

Accident Analysis and Prevention 35 (2003) 253-259



www.elsevier.com/locate/aap

Applying the random effect negative binomial model to examine traffic accident occurrence at signalized intersections

Hoong Chor Chin*, Mohammed Abdul Quddus

Department of Civil Engineering, The National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore Received 15 December 2000; received in revised form 20 November 2001; accepted 26 November 2001

Abstract

Poisson and negative binomial (NB) models have been used to analyze traffic accident occurrence at intersections for several years. There are however, limitations in the use of such models. The Poisson model requires the variance-to-mean ratio of the accident data to be about 1. Both the Poisson and the NB models require the accident data to be uncorrelated in time. Due to unobserved heterogeneity and serial correlation in the accident data, both models seem to be inappropriate. A more suitable alternative is the random effect negative binomial (RENB) model, which by treating the data in a time-series cross-section panel, will be able to deal with the spatial and temporal effects in the data.

This paper describes the use of RENB model to identify the elements that affect intersection safety. To establish the suitability of the model, several goodness-of-fit statistics are used. The model is then applied to investigate the relationship between accident occurrence and the geometric, traffic and control characteristics of signalized intersections in Singapore. The results showed that 11 variables significantly affected the safety at the intersections. The total approach volumes, the numbers of phases per cycle, the uncontrolled left-turn lane and the presence of a surveillance camera are among the variables that are the highly significant.

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Keywords: Accident occurrence; Signalized intersections; Intersection characteristics; Random effect negative binomial; Goodness-of-fit

1. Introduction

Traffic accidents at urban intersections place a huge burden on society. Sabey and Taylor (1980) have suggested that roadway characteristics, such as geometric design elements, traffic control measures, and traffic demand patterns contribute to about 30% of all traffic accidents, either alone or in combination with human, vehicular or environmental factors. Rumar (1985), citing previous studies, has concluded that roadway characteristics alone contribute to only a small proportion of traffic accidents and that accident occurrence was mainly through the errors in road user interaction with other factors, especially those of the road environment. The usual practice to understand the interaction between geometric and traffic factors with accident causation is to establish a relationship between accidents occurrence and intersection characteristics. Several approaches have been used to establish the relationship and these include the multiple linear regression models, Poisson regression models and negative binomial (NB) regression models.

Multiple linear regression (MLR) models have been used widely in traffic accident studies (Jovanis and Chang, 1986;

Joshua and Garber, 1990; Miaou and Lum, 1993a). However, a number of researchers (Zegeer et al., 1990; Miaou and Lum, 1993a; Jovanis and Chang, 1986) have highlighted several limitations in the MLR models to describe adequately the random, discrete, nonnegative and sporadic accident data. These include the presence of undesirable statistical properties, such as the possibility of negative accident counts, and the lack of distributional properties, such as the condition of normally distributed accident occurrence. Besides being unable to give appropriate statistical inferences about accident occurrence, MLR models may also give test statistics that can be questionable.

Since accident occurrences are necessarily discrete, often sporadic and more likely random events, the Poisson regression models appear to be more suitable than the MLR models. The Poisson process as binomial limit seems to fit exactly the sense of the word "accident" as a completely fortuitous event (Haight, 1967). In a number of studies in the recent years (Joshua and Garber, 1990; Jones et al., 1991; Miaou et al., 1992; Miaou and Lum, 1993a,b; Kulmala, 1994; Maycock and Hall, 1984), Poisson regression models have been used to establish statistical relationships between traffic accidents and contributing factors of the road.

The Poisson regression model has some potential problems. One important constraint is that the mean must be

^{*} Corresponding author. Tel.: +65-874-2550; fax: +65-779-1635. *E-mail address:* cvechc@leonis.nus.edu.sg (H.C. Chin).

equal to the variance. If this assumption is not valid, the standard errors, usually estimated by the maximum likelihood (ML) method, will be biased and the test statistics derived from the model will be incorrect. Many researchers have modified the simple Poisson assumption by assuming that the parameter is distributed, usually in a Pearson type III distribution. A historical and bibliographical account of the problem associated with the use of the Poisson model has been well documented (Haight, 1964). In a number of recent studies (Miaou, 1994; Shankar et al., 1995; Vogt and Bared, 1998), the accident data were found to be significantly overdispersed, i.e. the variance is much greater than the mean. This will result in incorrect estimation of the likelihood of accident occurrence.

In overcoming the problem of overdispersion, several researchers, like Miaou (1994), Kulmala (1995), Shankar et al. (1995), Poch and Mannering (1996), and Abdel-Aty and Radwan (2000) have employed the NB distribution instead of the Poisson. By relaxing the condition of mean equals to variance, NB regression models are more suitable in describing discrete and nonnegative events. To establish the NB regression models, a stochastic component is introduced into the relationship between traffic accident and covariates.

The NB regression models have more desirable properties than the Poisson to describe the relationship between accident occurrence and the site geometric characteristics. Furthermore as a mix of the Poisson and gamma distributions, NB models can also deal with between-sites variations (Kulmala, 1995). However, they suffer from the limitation that time variations are not well considered. Consequently, the estimates of the standard error in the regression coefficients may be underestimated and the *t*-ratios may be inflated. Shankar et al. (1998) have attempted to overcome this problem by introducing in the NB models, a trend variable to take into account temporal variability in the accident data. However, if heterogeneity in the accident counts exists in the different observations, it is unlikely that variables introduced will capture all the unobserved heterogeneity effects.

One way to overcome this limitation is to analyze the accident data in panels and to consider separate persistent individual effects in the NB models, as suggested by Hausman et al. (1984) in their study of patent applications. Hausman et al. considered both the fixed and random definitions of the individual effects; the former does not allow group-specific variations. In employing the model in what may be its first application in traffic accident studies, Shankar et al. (1998) have indicated that the random effect negative binomial (RENB) model may be more appropriate, because geometric and traffic variables are likely to have location-specific effects. With his study, it appears that the RENB models can significantly improve the explanatory power of accident models.

This paper describes the use of RENB models to examine the relationship between accident occurrence and the characteristics of signalized intersections in Singapore. Using accident data at signalized intersections in the southwestern part of Singapore, geometric, traffic and control factors affecting safety are identified. Several goodness-of-fit statistics are also used to evaluate the suitability of the model.

2. Methodology

2.1. Model description

In dealing with accident data that are discrete and nonnegative, as well as possibly random and sporadic, it may be natural to model the accident occurrence using a Poisson or NB regression model. Regardless of whether the basic model adopted is a Poisson model or a NB model, it presupposes that the accident counts in an intersection *i*, for any year *t* are independent. However, as the data contain location-specific effects and are likely to be serially correlated, it is best to treat them in a time-series cross-section panel with *M* location groups and *T* periods. Without treating the data as such, the estimated standard errors of the regression coefficients will be underestimated since each observation actually contributes less information than is assumed. Consequently, the *t*-ratios will be inflated and incorrect statistical inferences will be made.

If spatial effects exist in the data, the RENB model can be adopted by introducing a random location-specific effects term into the relationship between the expected numbers of accidents ($\tilde{\mu}_{it}$) and the covariates, X_{it} , of an observation unit i in a given time period t, i.e.

$$\tilde{\mu}_{it} = \mu_{it}\delta_i \tag{1}$$

where δ_i is a random location-specific effects. To ensure a positive value, the term $\tilde{\mu}_{it}$ can be rewritten as

$$\tilde{\mu}_{it} = \mu_{it}\delta_i = \exp(X_{it}\boldsymbol{\beta} + \mu_i) \tag{2}$$

where β is the coefficient vector to be estimated, μ_i the random effects across location and $\exp(\mu_i)$ is gamma distributed with mean 1 and variance k, where k is also the overdispersion parameter in the NB model. The number of accidents at an intersection i for a given year t, i.e. n_{it} is independently and identically NB distributed with parameters $\delta_i \mu_{it}$ and ϕ_i , where $\mu_{it} = \exp(X_{it}\beta)$. Hence n_{it} has mean $\delta_i \mu_{it}/\phi_i$ and the variance $(\delta_i \mu_{it}/\phi_i)/z$, where $z = 1/(1 + \delta_i/\phi_i)$. Additionally, in order to account for the variation of location over time, z is assumed to be a beta-distributed random variable with distributional parameters (a, b). Using the results from the derivation of Hausman et al. (1984), the probability density function of the RENB model for the ith intersection will be

$$P(n_{i1}, ..., n_{iT} | X_{i1}, ..., X_{iT}) = \frac{\sqrt{a+b}\sqrt{a+\sum_{T}\mu_{it}}\sqrt{b+\sum_{T}n_{it}}}{\sqrt{a}\sqrt{b}\sqrt{a+b+\sum_{T}\mu_{it}} + \sum_{T}n_{it}} \prod_{T} \frac{\sqrt{\mu_{it}+n_{it}}}{\sqrt{\mu_{it}}\sqrt{n_{it}+1}}$$
(3)

The parameters a, b and the coefficient vector β can be estimated using any standard ML algorithms (Cameron and Trivedi, 1998). In this RENB model, the random effect is added to the NB model by assuming that the overdispersion parameter is randomly distributed across groups. This formulation is better able to account for the unobserved heterogeneity across locations and time.

2.2. Model evaluation

The first step of model evaluation is to examine the significance of the variables included in the model. The estimated regression coefficients (β) for each explanatory variable should be statistically significant. Typically, the *t*-test is used to examine the significance of the coefficients. Besides this, engineering and intuitive judgment should be able to confirm the validity and practicality of the sign of each covariate and the rough magnitude of each estimated coefficient.

Furthermore, to evaluate if a model has sufficient explanatory and predictive power, some goodness-of-fit measures are used. The measures adopted are the log-likelihood ratio index and log-likelihood R^2 ratio, which will be described in the following.

2.2.1. Ratio of log-likelihood index (ρ^2)

As suggested by Ben-Akiva and Lerman (1985), a suitable goodness-of-fit statistic for random, discrete and sporadic count data model that is similar to the coefficient of multiple determinations in MLR models is the log-likelihood ratio index, ρ^2 , given by

$$\rho^2 = 1 - \frac{l(\boldsymbol{\beta})}{l(0)} \tag{4}$$

where $l(\beta)$ is the log-likelihood value of the fitted model, and l(0) is log-likelihood value of the zero model. Like the R^2 statistic, it has the undesirable characteristic that for the same data set, it will increase whenever new variables are added to the model. This disadvantage can be overcome by incorporating a correction for the number of covariates, m, to give the adjusted log-likelihood ratio index (Ben-Akiva and Lerman, 1985)

$$\bar{\rho}^2 = 1 - \frac{l(\beta) - m}{l(0)} \tag{5}$$

2.2.2. Ratio of log-likelihood R^2 (R_F^2)

In order to measure how much the model can explain the systematic (non-random) variation in the response variable, Kulmala (1995) has suggested the use of the log-likelihood R^2 ratio as a goodness-of-fit statistic to test the fitted model against both the zero model and full or saturated model. The log-likelihood ratio, denoted as $R_{\rm F}^2$, is given by

$$R_{\rm F}^2 = \frac{\frac{{\rm SD}^0}{n-2} - \frac{{\rm SD}^{\rm F}}{n-m-1}}{\frac{{\rm SD}^0}{n-2} - \frac{{\rm SD}^{\rm F}_{\rm E}}{n-m}}$$
(6)

where SD^0 and SD^F are the scale deviance value for the zero model and fitted model, respectively, SD_E^F the expected value of scale deviance and m is the number of parameters in the fitted model (Fridstrøm et al., 1995). In cases where the data sets have high expected values in the response variables, it may be to determine R_F^2 , since SD_E^F is asymptotically distributed following a χ^2 distribution with (n-m-1) degrees of freedom and is approximately equal to the sample size n (MT). Maycock and Hall (1984) have suggested that this is valid if $\mu_{it} \geq 0.5$. When $\mu_{it} < 0.5$, SD_E^F will fall rapidly to zero and it must be computed from the model predicted values (for a thorough discussion on the expected value of scaled deviance, refer to Kulmala (1995) and Maycock and Hall (1984)).

3. Model development

3.1. Data

To establish a suitable statistical model that examines the relationship between accident frequencies and the geometric, traffic and regulatory control characteristics of signalized intersections, a total of 52 four-legged intersections in the Southwestern part of Singapore were used. The number of intersections may appear small but it covers quite a large area of city-state Singapore accounting for more than 15% of such intersections. Accident data from year 1992 to 1999, making a total of 832 observations were used in the analysis. They accounted for about 3000 accidents in which 3% were fatal, 5% resulted in serious injuries and the rest in slight injuries.

For each observation, a total of 32 possible explanatory variables were considered. Traffic volumes, including total approach volume and right-turn volume, were obtained from loop detectors at the sites maintained by the Land Transport Authority (LTA). Geometric elements were either obtained from LTA records or site measurements. These include approach curvature, sight distance to intersection, road width, median width, left-turn length on slip roads, distances of up stream and down stream bus stops from intersection, uncontrolled left-turn lane, exclusive right-turn lane, acceleration section and the presence of overhead bridge near intersection. Regulatory control measures such as existence of surveillance camera, signal control types and signal timing plan were obtained from the site visit and file information of the intersection. A sample summary statistics of variables is presented on Table 1.

3.2. Model estimation

To avoid the problem of multicollinearity between variables that may bias the standard error of the coefficients and hence result in wrong signs or implausible magnitudes in the coefficients, only the most significant variable was taken from among the highly correlated variables in the

Table 1
A sample summary statistics of variables included in the model

Variable	Minimum	Maximum	Mean	S.D.
Total annual accidents in each road (dependent variable)	0	38	3.46	4.729
Total approach volume in thousand (ADT)	1.416	53.84	21.211	12.663
Right-turn volume in thousand (ADT)	0.5	28.78	8.23	5.30
Intersection sight distance (m)	65	400	304	127
Diagonal distance on left-turn slip road (m)	0	52.5	13.47	7.86
Distance of upstream bus stop from intersection (m)	5.36	400	214.65	89.66
Distance of downstream bus stop from intersection (m)	45.73	495.5	208.48	102.15
Distance of pedestrian crossing from the intersection (m)	64.75	500	291.35	161.14
Approach road width (m)	7.2	36	20.83	6.84
Number of lanes	2	10	5.79	1.9
Cycle time (s)	100	140	117.69	12.81
Red-time in pedestrian crossing (s)	40	118.5	75.49	15.38
Approach speed limit (km/h)	<40	>80	52.00	6.56
Number of phases per cycle	2	5	3.5	0.67
Bus stops at approach road	0	4	2	1
Number of bus bays	0	4	3	1
Uncontrolled left-turn lane	0	1	0.779	0.415
Exclusive right-turn lane	0	1	0.596	0.49
Surveillance camera	0	1	0.22	0.415
Pedestrian refuges	0	1	0.26	0.44
Acceleration section	0	1	0.28	0.43
Existence of median	0	1	0.79	0.41
Median width greater than 2 m	0	1	0.24	0.43
Curvature on approach road	0	1	0.39	0.49

model. For example, because the presence of curvature and the intersection sight distance were found to be correlated, the former was dropped, as it was less significant than the latter.

The RENB regression of the total annual accident frequency on the intersection geometric, traffic and regulatory control characteristics was estimated by ML algorithm. Beginning with the 32 explanatory variables in the data set, each variable was tested for statistical significance and the

insignificant ones were eliminated one by one. Incidence rate ratios (IRR), i.e. $\exp(\beta)$ were calculated to facilitate interpretation of the variables included in the model. The final set of variables, their coefficients, corresponding IRR and t-statistics together with the P-values for the hypotheses that $\beta = 0$ are shown in Table 2.

For the final model, the log-likelihood ratio index (ρ^2) and the adjusted log-likelihood ratio index ($\bar{\rho}^2$) were found to be 0.318 and 0.314, respectively. This indicates that the

Table 2
RENB model for total annual accident frequencies

Explanatory variable	Estimated coefficient (IRR)	t-Statistic (P-value)
Total approach volume in thousand (ADT)	0.0071 (1.01)	2.712 (0.0067)
Right-turn volume in thousand (ADT)	0.0101 (1.01)	1.516 (0.1296)
Uncontrolled left-turn lane (yes 1, otherwise 0)	0.3052 (1.36)	3.520 (0.0004)
Acceleration section on left-turn lane (yes 1, otherwise 0)	-0.2783 (0.76)	-2.113(0.0346)
Intersection sight distance (m)	0.0006 (1.00)	3.141 (0.0017)
Median width greater than 2 m (yes 1, otherwise 0)	0.1947 (1.21)	2.462 (0.0138)
Number of bus stops	0.0556 (1.06)	1.592 (0.1114)
Number of bus bays	-0.0492 (0.95)	-1.738 (0.082)
Number of phases per cycle	0.1108 (1.12)	3.073 (0.0021)
Existence of surveillance camera (yes 1, otherwise 0)	0.2438 (1.28)	3.858 (0.0001)
Signal control type (adaptive 1, pre-timed 0)	-0.0522 (0.95)	-0.767 (0.4428)
Parameter, a	159.82	
Parameter, b	204.89	
Total number of observations	832	
Function of log-likelihood at convergence, $l(\beta)$	-2001.792	
Function of log-likelihood at zero, $l(0)$	-2935.17	
Ratio of log-likelihood index (ρ^2) and adjusted log-likelihood index ($\bar{\rho}^2$)	0.318, 0.314	
Ratio of log-likelihood R^2 , R_F^2	0.65	

estimated model has sufficient explanatory and predictive power.

As $\mu_{it} > 0.5$, SD_E is approximately equal to sample size n as discussed earlier. Judging from the R_F^2 value, the RENB model explains about 65% of the systematic variations in the number of accidents at the intersections and this may be considered to be quite high.

3.3. Model interpretation

An examination of Table 2 shows that eight variables have positive impact whereas three variables have negative impact on total accident frequency at intersection. All the selected variables, except one, are highly significant (highest P=0.1296) in the RENB model. Engineering and intuitive judgments also confirmed the validity and practicality of signs. Moreover, IRR can be used to interpret the variables. If the IRR of a given variable is much less than 1.0, then an increase in value of the variable is associated with a significant improvement in safety. Conversely, if the IRR is much greater than 1.0, an increase in the value of the variable is associated with a significant decline in safety. Otherwise, the variable has no effect on safety (Olmstead, 2001). The significance and interpretation of the variables are described in the following.

3.3.1. Significance of variable in the model

Among the traffic variables, the most significant one affecting intersection safety is the total approach volume (t = 2.712, P = 0.0067). Exposure to accidents is likely to depend on traffic volume. As volume increases, there are also fewer available gaps for the right-turn opposing traffic¹ as well as left-turn merging traffic. As a result of fewer turning opportunities, drivers may be more willing to take risks when making the turn. Similarly, higher right-turn volumes will result in more conflicts between right-turn and straight-through vehicles (t = 1.516, P = 0.1296). Without taking into account the temporal effects, several studies, e.g. Kulmala (1995), Abdel-Aty and Radwan (2000), Poch and Mannering (1996), have also indicated that traffic volume increases accident occurrence.

Among the geometric design elements, several variables are found to contribute to accident occurrence at intersections significantly. The coefficient estimates show that having an uncontrolled left-turn lane increases accident occurrence (t=3.52, P=0.0004). The uncontrolled lane, which allows left-turn vehicles to merge into the cross traffic stream, increases the likelihood of accidents, perhaps of the sideswipe and head-to-side types. One reason is that the uncontrolled lane gives greater opportunities for drivers to merge into the cross-stream traffic when the latter has right-of-way. However, by providing a longer section on the left-turn lane for acceleration prior to the merge, there appears to be some improvements in safety (t=-2.113,

P=0.0346). The IRR for the acceleration section dummy is 0.76, indicating that all other things being equal, an intersection approach with acceleration section is associated with a 24% reduction in total annual accidents. This is highly significant. This may be because the acceleration section reduces the speed differential between the left-turn merging and cross traffic as well as allows vehicles to take up better positions prior to the merge.

Intersection sight distance, which ranges from 65 to $400 \,\mathrm{m}$ in the sample, is associated with higher accidents (t = 3.141, P = 0.0017). This may seem surprising at first since higher risks are expected with restrictive sight distances, as suggested in studies, such as Poch and Mannering (1996). However, such cases may be confined to very short sight distances, possibly below the observed range in the sample. For the range covered, increases in the sight distances may allow drivers to have greater freedom of maneuver and may increase their vehicle speeds thus resulting in possibly greater accident frequencies and severity risks. Kulmala (1995) also found similar results in his study on four-legged intersections.

Approach median widths greater than 2 m are associated with high accidents (t = 2.462, P = 0.0138). Wider median widths usually come from larger intersections and they allow greater degrees of spatial freedom for right-turning vehicles. Near the stop line, wider median widths may also create more conflicts as the number of conflict points is higher and movements of through vehicles are less channelized.

The presence of bus stops near the intersection is found to increase the total accident frequency at the intersection (t = 1.592, P = 0.1114). The presence of bus stops effectively decreases the width of the approach road, resulting in higher conflicts because standing buses may become obstacles to the moving traffic. However, there is a slight decrease in accident likelihood when bays are provided at the bus stops (t = -1.738, P = 0.082). Since bus bays separate the stationary buses from the other moving vehicles, exposure to accidents may be reduced or eliminated. From the computed IRR value, an intersection with a higher number of bays is linked with 5% reduction in total accidents per year.

The number of phases per cycle is the most significant variable among the traffic control factors that affect the safety of intersections. Having a higher number of phases per cycle may increase the number of accidents (t=3.073, P=0.0021). This is not surprising since most accidents occur during the phase change periods. In a similar study, Poch and Mannering (1996) have found that eight-phase signal controls increase rear-end and approach turn accidents.

The presence of a surveillance camera along the approach is found to associate with an increase in the total accident frequency on the approach (t = 3.858, P = 0.0001). At first glance, this may seem contradictory to findings elsewhere (e.g. Elvik et al., 1989), where the impact of cameras has resulted in reduction in injury accidents. It may be reasoned that the positive correlation may be because these intersections already have high accident rates even before the

¹ In Singapore, driving is on the left side of the road.

cameras were installed. However, unlike in many countries where surveillance cameras are installed at high-accident locations, in Singapore these cameras, introduced some 15 years ago, have been deployed primarily to discourage red running rather than to reduce accidents (Chin, 1989). Although an effective deterrent for red-runners, the surveillance camera may increase head-to-rear accidents on the approach due to greater traffic turbulence. Thus, an overall higher accident rate may have resulted from the presence of the camera.

The signalized intersections may be under adaptive or pre-timed signal control. From the model results, adaptive signal control appears to have some effect on reducing accident occurrence (t = -0.767, P = 0.4428). The IRR value for this dummy variable is 0.95 indicating that an intersection with adaptive signal control is associated with a 5% reduction in total accidents per year, all other things being equal. When an intersection is under adaptive control, the number of long gaps in the opposing traffic may be reduced and in turn this forces the traffic to follow a more regular discharge pattern with well-defined right-of-way. Consequently, conflicts between traffic streams may be reduced and these are generally limited to periods during phase changes. This may contribute to reducing cross-traffic accidents and appears to reinforce the findings of Poch and Mannering (1996), who found that intersections with signal control are safer than those without signal control.

4. Conclusion

In any study to establish accident occurrence models, accident data need to be analyzed in a systematic way. If both the Poisson regression and NB models fail to take into account location-specific effects and/or serial correlation in time of the accident counts, the RENB model may be used to capture all unobserved heterogeneity. By treating the data in time-series cross-section panels, the RENB model can explicitly account for the unobserved time and space effects. Judging from the values of various goodness-of-fit statistics, the appropriateness of the RENB model can be confirmed.

The RENB model is applied to identify the geometric elements, traffic factors and traffic control measures that may influence total accident frequency at signalized intersections in Singapore. It was found that total approach volumes, right-turn volumes, the presence of uncontrolled left-turn lane, median widths above 2 m, the presence of bus stops, intersection sight distance together with the presence of a surveillance camera and the number of phases per cycle are associated with higher total accident occurrence. On the other hand, the presence of an acceleration section and the provision of bus bays as well as the use of adaptive signal control tend to point to lower total accident occurrence. These findings may, however, be limited by the relatively small sample size used.

Although the RENB model can be elegantly developed and perhaps statistically satisfied, there remains the unresolved question of using it as a means of predicting accidents or assessing safety impact. Identification of significant explanatory variables explains only some association with accident occurrence and may not imply accident causation.

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