Ejercicio 1

a) $\frac{dY}{dt} = 15 \longrightarrow dy = 15\,dt$

$\int dy = \int 15\,dt$

$Y = 15t + A$

b) $\frac{dY}{dt} + 5 = 0 \longrightarrow dy = -5\,dt$

$\int dy = \int -5\,dt$

$y = -5t + A$

c) $\frac{dY}{dt} - 6Y = 18 \longrightarrow dy = 6(3+Y)\,dt$

S. Particular (Y_P)

$\dot{Y} = 0 \to -6Y = 18$

$Y = -3$

S. Complementaria (Y_C)

$r - 6 = 0$

$r = 6$

$Y = Ae^{6t}$

∴ $Y_G = Ae^{6t} - 3$

$\frac{dY}{(3+Y)} = 6\,dt$

$\int \frac{dY}{(3+Y)} = \int 6\,dt$

$\ln(3+Y) = 6t + A$

$3 + Y = e^{6t+A}$

∴ $Y = Ae^{6t} - 3$

d) $\frac{dY}{dt} + 4tY = 6t \longrightarrow dy = (3+2Y)2t\,dt$

$\frac{dy}{(3+2Y)} = 2t\,dt$

$\int \frac{dy}{(3+2Y)} = \int 2t\,dt$ $U = 3+2Y$, $dU = 2dy$, $\frac{dU}{2} = dy$

$\int \frac{dU}{2U} = \int 2t\,dt$

$\ln U = 2\cdot\frac{2t^2}{2} = 2t^2 + A$

$3 + 2Y = U = e^{2t^2+A} = Be^{2t^2}$

∴ $Y = Ce^{2t^2} - 1.5$

e) $2\frac{dY}{dt} - 2t^2Y = 9t^2 \longrightarrow dy = (4.5+Y)t^2\,dt$

$\frac{dy}{(4.5+Y)} = t^2\,dt$ $4.5 + Y = U$, $dY = dU$

$\int \frac{dU}{U} = \int t^2\,dt$

$\ln U = \frac{t^3}{3} + A$

$4.5 + Y = U = e^{\frac{t^3}{3}+A} = Be^{\frac{t^3}{3}}$

∴ $Y = Be^{\frac{t^3}{3}} - 4.5$

F) $\frac{dY}{dt} - 2tY = e^{t^2}$

FI: $e^{\int -2t\,dt} = e^{-t^2}$

$e^{-t^2}\frac{dY}{dt} - 2tYe^{-t^2} = e^{t^2}e^{-t^2}$

$\frac{dYe^{-t^2}}{dt} = 1$

$\int dYe^{-t^2} = \int dt$

$Ye^{-t^2} = t + A$

∴ $Y = (t+A)e^{t^2}$

$f(x) = u\cdot v$

$f'(x) = u'v + uv'$

$\to f(x) = Y\cdot e^{-t^2}$

$f'(x) = \underbrace{\dot{Y}}_{u'}\underbrace{e^{-t^2}}_{v} + \underbrace{e^{-t^2}(-2t)}_{v'}\underbrace{Y}_{u}$

g) $(t+5)\frac{dY}{dt} - (Y+9)\frac{dt}{dt} = 0$

$\frac{dy}{dt} = \frac{Y+9}{t+5}$ $U = t+5$, $dU = dt$

$z = Y+9$, $dz = dy$

$\frac{dY}{Y+9} = \frac{dt}{t+5}$

$\int \frac{dz}{z} = \int \frac{dU}{U}$

$\ln z = \ln U + A$

$\ln z - \ln U = A$

$\ln\left(\frac{z}{U}\right) = A$

$\frac{z}{U} = B$

$\frac{Y+9}{t+5} = B$

∴ $Y = B(t+5) - 9$

h) $Y^2(t^3+1)\frac{dy}{dt} + t^2(Y^3-5)\frac{dt}{dt} = 0$

$\frac{dY}{dt} = \frac{-t^2(Y^3-5)}{Y^2(t^3+1)}$ $U = t^3+1$, $\frac{dU}{3} = t^2dt$

$z = Y^3 - 5$, $\frac{dz}{3} = 2dY$

$\frac{Y^2\,dY}{(Y^3-5)} = \frac{-t^2\,dt}{(t^3+1)}$

$\int \frac{dz}{\cancel{3}z} = \int -\frac{dU}{\cancel{3}U}$

$\ln z = -\ln U + A$

$\ln(z\cdot U) = A$

$z\cdot U = B$

$(Y^3-5)(t^3+1) = B$

∴ $Y = \left(\frac{B}{t^3+1} + 5\right)^{1/3}$

EDO de segundo Orden

$Y'' + b_1Y' + b_2Y = a$

$Y_G = Y_C + Y_P$

$Y_C = A_1e^{r_1t} + A_2e^{r_2t}$

$Y'' + b_1Y' + b_2Y = a$

$r^2 + b_1r + b_2 = 0$

$r_1, r_2 = \frac{-b_1 \pm \sqrt{b_1^2 + 4b_2}}{2}$

$Y_P = \begin{cases} \frac{a}{b_2} & ;\ b_2 \neq 0 \\ \frac{at}{b_1} & ;\ b_2 = 0,\ b_1 \neq 0 \\ \frac{at^2}{2} & ;\ b_2 = b_1 = 0 \end{cases}$

i) $Y'' - 5Y' + 4Y = 2$

$Y_P = \frac{2}{4} = \frac{1}{2} = 0.5$

$Y_C \to r^2 - 5r + 4 = 0$

$\boxed{r = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)}}{2},\ r = \frac{5\pm3}{2},\ r = 1, 4}$

$(r-4)(r-1) = 0 \to r = 1, 4$

$Y_C = A_1e^{t} + A_2e^{4t}$

∴ $Y_G = A_1e^{t} + A_2e^{4t} + 0.5$

j) $Y'' + 3Y' = 12$

$Y_P = \frac{12}{3}t = 4t$

$Y_C \to r^2 + 3r = 0$

$r(r+3) = 0$

$r = 0, -3$

$Y_C = A_1e^{0t} + A_2e^{-3t} = A_1 + A_2e^{-3t}$

∴ $Y_G = A_1 + A_2e^{-3t} + 4t$

k) $Y'' = 16$

$Y_P = \frac{16t^2}{2} = 8t^2$

$Y_C \to r^2 = 0$

$r = 0$

$Y_C = A_1e^{0t} + A_2e^{0t} = A_1 + A_2 = C$

∴ $Y_G = 8t^2 + C$

$Y'' = 16 \xrightarrow{\int} Y' = 16t + C \xrightarrow{\int} Y = 8t^2 + C$

l) $Y'' + 3Y' - 10Y = 7e^t$; $Y(0) = -\frac{35}{36}$, $Y'(0) = -\frac{5}{36}$

$Y_P = -0.7te^t$

$Y_C \to r^2 + 3r - 10 = 0$

$(r-2)(r+5) = 0$

$r = 2, -5$

$Y_C = A_1e^{2t} + A_2e^{-5t}$

$Y_G = A_1e^{2t} + A_2e^{-5t} - 0.7te^t$

$t = 0 \to$ $Y(0) = -\frac{35}{36} = A_1 + A_2 \to A_2 = -A_1 - \frac{35}{36}$

$Y'(0) = -\frac{5}{36} = 2A_1 - 5A_2 - \frac{7}{10} = 2A_1 - 5\left(A_1 - \frac{35}{36}\right) - \frac{7}{10}$

$Y' = 2A_1e^{2t} + (-5)A_2e^{-5t} - 0.7e^t - 0.7te^t$

$Y'(0) = 2A_1 - 5A_2 - 0.7$

$-\frac{5}{36} = 7A_1 + \frac{35.5}{36} - \frac{7}{10}$

$A_1 = \frac{-\frac{5}{36} - \frac{35.5}{36} + \frac{7}{10}}{7} = -\frac{43}{70}$

$A_2 = -\left(-\frac{43}{70}\right) - \frac{35}{36} = -\frac{451}{1260}$

∴ $Y_P = -\frac{43}{70}e^{2t} - \frac{451}{1260}e^{5t} - \frac{7te^t}{10}$

m) $2tY'' - Y' + \frac{1}{Y'} = 0$

$u = Y' \longrightarrow u' = Y''$

$2tu' - u + \frac{1}{u} = 0$

$u' = \frac{u^2-1}{2ut} = \frac{\partial u}{\partial t}$

$\frac{2u\,\partial u}{u^2-1} = \frac{1}{t}\,dt$

$\ln(u^2-1) = \ln t + A$

$\ln\left(\frac{u^2-1}{t}\right) = A$

$\frac{u^2-1}{t} = B$

$u = \pm\sqrt{Bt+1}$

$\partial y = \pm\sqrt{Bt+1}\,\partial t$

$Y = \frac{1}{B}\int z^{1/2}\,dz$

∴ $Y = \pm\frac{1}{B}(Bt+1)^{3/2} + C_2$

n) $2YY'' = 1 + (Y')^2$

$u = Y' \to Y'' = u'$

$\frac{\partial^2 Y}{\partial t^2} = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial t} = \frac{\partial u}{\partial Y}u$

$Y'' = u\frac{\partial u}{\partial Y}$

$2Yu\frac{\partial u}{\partial Y} = 1 + u^2$

$\frac{2u\,\partial u}{1+u^2} = \frac{\partial Y}{Y}$

$\ln(1+u^2) = \ln Y + C$

$1 + u^2 = BY$

$u = \pm\sqrt{BY-1}$

$\int \frac{dY}{\sqrt{BY-1}} = \int \pm dt$

$\frac{2\sqrt{BY-1}}{B} = \pm t + C$

∴ $Y = \frac{1}{B} + \frac{B(t+C)^2}{4}$

Ejercicio 2

$-4(5+Y)$

EDO 1er Orden

a) $\frac{dY}{dt} + 4Y = -20 \longrightarrow \frac{dY}{5+Y} = -4\,dt$

$\ln(5+Y) = -4t + A$

$5 + Y = Be^{-4t}$

Solución General (SG): $Y = Be^{-4t} - 5$

Condición inicial (CI)

$t = 0 \to Y(0) = 10 = B - 5 \to B = 15$

Solución Específica (SE)

∴ $Y = 15e^{-4t} - 5$

b) $\frac{dY}{dt} = 3Y \longrightarrow \frac{dY}{Y} = 3dt$

$\ln Y = 3t + A$

SG: $Y = Be^{3t}$

CI: $t = 0 \to Y(0) = 2 = B$

SE: $Y = 2e^{3t}$

c) $\frac{dY}{dt} + 3Y = 6t$

$FI = e^{\int 3dt} = e^{3t}$

$e^{3t}\dot{Y} + 3e^{3t}Y = 6te^{3t}$

$\int dYe^{3t} = \int 6te^{3t}\,dt$

$Ye^{3t} = 6\int te^{3t}$

$\int fg' = fg - \int f'g$

$f = t$, $g' = e^{3t}$

$f' = 1$, $g = \frac{e^{3t}}{3}$

$u = 3t$, $\frac{du}{3} = dt$, $\int \frac{e^u}{3}\,du$

e^u, $\ln e$, e^u

$\int te^{3t} = \frac{te^{3t}}{3} - \int \frac{e^{3t}}{3}\,dt$

$= \frac{te^{3t}}{3} - \frac{e^{3t}}{9} + A$

$Ye^{3t} = \cancel{6}^{\,2}\left(\frac{te^{3t}}{\cancel{3}} - \frac{e^{3t}}{\cancel{9}_{\,3}}\right) + A$

$= 2te^{3t} - \frac{e^{3t}}{3} + A$

$= \frac{(3t-1)2e^{3t}}{3} + A$

SG: $Y = Be^{-3t} + 2t - \frac{2}{3}$

CI: $t = 0 \to Y(0) = \frac{1}{3} = B - \frac{1}{3} \to B = 1$

SE: $Y = e^{3t} + 2t - \frac{2}{3}$

EDO de Segundo Orden

a) $Y_P = 36$

$Y_C \to r^2 + r + 1/4 = 0$

$(r + 1/2)^2 = 0 \to r = -1/2, -1/2$

$Y_C = A_1e^{-0.5t} + A_2e^{-0.5t}$

SG: $Y = A_1e^{-0.5t} + A_2e^{-0.5t} + 36$

CI: $A_1 = -6$, $A_2 = 12$

SE: $Y = -6e^{-0.5t} + 12e^{-0.5t} + 36$

b) $Y_P = -7$

$Y_C = A_1e^{5t} + A_2e^{-t} - 7$

SG: $Y = A_1e^{5t} + A_2e^{-t} - 7$

CI: $A_1 = 3$, $A_2 = 9$

SE: $Y = 3e^{5t} + 9e^{-t} - 7$

c) SG: $Y = A_1 + A_2e^{0.5t} - 26t$

CI: $A_2 = 13$, $A_1 = 4$

SP: $Y = 4 + 13e^{0.5t} - 26t$

d) $Y'' + 2Y' + 10Y = 80$ $Y(0) = 10$ $Y'(0) = 13$

$Y_P = \frac{80}{10} = 8$

$Y_C \to r^2 + 2r + 10 = 0$

$r_1, r_2 = \frac{-2 \pm \sqrt{2^2 - 4\cdot10}}{2} = -1 \pm 3i$

raíces complejas

Por relación de euler

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

En nuestro ejercicio

$Y_C = A_1e^{(-1+3i)t} + A_2e^{(-1-3i)t}$

$Y_C = A_1e^{-t}e^{3it} + A_2e^{-t}e^{-3it}$

$Y_C = e^{-t}(A_1e^{3it} + A_2e^{-3it})$

$Y_C = e^{-t}[A_1(\cos 3t + i\sin 3t) + A_2(\cos 3t - i\sin 3t)]$

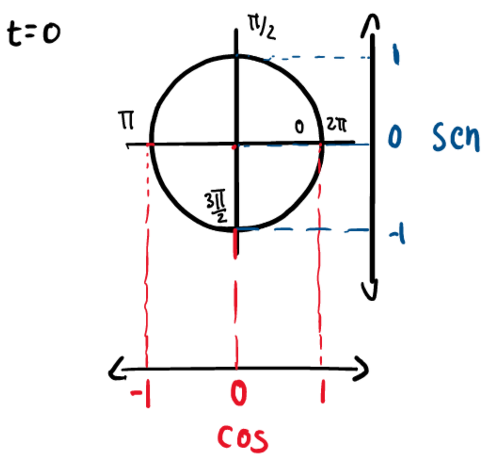
$Y_C = e^{-t}[(A_1+A_2)\cos 3t + i(A_1 - A_2)\sin 3t]$

$Y_C = e^{-t}[A_3\cos 3t + A_4\sin 3t]$

SG: $Y = e^{-t}[A_3\cos 3t + A_4\sin 3t] + 8$

CI:

$t = 0$



$Y(0) = 10 = 1[A_3\cos 0 + A_4\sin 0] + 8$

$Y(0) = 10 = A_3 + 0 + 8$

$2 = A_3$

$Y' = e^{-t}(-3.2\sin 3t + 3A_4\cos 3t) - e^{-t}(2\cos 3t + A_4\sin 3t)$

$Y'(0) = 1(-3.2\cdot 0 + 3A_4) - (2 + A_4\,0)$

$Y'(0) = 13 = 3A_4 - 2 \longrightarrow A_4 = 5$

SE: $Y = e^{-t}(2\cos 3t + 5\sin 3t) + 8$

Ejercicio 3

$\frac{dP_t}{dt} = bM_0 - bKP_tQ$

$\hat{P}_t = P_t - P_e$

$\frac{\partial \hat{P}_t}{\partial t} = \frac{\partial P_t}{\partial t} - \frac{\partial P_e}{\partial t}$

En equilibrio $\frac{\partial P_e}{\partial t} = 0$

$\frac{d\hat{P}_t}{dt} = \frac{\partial P_t}{\partial t} = bM_0 - bKP_tQ$

Dado el equilibrio

$M_0 = M_d = KP_eQ \to M_0 - KP_eQ = 0$

Y

$b(M_0 - KP_eQ) = 0$

$\frac{\partial \hat{P}}{\partial t} = -bKQ\hat{P} \longrightarrow \frac{\partial \hat{P}}{\hat{P}} = -bKQ\,dt$

$\ln \hat{P} = -bKQt + C$

∴ $\hat{P} = Ae^{-bKQt}$

Cuando $t \to \infty$, entonces

$\hat{P} \to 0$