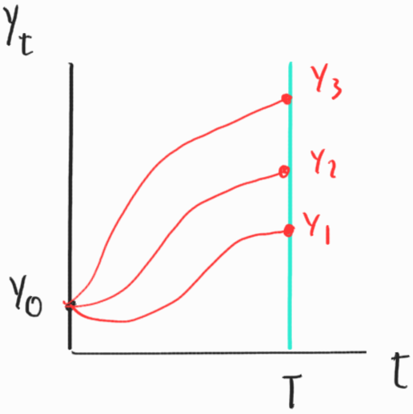
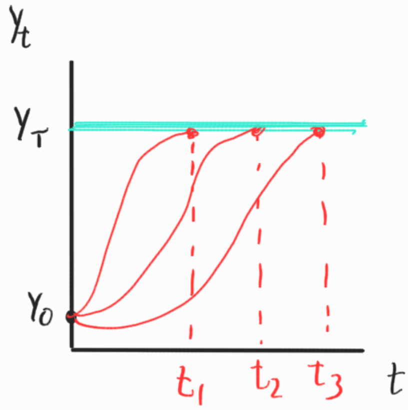
Condiciones de transversalidad

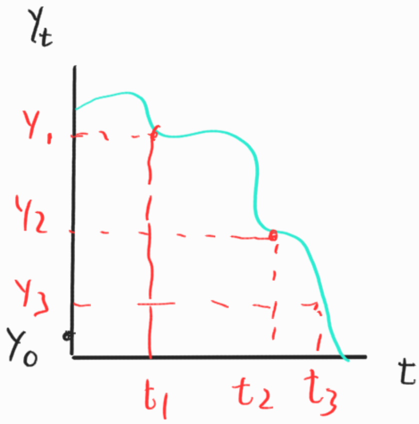
Caso I



Caso II



Caso III



Horizonte de tiempo Finito

$$V(y) = \int_0^T f(.)\,dt \longrightarrow \boxed{[f_{\dot{y}}]_{t=T}\,\Delta Y_T + [f - \dot{y} f_{\dot{y}}]_{t=T}\,\Delta T = 0}$$

Horizonte temporal Fijo

T , dado

Y_T , libre

$$\boxed{[f_{\dot{y}}]_{t=T} = 0}$$

Valor terminal Fijo

T , libre

Y_T , dado

$$\boxed{[f - \dot{y} f_{\dot{y}}]_{t=T} = 0}$$

Curva terminal

Y, T libre, $Y_{(T)} = f_{(t)} = \Phi_{(T)}$

$$\boxed{[f + (\Phi' - \dot{y}) f_{\dot{y}}]_{t=T} = 0}$$

Horizonte de tiempo Infinito

$$V(y) = \int_0^\infty f(.)\,dt \longrightarrow \boxed{\lim_{t\to\infty}\left([f_{\dot{y}}]_{t=T}\,\Delta Y_T + [f - \dot{y} f_{\dot{y}}]_{t=T}\,\Delta T\right) = 0}$$

$Y_{(T)}$ libre, o no te dice nada

$$\lim_{t\to\infty} [f_{\dot{y}}]_{t=T}$$

T , libre

$$\lim_{t\to\infty} [f - \dot{y} f_{\dot{y}}]_{t=T}$$

Ejercicio 1

1a) $V(y)=\int_0^2 (t^2-\dot{y}^2)dt$, $y_{(0)}=4$, $y_{(2)}$ libre

Datos

* $f(.) = f(t,\dot{y})$
* $y_{(2)}$ libre $\longrightarrow$ CT ① ② ③

Desarrollo

* $\left.\begin{array}{l} f_y = 0 \\ f_{\dot{y}} = 2\dot{y} \\ f_{\dot{y}t} = 2\ddot{y} \end{array}\right] \longrightarrow 2\ddot{y} = 0 \xrightarrow{\int} 2\dot{y} = H_1 \xrightarrow{\int} y_{(t)} = \frac{H_1}{2}t + H_2$

* Condiciones iniciales y finales

$t=0 \to y_{(0)} = 4 = 0 + H_2 \quad \to \boxed{H_2 = 4}$

CT: $[f_{\dot{y}}]_{t=2} = 0 \to 2\dot{y} = 0 \quad \to \boxed{H_1 = 0}$

* Solución específica

$\therefore y^*_{(t)} = 4$

1b) $V(y)=\int_0^T (t^2-\dot{y}^2)dt$, $y_{(0)}=4$, $y_{(T)}=5$, T libre

Datos

* $f(.) = f(t,\dot{y})$

* T libre $\longrightarrow$ CT ① ② ③

Desarrollo

* $\left.\begin{array}{l} f_y = 0 \\ f_{\dot{y}} = 2\dot{y} \end{array}\right] \to 2\dot{y} = H_1 \longrightarrow y_{(t)} = \frac{H_1}{2}t + H_2$

$\dot{y} = H_1/2 = C_1 \to y = C_1 t + C_2$

* Condiciones iniciales y finales

$\checkmark t=0 \to y_{(0)} = 4 = 0 + H_2 \quad \to \boxed{H_2 = 4}$

CT: $[f_{(.)} - \dot{y} f_{\dot{y}}]_{t=T} = 0$

$H_1 t + H_2$

$\dot{y} = H_1$

$H = T^{1/2}$

$(T + \dot{y}^2) - \dot{y}(2\dot{y}) = 0$

$\dot{y} = T^{1/2}$

$\dot{y} = C_1$

$2\dot{y} = H_1 \to \dot{y} = \frac{H_1}{2}$

$\boxed{H_1 = 2T^{1/2}}$

$C_1 = T^{1/2}$

$\checkmark t = T \to y_{(T)} = 5 = \left(\frac{2T}{2}\right)^{1/2} T + 4$

$5 = (T^{1/2})T + 4$

$T = 1 \to H_1 = 2(1)^{1/2} \to \boxed{H_1 = 2}$

$T = 1 \to C_1 = 1 \to y^*_{(t)} = t + 4$

* SE: $\therefore y^*_{(t)} = t + 4$

1c) $V(y) = -\int_0^T (1+\dot{y}^2)^{0.5} dt$, $y_{(0)} = 1$, $y_{(T)} = 2 - 3T$

Datos

* $f(.) = f(\dot{y})$
* T, y libre $\longrightarrow$ CT ① ② ③

Desarrollo

* $f_{\dot{y}\dot{y}}\ddot{y} = 0 \longrightarrow \ddot{y} = 0 \longrightarrow y_{(t)} = H_1 t + H_2$

* Condiciones iniciales y Finales

✓ CT: $[f+(\Phi'-\dot{y})f_{\dot{y}}]_{t=T}=0$

$$f_{(T)}=\Phi_{(T)}=2-3T \longrightarrow \Phi'_{(T)}=-3$$

$$\left[-(1+\dot{y}^2)^{1/2}+(-3-\dot{y})(-\dot{y})(1+\dot{y}^2)^{-1/2}\right]=0$$

$$\times\ (1+\dot{y}^2)^{-1/2}$$

$$\left[-1+\frac{3\dot{y}+\dot{y}^2}{1+\dot{y}^2}\right]=0$$

$$[-1+3\dot{y}]=0$$

$$SG:\ y_{(t)}=(1/3)t+H_1$$

✓ $t=0 \rightarrow y_{(0)}=1=0+H_1 \rightarrow \boxed{H_1=1}$

$\therefore$ SE: $y^*_{(t)}=(1/3)t+1$

1d) $\text{Min } V(y)=\int_0^{\infty} e^{-\rho t}(y^2+ay+b\dot{y}+c\dot{y}^2)\,dt$

st $y_{(0)}=d \qquad (a,b,c,d,\rho>0)$

Datos

* $f_{(.)}=f(y,\dot{y})$

* CTF CTI , 1 2

Desarrollo

* $f_y=(2y+a)e^{-\rho t}$

$f_{\dot{y}}=(2c\dot{y}+b)e^{-\rho t}$

$f_{\dot{y}t}=2c\ddot{y}e^{-\rho t}-2c\rho\dot{y}e^{-\rho t}-b\rho e^{-\rho t}$

$$2y+a=2c\ddot{y}-2c\rho\dot{y}-b\rho$$

$$2c\ddot{y}-2c\rho\dot{y}-2y=a+\rho b$$

$$\ddot{y}-\rho\dot{y}-y/c=(a+\rho b)/2c$$

* SP: $y_p=-(a+\rho b)/2c$

SC: $r^2-\rho r-1/c=0$

$$r=\frac{\rho\pm\sqrt{\rho^2-4/c}}{2}$$

$r_1 \quad r_2$

$$y=A_1e^{r_1t}+A_2e^{r_2t}$$

$$SG:\ y_{(t)}=A_1e^{r_1t}+A_2e^{r_2t}+y_p$$

* Condiciones iniciales y finales

✓ $t=0 \rightarrow y_{(0)}=d=A_1+A_2+y_p \quad \ldots\ \alpha$

✓ CT: $\lim_{t\to\infty}[f_{\dot{y}}]_{t=T}=0$

$$\lim_{t\to\infty}\left[(2c\dot{y}+b)e^{-\rho t}\right]_{t=T}=0$$

$$\boxed{\begin{aligned} Y &= A_1 e^{r_1 T} + A_2 e^{r_2 T} + Y_p \\ \dot{Y} &= A_1 r_1 e^{r_1 T} + A_2 r_2 e^{r_2 T} \end{aligned}}$$

$$\lim_{t\to\infty} \left[(2c(A_1 r_1 e^{r_1 T} + A_2 r_2 e^{r_2 T}) + b) e^{-\rho T} \right]_{t=T} = 0$$

$$\lim_{t\to\infty} \left[2cA_1 r_1 e^{(r_1-\rho)T} + \cancel{2cA_2 r_2 e^{(r_2-\rho)T}} + \cancel{b e^{-\rho T}} \right]_{t=T} = 0$$

¿Qué términos convergen?

$$\text{Si } \lim_{t\to\infty} [F_{\dot{y}}]_{t=T} = 0 \rightarrow A_1 = 0$$

$$Y_{(t)} = A_2 e^{r_2 t} + Y_p$$

→ Retomando α

$$A_2 = d - Y_p$$

$$\therefore Y^*_{(t)} = \left[d + \left(\frac{a+\rho b}{2c} \right) \right] e^{r_2 t} - \left(\frac{a+\rho b}{2c} \right)$$

$$A = e^{-zt} \qquad B = e^{zt}$$

$$\lim_{t\to\infty} A = 0$$

$$\lim_{t\to\infty} B \neq 0$$

Diagrama de Fases

Trazar la curvas que son soluciones del sistema, es decir, graficar las curvas formadas por los puntos $(X_{(t)}, Y_{(t)})$

$\dot{X}$: crecimiento de X cuando crece t

$\dot{X} = f(x,y)$: determina la trayectoria de la partícula con respecto a X

lo mismo para $\dot{Y}$

Pasos Generales

1) Despejar la curva

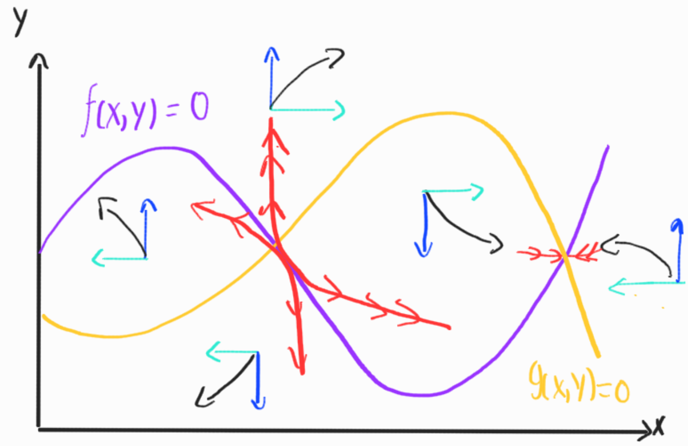
$\dot{X} = 0 \rightarrow \boxed{f(x,y) = 0}$

$\dot{Y} = 0 \rightarrow \boxed{g(x,y) = 0}$

2) Determinar las regiones

$$\frac{\partial \dot{X}}{\partial X}, \frac{\partial \dot{Y}}{\partial Y}, \frac{\partial \dot{X}}{\partial Y}, \frac{\partial \dot{Y}}{\partial X} \quad \begin{matrix} > 0 \\ < 0 \end{matrix}$$

3) Graficar



Ejercicio 2a

$$\dot{X} = -X + 2Y$$
$$\dot{Y} = -3Y$$

$$(|A - \lambda I|)V = 0$$

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix} \quad ; \quad -\lambda I = \begin{pmatrix} -\lambda & 0 \\ 0 & -\lambda \end{pmatrix}$$

$$\begin{vmatrix} -1-\lambda & 2 \\ 0 & -3-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(-3-\lambda) - (0)(2) = 0$$

$$\left.\begin{array}{l} \lambda_1 = -3 \\ \lambda_2 = -1 \end{array}\right\} \text{Valores Propios}$$

$\underline{\lambda_1 = -3}$

$$\begin{pmatrix} -1-(-3) & 2 \\ 0 & -3-(-3) \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$2v_1 + 2v_2 = 0 \quad \dots (1)$$
$$0v_1 + 0v_2 = 0 \quad \dots (2)$$

$$\text{Si } v_1 = 1 \to v_2 = -1 \quad \dots \alpha$$

$\underline{\lambda_2 = -1}$

$$\begin{pmatrix} -1-(-1) & 2 \\ 0 & -3-(-1) \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$0v_1 + 2v_2 = 0$$
$$0v_1 - 2v_2 = 0$$

$$\text{Si } v_2 = 0 \to v_1 = 1 \quad \dots \beta$$

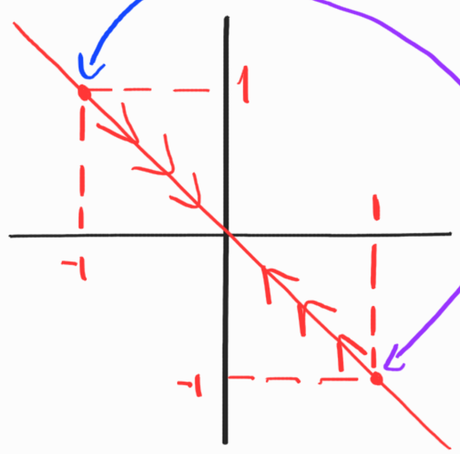
$$\to \begin{pmatrix} X \\ Y \end{pmatrix} = C_1 \overset{\alpha}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} e^{-3t} + C_2 \overset{\beta}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} e^{-t}$$

$$X = C_1 e^{-3t} + C_2 e^{-t}$$
$$Y = -C_1 e^{-3t}$$

$\to$ Solución Fácil para graficar:

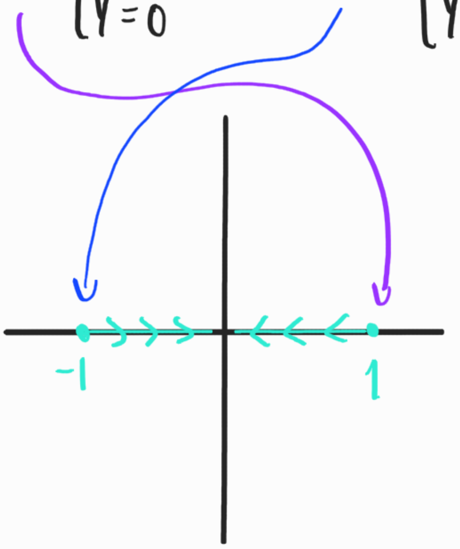
Paso 1: $C_1 = \pm 1 \to C_2 = 0$

$$C_1 = 1 \begin{cases} X = e^{-3t} \\ Y = -e^{3t} \end{cases} ; \quad C_1 = -1 \begin{cases} X = -e^{-3t} \\ Y = e^{-3t} \end{cases}$$

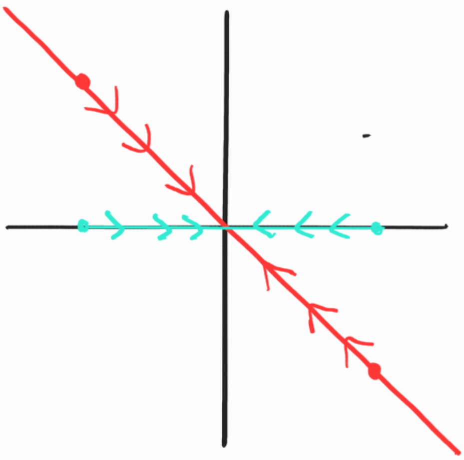


Paso 2: $C_1=0 \to C_2=\pm 1$

$$C_2=1 \begin{cases} X=e^{-t} \\ Y=0 \end{cases} \qquad C_2=1 \begin{cases} X=-e^{-t} \\ Y=0 \end{cases}$$



-> Uniendo Gráficos



-> Flujos del diagrama de fases

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \underbrace{C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t}}_{\substack{\text{Dominante} \\ \text{cuando} \\ t \to -\infty}} + \underbrace{C_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}}_{\substack{\text{Dominante} \\ \text{cuando} \\ t \to \infty}}$$

En términos absolutos $|-3|>|-1|$, por lo tanto los flujos del diagrama de fases será "paralelo" a $|-3|$

Si lo vemos como vectores, los vector resultantes serán los flujos del diagrama de Fases. Este vector resultante será "paralelo" a $|-3|$

Entonces, desde $-\infty$ será paralelo a $|-3|$ y hasta será pegado a $|-1|$ y como la solución es convergente, todo apuntará al origen.