{u}\,y\,dy = 0$

$y\,dy(\cancel{-u} + \cancel{u} + \sqrt{u}) - y^2du = 0 \rightarrow \sqrt{u}\,y\,dy - y^2du = 0$

$\sqrt{u}\,y\,dy = y^2du \rightarrow \int\frac{y\,dy}{y^2} = \int\frac{du}{\sqrt{u}} = \ln y = 2\sqrt{u} + C$

$\ln y = 2\sqrt{\frac{x}{y}} + C$

$y = Ce^{2\left(\frac{x}{y}\right)^{1/2}}$

$y = ux$ $\qquad u = \frac{y}{x}$

$dy = u\,dx + x\,du$

13.- $(x + ye^{y/x})dx - xe^{y/x}dy = 0 \qquad y(1) = 0$

$x + uxe^{u}dx - xe^{u}(u\,dx + x\,du) = 0$

$x\,dx + uxe^{u}dx - uxe^{u}dx - x^2e^{u}du = 0$

$x\,dx(1 + \cancel{ue^{u}} - \cancel{ue^{u}}) - x^2e^{u}du = 0 \rightarrow x\,dx - x^2e^{u}du = 0$

$x\,dx = x^2e^{u}du \Rightarrow \int \frac{dx}{x} = \int e^{u}du$

$\rightarrow \ln x = e^{u} + C$

$\rightarrow \ln x - e^{y/x} = C \rightarrow \ln|x| = e^{y/x} - 1$

17.- $\frac{dy}{dx} = y(xy^3 - 1) \rightarrow y' - y = xy^4 \rightarrow u = y^{-3} \rightarrow \frac{du}{dx} + u = -e^{t}$

$e^{x}u = -\frac{1}{2}e^{2x} + C$

ó

$y^{-1} = -\frac{1}{2}e^{x} + Ce^{-x}$

21.- $x^2\frac{dy}{dx} - 2xy = 3y^4, \quad y(1) = 1/2$

$y' - \frac{2}{x}y = \frac{3}{x^2}y^4 \rightarrow u = y^{-3} \rightarrow \frac{du}{dx} + \frac{6}{x}u = -\frac{9}{x^2}$

$x^6u = -\frac{9}{5}x^5 + C \quad$ ó $\quad y^{-3} = -\frac{9}{5}x^{-1} + Cx^{-6}$

$y(1) = 1/2 \rightarrow C = \frac{49}{5} \rightarrow y^{-3} = -\frac{9}{5}x^{-1} + \frac{49}{5}x^{-6}$

25.- $\frac{dy}{dx} = \tan^2(x+y)$ $\quad u = x+y \to \frac{du}{dx} = 1 + \frac{dy}{dx}$

$\to \frac{du}{dx} - 1 = \tan^2 u$ ó $\cos^2 u \, du = dx \therefore \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u = x + c$ ó $2u + \operatorname{sen} 2u = 4x + C$ $\quad 2(x+y) + \operatorname{sen} 2(x+y) = 4x + C$ ó $2y + \operatorname{sen} 2(x+y) = 2x + c$

29.- $\frac{dy}{dx} = \cos(x+y)$, $y(0) = \pi/4$

$u = x+y \quad \frac{du}{dx} = 1 + \frac{dy}{dx} \to \frac{du}{dx} - 1 = \cos u$

$\frac{1}{1+\cos u} du = dx$

$\frac{1}{1+\cos u} = \frac{1-\cos u}{1-\cos^2 u} = \frac{1-\cos u}{\operatorname{sen}^2 u} = \csc^2 u - \csc u \cot u$

$\therefore \csc(x+y) - \cot(x+y) = x + \sqrt{2} - 1$

38.- a) $y = x \to y = -x$ son soluciones y no miembros de la familia $y = x \operatorname{sen} \ln x + c$

b) $x = 5 \quad y = 0 \quad \operatorname{sen}^{-1}\frac{y}{x} = \ln x + c$

$\operatorname{sen}^{-1} 0 = \ln 5 + c$ ó $c = -\ln 5 \therefore \operatorname{sen}^{-1}\frac{y}{x} = \ln x - \ln 5 = \ln \frac{x}{5}$

se debe tener $-\frac{\pi}{2} \le \ln\frac{x}{5} < \frac{\pi}{2}$

$e^{-\frac{\pi}{2}} \le \frac{x}{5} \le e^{\pi/2}$

$5e^{-\pi/2} \le x \le 5e^{\pi/2}$

Ejercicios 2.6

1.- $y' = 2x - 3y + 1, \quad y(1) = 5 \quad ; \quad y(1.2)$

$f(x,y) = 2x - 3y + 1 \rightarrow h = 0.1$

$y_{n+1} = y_n + 0.1(2x_n - 3y_n + 1) = 0.2x_n + 0.7y_n + 0.1$

$y(1.1) = y_1 = 0.2(1) + 0.7(5) + 0.1 = 3.8$

$y(1.2) = y_2 = 0.2(1.1) + 0.7(3.8) + 0.1 = 2.98$

Para $h = 0.05$

$y_{n+1} = y_n + 0.05(2x_n - 3y_n + 1) = 0.1x_n + 0.85y_n + 0.05$

$y(1.05) = y_1 = 0.1(1) + 0.85(5) + 0.05 = 4.4$

$y(1.2 = y_4 = 0.1(1.15) + 0.85(3.4775) + 0.05 = 3.115$

5.- $y' = e^{-y} \quad , y(0) = 0 \quad ; \quad y(0.5)$

$h = 0.1$

X_n	Y_n
0.0	0.50
0.1	0.52
0.2	0.54
0.3	0.55
0.4	0.56
0.5	0.56

$h = 0.05$

X_n	Y_n
0.0	0.0
0.1	0.09
0.2	0.18
0.3	0.26
0.4	0.34
0.5	0.41

9.- $y' = xy^2 - \frac{y}{x} \quad , y(1) = 1 \quad ; \quad y(1.5)$

$h = 0.1$

X_n	Y_n
1.0	1.0
1.1	1.0
1.2	1.01
1.3	1.05
1.4	1.12
1.5	1.21

$h = 0.05$

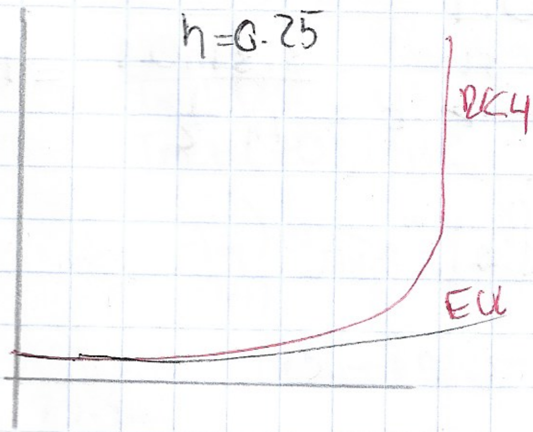
X_n	Y_n
1.0	1.00
1.05	1.00
1.1	1.00
1.15	1.01
1.2	1.02
1.25	1.05
1.3	1.07
1.35	1.11
1.45	1.20
1.5	1.26

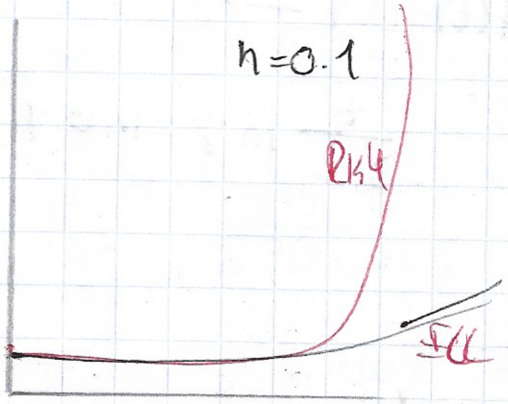
h = 0.1 h = 0.05

13 -

X_n	Y_n
0.0	1.0
0.1	1.0
0.2	1.02
0.3	1.06
0.4	1.12
0.5	1.23
0.6	1.38
0.7	1.61
0.8	1.97
0.9	2.6
1.0	3.8

X_n	Y_n
0.0	1.0
0.05	1.0
0.1	1.0
0.15	1.01
0.2	1.03
0.25	1.05
0.3	1.07
0.35	1.11
0.4	1.15
0.45	1.21
0.5	1.27
0.6	1.35
0.65	1.46
0.7	1.58
0.75	1.75
0.8	1.96
0.85	2.25
0.95	2.66
0.1	5.93





Ejercicios 3.1

1.-

$P = P(t)$ es la población en el tiempo t

P_0 = población inicial

$$\frac{dP}{dt} = kP \longrightarrow P = P_0 e^{kt}$$

$$P(5) = 2P_0 \longrightarrow k = \frac{1}{5}\ln 2 \rightarrow P = P_0 e^{\frac{\ln 2 t}{5}} \rightarrow P(t) = 3 = e^{\frac{\ln 2 t}{5}}$$

$$\ln 3 = \frac{\ln 2 t}{5} \qquad t = \frac{5 \ln 3}{\ln 2} = 7.92 \text{ años}$$

5.-

$A = A(t)$ = es la cantidad de plomo en el tiempo t

$$\frac{dA}{dt} = kA \quad y \quad A(0) = 1 \Longrightarrow A = e^{kt}$$

$$A(3.3) = \frac{1}{2} \Longrightarrow k = \frac{1}{3.3}\ln\frac{1}{2} \rightarrow$$ Cuando el 90 % de plomo decae, solo 0.1 grs permanece

$$A(t) = 0.1 \longrightarrow e^{t \frac{1}{3.33}\ln\frac{1}{2}} = 0.1$$

$$\frac{t}{3.3}\ln\frac{1}{2} = \ln 0.1 \longrightarrow t = \frac{3.3 \ln 0.1}{\ln\frac{1}{2}} = 13.6.5 \text{ hrs}$$

9.- $I = I(t)$ es la intensidad
t es el grosor

$$I(0) = I_0 \rightarrow \frac{dI}{dt} = kI \rightarrow I(3) = 0.25 I_0$$

$$I = I_0 e^{kt} \rightarrow k = \frac{1}{3} \ln 0.25 \rightarrow I(15) = 0.00098 I_0.$$

$$\frac{dS}{dt} = rS \rightarrow S = S_0 e^{rt} \rightarrow S(0) = S_0$$

a) $S_0 = 5000 \rightarrow r = 5.75\% \therefore S(5) = 6665.45$
b) $S(t) = 10,000 \rightarrow t = 12$ años
c) $S = 6651.82$

13.-

$$\frac{dT}{dt} = k(T-5) \rightarrow T = 5 + ce^{kt} \rightarrow T(1) = 55^{\circ} \rightarrow T(5) = 30^{\circ}$$

$$k = -\frac{1}{4} \ln 2 \rightarrow = 59.4611 \rightarrow T(0) = 64.4611^{\circ}$$

17.-

$$\frac{dT}{dt} = k(T - T_m) \rightarrow T(t) = T_m + ce^{kt} \rightarrow T(0) = c = 70 - T_m$$

$$T(t) = T_m + (70 - T_m) e^{kt}$$

$$T\left(\tfrac{1}{2}\right) = T_m + (70 - T_m) e^{k/2} = 110$$
$$T(1) = T_m + (70 - T_m) e^{k} = 145$$

$$e^{k/2} = \frac{(110 - T_m)}{(70 - T_m)} \rightarrow e^{k} = \left(e^{\frac{k}{2}}\right)^2 = \left(\frac{110 - T_m}{70 - T_m}\right)^2 = \frac{145 - T_m}{70 - T_m}$$

$$\left(\frac{110 - T_m}{70 - T_m}\right)^2 = 145 - T_m \rightarrow 12100 - 220 T_m + T_m^2 = 10150 - 215 T_m + T_m^2$$

$$T_m = 390$$

21.- $\frac{dA}{dt} = 4 - \frac{A}{50} \longrightarrow A = 200 + ce^{-\frac{t}{50}}$

$A(0) = 30 \longrightarrow c = -170\, e^{-\frac{t}{50}}$

$A = 200 - 170 e^{-\frac{t}{50}}$

25.- $\frac{dA}{dt} = 10 - \frac{10A}{500-(10-5)t} = 10 - \frac{2A}{100-t}$

$A = 1000 - 10t + c(100-t)^2 \longrightarrow A(0) = 0 \longrightarrow c = -\frac{1}{10}$

29.- $L \frac{di}{dt} + Ri = E(t) \rightarrow L = 0.1 \quad R = 50$

$E(t) = 50 \rightarrow i = \frac{3}{5} + ce^{-500t} \rightarrow i(0) = c$

$c = -\frac{3}{5}$

33.-

$0 \leq t \leq 20 \rightarrow 20 \frac{di}{dt} + 2i = 120$

$\frac{d}{dt}(e^{\frac{t}{10}} i) = 6e^{\frac{t}{10}} \rightarrow i = 60 + ce^{-\frac{t}{10}}$

$i(0) = 0 \rightarrow c = -60 \rightarrow i = 60 - 60e^{-\frac{t}{10}}$

$t > 20 \longrightarrow 20 \frac{di}{dt} + 2i = 0 \rightarrow i = c_2 e^{-\frac{t}{10}}$

$$i(t) = \begin{cases} 60 - 60 e^{-\frac{t}{10}} & ; \ 0 \leq t \leq 20 \\ 60(e^2 - 1) e^{-\frac{t}{10}} & t > 20 \end{cases}$$

37.- $m\frac{dv}{dt} = -mg - kv$

$\downarrow$

$v(t) = -\frac{mg}{k} + \left(300 + \frac{mg}{k}\right) e^{-\frac{kt}{m}}$

$s(0) = 0$

$\hookrightarrow s(t) = -\frac{mg}{k} t + \frac{m}{k}\left(300 + \frac{mg}{k}\right)\left(1 - e^{-\frac{kt}{m}}\right)$

$k = 0.0025, \quad m = 0.5 \rightarrow g = 32$

$s(t) = 1,340,000 - 6,400t - 1,340,000\, e^{-0.005t}$

$v(t) = -6,400 + 6,700\, e^{-0.005t}$

$v = 0, \quad t_a = 9.162$

$s(9.162) = 1363.79 \text{ ft}$

41.- a) $\frac{dP}{dt} = (k_1 - k_2)P \rightarrow P = P_0 e^{(k_1 - k_2)t}$ cuando $P_0 = P(0)$

b) $k_1 > k_2 \therefore P = P_0 \rightarrow k_1 < k_2 \rightarrow P \rightarrow 0$

45.-

$0 \leq t < 4, \; 6 \leq t < 10 \;$ y $\; 12 \leq t < 16$

$E(t) = 0$

$\hookrightarrow \frac{dE}{dt} = -\frac{E}{RC} \longrightarrow E = ke^{-\frac{t}{RC}}$

$E(4) = E(10) = E(16) = 12 \longrightarrow k = 12e^{\frac{4}{RC}}, \; k = 12e^{\frac{10}{RC}}, \; k\, 12\, e^{\frac{16}{RC}}$

$k = 12e^{\frac{22}{RC}}$

$E(t) = E(0) = 0 \longrightarrow 0 \leq t < 4, \; 6 \leq t < 10, \; 12 \leq t < 16$

$E(t) = E(4) = 12e^{\frac{4-t}{RC}} \rightarrow 4 \leq t < 6$

$E(t) = (10) = 12e^{\frac{10-t}{RC}} \longrightarrow 10 \leq t < 12$

$E(t) = (16) = 12e^{\frac{16-t}{RC}} \longrightarrow 16 \leq t < 18$