Solucionario 5

lunes, febrero 15, 2021 7:35 PM

Diagrama de Fases

Trazar las curvas que son soluciones del sistema; es decir, graficar las curvas formadas por los puntos $(X(t), Y(t))$.

$\dot{X}$: Crecimiento de X cuando crece t

$\dot{X} = f(X,Y)$: determina la trayectoria de la partícula con respecto a X

Lo mismo para $\dot{Y}$.

Pasos Generales

1) Despejar la curva

$\dot{X}=0 \longrightarrow \boxed{f(x,y)=0}$

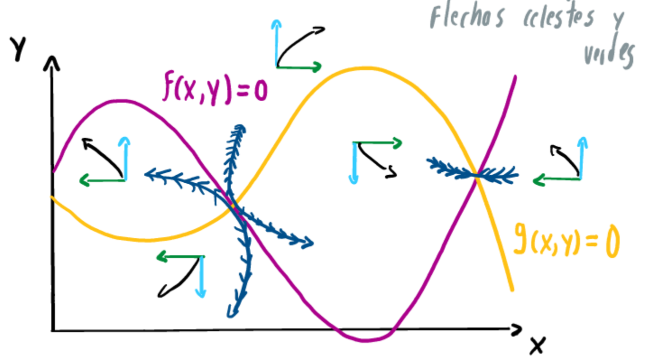
$\dot{Y}=0 \longrightarrow \boxed{g(x,y)=0}$

2) Determinar las regiones :

$\frac{\partial \dot{X}}{\partial X}, \frac{\partial \dot{Y}}{\partial Y}, \frac{\partial \dot{Y}}{\partial X}, \frac{\partial \dot{X}}{\partial Y} \quad \begin{matrix} >0 \\ <0 \end{matrix}$

3) Graficar

Acá pueden preguntar las Flechas celestes y verdes



Para un desarrollo matemático del diagrama de fases, revisar el desarrollo del ejercicio 2 de la PD 2

Ejercicio 1

Pasos:

① $\dot{y} = ay + bx + h \to \dot{y} = 0 \to 0 = ay + bx + h$

$Y = -\frac{b}{a}X - \frac{h}{a} \quad ; \quad \frac{\partial Y}{\partial X} = -\frac{b}{a} = m_Y$

$\dot{x} = cy + dx + K \to \dot{x} = 0 \to 0 = cy + dx + K$

$Y = -\frac{d}{c}X - \frac{K}{c} \quad ; \quad \frac{\partial Y}{\partial X} = -\frac{d}{c} = m_X$

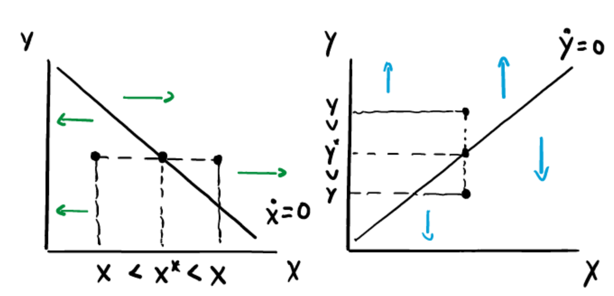
② $\frac{\partial \dot{Y}}{\partial Y} = a \quad ; \quad \frac{\partial \dot{X}}{\partial X} = d$

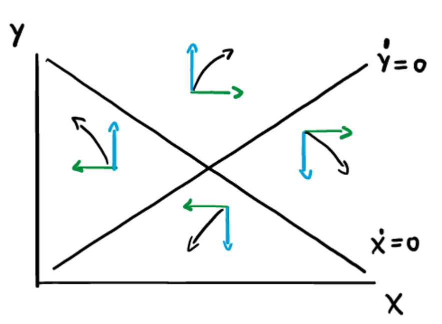
2a) $a, c, d > 0$ y $b < 0$

$\frac{\partial \dot{X}}{\partial X} > 0 \left[\begin{matrix} X > X^* \longrightarrow X \quad \rightarrow \\ X < X^* \longrightarrow X \quad \leftarrow \end{matrix} \right.$ $m_Y > 0$

$\frac{\partial \dot{Y}}{\partial Y} > 0 \left[\begin{matrix} Y > Y^* \longrightarrow Y \quad \uparrow \\ Y < Y^* \longrightarrow Y \quad \downarrow \end{matrix} \right.$ $m_X < 0$

3a)



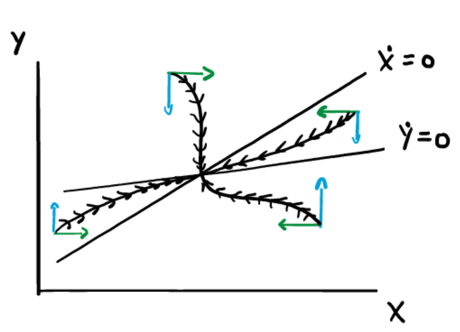


2b) $b, c > 0$ y $a, d < 0$

$\frac{\partial \dot{X}}{\partial X} < 0 \left[\begin{matrix} X > X^* \longrightarrow X \quad \leftarrow \\ X < X^* \longrightarrow X \quad \rightarrow \end{matrix} \right.$ $m_Y > 0$

$\frac{\partial \dot{Y}}{\partial Y} < 0 \left[\begin{matrix} Y > Y^* \longrightarrow Y \quad \downarrow \\ Y < Y^* \longrightarrow Y \quad \uparrow \end{matrix} \right.$ $m_X > 0$

3b)

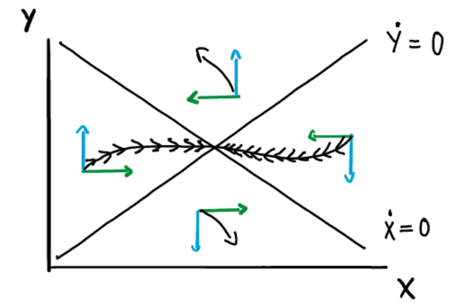


2c) $a > 0$ y $b, c, d < 0$

$\frac{\partial \dot{X}}{\partial X} < 0 \left[\begin{matrix} X > X^* \longrightarrow X \quad \leftarrow \\ X < X^* \longrightarrow X \quad \rightarrow \end{matrix} \right.$ $m_Y > 0$

$\frac{\partial \dot{Y}}{\partial Y} > 0 \left[\begin{matrix} Y > Y^* \longrightarrow Y \quad \uparrow \\ Y < Y^* \longrightarrow Y \quad \downarrow \end{matrix} \right.$ $m_Y < 0$

3c)

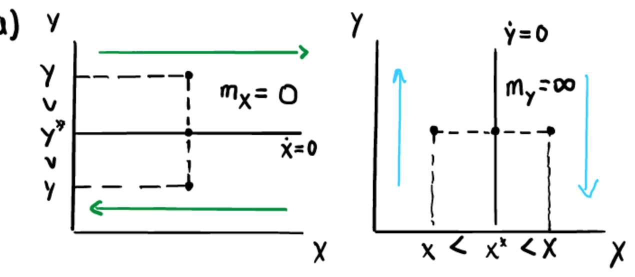


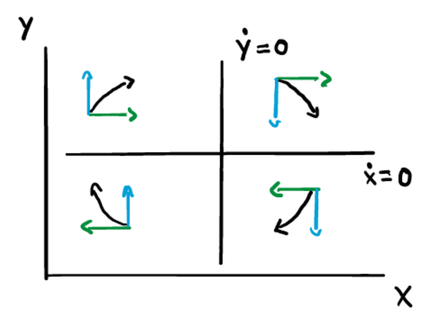
2d) $c > 0$, $b < 0$ y $a, d = 0$

$\frac{\partial \dot{X}}{\partial X} = 0 \to \frac{\partial \dot{X}}{\partial Y} = c > 0 \left[\begin{matrix} Y > Y^* \longrightarrow X \quad \rightarrow \\ Y < Y^* \longrightarrow X \quad \leftarrow \end{matrix} \right.$

$\frac{\partial \dot{Y}}{\partial Y} = 0 \to \frac{\partial \dot{Y}}{\partial X} = b < 0 \left[\begin{matrix} X > X^* \longrightarrow Y \quad \downarrow \\ X < X^* \longrightarrow Y \quad \uparrow \end{matrix} \right.$

3a)





Ejercicio 2

a) $\dot{Y} = -3Y + Y^2 + 2$

Pasos :

1. $\dot{Y} = 0 \to 0 = -3Y + Y^2 + 2 \to Y_1 = 2$, $Y_2 = 1$

2. $\frac{\partial \dot{Y}}{\partial Y} = -3 + 2Y = 0$

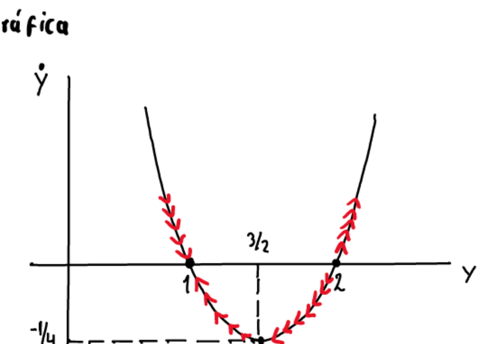
$Y = \frac{3}{2} \longrightarrow \dot{Y} \longrightarrow \dot{Y} = -\frac{1}{4}$ (Puntos críticos)

* $\frac{\partial^2 \dot{Y}}{\partial Y^2} = 2 > 0$

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=2} = 1 > 0$: inestable

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=1} = -1 < 0$: estable

3. Gráfica



b) $\dot{Y} = 8Y - 2Y^2$

Pasos :

1. $\dot{Y} = 0 \longrightarrow 0 = 8Y - 2Y^2 \longrightarrow Y(Y-4) = 0 \longrightarrow Y_1 = 0$, $Y_2 = 4$

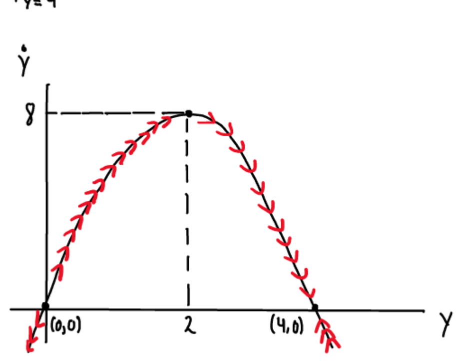
2. $\frac{\partial \dot{Y}}{\partial Y} = 8 - 4Y = 0 \longrightarrow Y = 2 \longrightarrow \dot{Y} = 8$

$\frac{\partial^2 \dot{Y}}{\partial Y^2} = -4 < 0$ max

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=0} = 8 > 0$ inestable

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=4} = -8 < 0$ estable

3.



Ejercicio 3

b) $\dot{X} = Y - X^2 + 3$
$\dot{Y} = Y - X + 1$

Pasos :

1. $\dot{X} = 0 = Y - X^2 + 3 \longrightarrow Y = X^2 - 3$ (Parábola)

$\dot{Y} = 0 = Y - X + 1 \longrightarrow Y = X - 1$ (Recta)

intersección

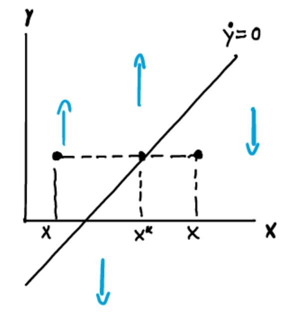
$X^2 - 3 = X - 1 \to X^2 - X - 2 = 0$

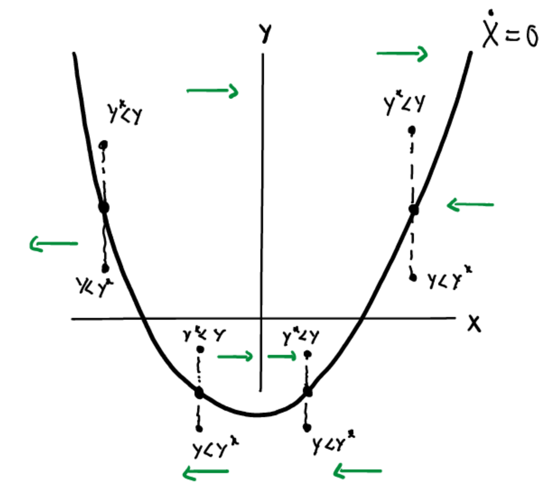
$X_1 = 2 \longrightarrow Y_1 = 1$
$X_2 = -1 \longrightarrow Y_2 = -2$

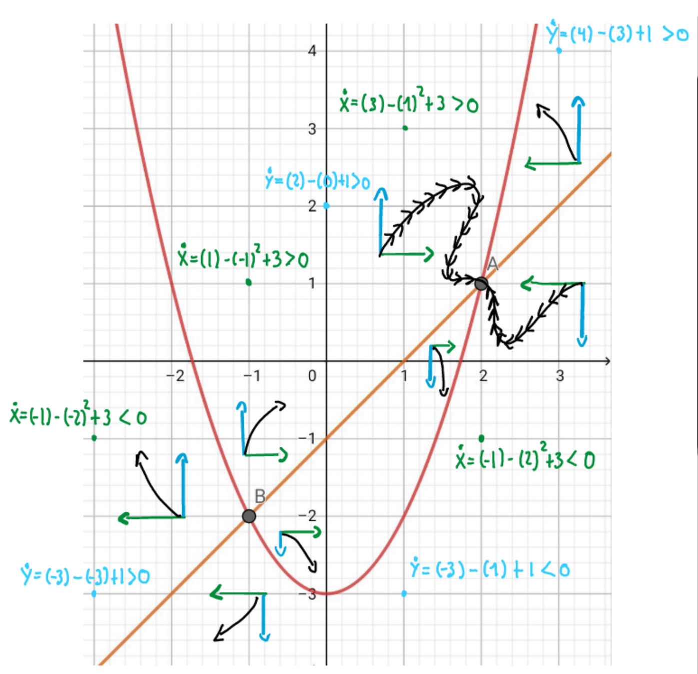
2. $\frac{\partial \dot{X}}{\partial X} = -2X \to \frac{\partial \dot{X}}{\partial Y} = 1 > 0 \left[\begin{matrix} Y > Y^* \to X \quad \rightarrow \\ Y < Y^* \to X \quad \leftarrow \end{matrix} \right.$

$\frac{\partial \dot{Y}}{\partial Y} = 1 \to \frac{\partial \dot{Y}}{\partial X} = -1 < 0 \left[\begin{matrix} X > X^* \to Y \quad \downarrow \\ X < X^* \to Y \quad \uparrow \end{matrix} \right.$

3.







c) $\dot{X} = Y - X^3$
$\dot{Y} = 1 - XY$

Pasos :

1. $\dot{X} = 0 = Y - X^3 \longrightarrow Y = X^3$

$\dot{Y} = 0 = 1 - XY \longrightarrow Y = \frac{1}{X}$

Intersección

$X^3 = \frac{1}{X} \longrightarrow X^4 = 1 \longrightarrow X^4 - 1 = 0$

$(X^2+1)(X^2-1) = 0$

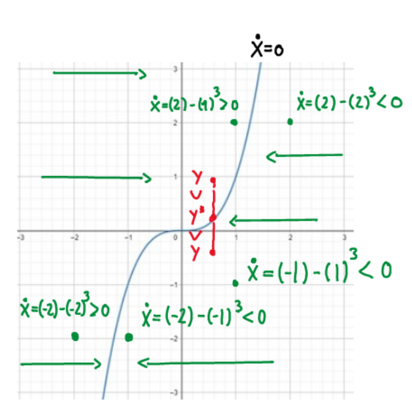
$(X^2+1)(X+1)(X-1) = 0$

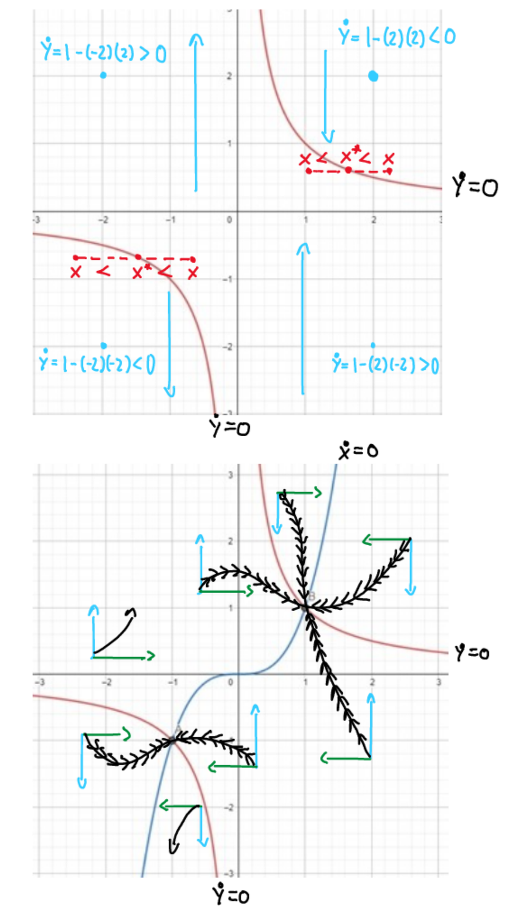
$X_1 = 1 \longrightarrow Y_1 = 1$
$X_2 = -1 \longrightarrow Y_2 = -1$
$X_3 = i$
$X_4 = -i$ ✗

2. $\frac{\partial \dot{X}}{\partial Y} = 1 > 0 \left[\begin{matrix} Y > Y^* \to X \quad \rightarrow \\ Y < Y^* \to X \quad \leftarrow \end{matrix} \right.$

$\frac{\partial \dot{Y}}{\partial X} = -Y \left[\begin{matrix} Y > 0 \to < 0 \left[\begin{matrix} X > X^* \longrightarrow Y \quad \downarrow \\ X < X^* \longrightarrow Y \quad \uparrow \end{matrix} \right. \\ Y < 0 \to > 0 \left[\begin{matrix} X > X^* \longrightarrow Y \quad \uparrow \\ X < X^* \longrightarrow Y \quad \downarrow \end{matrix} \right. \end{matrix} \right.$

3.





Ejercicio 5

$\dot{P} = \alpha[D(P) - O(P^e)] \qquad \dot{P}^e = \beta(P - P^e)$

Paso 1 :

1. $\dot{P} = 0 = \alpha[D(P) - O(P^e)] \longrightarrow D(P) - O(P^e) = 0$

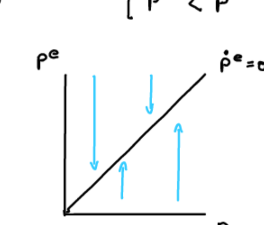
Por teorema de la función implícita

$D'(P)\,dP - O'(P^e)\,dP^e = 0$

$\frac{dP^e}{dP} = \frac{D'(P)}{O'(P^e)} \begin{matrix} <0 \\ >0 \end{matrix} < 0$

$\dot{P}^e = 0 = \beta(P - P^e) \longrightarrow P = P^e \longrightarrow \frac{dP^e}{dP} = 1$

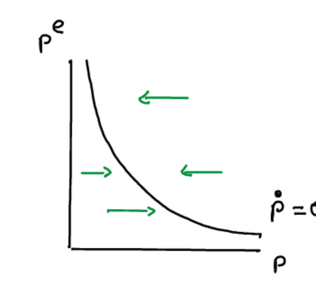
2. * $\frac{\partial \dot{P}^e}{\partial P^e} < 0 \left[\begin{matrix} P^e > P^{e*} \longrightarrow P^e \quad \downarrow \\ P^e < P^{e*} \longrightarrow P^e \quad \uparrow \end{matrix} \right.$



* $\frac{\partial \dot{P}}{\partial P} = \frac{\partial \dot{P}}{\partial D(P)} \cdot \frac{\partial D(P)}{\partial P}$

$\frac{\partial \dot{P}}{\partial D(P)} \to \alpha \to > 0$; $\frac{\partial D(P)}{\partial P} \to 0 <$

$\frac{\partial \dot{P}}{\partial P} < 0 \left[\begin{matrix} P > P^* \longrightarrow P \quad \leftarrow \\ P < P^* \longrightarrow P \quad \rightarrow \end{matrix} \right.$



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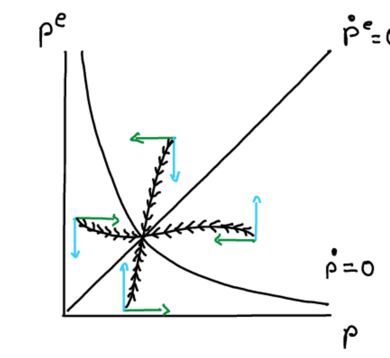


Ejercicio 6

1. $\dot{Y} = 0 \longrightarrow Y = E(Y - T, r) + G$

$dY = \frac{\partial E}{\partial Y}dY + \frac{\partial E}{\partial r}dr$

$dY = E_Y\,dY + E_r\,dr$

$(1 - E_Y)\,dY = E_r\,dr$

$\frac{dY}{dr} = \frac{(1 - E_Y)}{E_Y} < 0$: IS

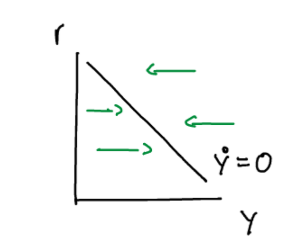
$\dot{r} = 0 \longrightarrow L(Y, r) = M/P$

$dL = \frac{\partial L}{\partial Y}dY + \frac{\partial L}{\partial r}dr = 0$

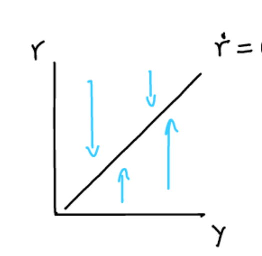
$dL = L_Y\,dY + L_r\,dr = 0$

$\frac{dY}{dr} = \frac{-L_r}{L_Y} > 0$: LM

2. $\frac{\partial \dot{Y}}{\partial Y} = -1 < 0 \left[\begin{matrix} Y > Y^* \longrightarrow Y \quad \leftarrow \\ Y < Y^* \longrightarrow Y \quad \rightarrow \end{matrix} \right.$



$\frac{\partial \dot{r}}{\partial r} = L_r < 0 \left[\begin{matrix} r > r^* \longrightarrow r \quad \downarrow \\ r < r^* \longrightarrow r \quad \uparrow \end{matrix} \right.$



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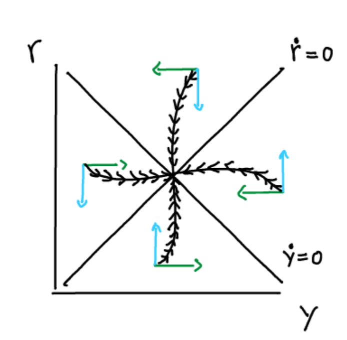

Ejercicio 7

$f(.) = \ln(c)e^{-\rho t} = \ln(AK - \dot{K} - \delta K)e^{-\rho t}$

a) Condiciones necesarias : Condición de Euler

* $f_K = \frac{1}{c}(A-\delta)e^{-\rho t}$

$f_{\dot{K}} = \frac{1}{c}(-1)e^{-\rho t}$

$\frac{\partial f_{\dot{K}}}{\partial t} = -\frac{\dot{c}}{c^2}(-1)e^{-\rho t} + \frac{\rho}{c}e^{-\rho t}$

$f_K = \frac{\partial f_{\dot{K}}}{\partial t}$

$\frac{1}{c}(A-\delta)e^{-\rho t} = \frac{\dot{c}}{c^2}e^{-\rho t} + \frac{\rho}{c}e^{-\rho t}$

$\dot{c} = (A - \delta - \rho)c$

∴ $C_{(t)} = H_1 e^{(A-\delta-\rho)t}$

* $\dot{K} = AK - \delta K - c$

$\dot{K} - (A - \delta)K = -c$

FI: $e^{\int -(A-\delta)dt} = e^{-(A-\delta)t}$

$Ke^{-(A-\delta)t} = -\int ce^{-(A-\delta)t}dt$

$c = H_1 e^{(A-\delta-\rho)t} \longrightarrow ce^{-(A-\delta)t} = H_1 e^{-\rho t}$

$Ke^{-(A-\delta)t} = -\int H_1 e^{-\rho t}$

∴ $K_{(t)} = \frac{H_1}{\rho}e^{(A-\delta-\rho)t} + H_2 e^{(A-\delta)t}$

b) Condiciones de transversalidad

$t = 0 \to K_{(0)} = 10 = \frac{H_1}{\rho} + H_2$

$t = \infty \neq t = T$

$\to \lim_{t\to\infty} [f_{\dot{Y}}] = 0$, valor terminal desconocido

$\lim_{t\to\infty} [Y_{(t)}] = Y_\infty$, " " conocido

El segundo es una alternativa del primero, solo cuando el horizonte de tiempo es infininito

$\lim_{t\to\infty} K = \frac{H_1}{\rho}e^{(A-\delta-\rho)t} + H_2 e^{(A-\delta)t} = 0$

$e^{(A-\delta-\rho)t} \to 0$

$H_2 \underbrace{e^{(A-\delta)t}}_{\infty} = 0$

$H_2 = 0$

$10 = \frac{H_1}{\rho} + H_2 \longrightarrow H_1 = 10\rho$

∴ $K^*_{(t)} = 10\rho e^{(A-\delta-\rho)t}$

$C^*_{(t)} = 10\rho e^{(A-\delta-\rho)t}$

c) Condiciones suficientes

$\frac{\partial f}{\partial K} = \frac{1}{c}(A-\delta)e^{-\rho t} \left[\begin{matrix} f_{KK} = -c^{-2}(A-\delta)^2 e^{-\rho t} \\ f_{K\dot{K}} = c^{-2}(A-\delta)e^{-\rho t} \end{matrix} \right.$

$\frac{\partial f}{\partial \dot{K}} = -\frac{1}{c}e^{-\rho t} \left[\begin{matrix} f_{\dot{K}K} = c^{-2}(A-\delta)e^{-\rho t} \\ f_{\dot{K}\dot{K}} = -c^{-2}e^{-\rho t} \end{matrix} \right.$

$$H = \begin{bmatrix} -c^{-2}e^{-\rho t} & c^{-2}(A-\delta)e^{-\rho t} \\ c^{-2}(A-\delta)e^{-\rho t} & -c^{-2}(A-\delta)^2 e^{-\rho t} \end{bmatrix}$$

$H_1 = -c^{-2}e^{-\rho t} < 0$

$H_2 = c^{-4}(A-\delta)^2 e^{-2\rho t} - c^{-4}(A-\delta)^2 e^{-2\rho t} = 0$

∴ Función concava $\longrightarrow$ Max local

d) Diagrama de Fases

1. $\dot{c} = 0 \longrightarrow (A - \delta - \rho)c = 0$
$c = 0$

$\dot{K} = 0 \longrightarrow AK - \delta K - c = 0$
$c = (A - \delta)K$

2. $\frac{\partial \dot{c}}{\partial c} = (A - \delta - \rho) < 0 \left[\begin{matrix} c > c^* \longrightarrow c \quad \downarrow \\ c < c^* \longrightarrow c \quad \uparrow \end{matrix} \right.$

$\frac{\partial \dot{K}}{K} = (A - \delta) > 0 \left[\begin{matrix} K > K^* \longrightarrow K \quad \rightarrow \\ K < K^* \longrightarrow K \quad \leftarrow \end{matrix} \right.$

3.