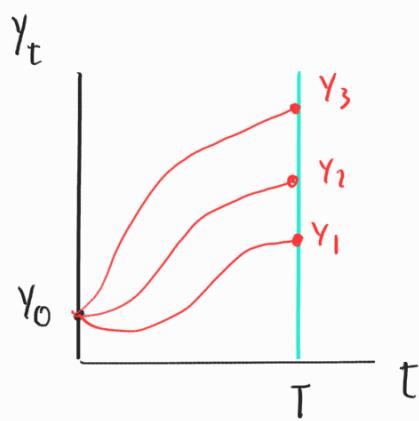
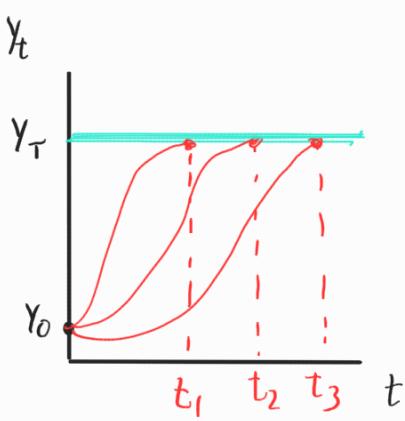


Condiciones de transversalidad

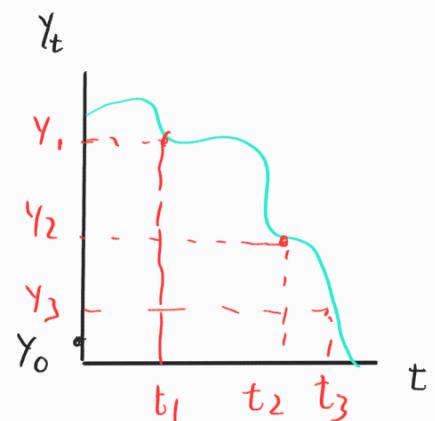
Caso I



Caso II



Caso III



Horizonte de tiempo Finito

$$V(y) = \int_0^T f(\cdot) dt \rightarrow \left[[f_y]_{t=T} \Delta y_T + [f - \dot{y}f_y]_{t=T} \Delta T = 0 \right]$$

Horizonte temporal Fijo

T , dado
 y_T , libre

Valor terminal Fijo

T , libre
 y_T , dado

Curva terminal

y_T libre, $y_{(T)} = f_{(T)} = \emptyset$

$$[f_y]_{t=T} = 0$$

$$[f - \dot{y}f_y]_{t=T} = 0$$

$$[f + (\Phi' - \dot{y})f_y]_{t=T} = 0$$

Horizonte de tiempo Infinito

$$V(y) = \int_0^\infty f(\cdot) dt \rightarrow \left[\lim_{t \rightarrow \infty} \left([f_y]_{t=T} \Delta y_T + [f - \dot{y}f_y]_{t=T} \Delta T \right) = 0 \right]$$

$y_{(T)}$ libre, o no te dice nada
 $\lim_{t \rightarrow \infty} [f_y]_{t=T}$

T , libre
 $\lim_{t \rightarrow \infty} [f - \dot{y}f_y]_{t=T}$

Ejercicio 1

1a) $V(y) = \int_0^2 (t^2 - y^2) dt$, $y(0) = 4$, $y(T)$ libre

Datos

* $f(.) = f(t, \dot{y})$

* $y(T)$ libre \rightarrow CT ① ② ③

Desarrollo

* $f_y = 0$
 $f_{\dot{y}} = 2\dot{y}$
 $f_{\ddot{y}t} = 2\ddot{y}$

$$\rightarrow 2\ddot{y} = 0 \rightarrow 2\dot{y} = H_1$$

$$\int \downarrow$$

$$y(t) = \frac{H_1}{2}t + H_2$$

* Condiciones iniciales y finales

$$t=0 \rightarrow y(0) = 4 = 0 + H_2 \rightarrow H_2 = 4$$

$$CT: [f_{\dot{y}}]_{t=2} = 0 \rightarrow 2\dot{y} = 0 \rightarrow H_1 = 0$$

* Solución específica

$$y(t) = 4 \cancel{\text{X}}$$

1b) $V(y) = \int_0^T (t^2 - y^2) dt$, $y(0) = 4$, $y(T) = 5$, T libre

Datos

* $f(.) = f(t, \dot{y})$

* T libre \rightarrow CT ① ② ③

Desarrollo

* $f_y = 0$
 $f_{\dot{y}} = 2\dot{y}$

$$\rightarrow 2\dot{y} = H_1 \rightarrow y(t) = \frac{H_1}{2}t + H_2$$

$$\dot{y} = H_1/2 = C_1 \rightarrow y = C_1 t + C_2$$

* Condiciones iniciales y finales

$$\sqrt{t=0} \rightarrow y(0) = 4 = 0 + H_2 \rightarrow H_2 = 4$$

$$CT: [f_{\dot{y}} - \dot{y}f_y]_{t=T} = 0 \quad H_1 t + H_2$$

$$(T + \dot{y}^2) - \dot{y}(2\dot{y}) = 0 \quad \dot{y} = H_1$$

$$\dot{y} = T^{1/2} \quad H = T^{1/2}$$

$$\dot{y} = H_1 \rightarrow \dot{y} = \frac{H_1}{2}$$

$$H_1 = 2T^{1/2}$$

$$C_1 = T^{1/2}$$

$$\sqrt{t=T} \rightarrow y(T) = 5 = (2T)^{1/2}T + 4$$

$$S = (T^{1/2})T + 4 \quad T = 1 \rightarrow H_1 = 2 \cancel{11} \rightarrow H_1 = 2$$

$$y(t) = t + 4 \cancel{11}$$

$$* SE: y(t) = t + 4 \cancel{11}$$

$$1c) V(y) = - \int_0^T (1 + \dot{y}^2)^{0.5} dt, \quad y(0) = 1, \quad y(T) = 2 - 3T$$

Datos

* $f(.) = f(\dot{y})$

* T, y libre \rightarrow CT ① ② ③

Desarrollo

* $f_{\dot{y}\dot{y}} \ddot{y} = 0 \rightarrow \ddot{y} = 0 \rightarrow y(t) = H_1 t + H_2$

* Condiciones iniciales y finales

$$\checkmark CT : \left[f + (\ddot{\Phi} - \dot{y}) f_y \right]_{t=T} = 0$$

$$f_{(T)} = \ddot{\Phi}_{(T)} = 2 - 3T \rightarrow \ddot{\Phi}_{(T)} = -3$$

$$\begin{cases} \left[-(1+\dot{y}^2)^{-1/2} + (-3-\dot{y})(-\dot{y})(1+\dot{y}^2)^{-1/2} \right] = 0 \\ \times (1+\dot{y}^2)^{-1/2} \\ \left[-1 + \frac{3\dot{y} + \dot{y}^2}{1+\dot{y}^2} \right] = 0 \end{cases}$$

$$[-1 + 3\dot{y}] = 0$$

$$SG: Y_{(t)} = (1/3)t + H_1$$

$$\checkmark t=0 \rightarrow Y_{(0)} = 1 = 0 + H_1 \rightarrow H_1 = 1$$

$$SE: Y_{(t)} = (1/3)t + 1$$

$$Id) Min V(Y) = \int_0^\infty e^{-pt} (Y^2 + aY + b\dot{y} + \dot{Y}^2) dt$$

$$st \quad Y_{(0)} = d \quad (a, b, c, d, p > 0)$$

Datos

$$* f(.) = f(y, \dot{y})$$

\textcircled{CTF}

\textcircled{CTI}

①

②

Desarrollo

$$* f_y = (2Y + a)e^{-pt}$$

$$f_{\dot{y}} = (2C\dot{Y} + b)e^{-pt}$$

$$f_{\ddot{y}t} = 2CYe^{-pt} - 2CPY\dot{e}^{-pt} - bPe^{-pt}$$

$$2Y + a = 2CY - 2CPY - bP$$

$$2C\overset{\circ}{Y} - 2CP\overset{\circ}{Y} - 2Y = a + Pb$$

$$\overset{\circ}{Y} - P\overset{\circ}{Y} - Y_C = (a + Pb)/2C$$

$$* SP: Y_p = -(a + Pb)/2C$$

$$SC: r^2 - Pr - Y_C = 0$$

$$r = \frac{P \pm \sqrt{P^2 - 4Y_C}}{2}$$

$$r_1 \quad r_2$$

$$Y = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

$$SG: Y_{(t)} = A_1 e^{r_1 t} + A_2 e^{r_2 t} + Y_p$$

* Condiciones iniciales y finales

$$/ t=0 \rightarrow Y_{(0)} = d = A_1 + A_2 + Y_p \dots \propto$$

$$/ CT: \lim_{t \rightarrow \infty} [f_y]_{t=T} = 0$$

$$\lim_{t \rightarrow \infty} [(2C\dot{Y} + b)e^{-pt}]_{t=T} = 0$$

$$\left\{ \begin{array}{l} Y = A_1 e^{r_1 t} + A_2 e^{r_2 t} + Y_p \\ \downarrow \\ \dot{Y} = A_1 r_1 e^{r_1 t} + A_2 r_2 e^{r_2 t} \end{array} \right.$$

$$\lim_{t \rightarrow \infty} \left[(2c(A_1 r_1 e^{r_1 t} + A_2 r_2 e^{r_2 t}) + b) e^{-pt} \right] = 0$$

$t=T$

$$\lim_{t \rightarrow \infty} \left[2c(A_1 r_1 e^{(r_1-p)t} + A_2 r_2 e^{(r_2-p)t}) + be^{-pt} \right] = 0$$

$t=T$

¿Qué términos convergen?

$$\text{Si } \lim_{t \rightarrow \infty} \left[f_y \right] = 0 \rightarrow A_1 = 0$$

$$Y(t) = A_2 e^{r_2 t} + Y_p$$

→ Retomando α

$$A_2 = d - Y_p$$

$$Y(t) = \left[d + \left(\frac{a+pb}{2c} \right) \right] e^{r_2 t} - \left(\frac{a+pb}{2c} \right)$$

$$A = e^{-zt}$$

$$\beta = e^{zt}$$

$$\lim_{t \rightarrow \infty} A = 0$$

$$\lim_{t \rightarrow \infty} \beta \neq 0$$

Diagrama de Fases

Trazar las curvas que son soluciones del sistema, es decir, graficar las curvas formadas por los puntos $(X_{(t)}, Y_{(t)})$

\dot{X} : crecimiento de X cuando crece t

$\dot{X} = f(x, y)$: determina la trayectoria de la partícula con respecto a X

lo mismo para \dot{Y}

Pasos Generales

1) Despejar la curva

$$\dot{X} = 0 \rightarrow f(x, y) = 0$$

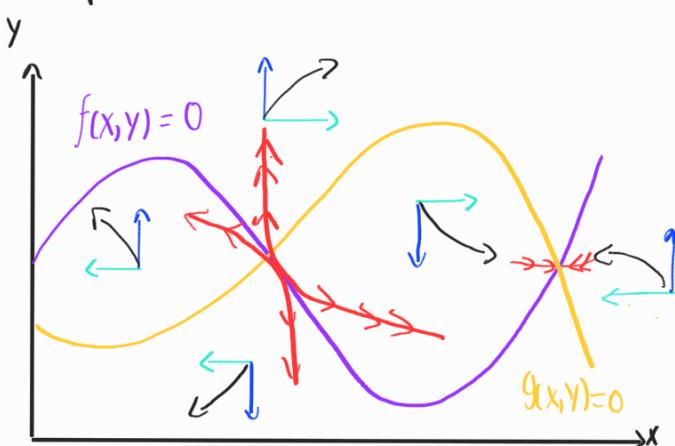
$$\dot{Y} = 0 \rightarrow g(x, y) = 0$$

2) Determinar las regiones

$$\frac{\partial \dot{X}}{\partial X}, \frac{\partial \dot{Y}}{\partial Y}, \frac{\partial \dot{X}}{\partial Y}, \frac{\partial \dot{Y}}{\partial X} > 0$$

$$\frac{\partial \dot{X}}{\partial X}, \frac{\partial \dot{Y}}{\partial Y}, \frac{\partial \dot{X}}{\partial Y}, \frac{\partial \dot{Y}}{\partial X} < 0$$

3) Graficar



Ejercicio 2a

$$\begin{aligned}\dot{X} &= -X + 2Y \\ \dot{Y} &= -3Y\end{aligned}$$

$$(A - \lambda I) V = 0$$

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix} \quad ; \quad -\lambda I = \begin{pmatrix} -\lambda & 0 \\ 0 & -\lambda \end{pmatrix}$$

$$\begin{vmatrix} -1-\lambda & 2 \\ 0 & -3-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(-3-\lambda) - (0)(2) = 0$$

$$\left. \begin{array}{l} \lambda_1 = -3 \\ \lambda_2 = -1 \end{array} \right\} \begin{array}{l} \text{Valores} \\ \text{Propios} \end{array}$$

$$\underline{\lambda_1 = -3}$$

$$\begin{pmatrix} -1-(-3) & 2 \\ 0 & -3-(-3) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$2V_1 + 2V_2 = 0 \dots (1)$$

$$0V_1 + 0V_2 = 0 \dots (2)$$

$$\text{Si } V_1 = 1 \rightarrow V_2 = -1 \dots \alpha$$

$$\begin{pmatrix} \lambda_2 = -1 & \\ -1-(-1) & 2 \\ 0 & -3-(-1) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$0V_1 + 2V_2 = 0$$

$$0V_1 - 2V_2 = 0$$

$$\text{Si } V_2 = 0 \rightarrow V_1 = 1 \dots \beta$$

$$\rightarrow \begin{pmatrix} X \\ Y \end{pmatrix} = C_1 \begin{pmatrix} \alpha \\ 1 \end{pmatrix} e^{-3t} + C_2 \begin{pmatrix} \beta \\ 0 \end{pmatrix} e^{-t}$$

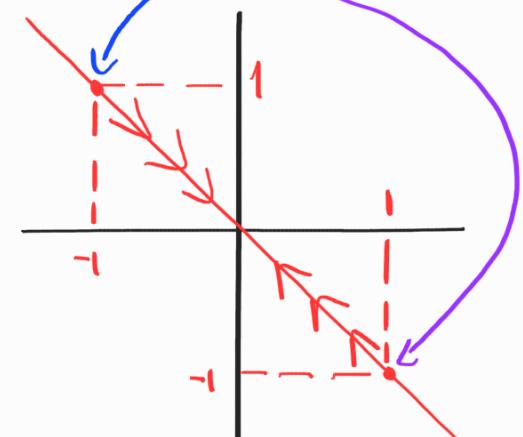
$$X = C_1 e^{-3t} + C_2 e^{-t}$$

$$Y = -C_1 e^{-3t}$$

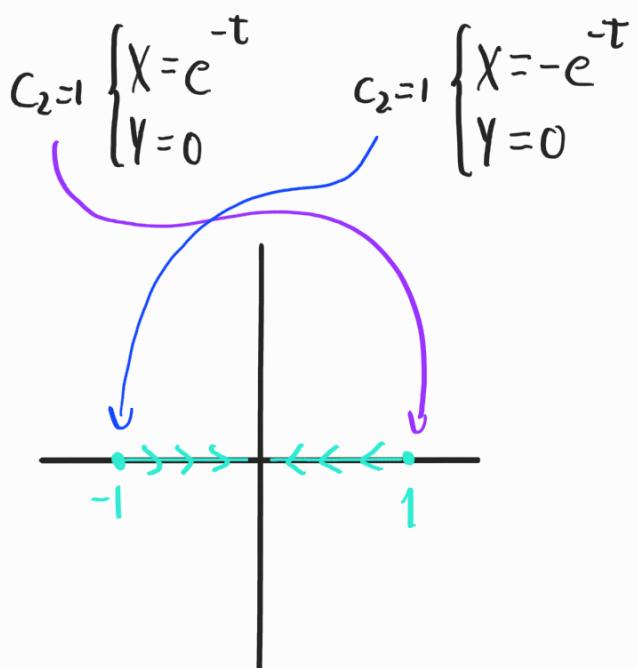
→ Solución Fácil para graficar :

$$\text{Paso 1: } C_1 = \pm 1 \rightarrow C_2 = 0$$

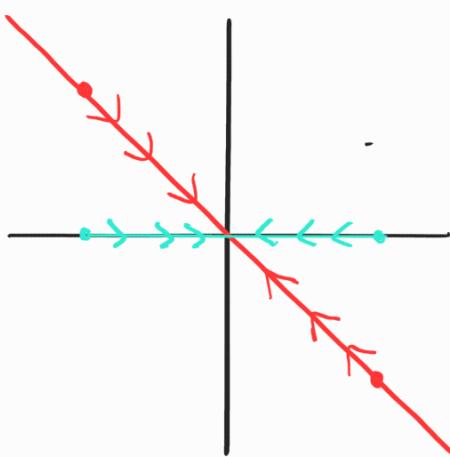
$$C_1 = 1 \left\{ \begin{array}{l} X = e^{-3t} \\ Y = -e^{3t} \end{array} \right. ; \quad C_1 = -1 \left\{ \begin{array}{l} X = -e^{-3t} \\ Y = e^{-3t} \end{array} \right.$$



Paso 2: $C_1=0 \rightarrow C_2=\pm 1$



→ Uniendo Gráficos



→ Flujos del diagrama de fases

$$\begin{pmatrix} X \\ Y \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + C_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$$

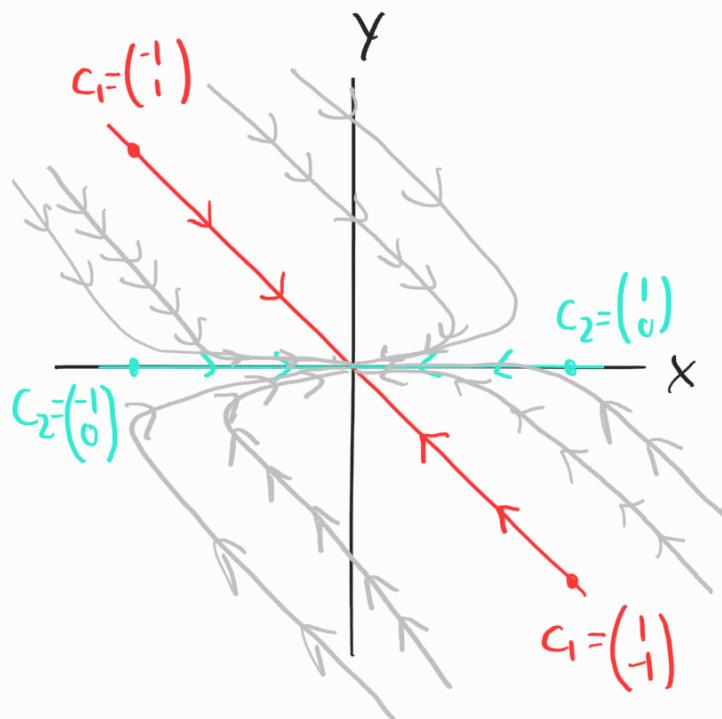
$\underbrace{\qquad\qquad}_{\text{Dominante cuando } t \rightarrow -\infty}$
 $\underbrace{\qquad\qquad}_{\text{Dominante cuando } t \rightarrow \infty}$

En términos absolutos $| -3 | > | -1 |$, por lo tanto los flujos del diagrama de fases será "Paralelo" a $| -3 |$

Si lo vemos como vectores, los vector resultantes serán los flujos del diagrama de fases. Este vector resultante será "paralelo" a $| -3 |$



Entonces, desde $-\infty$ será paralelo a $| -3 |$ y desde ∞ será pegado a $| -1 |$ y como la solución es convergente, todo apuntará al origen.



Paso 1: $\dot{x} = 0 \rightarrow -x + 2y = 0 \rightarrow y = 0.5x$

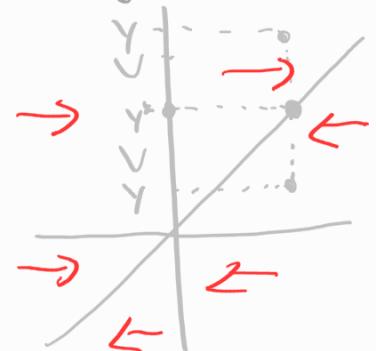
$\dot{y} = 0 \rightarrow -3y = 0 \rightarrow y = 0$

Paso 2:

$$\frac{\partial \dot{x}}{\partial x} = -1 < 0 \quad \begin{cases} x > x^* \rightarrow x \leftarrow \\ x < x^* \rightarrow x \rightarrow \end{cases}$$

$$\frac{\partial \dot{x}}{\partial y} = 2 > 0 \quad \begin{cases} y > y^* \rightarrow x \rightarrow \\ y < y^* \rightarrow x \leftarrow \end{cases}$$

$$\frac{\partial \dot{y}}{\partial y} = -3 < 0 \quad \begin{cases} y > y^* \rightarrow y \downarrow \\ y < y^* \rightarrow y \uparrow \end{cases}$$



Paso 3:

