Package **Pyliferisk**

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Abstract: A python library for life actuarial calculations. Simple, powerful and easy-to-use.

This document aims to be the only necessary and authoritative source of information about pyliferisk, usable as a comprehensive reference, an user guide, and tutorial all-in-one. It also includes a sample of illustrative and common examples of actuarial calculations.

Pyliferisk is compatible with both version Python 3.x and 2.7 and has no dependencies other than the Python Standard Library, making it amazingly fast.

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1 Introduction

Pyliferisk is an open library written in Python for life and actuarial calculation contracts, based on commonly used methodologies among actuaries (International Actuarial Notation).

This library is able to cover all life contingencies risks (since the actuarial formulas follow the International Actuarial Notation), as well as to support the main insurance products: Term Life, Whole Life, Annuities and Universal Life.
Additionally, the library can be easily tailored to any particular or local specifications, since Python is a very intuitive language.

It is ideal not only for academic purposes but also for professional use by actuaries (implementation of premiums and reserves modules) or by auditors (validation of reserves or capital risk models such as parallel runs).

This library is distributed as a single file module and has no dependencies other than the Python Standard Library, making it amazingly fast. Additionally, the package includes several life mortality tables (`pyliferisk.mortalitytables`), mainly extracted from academic textbooks. Nevertheless, additional libraries as Numpy or Pandas may be required for increasing functionality, such as cash flow operations, random number generation, interpolation, etc.

You can find also examples for different contracts in the examples section.

Why Python?

Because computing plays an important role in the actuarial profession, but actuaries are not programmers. Python is friendly and easy to learn.

Nowadays, programming is becoming an indispensable skill for actuaries. Python is a clear, readable syntax, powerful and fast language. Easy to learn, especially when you are not used to coding. This language lets you write quickly the code you need, without cumbersome rules or variable predefined tasks. It is clear, forget ending with commas and using curly brackets in functions.

For European actuaries, Solvency II opens a big opportunity. The new requirements transform into agility, transversality, and auditability. The internal model is not only software, but it should also be an internal process used extensively where all parts must walk hand in hand.

Python 3 and 2.7

Pyliferisk is compatible with both version: 3.x and 2.7

Potential uses

This library may be used in tariff processes, in the design phase of new products such as profit testing or estimation of future benefits. Other uses include:

- Auditing purposes tool
- Assumption calibrations, back-testing, etc..
- Replicate the main calculations of the internal model for implementation in pricing, product approval, reserving, etc..
- Perform small reports (output format may be xml, xls, etc...)

If you find something that Python cannot do, or if you need the performance advantage of low-level code, you can write or reuse extension modules in C or C++. For reusing R implementations, with the library `rpy2` is possible run applications built in R.

Installation

Once Python is running, just install this library with `pip install`

```
> pip install pyliferisk
```

Then, to import this library in projects is automatic as usually:

Option 1:

```python
from pyliferisk import *
from pyliferisk.mortalitytables import *

tariff = MortalityTable(nt=GKM95)
```

Option 2:

```python
import pyliferisk as life
import pyliferisk.mortalitytables as mort

tariff = life.MortalityTable(nt=mort.GKM95)
```

Option 3, if only like to use specific functions:

```python
from pyliferisk import MortalityTable
from pyliferisk.mortalitytables import GKM95

tariff = MortalityTable(nt=GKM95)
```

Update library

Run this command from terminal:

```
> pip install pyliferisk --upgrade
```

License

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2 Pyliferisk: List of formulas

The names of the formulas follow the International Actuarial Notation and are easily guessable (qx, lx...), with a few exceptions regarding special characters.

2.1 Biometric functions: *Class* MortalityTable

The Instance Variables for the `MortalityTable()` class are:

- `nt` = The actuarial table used to perform life contingencies calculations. Syntax: nt = GKM95 (Note: GKM95 must be created previously or be included in mortalitytables.py)
  Example:

```
tariff = MortalityTable(nt=GKM95)
```

- `perc` = Optional variable to indicate the percentage of mortality to be applied. Syntax: perc=85 . Variable `perc` can be omitted, in this case it will be 100 by default.
  Example:

```
experience = MortalityTable(nt=GKM95, perc=85)
```

Once define the variables for your mortality table, all available biometric functions are the following:

2.1.1 *method* `.qx[x]`

Description	Returns the probability that a life aged x dies before 1 year With the convention: the true probability is qx/1000
Actuarial notation	q_x
Usage	`mt.qx[x]`
Args	x: the age as integer number.
Example	`tariff = MortalityTable(nt=SPAININE2004)` `tariff.qx[50]`

2.1.2 *method* `.lx[x]`

Description	Returns the number of survivors at beginning of age x
Actuarial notation	l_x
Usage	`mt.lx[x]`
Args	x: the age as integer number.
Example	`tariff.lx[50]`

2.1.3 *method* `.w`

Description	ultimate age (lw = 0)
Actuarial notation	w
Usage	`mt.w`
Example	`tariff.w`

2.1.4 *method* `mt.dx[x]`

Description	Returns the number of dying at beginning of age x
Actuarial notation	d_x
Usage	`mt.dx[x]`
Args	x: the age as integer number.
Example	`tariff.dx[50]`

2.1.5 *method* `.ex[x]`

Description	Returns the curtate expectation of life. Life expectancy
Actuarial notation	e_x
Usage	`mt.ex[x]`
Args	self: the mortality table x: the age as integer number.
Example	`tariff.ex[50]`

Example:

Print the omega (limiting age) of the both mortality tables and the qx at 50 years-old:

```python
#!/usr/bin/python
from pyliferisk import MortalityTable
from pyliferisk.mortalitytables import SPAININE2004, GKM95

tariff = MortalityTable(nt=SPAININE2004)
experience = MortalityTable(nt=GKM95, perc=85)

# Print the omega (limiting age) of the both tables:
print(tariff.w)
print(experience.w)

# Print the qx at 50 years old:
print(tariff.qx[50] / 1000)
print(experience.qx[50] / 1000)
```

Return the following results:

```
101
121
0.003113
0.003662395
```

2.2 Actuarial Present Value: *Class* Actuarial

The Present Value of the benefit payment is a function of time of death given a survival model and an interest rate.

The Instance Variables for the `Actuarial()` class are:

- `nt` = The actuarial table used to perform life contingencies calculations.
  Syntax: nt=GKM95 (Note: GKM95 must be included in mortalitytables.py)
- `i` = interest rate. The effective rate of interest, namely, the total interest earned in a year.
  Syntax: i=0.02
- `perc` = Optional variable to indicate the percentage of mortality to be applied.
  Syntax: perc=85 . Variable `perc` can be omitted, in this case, it will be 100 by default.

```
tariff = Actuarial(nt=GKM95, i=0.05)
```

2.2.1 *formula* `Ax()`

Description	Returns the Expected Present Value (EPV) of a whole life insurance (i.e. net single premium). It is also commonly referred to as the Actuarial Value or Actuarial Present Value.
Actuarial notation	A_x
Usage	`Ax(mt, x)`
Args	mt: Mortality table. x: the age as integer number.
Example	`mt = Actuarial(nt=SPAININE2004, i=0.02)` `Ax(mt, 50)`

2.2.2 *formula* `Axn()`

Description	Returns the EPV (net single premium) of a term insurance.
Actuarial notation	$A^{1}_{x:\overline{n}\rceil}$
Usage	`Axn(mt, x, n)`
Args	mt: Mortality table. x: the age as integer number. n: period in years.
Example	`mt = Actuarial(nt=SPAININE2004, i=0.02)` `Axn(mt, 50, 10)`

2.2.3 *formula* `qAx()`

Description This function evaluates the APV of a geometrically increasing annual annuity-due.

Actuarial notation ${}_qA_x$

Usage `Axn(mt, x, q)`

Args
mt: Mortality table.
x: the age as integer number.
q: increase rate.

Example
```
mt = Actuarial(nt=SPAININE2004, i=0.02)
Axn(mt, 50, 0.03)
```

2.2.4 *formula* `qAxn()`

Description This function evaluates the APV of a geometrically increasing Term insurance.

Actuarial notation ${}_qA_{x:\overline{n}|}$

Usage `qAxn(nt, x, n, q)`

Args
mt: Mortality table.
x: the age as integer number.
n: period in years.
q: increase rate.

Example
```
mt = Actuarial(nt=SPAININE2004, i=0.02)
Axn(mt, 50, 10, 0.03)
```

2.2.5 *formula* `AExn()`

Description Returns the EPV of an endowment insurance.
An endowment insurance provides a combination of a term insurance and a pure endowment

Actuarial notation $A_{x:\overline{n}|}$

Usage `AExn(mt, x, n)`

Args
mt: Mortality table.
x: the age as integer number.
n: period in years.

Example
```
mt = Actuarial(nt=SPAININE2004, i=0.02)
AExn(mt, 50, 10)
```

Syntax:

Notation	Description	Syntax
A_x	whole-life death insurance	`Ax(nt, x)`
$A^1_{x:\overline{n}|}$	Term insurance	`Axn(nt, x, n)`
$A_{x:\overline{n}|}$	Endowment insurance	`AExn(nt, x, n)`
${}_qA_x$	Increasing whole-life	`qAx(nt, x, n)`
${}_qA_{x:\overline{n}|}$	Increasing Term insurance	`qAxn(nt, x, n, q)`

Examples:

1) A whole-life single premium:

```
#!/usr/bin/python
from pyliferisk import Actuarial, Ax
from pyliferisk.mortalitytables import GKM95

mt = Actuarial(nt=GKM95, i=0.02)
x = 50           #age
C = 1000         #capital

print(Ax(mt, x) * C)
```

```
589.0804423991423
```

2) A term insurance single premium:

```
#!/usr/bin/python
from pyliferisk import Actuarial, Axn
from pyliferisk.mortalitytables import GKM95

mt = Actuarial(nt=GKM95, i=0.03)
x = 40           #age
n = 20           #horizon
C = 10000        #capital

print(Axn(mt, x, n) * C)
```

```
646.1486398262324
```

3) Example 2 with cashflow approach:

```
#!/usr/bin/python
from pyliferisk import MortalityTable
from pyliferisk.mortalitytables import GKM95
import numpy as np

mt = MortalityTable(nt=GKM95)
x = 40          #age
n = 20          #horizon
C = 10000       #capital
i = 0.03        #interest rate

payments = []
for t in range(0, n):
    payments.append((mt.lx[x+t] - mt.lx[x+t+1]) / mt.lx[x] * C)

discount_factor = []
for y in range(0, n):
    discount_factor.append(1 / (1 + i) ** (y + 0.5))

print(np.dot(discount_factor, payments).round(2))
```

```
646.1486398262326
```

To print the results year by year:

```
print('{0:5} {1:10} {2:10}'.format(' t','factor','payment'))

for t in range(0,n):
        print('{0:2} {1:10} {2:10}'.format(t, np.around(discount_factor[t
            ],5), np.around(payments[t],4)))
```

```
 t     factor      payment
 0     0.98533      18.694
 1     0.95663     19.9456
 2     0.92877     21.3621
 3     0.90172     22.9574
 4     0.87545     24.7629
 5     0.84995      26.815
 6      0.8252     29.1475
 7     0.80116     31.7959
 8     0.77783     34.7884
 9     0.75517     38.1576
10     0.73318     41.9304
11     0.71182     46.1285
12     0.69109     50.7779
13     0.67096     55.8959
14     0.65142     61.4878
15     0.63245     67.5536
16     0.61402     74.0921
17     0.59614      81.095
18     0.57878     88.5512
19     0.56192     96.4435
```

2.3 Annuity formula

A life annuity refers to a series of payments to an individual as long as the individual is alive on the payment date. It may be temporary or payable for whole-life. The payment intervals may commence immediately or deferred. The payment may be due at the beginnings of the intervals (annuity due) or at the end (annuity immediate).

2.3.1 *function* `annuity()`

Description	Returns the actuarial present value of annuity payments	
Usage	`annuity(mt, x, n, 0/1, m=1,['a'/'g',q], -d)`	
Args		
1.	mt	the mortality
2.	x	The age of the insured
3.	n	The horizon (term of insurance) in years or payment duration or
	or `'w'`	w as whole-life. Also 99 is defined as whole-life.
4.	0	annuity-immediate
	or 1	annuity-due
optional args:		
–	m:	Number of fractional payments per period. If missing, m is set as 1
–	[`'a'`, q]	Arithmetically increasing
	or [`'g'`, q]	Geometrically increasing
–	-d	The deferring period in years (as negative integer)

Examples:

1) The present value of a 5-year (financial) annuity with nominal annual interest rate 12% and monthly payments of $100 is:

```python
#!/usr/bin/python
from pyliferisk import *
import numpy as np

mt = Actuarial(nt=FIN, i=0.12/12) #FIN = Financial table

n = 5 * 12
C = 100

print(annuity(mt, 0, n, 1) * C) #replace age 'x' by 0
```

Returns:

```
4495.503840622397
```

The equivalent formula in Excel is: `PV(12%/12,12*20,500,,0)`

2) Premium calculation: A Life Temporal insurance for a male, 30 years-old and a horizon of 10 years, fixed annual premium (GKM95, interest 6%):

Actuarial equivalence: $\pi^{\ 1}_{30:\overline{10}|} = 1000 \cdot \dfrac{A^{\ 1}_{30:\overline{10}|}}{\ddot{a}_{30:\overline{10}|}}$

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import GKM95

nt = Actuarial(nt=GKM95, i=0.06)
x = 30
n = 10
C = 1000

print(C * (Axn(nt, x, n) / annuity(nt, x, n, 0)))
```

```
1.398266715155939
```

3) Reserving a life risk insurance with regular premium. Applying the equivalence principle, where: ${}_0V_x = A_x - \pi \cdot \ddot{a}_x = 0$ and: $\pi = \dfrac{A^{\ 1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$. At time t: ${}_tV_x = A^{\ 1}_{40+t:\overline{10-t}|} - \pi \cdot \ddot{a}_{40+t:\overline{10-t}|}$

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import GKM95

nt = Actuarial(nt=GKM95, i=0.03)
x = 40
n = 20
Cm = 100000
Premium = Cm * Axn(nt, x, n) / annuity(nt, x, n, 0)  #fixed premium

def Reserve(t):
    return round(Cm * Axn(nt, x+t, n-t)
                           - Premium * annuity(nt, x+t, n-t, 0),2)

for t in range(0, n+1):
    print(t, Reserve(t))
```

```
0 0.0
1 257.11
2 509.4
3 755.01
4 991.91
5 1217.65
6 1429.31
7 1623.49
8 1796.3
9 1943.34
10 2059.65
11 2139.73
12 2177.5
13 2166.2
14 2098.4
15 1966.05
16 1760.38
17 1471.86
18 1090.07
19 603.61
20 0.0
```

Actuarial notation vs. Syntax formula

Notation	Description	Syntax
$\ddot{a}_{x:\overline{n}\rceil}$	n-year temporary life annuity-due	`annuity(nt,x,n,0)`
$a_{x:\overline{n}\rceil}$	n-year temporary life annuity	`annuity(nt,x,n,1)`
$\ddot{a}^{(m)}_{x:\overline{n}\rceil}$	n-year annuity-due m-monthly payments	`annuity(nt,x,n,0,m)`
$\ddot{a}_x$	whole life annuity-due	`annuity(nt,x,'w',0)`
a_x	whole life annuity	`annuity(nt,x,'w',1)`
$\ddot{a}^{(m)}_x$	whole life annuity-due m-monthly	`annuity(nt,x,'w',0,m)`
$a^{(m)}_x$	whole life annuity m-monthly	`annuity(nt,x,'w',1,m)`
${}_{n\|}\ddot{a}_x$	d-year deferred whole life annuity-due	`annuity(nt,x,n,0,-d)`
${}_{n\|}a_x$	d-year deferred whole life annuity	`annuity(nt,x,n,1,-d)`
${}_{n\|}\ddot{a}^{(m)}_{x:\overline{n}\rceil}$	d-year deferred n-year temporal annuity-due m-monthly payments	`annuity(nt,x,n,0,m,-d)`
${}_{n\|}\ddot{a}_x$	d-year deferred whole life annuity-due	`annuity(nt,x,'w',0,-d)`
${}_{n\|}a_x$	d-year deferred whole life annuity	`annuity(nt,x,'w',1,-d)`
	Increasing annuities (a sample of them)	
$(Ia)_x$	arithmetically increasing whole-life annuity	`annuity(nt,x,'w',1,['a',c])`
${}^{q}(I\ddot{a})_{x:\overline{n}\rceil}$	arithmetically increasing n-year temporal annuity-due	`annuity(nt,x,n,0,['a',c])`
${}^{q}\ddot{a}_x$	geometrically increasing whole-life annuity-due	`annuity(nt,x,'w',0,['g',c])`
${}_{n\|}a_x$	d-year deferred geometrically increasing whole-life annuity-due	`annuity(nt,x,'w',0,['g',c],-d)`
${}^{q}_{n\|}\ddot{a}^{(m)}_{x:\overline{n}\rceil}$	d-year deferred geometrically increasing n-year temporal annuity-due m-monthly payments	`annuity(nt,x,n,0,m,['g',c],-d)`

2.4 Pure endowment: Deferred capital

2.4.1 *formula* `nEx()`

Description	Returns the EPV of a pure endowment (deferred capital). Pure endowment benefits are conditional on the survival of the policyholder.
Actuarial notation	${}_nE_x$
Usage	`nEx(mt, x, n)`
Args	mt: Mortality table. x: the age as integer number. n: period in years.
Example	`mt = Actuarial(nt=SPAININE2004, i=0.02)` `nEx(mt, 50, 10)`

Syntax:

Notation	Description	Syntax
$A_{x:\overset{1}{\overline{n}\rceil}}$	Pure endowment (Deferred capital)	`nEx(nt, x, n)`
${}_nE_x$	EPV of a pure endowment (deferred capital)	`nEx(nt, x, n)`

Example:

A deferred capital premium calculation:

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import GKM80

C = 1000
x = 60
n = 25
exp = 0.2 / 100          # expenses over capital
com =  0.10              # commision over premium

mt = Actuarial(nt=GKM80, i=0.025)

def Premium(mt, x, n):
  return (nEx(mt, x, n) + Axn(mt, x, n)) / annuity(mt, x, n, 0) * C

print((Premium(mt, x, n) + C * exp) / (1 - com))
```

Returns:

```
58.8146911381352
```

2.5 Commutation factors

Nowadays, the standard techniques for actuarial calculations use cashflow projections. In fact, the use of interest rates curves is required for the calculation of the technical provisions for insurance obligations (such as risk-free interest rate term structures in Solvency II framework) where commutation factors are not adequate.

Despite commutation factors may seem quite prehistoric, it may be useful for academic purposes or replicating older products exactly "to the letter". For this reason, the pyliferisk library includes a list of commutations factors (`Dx, Nx, Cx, Mx`) in order to facilitate the migration from older actuarial software (such as Cobol or Cactus) to Python, or for simple calculations (where your model shouldn't be subject to future changes). If your goal is to reduce the timing, there are a lot of other aspects where you can save time.

```python
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import GKM95

tariff = Actuarial(nt=GKM95, i=0.02)

print(tariff.Dx[50])
print(tariff.Nx[50])
print(tariff.Cx[50])
print(tariff.Mx[50])
```

Returns:

```
35633.87396668312
844266.356048293
109.8353338458543
19269.483448767027
```

2.6 Other functions

Other functions can be derived from the lx figures. Anyway, the library include the following additional formulas:

- `px(mt, x)`: Returns the probability of surviving within 1 year (p_x).
- `tpx(mt, x, t)`: Returns the probability that x will survive within t years (${}_tp_x$).
- `tqx(mt, x, t)`: Returns the probability to die within n years at age x (${}_tq_x$).
- `tqxn(mt, x, n, t)`: Probability to die in n years being alive at age x (${}_{n|}q_x$).
- `mx(mt, x)`: Returns the central mortality rate (m_x).

2.7 Help (Documentation strings)

According Python PEP257 convention, formulas includes a documentation string (also known as docstrings):

```
#!/usr/bin/python
from pyliferisk import *

print(qx.__doc__)
```

Returns:

```
 qx: Returns the probability that a life aged x dies before 1 year
           With the convention: the true probability is qx/1000
    Args:
        mt: the mortality table
        x: the age as integer number.
```

3 Mortality tables

The package includes a sample of life mortality tables (`mortalitytables.py`), mainly extracted from academic textbooks or contributions. It's possible to import tables of an external source, i.e. txt or csv files.

Tables must be imported for use, either all (`import *`) or only which you will use, example:

```
from pyliferisk.mortalitytables import GKM95, UK43
```

Notes:

- The probability is qx * 1000.
- The first item indicates the age when the table starts. For example, UK43 table is 0 for the first 30 ages.

There is a financial table (called `FIN`) for financial annuities which doesn't include mortality.

3.1 Adding tables

Tables are added in list format where first item indicates the age when table starts. ie:

SCOT_DLT_00_02_M = [0, 0.006205, 0.000328, 0.00026 ....]

In the SOA repository (`http://mort.soa.org`) is available a variety (over 2,500) of rate tables of interest to actuaries: SOA experience mortality and lapse tables, regulatory valuation tables, population tables and various international tables).

3.2 *Class* .view()

Instruction `.view()` provides a view of the main variables (`qx, lx, dx, ex, Dx, Nx, Cx, Mx, nEx`) of any mortality table. The default view is lx for 10-years. Usage example:

```
mt = MortalityTable(nt=INM05)
mt.view()
```

```
[x=0]   lx=100000.0
[x=1]   lx=99564.1
[x=2]   lx=99530.7460265
[x=3]   lx=99506.659586
[x=4]   lx=99488.4498673
[x=5]   lx=99473.4271113
[x=6]   lx=99460.6945127
[x=7]   lx=99448.1624651
[x=8]   lx=99435.5325485
[x=9]   lx=99421.9098806
[x=10]   lx=99409.3827199
Total number of rows for lx = 107
```

Other view (qx from 60 to 65 years-old):

```
mt = MortalityTable(nt=INM05)
mt.view(60, 65, var = 'qx')
```

```
[x=60]   qx=9.958
[x=61]   qx=10.736
[x=62]   qx=11.199
[x=63]   qx=12.455
[x=64]   qx=13.861
[x=65]   qx=15.224
Total number of rows for qx = 106
```

4 Examples

This section includes a list of examples using `pyliferisk`.

4.1 Example 1. Cashflow calculation: Annuity-immediate Geometrically increasing

Prospective Reserves look forward, as the expected present value of future outgo less the expected present value of the future income. Using NumPy library for discount them.
Life annuity immediate geometrically increasing for a male 67 years-old, as single premium (PASEM2010, interest 5%):

```
1  #!/usr/bin/python
2  import pyliferisk as life
3  from pyliferisk.mortalitytables import SPAIN_PASEM2010M
4  import numpy as np
5  
6  mt = life.MortalityTable(nt=SPAIN_PASEM2010M)
7  
8  age = 67
9  initial_payment = 8000
10 incr = 0.03                            # increment
11 i = 0.05                               # interest rate
12 
13 discount_factor = []
14 for y in range(0, mt.w-age):
15         discount_factor.append(1 / (1 + i) ** (y + 1))
16 
17 payments = [initial_payment]
18 for x in range(0, mt.w-age-1):
19         payments.append(payments[x] * (1 + incr) * (1 - mt.qx[age+x] /
               1000))
20 
21 print('Premium:', np.dot(payments, discount_factor).round(2))
```

```
Premium: 967183.635
```

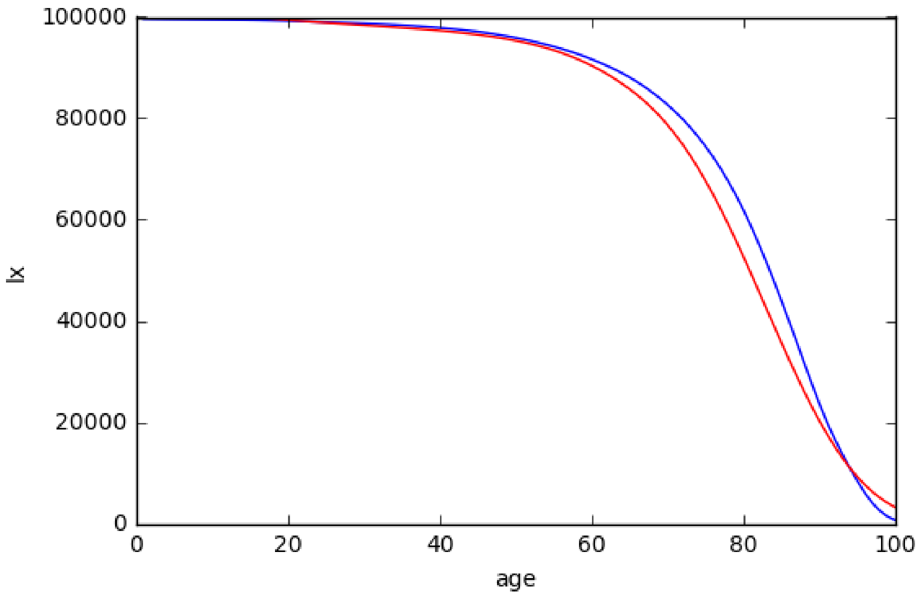
There are different ways of discounting (see previous line 15). In case of:

- not discount first year: the exponent should be only `y`.
- discount first year: the exponent should be `y+1`.
- discount half year: the exponent should be `y+0.5`.

4.2 Example 2. Drawing graph with matplotlib

Plotting a surviving graph with pyplot library:

```
#!/usr/bin/python
import matplotlib.pyplot as plt
import pyliferisk as life
from pyliferisk.mortalitytables import SPAININE2004, GKM95

tariff = life.MortalityTable(nt=SPAININE2004)
experience = life.MortalityTable(nt=GKM95, perc=75)
x = range(0, tariff.w)
y = tariff.lx[:tariff.w]
z = experience.lx[:tariff.w]
plt.plot(x,y, color = 'blue')
plt.plot(x,z, color = 'red')
plt.ylabel('lx')
plt.xlabel('age')
```



4.3 Example 3. Obtain data from plain text file

Term Life with maturity capital benefit and similar death capital.

```python
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import SPAININE2004, GKM95
import csv

mt = Actuarial(nt=GKM80, i=0.04)

def single_risk_premium(x, n):
   return nEx(mt, x, n) + Axn(mt, x, n)

def annual_risk_premium(x, n):
   return (single_risk_premium(x, n) / annuity(mt, x, n, 0))

SingleRiskPrem=[]
AnnualRiskPrem=[]

columns = '{0:8} {1:2} {2:9} {3:8} {4:10} {5:10}'
print(columns.format('Contract', 'Age', 'Duration', 'Capital', 'Single Pr', 'Annual Pr'))
print('--' * 26)

with open('colective.csv', 'r') as file:
    colective = csv.DictReader(file, delimiter=';')
    for row in colective:
        age = int(row['age'])
        dur = int(row['duration'])
        capital = int(row['capital'])
        single_premium = round(capital * single_risk_premium(age, dur), 2)
        annual_premium = round(capital * annual_risk_premium(age, dur), 2)
        AnnualRiskPrem.append(annual_premium)
        print(columns.format(row['N_pol'], age, dur, capital, single_premium, annual_premium))

print('--'*26)
print('Total Annual Premium:', sum(AnnualRiskPrem))
```

```
Contract Age Duration  Capital  Single Pr   Annual Pr
----------------------------------------------------
00001    42         10  2000000 1362199.46  163939.94
00002    35         15  1500000  842908.92    73904.7
00003    51         12  1000000  643890.86   69150.37
00004    31          5  3500000 2878725.12  623332.65
00005    37         20  2000000  943160.32   68500.76
----------------------------------------------------
Total Annual Premium: 998828.42
```

For example, this run spent 0m0.024s in a MacBook Pro 2,6 GHz Intel Core i5 8 GB 1600 MHz DDR3.

4.4 Example 4. Obtain the Risk-free rate from MS Excel

Obtain the risk-free rate from the EIOPA Excel file at 31-12-2016 with Pandas. `Pandas` (Python Data Analysis) is an open source library providing high-performance, easy-to-use data structures, and data analysis tools. `http://pandas.pydata.org`

```python
#!/usr/bin/python
import pandas as pd

df = pd.read_excel('EIOPA_RFR_20161231_Term_Structures.xlsx', sheet_name=
    'RFR_spot_no_VA', skiprows=9, usecols='C:C', names=['Euro'])

print(df.head(20))
```

```
       Euro
0  -0.00302
1  -0.00261
2  -0.00208
3  -0.00123
4  -0.00024
5   0.00092
6   0.00215
7   0.00341
8   0.00461
9   0.00571
10  0.00671
11  0.00760
12  0.00841
13  0.00908
14  0.00958
15  0.00993
16  0.01019
17  0.01046
18  0.01077
19  0.01117
```

Even is possible to read directly from any site with the urllib2, Scrapy or BeautifulSoup4 (a screen-scraping libraries for parsing HTML and XML).

4.5 Example 5. Cashflow Calculation Reserving

Reserving for a Whole Life contract using a lineal interest rate and the risk-free rate obtained in the previous example.

Once again, the following two examples should be enough clear:

Using lineal interest rate

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import INM05
import numpy as np

tariff  = Actuarial(nt=INM05, i=0.05)
reserve = MortalityTable(nt=INM05)
age = 32                            # age
Cd = 3000                           # capital death
Premium = Cd * Ax(tariff, 25) / annuity(tariff, 25, 'w', 0) #fixed at age 25

qx_vector = []
px_vector=[]
for i in range(age, reserve.w + 1):
        qx = ((reserve.lx[i] - reserve.lx[i+1]) / reserve.lx[age])
        qx_vector.append(qx)
        qx_sum = sum(qx_vector)
        px_vector.append(1 - qx_sum)

def Reserve(i):
        discount_factor = []
        for y in range(0, reserve.w-age + 1):
                discount_factor.append(1 / ( 1 + i) ** (y + 1))

        APV_Premium = np.dot(Premium, px_vector)
        APV_Claims = np.dot(Cd, qx_vector)
        # Reserve = APV(Premium) - APV(Claim)
        return np.dot(discount_factor, np.subtract(APV_Claims, APV_Premium)).round(2)

print(Reserve(0.0191))
print(Reserve(0.0139))
```

```
820.8
1088.37
```

Including risk free rate curve

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import INM05
import numpy as np
import pandas as pd

rfr = pd.read_excel('EIOPA_RFR_20161231_Term_Structures.xlsx', sheet_name='RFR_spot_no_VA', skiprows=9, usecols='C:C', names=['Euro'])

tariff  = Actuarial(nt=INM05, i=0.05)
reserve = MortalityTable(nt=INM05)
x = 32                  # age
Cd = 3000               # capital death
Premium = Cd * Ax(tariff, 25) / annuity(tariff, 25, 'w', 0)

qx_vector = []
px_vector = []
for i in range(x,reserve.w + 1):
        qx = ((reserve.lx[i]-reserve.lx[i+1]) / reserve.lx[x])
        qx_vector.append(qx)
        qx_sum = sum(qx_vector)
        px_vector.append(1 - qx_sum)

def Reserve(i):
        discount_factor = []
        for y in range(0, reserve.w - x + 1):
                if isinstance(i,float):
                        discount_factor.append(1 / (1 + i) ** (y + 1))
                elif i == 'rfr':
                        discount_factor.append(1 / (1 + rfr['Euro'][y]) ** (y + 1))

        APV_Premium = np.dot(Premium, px_vector)
        APV_Claims = np.dot(Cd, qx_vector)
        return np.dot(discount_factor, np.subtract(APV_Claims, APV_Premium)).round(2)

print(Reserve(0.0191))
print(Reserve(0.0139))
print(Reserve('rfr'))
```

```
820.8
1088.37
544.31
```

4.6 Example 6. Cashflows in pandas

Replicating Example 1 with Pandas.

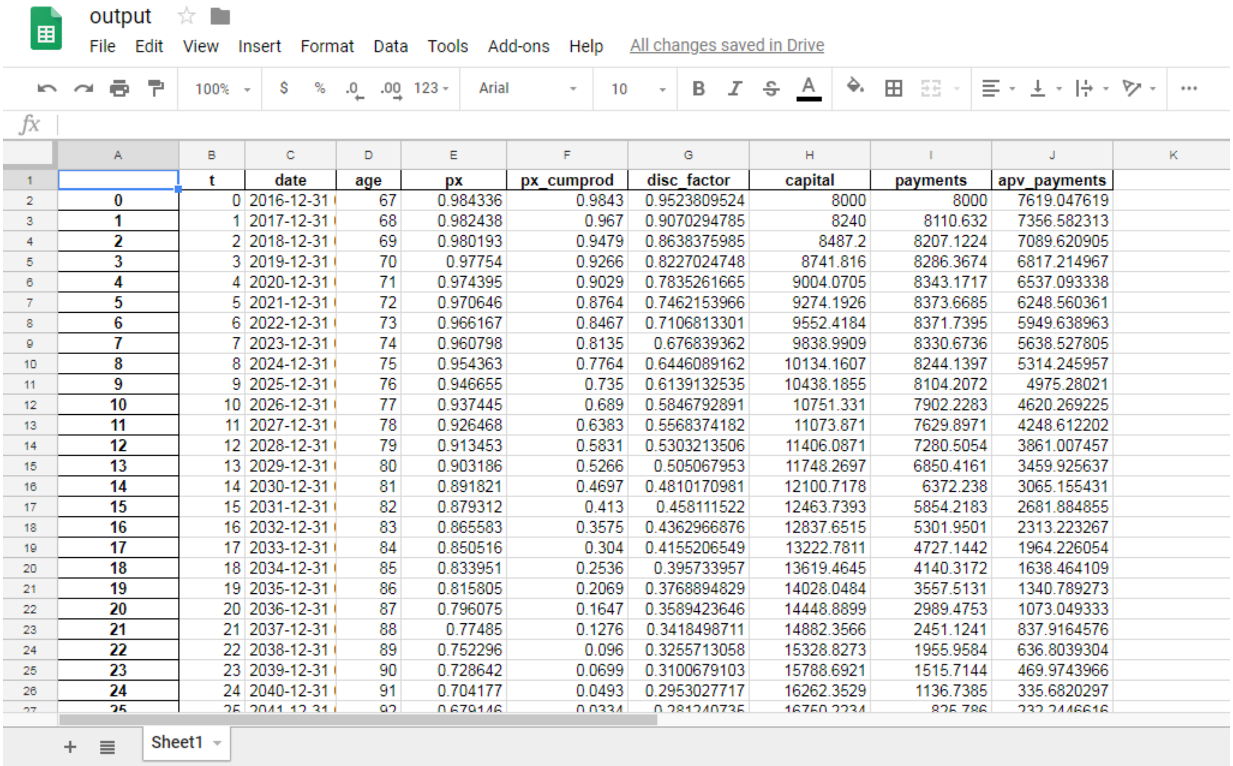
Life annuity immediate geometrically increasing for a male 67 years-old, as single premium (PASEM2010, interest 5%).

```
#!/usr/bin/python
from pyliferisk import *
from pyliferisk.mortalitytables import SPAIN_PASEM2010M
import pandas as pd

mt = MortalityTable(nt=SPAIN_PASEM2010M)

age = 67
initial_payment = 8000
incr = 0.03                              # increment annutie
i = 0.05                                 # interest rate

period = list(range(0, mt.w - age))

df = pd.DataFrame(data=period, columns=['t'])

df['date'] = pd.DataFrame(pd.date_range('1/1/2016', periods=len(period),
    freq='A'))
df['age'] = pd.Series(list(range(age, mt.w)))
df['px'] = df['t'].apply(lambda t: 1 - mt.qx[age+t] / 1000)
df['px_cumprod'] = df['px'].cumprod().round(4)
df['disc_factor'] = df['t'].apply(lambda t: 1 / (1 + i) ** (t + 1))
df['capital'] = df['t'].apply(lambda t: initial_payment * ((1 + incr) **
    t)).round(4)
df['payments'] = df['t'].apply(lambda t: initial_payment if t == 0 else
    df['capital'][t] * df['px_cumprod'][t-1]).round(4)
df['apv_payments'] = df['payments'] * df['disc_factor']
premium = df['apv_payments'].sum().round(2)
print(premium)
```

```
96717.79
```

Export the DataFrame to a MS Excel file:

```
writer = pd.ExcelWriter("output.xls")
df.to_excel(writer,'Sheet1')
writer.save()
```



	t	date	age	px	px_cumprod	disc_factor	capital	payments	apv_payments
0	0	2016-12-31	67	0.984336	0.9843	0.9523809524	8000	8000	7619.047619
1	1	2017-12-31	68	0.982438	0.967	0.9070294785	8240	8110.632	7356.582313
2	2	2018-12-31	69	0.980193	0.9479	0.8638375985	8487.2	8207.1224	7089.620905
3	3	2019-12-31	70	0.97754	0.9266	0.8227024748	8741.816	8286.3674	6817.214967
4	4	2020-12-31	71	0.974395	0.9029	0.7835261665	9004.0705	8343.1717	6537.093338
5	5	2021-12-31	72	0.970646	0.8764	0.7462153966	9274.1926	8373.6685	6248.560361
6	6	2022-12-31	73	0.966167	0.8467	0.7106813301	9552.4184	8371.7395	5949.638963
7	7	2023-12-31	74	0.960798	0.8135	0.676839362	9838.9909	8330.6736	5638.527805
8	8	2024-12-31	75	0.954363	0.7764	0.6446089162	10134.1607	8244.1397	5314.245957
9	9	2025-12-31	76	0.946655	0.735	0.6139132535	10438.1855	8104.2072	4975.28021
10	10	2026-12-31	77	0.937445	0.689	0.5846792891	10751.331	7902.2283	4620.269225
11	11	2027-12-31	78	0.926468	0.6383	0.5568374182	11073.871	7629.8971	4248.612202
12	12	2028-12-31	79	0.913453	0.5831	0.5303213506	11406.0871	7280.5054	3861.007457
13	13	2029-12-31	80	0.903186	0.5266	0.505067953	11748.2697	6850.4161	3459.925637
14	14	2030-12-31	81	0.891821	0.4697	0.4810170981	12100.7178	6372.238	3065.155431
15	15	2031-12-31	82	0.879312	0.413	0.458111522	12463.7393	5854.2183	2681.884855
16	16	2032-12-31	83	0.865583	0.3575	0.4362966876	12837.6515	5301.9501	2313.223267
17	17	2033-12-31	84	0.850516	0.304	0.4155206549	13222.7811	4727.1442	1964.226054
18	18	2034-12-31	85	0.833951	0.2536	0.395733957	13619.4645	4140.3172	1638.464109
19	19	2035-12-31	86	0.815805	0.2069	0.3768894829	14028.0484	3557.5131	1340.789273
20	20	2036-12-31	87	0.796075	0.1647	0.3589423646	14448.8899	2989.4753	1073.049333
21	21	2037-12-31	88	0.77485	0.1276	0.3418498711	14882.3566	2451.1241	837.9164576
22	22	2038-12-31	89	0.752296	0.096	0.3255713058	15328.8273	1955.9584	636.8039304
23	23	2039-12-31	90	0.728642	0.0699	0.3100679103	15788.6921	1515.7144	469.9743966
24	24	2040-12-31	91	0.704177	0.0493	0.2953027717	16262.3529	1136.7385	335.6820297

5 Books

The author has checked the library with examples from the following textbooks:

- Actuarial Mathematics for Life Contingent Risks (David C. M. Dickson, Mary R. Hardy and Howard R. Waters) Cambridge University Press, 2009.
- Actuarial Mathematics (2nd Edition), Bowers et al. Society of Actuaries, 1997.
- Matemática de los Seguros de Vida, (Gil Fana J.A., Heras Matínez A. and Vilar Zanón J.L.) Fundación Mapfre Estudios, 1999.

It will be documented in the examples folder. Contributions are greatly appreciated.

6 Acknowledgements

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7 To-Do's

- An example using a lx given
- An example with Generation Mortality Table