



**AJ INSTITUTE OF ENGINEERING & TECHNOLOGY**

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**DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING**

**LECTURE NOTES – MODULE 3**

**SUBJECT : SYSTEM SOFTWARE AND COMPILERS**

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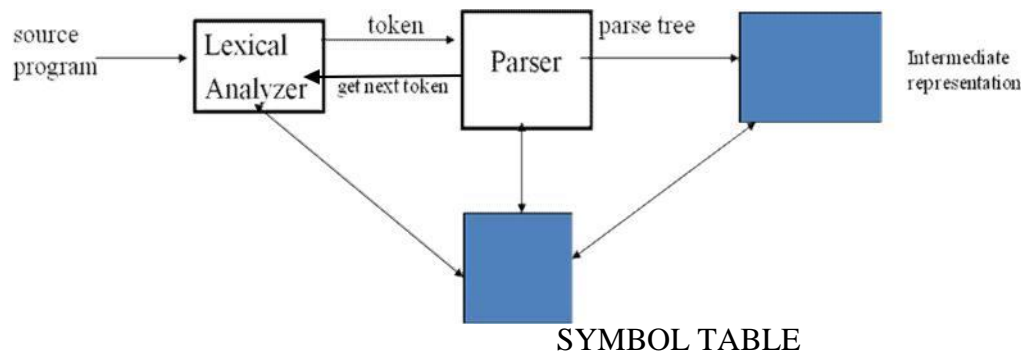
**Prepared By:-  
Mrs. Snitha Shetty,  
Asst. Professor, Dept. of CS&E**

## Module 4

**Syntax Analysis: Introduction, Role Of Parsers, Context Free Grammars, Writing a grammar, Top Down Parsers, Bottom-Up Parsers, Operator-Precedence Parsing Text book 2: Chapter 4 4.1 4.2 4.3 4.4 4.5 4.6 Text book 1 : 5.1.3 4.1 Introduction**

- *Syntax Analyzer* creates the syntactic structure of the given source program.
- This syntactic structure is mostly a *parse tree*.
- Syntax Analyzer is also known as *parser*.
- The syntax of a programming is described by a *context-free grammar (CFG)*. We will use BNF (Backus-Naur Form) notation in the description of CFGs.
- The syntax analyzer (parser) checks whether a given source program satisfies the rules implied by a context-free grammar or not.
  - If it satisfies, the parser creates the parse tree of that program.
  - Otherwise the parser gives the error messages.
- A context-free grammar
  - gives a precise syntactic specification of a programming language.
  - the design of the grammar is an initial phase of the design of a compiler.
  - a grammar can be directly converted into a parser by some tools.
- Parser works on a stream of tokens.
- The smallest item is a token.

Fig :Position Of Parser in Compiler model



- We categorize the parsers into two groups:
  1. **Top-Down Parser**
    2. the parse tree is created top to bottom, starting from the root.
  1. **Bottom-Up Parser**
    - the parse is created bottom to top; starting from the leaves
- Both top-down and bottom-up parsers scan the input from left to right (one symbol at a time).
- Efficient top-down and bottom-up parsers can be implemented only for sub-classes of context-free grammars.
  - LL for top-down parsing
  - LR for bottom-up parsing
- Common Programming errors can occur at many different levels.

1. Lexical errors: include misspelling of identifiers, keywords, or operators.
2. Syntactic errors : include misplaced semicolons or extra or missing braces.
3. Semantic errors: include type mismatches between operators and operands.
4. Logical errors: can be anything from incorrect reasoning on the part of the programmer.
5. Report the presence of errors clearly and accurately
5. Recover from each error quickly enough to detect subsequent errors.
6. Add minimal overhead to the processing of correct programs.
7. Panic-Mode Recovery
8. Phrase-Level Recovery
9. Error Productions
10. Global Correction

- On discovering an error, the parser discards input symbols one at a time until one of a designated set of Synchronizing tokens is found.
- Synchronizing tokens are usually delimiters.

Ex: semicolon or } whose role in the source program is clear and unambiguous.

- It often skips a considerable amount of input without checking it for additional errors.

Advantage:

Simplicity

Is guaranteed not to go into an infinite loop

### **Phrase-Level Recovery**

- A parser may perform local correction on the remaining input. i.e it may replace a prefix of the remaining input by some string that allows the parser to continue.

Ex: replace a comma by a semicolon, insert a missing semicolon

- Local correction is left to the compiler designer.
- It is used in several error-repairing compilers, as it can correct any input string.
- Difficulty in coping with the situations in which the actual error has occurred before the point of detection.
- We can augment the grammar for the language at hand with productions that generate the **erroneous constructs**.
- Then we can use the grammar augmented by these error productions to **Construct a parser**.
- If an error production is used by the parser, we can generate appropriate **error diagnostics** to indicate the erroneous construct that has been recognized in the input.

### **Global Correction**

- We use algorithms that perform minimal sequence of changes to obtain a globally least cost correction.

- Given an incorrect input string  $x$  and grammar  $G$ , these algorithms will find a parse tree for a related string  $y$ .
- Such that the number of insertions, deletions and changes of tokens required to transform  $x$  into  $y$  is as small as possible.
- It is too costly to implement in terms of time space, so these techniques only of theoretical interest.

## 4.2 Context-Free Grammars

- Inherently recursive structures of a programming language are defined by a context-free grammar.
- In a context-free grammar, we have:
  - A finite set of terminals (in our case, this will be the set of tokens)
  - A finite set of non-terminals (syntactic-variables)
  - A finite set of productions rules in the following form
    - $A \rightarrow D$  where  $A$  is a non-terminal and  $D$  is a string of terminals and non-terminals (including the empty string)
  - A start symbol (one of the non-terminal symbol)

### NOTATIONAL CONVENTIONS

#### 1. Symbols used for terminals are :

- Lower case letters early in the alphabet (such as  $a, b, c, \dots$ )
- Operator symbols (such as  $+, *, \dots$ )
- Punctuation symbols (such as parenthesis, comma and so on)
- The digits (0...9)
- Boldface strings and keywords (such as **id** or **if**) each of which represents a single terminal symbol

#### 2. Symbols used for non terminals are:

- Uppercase letters early in the alphabet (such as  $A, B, C, \dots$ )
- The letter  $S$ , which when it appears is usually the start symbol.
- Lowercase, italic names (such as *expr* or *stmt*).

#### 3. Lower case greek letters, $\alpha, \beta, \dots$ for example represent (possibly empty) strings of grammar symbols.

Example: using above notations list out terminals, non terminals and start symbol in the following example

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

$E \rightarrow E+E$

- $E+E$  derives from  $E$ 
  - we can replace  $E$  by  $E+E$
  - to be able to do this, we have to have a production rule  $E \rightarrow E+E$  in our grammar.

$E \rightarrow E+E \rightarrow id+E \rightarrow id+id$

- A sequence of replacements of non-terminal symbols is called a **derivation** of  $id+id$  from  $E$ .
- In general a derivation step is

$DAE \rightarrow DJ$  if there is a production rule  $A \rightarrow J$  in our grammar

where  $D$  and  $E$  are arbitrary strings of terminal and non-terminal symbols

$D_1 \rightarrow D_2 \rightarrow \dots \rightarrow D_n$  ( $D_n$  derives from  $D_1$  or  $D_1$  derives  $D_n$ )

- $\rightarrow$  : derives in one step
- $\rightarrow^*$  : derives in zero or more steps
- $\rightarrow^+$  : derives in one or more steps

### CFG – Terminology

- $L(G)$  is the *language of  $G$*  (the language generated by  $G$ ) which is a set of sentences.
- A *sentence of  $L(G)$*  is a string of terminal symbols of  $G$ .
- If  $S$  is the start symbol of  $G$  then

$Z$  is a sentence of  $L(G)$  iff  $S \rightarrow^* Z$  where  $Z$  is a string of terminals of  $G$ .

- If  $G$  is a context-free grammar,  $L(G)$  is a *context-free language*.
- Two grammars are *equivalent* if they produce the same language.
- $S \rightarrow^* D$  - If  $D$  contains non-terminals, it is called as a *sentential form* of  $G$ .

- If  $D$  does not contain non-terminals, it is called as a *sentence* of

$G$ .

### Derivation Example

$E \rightarrow -E \rightarrow -(E) \rightarrow -(E+E) \rightarrow -(id+E) \rightarrow -(id+id)$

OR

$E \rightarrow -E \rightarrow -(E) \rightarrow -(E+E) \rightarrow -(E+id) \rightarrow -(id+id)$

- At each derivation step, we can choose any of the non-terminal in the sentential form of  $G$  for the replacement.
- If we always choose the left-most non-terminal in each derivation step, this derivation is called as **left-most derivation**.
- If we always choose the right-most non-terminal in each derivation step, this derivation is called as **right-most derivation**.

## Left-Most and Right-Most Derivations

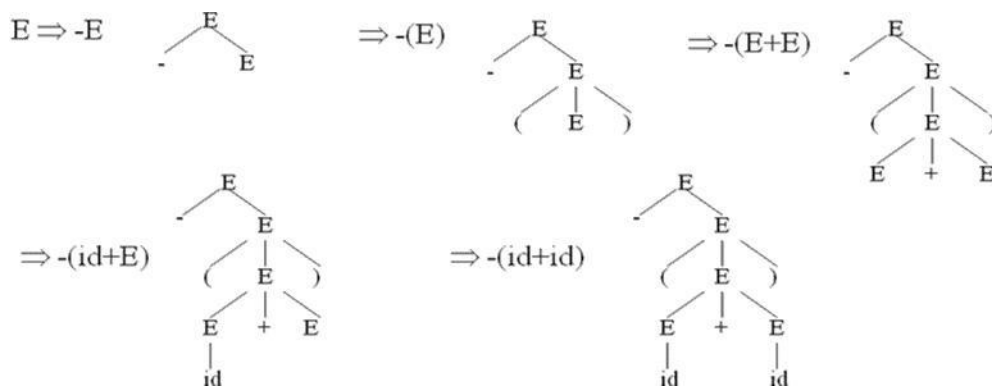
### Left-Most Derivation

$$E \xrightarrow{lm} -E \xrightarrow{lm} -(E) \xrightarrow{lm} -(E+E) \xrightarrow{lm} -(id+E) \xrightarrow{lm} -(id+id)$$

### Right-Most Derivation

$$E \xrightarrow{rm} -E \xrightarrow{rm} -(E) \xrightarrow{rm} -(E+E) \xrightarrow{rm} -(E+id) \xrightarrow{rm} -(id+id)$$

- We will see that the top-down parsers try to find the left-most derivation of the given source program.
- We will see that the bottom-up parsers try to find the right-most derivation of the given source program in the reverse order.
- Inner nodes of a parse tree are non-terminal symbols.
- The leaves of a parse tree are terminal symbols.
- A parse tree can be seen as a graphical representation of a derivation.



### Problems on derivation of a string with parse tree:

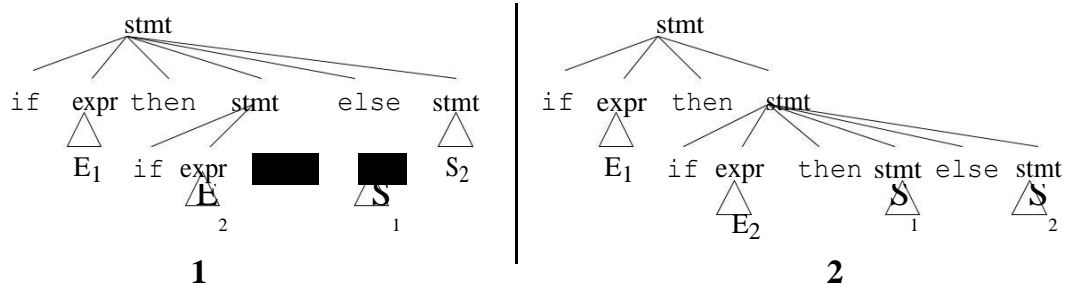
1. Consider the grammar  $S \rightarrow (L) \mid a$   
 $L \rightarrow L, S \mid S$ 
  - i. What are the terminals, non terminal and the start symbol?
  - ii. Find the parse tree for the following sentence
    - a. (a,a)
    - b. (a, (a, a))
    - c. (a, ((a,a),(a,a)))
  - iii. Construct LMD and RMD for each.
2. Do the above steps for the grammar  $S \rightarrow aS \mid aSbS \mid \square$  for the string “aaabaab”
  - A grammar produces more than one parse tree for a sentence is called as an *ambiguous* grammar.

- For most parsers, the grammar must be unambiguous.
- unambiguous grammar
  - ➔ unique selection of the parse tree for a sentence
- We should eliminate the ambiguity in the grammar during the design phase of the compiler.
- An ambiguous grammar should be written to eliminate the ambiguity.
- We have to prefer one of the parse trees of a sentence (generated by an ambiguous grammar) to disambiguate that grammar to restrict to this choice.
- EG:

### Ambiguity (cont.)

stmt  $\rightarrow$  if expr then stmt |  
if expr then stmt else stmt | otherstmts

if E<sub>1</sub> then if E<sub>2</sub> then S<sub>1</sub> else S<sub>2</sub>



- We prefer the second parse tree (else matches with closest if).
- So, we have to disambiguate our grammar to reflect this choice.
- The unambiguous grammar will be:
- stmt  $\rightarrow$  matchedstmt | unmatchedstmt

- $\text{matchedstmt} \rightarrow \text{if expr then matchedstmt else matchedstmt} \mid \text{otherstmts}$
- $\text{unmatchedstmt} \rightarrow \text{if expr then stmt} \mid$
- $\text{if expr then matchedstmt else unmatchedstmt}$

### Problems on ambiguous grammar:

Show that the following grammars are ambiguous grammar by constructing either 2 lmd or 2 rmd for the given string.

1.  $S \rightarrow S(S)S \mid \square$  with the string  $(( ) ( ) )$
2.  $S \rightarrow S+S \mid SS \mid (S) \mid S^* a$  with the string  $(a+a)^*a$
3.  $S \rightarrow aS \mid aSbS \mid \square$  with the string  $abab$

### Ambiguity – Operator Precedence

- Ambiguous grammars (because of ambiguous operators) can be disambiguated according to the precedence and associativity rules.

$E \rightarrow E+E \mid E^*E \mid E^{\wedge}E \mid \text{id} \mid (E)$

disambiguate the grammar

precedence:  $\wedge$  (right to left)  
 $*$  (left to right)  
 $+$  (left to right)

$E \rightarrow E+T \mid T$

$T \rightarrow T^*F \mid F$

$F \rightarrow G^{\wedge}F \mid G$

$G \rightarrow \text{id} \mid (E)$

### Left Recursion

- A grammar is *left recursive* if it has a non-terminal A such that there is a derivation.

+

$A \xrightarrow{+} AD$  for some string D

- Top-down parsing techniques **cannot** handle left-recursive grammars.
- So, we have to convert our left-recursive grammar into an equivalent grammar which is not left-recursive.
- The left-recursion may appear in a single step of the derivation (*immediate left-recursion*), or may appear in more than one step of the derivation.



**Immediate Left-Recursion**

$A \rightarrow A D \mid E$  where E does not start with A  
 eliminate immediate left recursion  
 $A \rightarrow E A$   
 $A \rightarrow D A \mid H$  an equivalent grammar  
 In general

**Left-Recursion – Problem**

- A grammar cannot be immediately left-recursive, but it still can be left-recursive.
- By just eliminating the immediate left-recursion, we may not get a grammar which is not left-recursive.

$S \rightarrow Aa \mid b$   
 $A \rightarrow Sc \mid d$  This grammar is not immediately left-recursive, but it is still left-recursive.

$S \rightarrow Aa \mid Sca$  or  
 $A \rightarrow Sc \mid Aac$  causes to a left-recursion

- So, we have to eliminate all left-recursions from our grammar

**Eliminate Left-Recursion – Algorithm**

- Arrange non-terminals in some order:  $A_1 \dots A_n$ 
  - **for i from 1 to n do {**
    - **for j from 1 to i-1 do {**
      - replace each production
      - $A_i \rightarrow A_j J$
      - by
      - $A_i \rightarrow D_1 J \mid \dots \mid D_k J$
      - where  $A_j \rightarrow D_1 \mid \dots \mid D_k$
    - }**
  - eliminate immediate left-recursions among  $A_i$  productions

**Example2:**

$S \rightarrow Aa \mid b$   
 $A \rightarrow Ac \mid Sd \mid f$   
 - Order of non-terminals: A,  
 S for A:  
 - we do not enter the inner loop.  
 - Eliminate the immediate left-recursion in A  
 $A \rightarrow SdA \mid fA$   
 $A \rightarrow cA \mid H$   
 for S:  
 - Replace  $S \rightarrow Aa$  with  $S \rightarrow SdAa \mid fAa$   
 So, we will have  $S \rightarrow SdAa \mid fAa \mid b$   
 - Eliminate the immediate left-recursion in S

$$S \rightarrow fA'aS' \mid bS'$$

$$S \rightarrow dA aS' \mid H$$

So, the resulting equivalent grammar which is not left-recursive is:

$$S \rightarrow fA'aS' \mid bS'$$

$$S \rightarrow dA aS' \mid H$$

$$A \rightarrow SdA \mid fA$$

$$A \rightarrow cA \mid H$$

### Problems of left recursion

1.  $S \rightarrow S(S)S \mid \square$
2.  $S \rightarrow S+S \mid SS \mid (S) \mid S^* a$
3.  $S \rightarrow SS+ \mid SS^* \mid a$
4.  $\text{bexpr} \rightarrow \text{bexpr or bterm} \mid \text{bterm bterm}$   
 $\rightarrow \text{bterm and bfactor} \mid \text{bfactor bfactor} \rightarrow$   
 $\text{not bfactor} \mid (\text{bexpr}) \mid \text{true} \mid \text{false}$
5.  $S \rightarrow (L) \mid a, L \rightarrow L,S \mid S$

### Left-Factoring

- A predictive parser (a top-down parser without backtracking) insists that the grammar must be *left-factored*.

grammar  $\rightarrow$  a new equivalent grammar suitable for predictive parsing

stmt  $\rightarrow$  if expr then stmt else stmt  $\mid$   
 if expr then stmt

- when we see if, we cannot now which production rule to choose to re-write *stmt* in the derivation.

- In general,

$A \rightarrow DE_1 \mid DE_2$  where D is non-empty and the first symbols of  $E_1$  and  $E_2$  (if they have one) are different.

- when processing D we cannot know whether expand

$A \rightarrow DE_1$  or

$A \rightarrow DE_2$

- But, if we re-write the grammar as follows

$A \rightarrow DA$

$A' \rightarrow E_1 \mid E_2$  so, we can immediately expand A to DA

### Left-Factoring – Algorithm

- For each non-terminal A with two or more alternatives (production rules) with a common non-empty prefix, let say

$A \rightarrow DE_1 \mid \dots \mid DE_n \mid J_1 \mid \dots \mid J_m$

convert it into ,

$A \rightarrow DA \mid J_1 \mid \dots \mid J_m$

$A \rightarrow E_1 \mid \dots \mid E_n$

### Left-Factoring – Example1

$A \rightarrow a\underline{b}B \mid a\underline{B} \mid cdg \mid cdeB \mid cdfB$

$$A \rightarrow aA' \mid cdg \mid cdeB \mid cdfB$$

$$A' \rightarrow bB \mid \overline{B} \quad \text{---} \quad \text{---}$$

$$A \rightarrow aA' \mid cdA''$$

$$A' \rightarrow bB \mid B$$

$$A'' \rightarrow g \mid eB \mid fB$$
**Example2**

$$A \rightarrow ad \mid a \mid ab \mid abc \mid b$$

$$A \rightarrow aA' \mid b$$

$$A' \rightarrow d \mid H \mid b \mid bc$$

$$A \rightarrow aA' \mid b$$

$$A' \rightarrow d \mid H \mid bA''$$

$$A'' \rightarrow H \mid c$$
**Problems on left factor**

$$1. S \rightarrow iEtS \mid iEtSeS \mid a, \quad E \rightarrow b$$

$$6. S \rightarrow 0S1 \mid 01$$

$$2. S \rightarrow S(S)S \mid \square$$

$$7. S \rightarrow S+S \mid |SS \mid (S) \mid S^* a$$

$$3. S \rightarrow aS \mid aSbS \mid \square$$

$$8. S \rightarrow (L) \mid a, L \rightarrow L,S \mid S$$

$$4. S \rightarrow SS+ \mid SS^* \mid a$$

$$5. bexpr \rightarrow bexpr \text{ or } bterm \mid bterm$$

$$9. rexpr \rightarrow rexpr + rterm \mid rterm$$

$$bterm \rightarrow bterm \text{ and } bfactor \mid bfactor$$

$$rterm \rightarrow rterm \quad rfactor \mid$$

$$rfactor$$

$$bfactor \rightarrow \text{not } bfactor \mid (bexpr) \mid \text{true} \mid \text{false}$$

$$rfactor \rightarrow rfactor^* \mid rprimary$$

$$rprimary \rightarrow a \mid b$$

$$\text{do both}$$

$$\text{leftfactor and left recursion}$$
**Non-Context Free Language Constructs**

- There are some language constructions in the programming languages which are not context-free. This means that, we cannot write a context-free grammar for these constructions.
- $L1 = \{ ZcZ \mid Z \text{ is in } (a \mid b)^* \}$  is not context-free
  - ➔ Declaring an identifier and checking whether it is declared or not later. We cannot do this with a context-free language. We need semantic analyzer (which is not context-free).
- $L2 = \{ a^n b^m c^n d^m \mid n \geq 1 \text{ and } m \geq 1 \}$  is not context-free
  - ➔ Declaring two functions (one with n parameters, the other one with m parameters), and then calling them with actual parameters.

First L stands for left to right scan

Second L stands for LMD

(1) stands for only one i/p symbol to predict the parser

(2) stands for k no. of i/p symbol to predict the parser

- The parse tree is created top to bottom.
- Top-down parser
  - Recursive-Descent Parsing
    - Backtracking is needed (If a choice of a production rule does not work, we backtrack to try other alternatives.)
    - It is a general parsing technique, but not widely used.
    - Not efficient
  - Predictive Parsing
    - no backtracking
    - efficient
    - Needs a special form of grammars (LL (1) grammars).
    - Recursive Predictive Parsing is a special form of Recursive Descent parsing without backtracking.

Non-Recursive (Table Driven) Predictive Parser is also known as LL (1) parser.

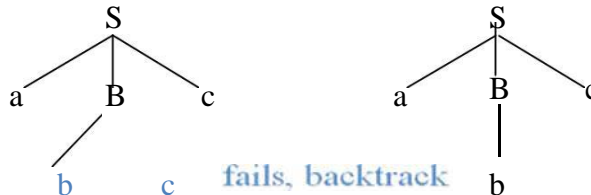
#### Recursive-Descent Parsing (uses Backtracking)

- Backtracking is needed.
- It tries to find the left-most derivation.

S  $\rightarrow$  aBc

B  $\rightarrow$  bc | b

input: abc



#### Predictive Parser

- When re-writing a non-terminal in a derivation step, a predictive parser can uniquely choose a production rule by just looking the current symbol in the input string.

A  $\rightarrow$  D<sub>1</sub> | ... | D<sub>n</sub>

input: ... a .....  
 ↑  
 current token

#### example

```
stmt  $\rightarrow$  if ..... |
        while ..... |
        begin ..... |
        for .....
```

- When we are trying to write the non-terminal *stmt*, if the current token is if we have to choose first production rule.

- When we are trying to write the non-terminal *stmt*, we can uniquely choose the production rule by just looking the current token.
- We eliminate the left recursion in the grammar, and left factor it. But it may not be suitable for predictive parsing (not LL(1) grammar).
- Non-Recursive predictive parsing is a table-driven parser.
- It is a top-down parser.
- It is also known as LL(1) Parser.

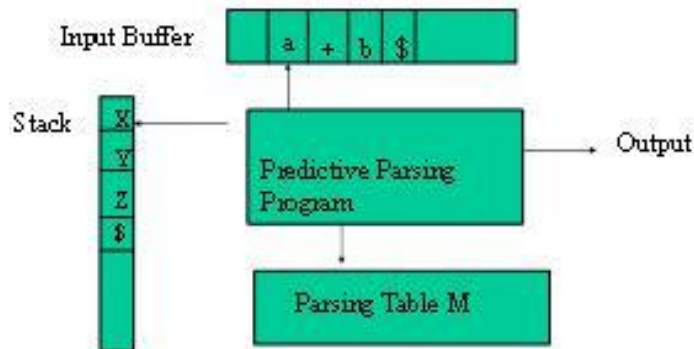


Fig: Model Of Non-Recursive predictive parsing

**LL(1) Parser****input buffer**

- our string to be parsed. We will assume that its end is marked with a special symbol \$.

**output**

- a production rule representing a step of the derivation sequence (left-most derivation) of the string in the input buffer.

**stack**

- contains the grammar symbols
- at the bottom of the stack, there is a special end marker symbol \$.
- initially the stack contains only the symbol \$ and the starting symbol S. \$\$  
     ← initial stack
- when the stack is emptied (ie. only \$ left in the stack), the parsing is completed.

**parsing table**

- a two-dimensional array  $M[A, a]$
- each row is a non-terminal symbol
- each column is a terminal symbol or the special symbol \$

each entry holds a production rule.

- Two functions are used in the construction of LL(1) parsing tables:
  - FIRST FOLLOW

- **FIRST(D)** is a set of the terminal symbols which occur as first symbols in strings derived from D where D is any string of grammar symbols.
- if D derives to H, then H is also in FIRST(D) .
- **FOLLOW(A)** is the set of the terminals which occur immediately after (follow) the *non-terminal* A in the strings derived from the starting symbol.
  - a terminal a is in FOLLOW(A) if  $S \xrightarrow{*} DAaE$
  - \$ is in FOLLOW(A) if  $S \xrightarrow{*} DA$
- If X is a terminal symbol  $\rightarrow \text{FIRST}(X) = \{X\}$
- If X is a non-terminal symbol and  $X \rightarrow H$  is a production rule  $\rightarrow H$  is in FIRST(X).
- If X is a non-terminal symbol and  $X \rightarrow Y_1 Y_2 \dots Y_n$  is a production rule
  - $\rightarrow$  if a terminal **a** in FIRST( $Y_i$ ) and H is in all FIRST( $Y_j$ ) for  $j=1, \dots, i-1$  then **a** is in FIRST(X).
  - $\rightarrow$  if H is in all FIRST( $Y_j$ ) for  $j=1, \dots, n$  then H is in FIRST(X).
- If X is H  $\rightarrow \text{FIRST}(X) = \{H\}$
- If X is  $Y_1 Y_2 \dots Y_n$   $\rightarrow$  if a terminal **a** in FIRST( $Y_i$ ) and H is in all FIRST( $Y_j$ ) for  $j=1, \dots, i-1$  then **a** is in FIRST(X).
  - $\rightarrow$  if H is in all FIRST( $Y_j$ ) for  $j=1, \dots, n$  then H is in FIRST(X).

#### Compute FOLLOW (for non-terminals)

- If S is the start symbol  $\rightarrow \$$  is in FOLLOW(S)
- if  $A \rightarrow DBE$  is a production rule  $\rightarrow$  everything in FIRST(E) is FOLLOW(B) except H
- If (  $A \rightarrow DB$  is a production rule ) or (  $A \rightarrow DBE$  is a production rule and H is in FIRST(E) )
  - $\rightarrow$  everything in FOLLOW(A) is in FOLLOW(B).

We apply these rules until nothing more can be added to any follow set.

#### LL(1) Parser – Parser Actions

- The symbol at the top of the stack (say X) and the current symbol in the input string (say a) determine the parser action.
- There are four possible parser actions.
  1. If X and a are \$  $\rightarrow$  parser halts (successful completion)
  2. If X and a are the same terminal symbol (different from \$)
    - $\rightarrow$  parser pops X from the stack, and moves the next symbol in the input buffer.
  3. If X is a non-terminal

→ parser looks at the parsing table entry  $M[X, a]$ . If  $M[X, a]$  holds a production rule  $X \rightarrow Y_1 Y_2 \dots Y_k$ , it pops  $X$  from the stack and pushes  $Y_k, Y_{k-1}, \dots, Y_1$  into the stack. The parser also outputs the production rule  $X \rightarrow Y_1 Y_2 \dots Y_k$  to represent a step of the derivation.

### Non-Recursive predictive parsing Algorithm

**METHOD:** Initially, the parser is in a configuration with  $w\$$  in the input buffer and the start symbol  $S$  of  $G$  on top of the stack, above  $\$$ . The program in Fig. 4.20 uses the predictive parsing table  $M$  to produce a predictive parse for the input. □

```

set  $ip$  to point to the first symbol of  $w$ ;
set  $X$  to the top stack symbol;
while (  $X \neq \$$  ) { /* stack is not empty */
    if (  $X$  is  $a$  ) pop the stack and advance  $ip$ ;
    else if (  $X$  is a terminal )  $error()$ ;
    else if (  $M[X, a]$  is an error entry )  $error()$ ;
    else if (  $M[X, a] = X \rightarrow Y_1 Y_2 \dots Y_k$  ) {
        output the production  $X \rightarrow Y_1 Y_2 \dots Y_k$ ;
        pop the stack;
        push  $Y_k, Y_{k-1}, \dots, Y_1$  onto the stack, with  $Y_1$  on top;
    }
    set  $X$  to the top stack symbol;
}

```

Figure 4.20: Predictive parsing algorithm

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CS416 Compiler Design

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### LL(1) Parser – Example1

S o aBa

LL (1) Parsing Table

B o bB | H

FIRST FUNCTION

FIRST(S) = {a}

FIRST(aBa) = {a}

FIRST(B) = {b}

FIRST(bB) = {b} FIRST(H) = {H}

FOLLOW FUNCTION

FOLLOW(S) = { $\$$ }

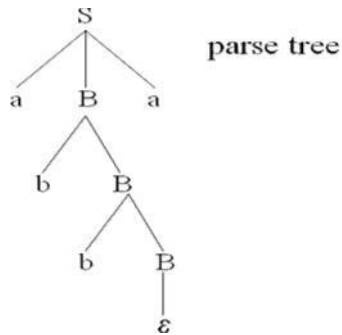
FOLLOW(B) = {a}

|       | a       | b       | \$ |
|-------|---------|---------|----|
| S     | S o aBa |         |    |
| B     | B o H   | B o bB  |    |
| stack | input   | output  |    |
| \$S   | abba\$  | S o aBa |    |
| \$aBa | abba\$  |         |    |
| \$aB  | bba\$   | B o bB  |    |
| \$aBb | bba\$   |         |    |

$\$aB$              $ba\$$              $B \circ bB$   
 $\$aBb$             $ba\$$   
 $\$aB$              $a\$$              $B \circ H$   
 $\$a$              $a\$$   
 $\$$              $\$$             accept, successful completion

Outputs:  $S \circ aBa$     $B \circ bB$     $B \circ bB$              $B \circ H$

Derivation(left-most):  $S \rightarrow aBa \rightarrow abBa \rightarrow abbBa \rightarrow abba$



### Example2

$E \rightarrow TE$  ,  
 $E \rightarrow +TE$  |  $H$   
 $T \rightarrow FT$  ,  
 $T \rightarrow *FT$  |  $H$   
 $F \rightarrow (E)$  |  $id$

Soln:

### FIRST, Example

$E \rightarrow TE$  ,  
 $E \rightarrow +TE$  |  $H$   
 $T \rightarrow FT$  ,  
 $T \rightarrow *FT$  |  $H$   
 $F \rightarrow (E)$  |  $id$

$FIRST(F) = \{(, id\}$   
 $FIRST(T) = \{*, H\}$   
 $FIRST(T) = \{(, id\}$   
 $FIRST(E) = \{+, H\}$   
 $FIRST(E) = \{(, id\}$   
 $FIRST(H) = \{H\}$

$FIRST(TE) = \{(, id\}$   
 $FIRST(+TE) = \{+\}$   
 $FIRST(H) = \{H\}$   
 $FIRST(FT) = \{(, id\}$   
 $FIRST(*FT) = \{*\}$   
 $FIRST((E)) = \{(\}$

$FIRST(id) = \{id\}$

### FOLLOW Example

$E \rightarrow TE$   
 $E \rightarrow +TE$  |  $H$   
 $T \rightarrow FT$   
 $T \rightarrow *FT$  |  $H$   
 $F \rightarrow (E)$  |  $id$

$FOLLOW(E) = \{ \$, ) \}$   
 $FOLLOW(E) = \{ \$, ) \}$   
 $FOLLOW(T) = \{ +, ), \$ \}$



$\text{FOLLOW}(T) = \{+, ), \$\}$

$\text{FOLLOW}(F) = \{+, *, ), \$\}$

### Constructing LL (1) Parsing Table – Algorithm

- for each production rule  $A \rightarrow D$  of a grammar  $G$ 
  - for each terminal  $a$  in  $\text{FIRST}(D)$   $\rightarrow$  add  $A \rightarrow D$  to  $M[A, a]$
  - If  $H$  in  $\text{FIRST}(D)$   $\rightarrow$  for each terminal  $a$  in  $\text{FOLLOW}(A)$  add  $A \rightarrow D$  to  $M[A, a]$
  - If  $H$  in  $\text{FIRST}(D)$  and  $\$$  in  $\text{FOLLOW}(A)$   $\rightarrow$  add  $A \rightarrow D$  to  $M[A, \$]$
  - All other undefined entries of the parsing table are error entries.

### Constructing LL (1) Parsing Table – Example

$E \rightarrow TE$   $\text{FIRST}(TE) = \{ (, \text{id} \}$   $\rightarrow$   $E \rightarrow TE$  into  $M[E, (]$  and  $M[E, \text{id}]$

$E \rightarrow +TE$   $\text{FIRST}(+TE) = \{ + \}$   $\rightarrow$   $E \rightarrow +TE$  into  $M[E, +]$

$E \rightarrow H$   $\text{FIRST}(H) = \{ H \}$   $\rightarrow$  none

but since  $H$  in  $\text{FIRST}(H)$  and  $\text{FOLLOW}(E) = \{ \$, ) \}$

$\rightarrow$   $E \rightarrow H$  into  $M[E, \$]$  and  $M[E, )]$

$T \rightarrow FT$   $\text{FIRST}(FT) = \{ (, \text{id} \}$   $\rightarrow$   $T \rightarrow FT$  into  $M[T, (]$  and  $M[T, \text{id}]$

$T \rightarrow *FT$   $\text{FIRST}(*FT) = \{ * \}$   $\rightarrow$   $T \rightarrow *FT$  into  $M[T, *]$

$T \rightarrow H$   $\text{FIRST}(H) = \{ H \}$   $\rightarrow$  none

but since  $H$  in  $\text{FIRST}(H)$

and  $\text{FOLLOW}(T) = \{ \$, ) , + \}$

$\rightarrow$   $T \rightarrow H$  into  $M[T, \$]$ ,  $M[T, )]$  and  $M[T, +]$

$F \rightarrow (E)$   $\text{FIRST}((E)) = \{ ( \}$   $\rightarrow$   $F \rightarrow (E)$  into  $M[F, (]$

$F \rightarrow \text{id}$   $\text{FIRST}(\text{id}) = \{ \text{id} \}$   $\rightarrow$   $F \rightarrow \text{id}$  into  $M[F, \text{id}]$

|   | id                        | + | *                   | (                   | ) | \$ |
|---|---------------------------|---|---------------------|---------------------|---|----|
| E | $E \rightarrow TE$        |   |                     |                     |   |    |
| E | $E \rightarrow +TE$       |   |                     |                     |   |    |
| T | $T \rightarrow FT$        |   |                     |                     |   |    |
| T | $T \rightarrow H$         |   | $T \rightarrow *FT$ |                     |   |    |
| F | $F \rightarrow \text{id}$ |   |                     | $F \rightarrow (E)$ |   |    |

| stack    | input   | output                    |
|----------|---------|---------------------------|
| \$E      | id+id\$ | $E \rightarrow TE$        |
| \$E T    | id+id\$ | $T \rightarrow FT$        |
| \$E T F  | id+id\$ | $F \rightarrow \text{id}$ |
| \$E T id | id+id\$ |                           |
| \$E T    | +id\$   | $T \rightarrow H$         |
| \$E      | +id\$   | $E \rightarrow +TE$       |
| \$E T+   | +id\$   |                           |

1.  $S \rightarrow S(S)S \mid \square$  with the string  $(( ))$
2.  $S \rightarrow + S S \mid |^* S S \mid a$  with the string  $“+^*aa a”$
3.  $S \rightarrow aSbS \mid bSaS \mid \square$  with the string  $“aabbbab”$
4.  $bexpr \rightarrow bexpr \text{ or } bterm \mid bterm$   
 $bterm \rightarrow bterm \text{ and } bfactor \mid bfactor$   
 $bfactor \rightarrow \text{not } bfactor \mid (bexpr) \mid \text{true} \mid \text{false}$   
string  $“\text{not}(\text{true or false})”$
5.  $S \rightarrow 0S1 \mid 01$  string  $“00011”$
6.  $S \rightarrow aB \mid aC \mid Sd \mid Se$   
 $B \rightarrow bBc \mid f$   
 $C \rightarrow g$
7.  $P \rightarrow Ra \mid Qba$   
 $R \rightarrow aba \mid caba \mid Rbc$   
 $Q \rightarrow bbc \mid bc$  string  $“cababca”$
8.  $S \rightarrow PQR$   
 $P \rightarrow a \mid Rb \mid \square$   
 $Q \rightarrow c \mid dP$   
 $\mid \square$   
 $R \rightarrow e \mid f$  string  $“adeb”$
9.  $E \rightarrow E+ T \mid T$   
 $T \rightarrow id \mid id[ ] \mid id[X]$   
 $X \rightarrow E, E \mid E$  string  $“id[id]”$
10.  $S \rightarrow (A) \mid 0 A \rightarrow SB$   
 $B \rightarrow ,SB \mid \square$  string  $“(0, (0,0))”$
11.  $S \rightarrow a \mid n \mid (T) T \rightarrow T, S \mid S \text{ String}$   
 $(a, (a, a)) \text{ String } ((a, a), n, (a), a)$

- A grammar whose parsing table has no multiply-defined entries is said to be LL (1) grammar. one input symbol used as a look-head symbol do determine parser action LL (1) left most derivation input scanned from left to right
- The parsing table of a grammar may contain more than one production rule. In this case, we say that it is not a LL (1) grammar.

So i Ct S E | a E o  
e S | H C o b

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|   | a     | b     | e                | i         | t\$ |       |
|---|-------|-------|------------------|-----------|-----|-------|
| S | S o a |       |                  | S o iCtSE |     |       |
| E |       |       | E o e S<br>E o H |           |     | E o H |
| C |       | C o b |                  |           |     |       |

two production rules for M[E, e]

Problem → ambiguity

- What do we have to do if the resulting parsing table contains multiply defined entries?
  - If we didn't eliminate left recursion, eliminate the left recursion in the grammar.
  - If the grammar is not left factored, we have to left factor the grammar.
  - If it's (new grammar's) parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.
- A left recursive grammar cannot be a LL (1) grammar.
  - $A \rightarrow AD \mid E$ 
    - any terminal that appears in FIRST(E) also appears FIRST(AD) because  $AD \overset{?}{\rightarrow} ED$ .
    - If E is H, any terminal that appears in FIRST(D) also appears in FIRST(AD) and FOLLOW(A).
- A grammar is not left factored, it cannot be a LL(1) grammar
  - $A \rightarrow DE_1 \mid DE_2$ 
    - any terminal that appears in FIRST( $DE_1$ ) also appears in FIRST( $DE_2$ ).
- An ambiguous grammar cannot be a LL (1) grammar.

### Properties of LL(1) Grammars

- A grammar G is LL(1) if and only if the following conditions hold for two distinctive production rules  $A \rightarrow D$  and  $A \rightarrow E$ 
  - Both D and E cannot derive strings starting with same terminals.
  - At most one of D and E can derive to H.
  - If E can derive to H, then D cannot derive to any string starting with a terminal in FOLLOW(A).
- An error may occur in the predictive parsing (LL(1) parsing)
  - if the terminal symbol on the top of stack does not match with the current input symbol.

- if the top of stack is a non-terminal A, the current input symbol is a, and the parsing table entry M[A, a] is empty.
- What should the parser do in an error case?
  - The parser should be able to give an error message (as much as possible meaningful error message).
  - It should be recovered from that error case, and it should be able to continue the parsing with the rest of the input.
- Panic-Mode Error Recovery
  - Skipping the input symbols until a synchronizing token is found.
- Phrase-Level Error Recovery
  - Each empty entry in the parsing table is filled with a pointer to a specific error routine to take care that error case.
- Error-Productions
  - If we have a good idea of the common errors that might be encountered, we can augment the grammar with productions that generate erroneous constructs.
  - When an error production is used by the parser, we can generate appropriate error diagnostics.
  - Since it is almost impossible to know all the errors that can be made by the programmers, this method is not practical.
- Global-Correction
  - Ideally, we would like a compiler to make as few changes as possible in processing incorrect inputs.
  - We have to globally analyze the input to find the error.
  - This is an expensive method, and it is not in practice.
- In panic-mode error recovery, we skip all the input symbols until a synchronizing token is found.
- What is the synchronizing token?
  - All the terminal-symbols in the follow set of a non-terminal can be used as a synchronizing token set for that non-terminal.
- So, a simple panic-mode error recovery for the LL(1) parsing:
  - All the empty entries are marked as ***synch*** to indicate that the parser will skip all the input symbols until a symbol in the follow set of the non-terminal A which on the top of the stack. Then the parser will pop that non-terminal A from the stack. The parsing continues from that state.
  - To handle unmatched terminal symbols, the parser pops that unmatched terminal symbol from the stack and it issues an error message saying that that unmatched terminal is inserted.

**Panic-Mode Error Recovery – Example****S**  $\circ$  **AbS** | **e** | **H****A**  $\circ$  **a** | **cAd****Soln:**

FIRST (S) = FIRST (A) = {a, c}

FIRST (A) = {a, c}

FOLLOW (S) = {\$}

FOLLOW (A) = {b, d}

|   | a             | b    | c             | d    | e           | \$          |
|---|---------------|------|---------------|------|-------------|-------------|
| S | S $\circ$ AbS | sync | S $\circ$ AbS | sync | S $\circ$ e | S $\circ$ H |
| A | A $\circ$ a   | sync | A $\circ$ cAd | sync | sync        | sync        |

**Eg:** input string “aab”stack   input   output\$S   aab\$   S  $\circ$  AbS\$SbA   aab\$   A  $\circ$  a

\$Sba   aab\$

\$Sb   ab\$   Error: missing b, inserted

\$S   ab\$   S  $\circ$  AbS\$SbA   ab\$   A  $\circ$  a

\$Sba   ab\$

\$Sb   b\$

\$S   \$   S  $\circ$  H

\$   \$   accept

**Eg:** Another input string “ceadb”stack   input   output\$S   ceadb\$   S  $\circ$  AbS\$SbA   ceadb\$   A  $\circ$  cAd

\$SbdAc   ceadb\$

\$SbdA   eadb\$   Error:unexpected e (illegal A)

(Remove all input tokens until first b or d, pop A)

\$Sbd   db\$

\$Sb   b\$

\$S   \$   S  $\circ$  H

\$   \$   accept

- Each empty entry in the parsing table is filled with a pointer to a special error routine which will take care that error

### 4.3 Bottom-Up Parsing

- A **bottom-up parser** creates the parse tree of the given input starting from leaves towards the root.
- A bottom-up parser tries to find the right-most derivation of the given input in the reverse order.

$S \xrightarrow{\text{m}} \dots \xrightarrow{\text{m}} \check{Y} Z$  (the right-most derivation of  $Z$ )  
 (the bottom-up parser finds the right-most derivation in the reverse order)

- Bottom-up parsing is also known as **shift-reduce parsing** because its two main actions are shift and reduce.
  - At each shift action, the current symbol in the input string is pushed to a stack.
  - At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will be replaced by the non-terminal at the left side of that production.
  - There are also two more actions: accept and error.

- A shift-reduce parser tries to reduce the given input string into the starting symbol.

a string  $\rightarrow$  the starting symbol  
 reduced to

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule.
- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order.

Rightmost Derivation:  $S \xrightarrow{\text{m}} \check{Y} Z$

$\text{m}$

Shift-Reduce Parser finds:  $Z \square \dots \square S$

$\text{m} \quad \text{m}$

#### Example

$S \rightarrow aABb$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

input string: aaabb

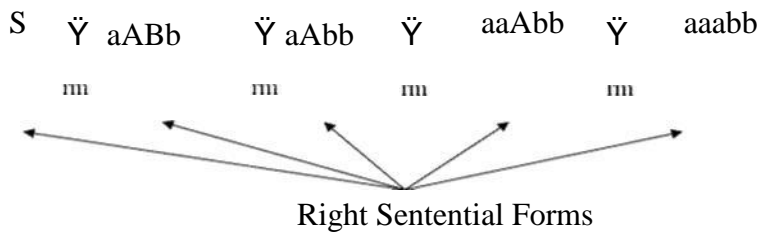
aaAbb

aAbb

aABb

S

reduction



- How do we know which substring to be replaced at each reduction step?

### Handle

- Informally, a **handle** of a string is a substring that matches the right side of a production rule.
  - But not every substring matches the right side of a production rule is handle
- A **handle** of a right sentential form  $\alpha J$  ( $\alpha \in \Sigma^*$ ) is a production rule  $A \rightarrow E$  and a position of  $J$  where the string  $E$  may be found and replaced by  $A$  to produce the previous right-sentential form in a rightmost derivation of  $J$ .

\*  
 $S \xrightarrow{*} \alpha DAZ \xrightarrow{*} \alpha DEZ$

$\equiv \quad \equiv$

- If the grammar is unambiguous, then every right-sentential form of the grammar has exactly one handle.
- We will see that  $Z$  is a string of terminals.

### Handle Pruning

- A right-most derivation in reverse can be obtained by **handle-pruning**.

$S = \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n = \omega$  ← input string

- Start from  $\gamma_n$ , find a handle  $A_n \rightarrow \beta_n$  in  $\gamma_n$ , and replace  $\beta_n$  in  $\gamma_n$  by  $A_n$  to get  $\gamma_{n-1}$ .
- Then find a handle  $A_{n-1} \rightarrow \beta_{n-1}$  in  $\gamma_{n-1}$ , and replace  $\beta_{n-1}$  in  $\gamma_{n-1}$  by  $A_{n-1}$  to get  $\gamma_{n-2}$ .
- Repeat this, until we reach  $S$ .

- x Handle pruning help in finding handle which will be reduced to a non terminal, that is the process of shift reduce parsing.

### A Shift-Reduce Parser

$E \rightarrow E+T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow (E) \mid id$

Right-Most Derivation of  $id+id*id$

$E \Rightarrow E+T \Rightarrow E+T * F \Rightarrow E+T * id \Rightarrow E+F * id$   
 $\Rightarrow E+id * id \Rightarrow T+id * id \Rightarrow F+id * id \Rightarrow id+id * id$

| <u>Right-Most Sentential Form</u> | <u>Reducing Production</u> |
|-----------------------------------|----------------------------|
| <u>id</u> +id*id                  | $F \rightarrow id$         |
| <u>F</u> +id*id                   | $T \rightarrow F$          |
| <u>T</u> +id*id                   | $E \rightarrow T$          |
| E+ <u>id</u> *id                  | $F \rightarrow id$         |
| E+ <u>F</u> *id                   | $T \rightarrow F$          |
| E+T* <u>id</u>                    | $F \rightarrow id$         |
| E+T* <u>F</u>                     | $T \rightarrow T * F$      |
| <u>E+T</u>                        | $E \rightarrow E+T$        |
| E                                 |                            |

Handles are red and underlined in the right-sentential forms.

### A Stack Implementation of A Shift-Reduce Parser

- There are four possible actions of a shift-parser action:
  - Shift** : The next input symbol is shifted onto the top of the stack.
  - Reduce**: Replace the handle on the top of the stack by the non-terminal.
  - Accept**: Successful completion of parsing.
  - Error**: Parser discovers a syntax error, and calls an error recovery routine.
- Initial stack just contains only the end-marker \$.
- The end of the input string is marked by the end-marker \$.

**Consider the following grams and parse the respective strings using shift-reduce parser.**

(1)  $E \rightarrow E+T \mid T$

$T \rightarrow T * F \mid F$

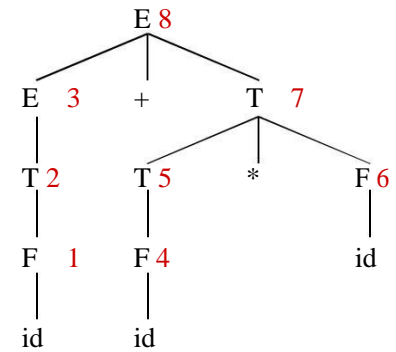
- If the incoming operator has more priority than in stack operator then perform shift.
- If in stack operator has same or less priority than the priority of incoming operator then perform reduce.



## A Stack Implementation of A Shift-Reduce Parser

| <u>Stack</u> | <u>Input</u> | <u>Action</u>     |
|--------------|--------------|-------------------|
| \$           | id+id*id\$   | shift             |
| \$id         | +id*id\$     | reduce by F o id  |
| \$F          | +id*id\$     | reduce by T o F   |
| \$T          | +id*id\$     | reduce by E o T   |
| \$E          | +id*id\$     | shift             |
| \$E+         | id*id\$      | shift             |
| \$E+id       | *id\$        | reduce by F o id  |
| \$E+F        | *id\$        | reduce by T o F   |
| \$E+T        | *id\$        | shift             |
| \$E+T*       | id\$         | shift             |
| \$E+T*id     | \$           | reduce by F o id  |
| \$E+T*F      | \$           | reduce by T o T*F |
| \$E+T        | \$           | reduce by E o E+T |
| \$E          | \$           | accept            |

Parse Tree



- (2)  $S \rightarrow TL$ ;  
 $T \rightarrow \text{int} \mid \text{float}$   
 $L \rightarrow L, \text{id} \mid \text{id}$   
 String is “int id, id;” do shift-reduce parser.
- (3)  $S \rightarrow (L) \mid a L$   
 $\rightarrow L, S \mid S$   
 String “(a,(a,a))” do shift-reduce parser.

## Shift reduce parser problem

- Take the grammar:
- Sentence  $\rightarrow$  NounPhrase VerbPhrase      NounPhrase  $\rightarrow$  Art Noun  
 VerbPhrase  $\rightarrow$  Verb | Adverb Verb      Art  $\rightarrow$  the | a | ...  
 Verb  $\rightarrow$  jumps | sings | ...      Noun  $\rightarrow$  dog | cat | ...

And the input: “the dog jumps”. Then the bottom up parsing is:

| Stack                   | Input Sequence         | ACTION                    |
|-------------------------|------------------------|---------------------------|
| \$                      | the dog jumps\$        | SHIFT word onto stack     |
| \$the                   | dog jumps\$            | REDUCE using grammar rule |
| <del>\$Art</del>        | <del>dog jumps\$</del> | <del>SHIFT..</del>        |
| \$Art dog               | jumps\$                | REDUCE..                  |
| \$Art Noun              | jumps\$                | REDUCE                    |
| \$NounPhrase            | jumps\$                | SHIFT                     |
| \$NounPhrase jumps      | \$                     | REDUCE                    |
| \$NounPhrase Verb       | \$                     | REDUCE                    |
| \$NounPhrase VerbPhrase | \$                     | REDUCE                    |
| \$Sentence              | \$                     | SUCCESS                   |

## 4.4 Shift-Reduce Parsers

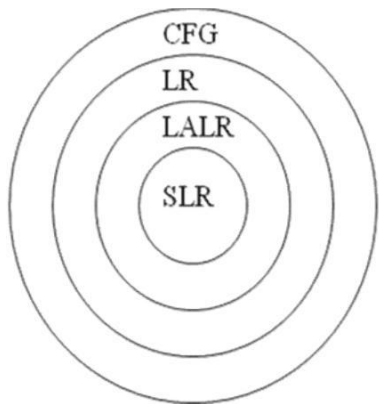
- There are two main categories of shift-reduce parsers

### 1. Operator-Precedence Parser

- Simple, but only a small class of grammars.
- **LR-Parsers**

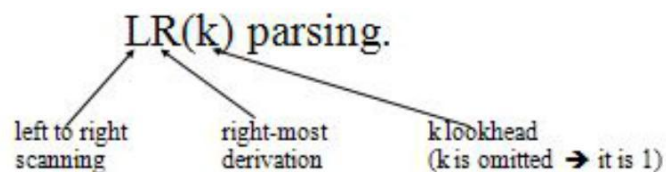
- Covers wide range of grammars.
  - SLR – simple LR parser
  - LR – most general LR parser
  - LALR – intermediate LR parser (lookahead LR parser)

SLR, LR and LALR work same, only their parsing tables are different



## LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:



- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.  

$$LL(1)\text{-Grammars} \subset LR(1)\text{-Grammars}$$
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

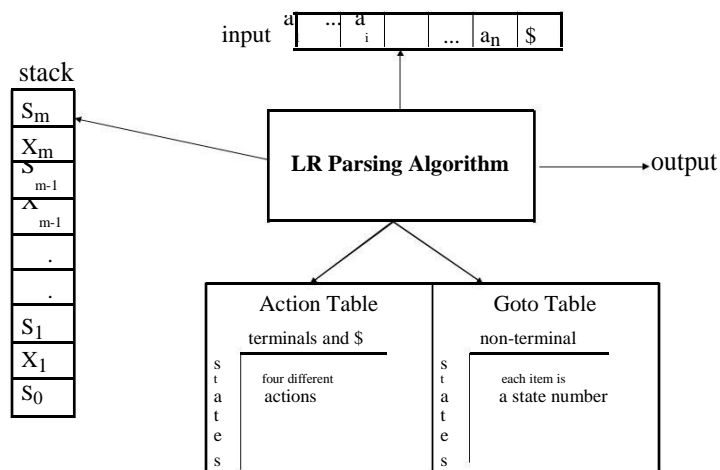
## LR Parsers

- **LR-Parsers**

- covers wide range of grammars.
- SLR – simple LR parser
- LR – most general LR parser (canonical LR)
- LALR – intermediate LR parser (look-head LR parser)

- SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.

### LR Parsing Algorithm



### A Configuration of LR Parsing Algorithm

- A configuration of a LR parsing is:

$$( \underbrace{S_0 X_1 S_1 \dots X_m S_m}_{\text{Stack}}, \underbrace{a_i a_{i+1} \dots a_n \$}_{\text{Rest of Input}} )$$

- $S_m$  and  $a_i$  decides the parser action by consulting the parsing action table. (*Initial Stack* contains just  $S_0$ )
- A configuration of a LR parsing represents the right sentential form:

$$X_1 \dots X_m a_i a_{i+1} \dots a_n \$$$

## Actions of A LR-Parser

1. **shift s** -- shifts the next input symbol and the state **s** onto the stack  
 $(S_0 X_1 S_1 \dots X_m S_m, a_i a_{i+1} \dots a_n \$) \rightarrow (S_0 X_1 S_1 \dots X_m S_m \textcolor{red}{a_i s}, a_{i+1} \dots a_n \$)$
2. **reduce AoE** (or **rn** where n is a production number)  
 – pop  $2|E| (=r)$  items from the stack;  
 – then push **A** and **s** where **s=goto[s<sub>m-r</sub>,A]**  
 $(S_0 X_1 S_1 \dots X_m S_m, a_i a_{i+1} \dots a_n \$) \rightarrow (S_0 X_1 S_1 \dots X_{m-r} \textcolor{red}{S_{m-r} A s}, a_i \dots a_n \$)$   
 – Output is the reducing production reduce AoE
3. **Accept** – Parsing successfully completed
4. **Error** -- Parser detected an error (an empty entry in the action table)

### Reduce Action

- pop  $2|E| (=r)$  items from the stack; let us assume that **E** =  $Y_1 Y_2 \dots Y_r$
- then push **A** and **s** where **s=goto[s<sub>m-r</sub>,A]**  
 $(S_0 X_1 S_1 \dots X_{m-r} S_{m-r} Y_1 S_{m-r} \dots Y_r S_m, a_i a_{i+1} \dots a_n \$)$   
 $\rightarrow (S_0 X_1 S_1 \dots X_{m-r} S_{m-r} A s, a_i \dots a_n \$)$
- In fact,  $Y_1 Y_2 \dots Y_r$  is a handle.  
 $X_1 \dots X_{m-r} A a_i \dots a_n \$ \sqsupset X_1 \dots X_m Y_1 \dots Y_r a_i a_{i+1} \dots a_n \$$

## Constructing SLR Parsing Tables – LR(0) Item

- An LR(0) item of a grammar G is a production of G a dot at the some position of the right side.
- Ex:     A → aBb                 Possible LR(0) Items:                 A → .aBb  
     (four different possibility) A → a.Bb  
     A → aB.b  
     A → aBb.
- Sets of LR(0) items will be the states of action and goto table of the SLR parser.
- A collection of sets of LR(0) items (the canonical LR(0) collection) is the basis for constructing SLR parsers.
- Augmented Grammar:**  
 G' is G with a new production rule S' → OS where S' is the new starting symbol.

## The Closure Operation

- If  $I$  is a set of LR(0) items for a grammar  $G$ , then  $\text{closure}(I)$  is the set of LR(0) items constructed from  $I$  by the two rules:
  1. Initially, every LR(0) item in  $I$  is added to  $\text{closure}(I)$ .
  2. If  $A \rightarrow D.BE$  is in  $\text{closure}(I)$  and  $B \rightarrow J$  is a production rule of  $G$ ; then  $B \rightarrow J$  will be in the  $\text{closure}(I)$ . We will apply this rule until no more new

**Example: I** LR(0) items can be added to closure(I).  
 $= \{$

## The Closure Operation -- Example

|                     |                                    |              |
|---------------------|------------------------------------|--------------|
| $E' \circ E$        | $\text{closure}(\{E' \circ E\}) =$ |              |
| $E \circ E+T$       | $\leftarrow \{E' \circ E$          | kernel items |
| $E \circ T$         | $E \circ E+T$                      |              |
| $T \circ T * F$     | $E \circ T$                        |              |
| $T \circ F$         | $T \circ T * F$                    |              |
| $F \circ (E)$       | $T \circ F$                        |              |
| $F \circ \text{id}$ | $F \circ (E)$                      |              |
|                     | $F \circ \text{id} \quad \}$       |              |

## Goto Operation

- If  $I$  is a set of LR(0) items and  $X$  is a grammar symbol (terminal or non-terminal), then  $\text{goto}(I, X)$  is defined as follows:
  - If  $A \circ D.XE$  in  $I$  then every item in  $\text{closure}(\{A \circ DX.E\})$  will be in  $\text{goto}(I, X)$ .

$$\{E' \circ E, E \circ E+T, E \circ T, T \circ T^*F, T \circ F, F \circ (E), F \circ id\}$$

$$\text{goto}(I, E) = \{ E' \circ E., E \circ E.+T \}$$

$$\text{goto}(I, T) = \{ E \circ T., T \circ T.*F \}$$

$$\text{goto}(I, F) = \{ T \circ F. \}$$

$$\text{goto}(I, () = \{ F \circ (.E), E \circ .E+T, E \circ .T, T \circ .T*F, T \circ .F, F \circ .(E), F \circ .id \}$$

$$\text{goto}(I, id) = \{ F \circ id. \}$$

### Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.
- Algorithm:**

C is { closure({S' o .S}) }

repeat the followings until no more set of LR(0) items can be added to C.

for each I in C and each grammar symbol X

if goto(I, X) is not empty and not in C

add goto(I, X) to C

- goto function is a DFA on the sets in C.

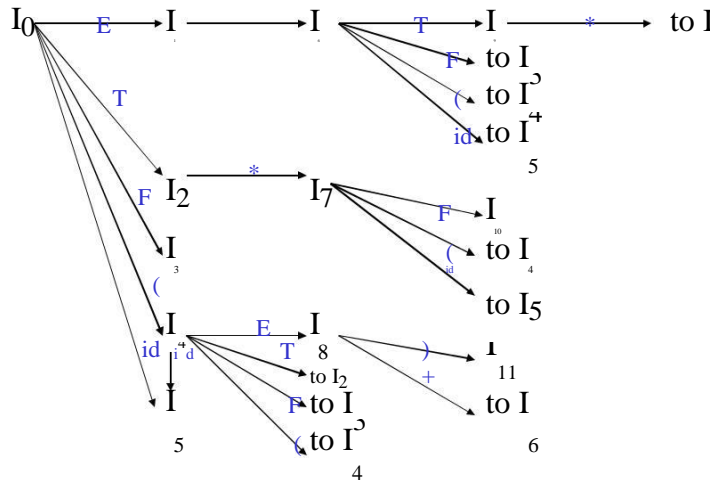
**The Canonical LR(0) Collection – Example**

|                          |                           |                           |                            |
|--------------------------|---------------------------|---------------------------|----------------------------|
| I <sub>0</sub> : E' o .E | I <sub>1</sub> : E' o E.  | I <sub>6</sub> : E o E+.T | I <sub>9</sub> : E o E+T.  |
| E o .E+T                 | E o E.+T                  | T o .T*F                  | T o T.*F                   |
| E o .T                   |                           | T o .F                    |                            |
| T o .T*F                 | I <sub>2</sub> : E o T.   | F o .(E)                  | I <sub>10</sub> : T o T*F. |
| T o .F                   | T o T.*F                  | F o .id                   |                            |
| F o .(E)                 |                           |                           |                            |
| F o .id                  | I <sub>3</sub> : T o F.   | I <sub>7</sub> : T o T*.F | I <sub>11</sub> : F o (E). |
|                          |                           | F o .(E)                  |                            |
|                          | I <sub>4</sub> : F o (.E) | F o .id                   |                            |
|                          | E o .E+T                  |                           |                            |
|                          | E o .T                    | I <sub>8</sub> : F o (E.) |                            |
|                          | T o .T*F                  | E o E.+T                  |                            |
|                          | T o .F                    |                           |                            |
|                          | F o .(E)                  |                           |                            |



F o .id  
Is: F o id.

## Transition Diagram (DFA) of Goto Function



## Constructing SLR Parsing Table (of an augmented grammar $G'$ )

1. Construct the canonical collection of sets of LR(0) items for  $G'$ .  $Cm\{I_0, \dots, I_n\}$
2. Create the parsing action table as follows
  - If  $a$  is a terminal,  $A \odot D.aE$  in  $I_i$  and  $goto(I_i, a) = I_j$  then  $action[i, a]$  is **shift j**.
  - If  $A \odot D.$  is in  $I_i$ , then  $action[i, a]$  is **reduce  $A \odot D$**  for all  $a$  in  $FOLLOW(A)$  where  $AZS'$ .
  - If  $S' \odot S.$  is in  $I_i$ , then  $action[i, \$]$  is **accept**.
  - If any conflicting actions generated by these rules, the grammar is not SLR(1).
3. Create the parsing goto table
  - for all non-terminals  $A$ , if  $goto(I_i, A) = I_j$  then  $goto[i, A] = j$
4. All entries not defined by (2) and (3) are errors.
5. Initial state of the parser contains  $S' \odot S$

## (SLR) Parsing Tables for Expression Grammar

1)  $E \rightarrow E+T$ 2)  $E \rightarrow T$ 3)  $T \rightarrow T * F$ 4)  $T \rightarrow F$ 5)  $F \rightarrow (E)$ 6)  $F \rightarrow id$ 

| Action Table |    |    |    |    |     | Goto Table |   |   |    |
|--------------|----|----|----|----|-----|------------|---|---|----|
| state        | id | +  | *  | (  | )   | \$         | E | T | F  |
| 0            | s5 |    |    | s4 |     |            | 1 | 2 | 3  |
| 1            |    | s6 |    |    |     | acc        |   |   |    |
| 2            |    | r2 | s7 |    | r2  | r2         |   |   |    |
| 3            |    | r4 | r4 |    | r4  | r4         |   |   |    |
| 4            | s5 |    |    | s4 |     |            | 8 | 2 | 3  |
| 5            |    | r6 | r6 |    | r6  | r6         |   |   |    |
| 6            | s5 |    |    | s4 |     |            |   | 9 | 3  |
| 7            | s5 |    |    | s4 |     |            |   |   | 10 |
| 8            |    | s6 |    |    | s11 |            |   |   |    |
| 9            |    | r1 | s7 |    | r1  | r1         |   |   |    |
| 10           |    | r3 | r3 |    | r3  | r3         |   |   |    |
| 11           |    | r5 | r5 |    | r5  | r5         |   |   |    |

**Actions of A (S)LR-Parser – Example**

| <u>stack</u> | <u>input</u> | <u>action</u> | <u>goto</u> | <u>parsing</u>  |
|--------------|--------------|---------------|-------------|---|
| \$0          | id*id+id\$   | [0,id]=s5     |             | shift 5   |
| \$0id5       | *id+id\$     | [5,*]=r6      | [0,F]=3     | reduce by Foid (pop 2 id  no. of symbols from stack and push F to the stack)    |
| \$0F3        | *id+id\$     | [3,*]=r4      | [0,T]=2     | reduce by ToF (pop 2 F  no. of symbols from stack and push T onto the stack)    |
| \$0T2        | *id+id\$     | [2,*]=s7      |             | shift 7   |
| \$0T2*7      | id+id\$      | [7,id]=s5     |             | shift 5   |
| \$0T2*7id5   | +id\$        | [5,+]=r6      | [7,F]=10    | reduce by Foid(pop 2 id  no. of symbols from stack and push F onto the stack)   |
| \$0T2*7F10   | +id\$        | [10,+]=r3     | [0,T]=2     | reduce by ToT*F(pop 2  T*F  no. of symbols from stack and push F on the stack)  |
| \$0T2        | +id\$        | [2,+]=r2      | [0,E]=1     | reduce by EoT (pop 2 T  no. of symbols from stack and push E onto the stack)    |
| \$0E1        | +id\$        | [1,+]=s6      |             | shift 6   |
| \$0E1+6      | id\$         | [6,id]=s5     |             | shift 5   |
| \$0E1+6id5   | \$           | [5,\$]=r6     | [6,F]=3     | reduce by Foid (pop 2 id  no. of symbols from stack and push F onto the stack)  |
| \$0E1+6F3    | \$           | [3,\$]=r4     | [6,F]=3     | reduce by ToF (pop 2 F  no. of symbols from stack and push T onto the stack)    |
| \$0E1+6T9    | \$           | [9,\$]=r1     | [0,E]=1     | reduce by EoE+T (pop 2  E+T  no. of symbols from stack and push F on the stack) |
| \$0E1        | \$           |               |             | accept  |

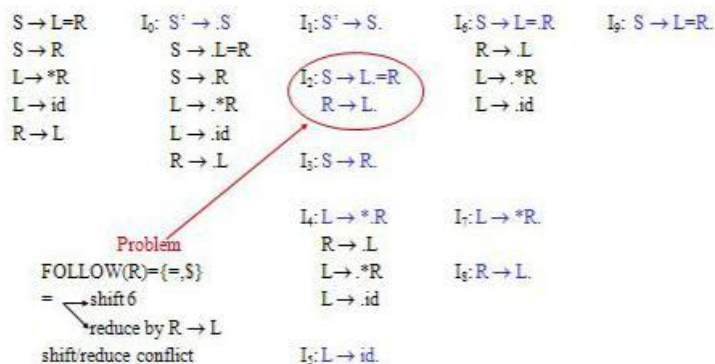
**SLR(1) Grammar**

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).
- Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

**shift/reduce and reduce/reduce conflicts**

- If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a **shift/reduce conflict**.
- If a state does not know whether it will make a reduction operation using the production rule i or j for a terminal, we say that there is a **reduce/reduce conflict**.
- If the SLR parsing table of a grammar G has a conflict, we say that that grammar is not SLR grammar.

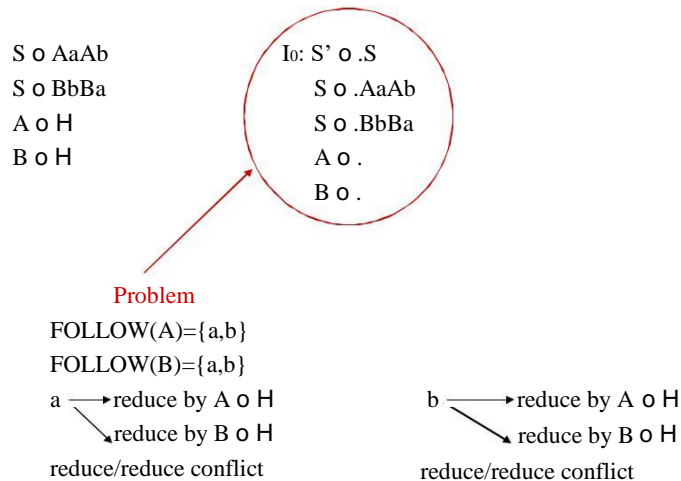
1.  $S \rightarrow SS+ \mid SS^* \mid a$  with the string “aa+a\*”
2.  $S \rightarrow (L) \mid a, L \rightarrow L, S \mid S$   
but not LL(1)  
 $S \rightarrow SA \mid$   
 $A \rightarrow a$
3.  $S \rightarrow aSb \mid ab$
4.  $S \rightarrow aSbS \mid bSaS \mid \square$
5.  $S \rightarrow E \# E$   
 $E \rightarrow T \mid E$   
 $T \rightarrow T \mid F$   
 $T \rightarrow F \mid F$   
 $E \rightarrow (E) \mid F \mid$   
 $i$
6.  $S \rightarrow +SS \mid *SS \mid a$  with the string “+\*aaa”
7. Show that following grammar is SLR(1)
8.  $X \rightarrow Xb \mid a$  parse the string “abb”
9. Given the grammar  $A \rightarrow (A) \mid a$  string “((a))”

**Conflict Example**

Construct parsing table for this. In this table there are 2 actions in one entry of the table which is why It is not a SLR(1) grammar.

**Another example for not SLR(1) grammar:**

### Conflict Example2



**Problems : show that following grammars are not SLR(1) by constructing parsing table.**

1. Show that  $S \rightarrow S(S)S \mid \square$  not SLR(1)

2. Show that  $S \rightarrow AaAb \mid BbBa$

$A \rightarrow \square$

$B \rightarrow \square$  is not SLR(1) but is LL(1)

### Question Bank

1. a. Construct SLR (1) parsing table for the given grammar.

**$E \rightarrow E + T \mid T$**

**$T \rightarrow T * F \mid F$**

**$F \rightarrow (E) \mid id$**

2. Discuss S attribute and L attribute with respect to SDD.  
3. Write the algorithm to eliminate left recursion and for left factoring.  
4. Show stack implementation for given grammar using shift reduce parsing technique.

**$S \rightarrow S + S$**

**$S \rightarrow S - S$**

**$S \rightarrow (S)$**

**$S \rightarrow a$**

**Input string is  $a1-(a2+a3)\$$**

5. What is the role of parser? Explain the different error recovery strategies.  
6. Using the operator-precedence parsing algorithm, construct the table and parse the input string  $id+id*id$ .  
7. What are the actions of a shift reduce parser. Design shift-reduce parser for the following grammar on the input 10201  $S \rightarrow 0 S 0 \mid 1 S 1 \mid 2$ .