# Mathematical Model to Measure Honey Production Variation in a Bee Colony Infected by *Varroa Destructor*

Joel Frayle Moreno*, Juan Pablo García Villa†, Sara García López‡, Jacobo Valderrama Rovira§
Department of Biomedical Engineering, University of Los Andes, Bogotá, Colombia
Emails: *j.frayle@uniandes.edu.co, †jp.garciav@uniandes.edu.co, ‡s.garcial@uniandes.edu.co, §j.valderramar@uniandes.edu.co

**Abstract**

The *Apis Mellifera* is the most widely distributed bee species in the world. Its task is pollination, a duty on which the survival of human beings depends. Pollination results in honey production, and it has been evidenced that infection by *Varroa* leads to a significant decrease in this production. This raises the question: What is the percentage reduction in honey production between a *Varroa*-infected hive and a healthy one over 90 days in Medellín? The result shows that at the end of the 90 days, a *Varroa*-infected hive experiences a 78.959% decrease in honey production. This response is consistent with the obtained results, as the number of healthy bees decreases over time while infected ones increase. Understanding that an infected bee does not produce the same amount of honey as a healthy one, the overall hive production declines, just as the parasite population first increases and then decreases as the number of bees available for infection drops. Keywords:
***Honey, Bees, Varroa destructor, Mite, Model***.

## I. Introduction

Bees such as Apis Mellifera, the most widely distributed species in the world, feed on the nectar and pollen of flowers, simultaneously performing pollination [1]. It has been evidenced that one of the main causes of bee mortality is infections. In the case of infection by Varroa destructor, this parasite results in the death of 85% of the bee colonies it colonizes [4]. Considering that 75% of the world's food crops depend on pollination to thrive [7] and that bees are expected to become extinct by 2032 [6], it is crucial to analyze how Varroa affects honey production, using this quantity as a factor to measure pollination and understand how bees are impacted by this parasite.

This is why studies such as that of Torres build models that show the relationship between the number of colonies and the survival rate of colonies infected with Varroa destructor, emphasizing the rate of impact depending on the life stage of the bees [9]. Similarly, the modeling by Romero-Leiton et al. highlights the relationship between honey production and stress conditions in both mature and immature bees [15]. Lastly, the model by Ricoy et al. not only considers Varroa infection and differentiation by stage but also distinguishes between male and female Varroa births.

Taking this into account, developing a mathematical model of a continuous system is a solid alternative, as it allows the correlation of key components—bees, honey, larvae, cells, and parasites—with factors such as mortality, growth, and infection. The objective of this model is to understand how the ectoparasite primarily affects the larval stage (which it reproduces in) and the adult stage (which feeds and transports it as an adult) through honey production. Honey production, in turn, reflects the bees' pollination capacity, recognizing them as the main agents in this process, which directly impacts humanity.

Therefore, the following question is posed: What is the percentage reduction in honey production between a Varroa-infected hive and a healthy one over 90 days in Medellín?

## II. Box Diagram

In Figure 1, the block diagram of the developed model is shown, along with its respective relationships.

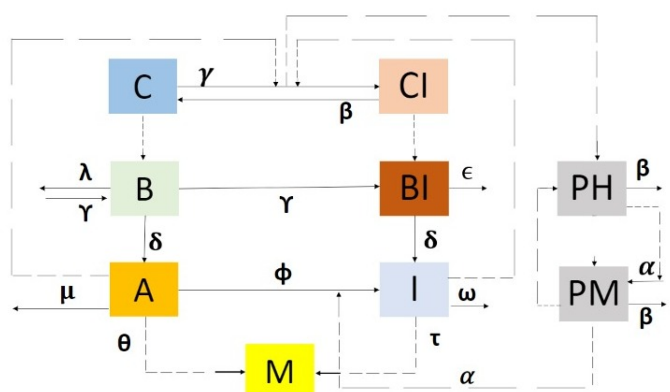


Figure 1: Block Diagram of the Model.

As seen in the block diagram, there are a total of 9 actors. (C) refers to the cells in the hive where the bees live, (CI) refers to the cells infected by parasites, (B) represents the larvae, (BI) the infected larvae, (A) the adult bees, (I) the infected adult bees, (M) the honey produced in the hive by all bees, (PM) the female parasites, and (PH) the male parasites.

Regarding the model parameters, these are divided into 13. ($\gamma$) represents the birth rate of bees, ($\Phi$) the infection rate of bees, ($\mu$) the mortality rate of healthy bees, ($\tau$) the honey production rate of infected bees, and ($\beta$) denotes two events: both the mortality rate of mites and the recovery rate of cells, as a mite's death means that the cell it inhabited becomes clean. ($\omega$) is the mortality rate of infected bees, ($\lambda$) the mortality rate of healthy larvae, ($\epsilon$) the mortality rate of infected larvae, ($\theta$) the honey production rate of healthy bees, ($\alpha$) the number of female parasites per male, ($\delta$) the maturation rate of larvae, and ($\Omega$) the conversion factor between larvae and bees.

In the analysis of the block diagram, actors and parameters are written in parentheses for differentiation. The following behaviors are observed for each block (actor): (C) decreases based on the parameter ($\gamma$) due to the conversion from (C) to (CI) influenced by (A); it also increases based on ($\beta$) when (CI) returns to (C). On the other hand, (CI) increases based on ($\gamma$) due to the conversion from (C) to (CI) influenced by (I) and decreases based on ($\beta$) when a larva is no longer present in the cell. (B) decreases based on ($\lambda$) due to mortality and increases based on ($\gamma$) due to birth; it also decreases based on ($\gamma$) when infected, converting from (B) to (BI), and decreases based on ($epsilon$) when maturing from (B) to (A).

On the other hand, (BI) increases based on ($\gamma$) due to the infection of (B) and also indirectly due to (CI). Similarly, they die based on ($\epsilon$) and decrease based on ( $\delta$) when maturing from (BI) to (I). (A) increases at a rate of ($\delta$) due to the maturation of (B), decreases due to infection at a rate of ($\Phi$), which is influenced by (PM), and dies at a rate of ($\mu$). Meanwhile, (I) increases at a rate of ($\Phi$) due to infection from (A), mediated by (PM), and also grows at a rate of ($\delta$) due to the maturation of (BI), ultimately dying at a rate of ($\omega$).

(M) is determined by the honey production of healthy bees (A) and infected bees (I) through the parameters ($\Theta$) and ($\tau$), respectively. Finally, (PH) is related to (PM) through the parameter ($\alpha$) and decreases based on ($\beta$). (PM) follows a similar pattern, also decreasing based on ($\beta$) and being related to (PH) through ($\alpha$). The equations and parameters used are specified in the Annex section.

For this model to be consistent, some assumptions must be made. The first is that the larva transitions directly into an adult bee, which is biologically incorrect since there are other intermediate stages. A second assumption is that once a bee is infected, it dies. While this is true in most cases, *Varroa* does not always survive the disinfection processes, with some being more successful than others.

Additionally, another assumption is that infected bees have a constant production rate throughout the disinfection process. This is not entirely accurate, as honey production decreases as the bee loses more hemolymph. Furthermore, it is assumed that the entire bee population in the hive consists of worker bees and that the hive is treated as a closed system, considering only the bees within it and not those from other colonies. It is important to clarify that, in this model, the bee population will never reach zero, as 2500 bees are integrated into the system daily.

## III. Model Justification

In order to adequately answer the initial question, the proposed model includes a state variable that quantifies the amount of honey produced by healthy bees and the honey production by infected bees, all measured in milliliters. Additionally, the model reports the number of individuals in the different modeled populations. Since the amount of honey also depends on the number of bees, the model provides the necessary elements to measure the honey quantity over a given period.

By calculating the honey produced over the ninety days following the initial infection in both the healthy and infected populations, the total variation in honey production can be determined. In this sense, the model is useful for comparing different scenarios, with or without infection, and thus addressing the research question.

Regarding the chosen time frame, worker bees have a lifespan of less than six weeks, meaning that within a 0–90 day interval, this process can occur, allowing for vital dynamics that also influence honey production. Finally, the study was set in Medellín, as seasonal changes significantly impact honey production. Since Medellín does not experience drastic climate variations, it serves as an ideal location for this study.

## IV. Results and Analysis

In Figure 2, the results of the computational modeling of the behavior of the larvae and parasite population are presented. It is important to remember at this point that the parasites are linked to the larvae since it is through the cells where the larvae grow that the reproduction of the parasites occurs. The female parasite enters one of the larvae growth cells and lays 7 eggs, the first always being a male parasite egg, and the other 6 always correspond to female parasites, which will later attach to the larva and emerge from the cell attached to it. Meanwhile, the male parasite remains in the cell.

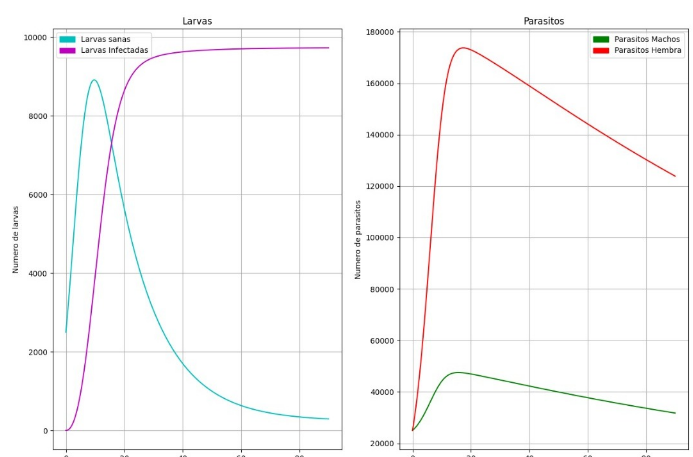


Figure 2: Simulated populations of larvae and parasites. The y-axis shows the number of individuals, and the x-axis shows days.

At this point, it can be observed that both healthy and infected larvae initially grow rapidly. It is important to note that the first parasites take as long to leave the cells as it takes for the larva to become an adult bee. This is evident in the upward trend of the healthy larvae and the delay in the rise of the infected bees. This downward trend continues, as there comes a point when it is more likely to leave a larva in an infected cell than in a healthy one, causing the growth of healthy larvae to decrease towards zero. On the other hand, the growth of the parasites begins exponentially. The growth of the females is much greater than that of the males due to their reproductive characteristics. As will be explained later, after 10 days, a point is reached where adult bee infection occurs, which causes the population to decrease, and as a result, the parasites also begin to decrease at this point. Now, in the results of Figure 3, another comparison is presented, where the population of adult bees is considered in a healthy population, meaning a colony without the presence of *Varroa destructor*, the healthy bees present within an infected colony with the presence of *Varroa*, and the infected bees, which are contaminated with the parasite.

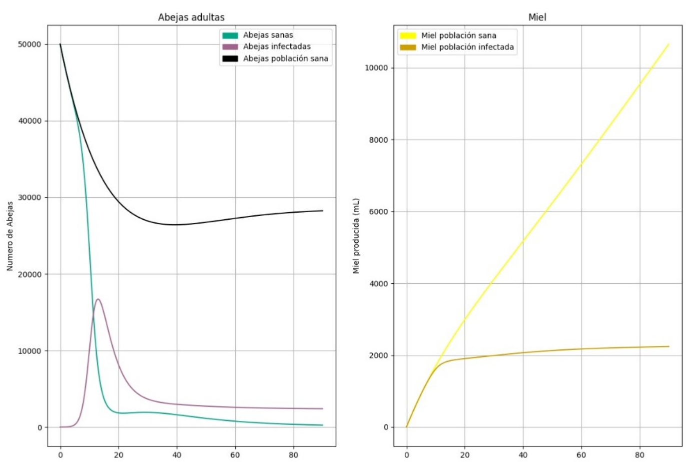


Figure 3: Simulated populations of adult bees and resulting honey production. The y-axis shows individuals or milliliters of honey, and the x-axis shows days.

In the graph presenting the adult bees, it can be observed that all populations, even the bees in the healthy population, show a decrease in population. This is due to the fact that the model had an initial parameter for the bees that was quite close to the population saturation point; however, after one generation of bees, their population begins to grow again. In comparison, we have the population of healthy bees within the infected population, which from approximately day 5, shows a more rapid and sustained decline until around day 15. At this point, its behavior changes and shows a slight increase, followed by a constant decrease towards 0. On the other hand, the bees infected with *Varroa* initially show an exponential growth until around day 10, after which they rapidly decline, stabilizing at 2500.

Let us focus on the fact that several of the populations mentioned earlier experience a significant decrease around day 10. This is due to something known as population collapse. In this scenario, the bees cannot survive the parasites, and due to their death, the mites also begin to die. This explains the previously observed sustained decrease in the mites and the changes in trend in the bees as well. When performing several models, the trend varies in the time it takes for the colony to collapse, but the collapse itself remains consistent. In this case, the healthy bees slowly decrease from day 10 until reaching 0, while the infected bees remain at the daily birth rate of bees.

In the appendices of this document, you will find the figure by V. Ratti, P. G. Kevan, and H. J. Eberl [11]. Their models are similar to ours, although this model includes a form of mite population control through the seasons, where mites tend to die at a higher rate than bees. However, this model reaches the same conclusion: the colony collapses after a certain period, with waves similar to those shown by the infected bees in figure 3, but repeated across the seasons with lesser magnitude.

Finally, we focus on answering our research question, for which we rely on the honey model from figure 3. It can be observed that the healthy population produces honey in a roughly linear fashion, meaning the daily honey production remains relatively constant over time. The mathematical model results show that after 90 days, the total honey produced by the colony of bees is 10647.1795 mL, while the infected population produces a total of 2240.2006 mL of honey.

We calculate the variation as follows:

$$\frac{\text{Healthy population honey} - \text{Infected population honey}}{\text{Healthy population honey}} * 100$$

That is:

$$\frac{10647.1795 - 2240.2006}{10647.1795} * 100 = 78.959\%$$

Therefore, according to the computational model, a colony of infected bees will produce $78.959\%$ less honey than the healthy population after 90 days from the first *Varroa* infection. Moreover, it can be said that an infection by *Varroa* can be diagnosed approximately 15 days after the first infection, as this is the point where the honey production of the infected population changes its slope.

It can be asserted that the above model is correct in a biological sense, correlates with similar works, and provides a meaningful value regarding the decrease in honey production.

## V. Future Work

For a future iteration, the model could consider the four stages of a bee's life cycle, where the factors found in the previously mentioned articles could be used to assess the impact of *Varroa* in each of these stages and consider the time (in days) that passes during each stage. Additionally, the disinfection rate of some of the existing treatments for the elimination of *Varroa* could be included, which would create a bidirectional flow between the infected and healthy blocks at any growth stage. Furthermore, it should be considered that in a hive, there are different types of bees, not just workers as assumed in this model. Finally, to make it more realistic, it should be taken into account that a bee can visit multiple hives, not just one as considered in the model, and the model should be applied within an apiary.

This model can be used by honey production companies to create rapid detection models for *Varroa*-infected colonies, improve the model to test the best methods for controlling the infection within the hive, and even develop control methods to prevent the spread of *Varroa* between hives in an apiary. Furthermore, although Medellín has a climate without distinct seasons, it does have winter and summer periods, which can influence honey production and the transmission of the infection. This is a limitation that can be improved.

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