a) $X_t = a X_{t-1} + b$

$X_1 = a X_0 + b$

$X_2 = a X_1 + b = a(aX_0 + b) + b = a^2 X_0 + ab + b$

$X_3 = a X_2 + b = a(a^2 X_0 + ab + b) + b$

$= a^3 X_0 + a^2 b + ab + b$

$X_1 = a X_0 + b$

$X_2 = a^2 X_0 + ab + b$

$X_3 = a^3 X_0 + a^2 b + ab + b$

$X_t = a^t X_0 + \underbrace{a^{t-1} b + a^{t-2} b + \ldots + a^2 b + ab}\, + b$

$S = a^{t-1} b + a^{t-2} b + \ldots + a^2 b + ab + b$

$aS = a^t b + a^{t-1} + \ldots + a^3 b + a^2 b + ab$

$S - aS = b - a^t b$

$S = \frac{b(1-a^t)}{1-a}$

$X_t = a^t X_0 + b\left(\frac{1-a^t}{1-a}\right)$

$\to X_t = a^t X_0 + b\left[\frac{1}{1-a} - \frac{a^t}{1-a}\right]$

$X_t = a^t X_0 + \frac{b}{1-a} - \frac{a^t b}{1-a}$

$\therefore X_t = \left(X_0 - \frac{b}{1-a}\right) a^t + \frac{b}{1-a}$

Alternativamente

$$X_t = aX_{t-1} + b \longrightarrow X_t = X_c + \bar{X}$$

Donde $\bar{X}$:

$$X_t = X_{t-1} = \bar{X} \qquad \bar{X}(1-a) = b$$

$$\bar{x} = a\bar{x} + b \qquad \bar{x} = \frac{b}{1-a}$$

Donde X_c:

$$X_1 - aX_0 = 0 \qquad X_3 = aX_2 = a^3X_0$$

$$X_1 = aX_0 \qquad \vdots$$

$$X_2 = aX_1 = a^2X_0 \qquad X_c = a^tX_0 = Aa^t$$

$$\therefore \; X_t = Aa^t + \frac{b}{1-a} \longrightarrow \text{Si } t=0 \rightarrow X_t = \left(X_0 - \frac{b}{1-a}\right)a^t + \frac{b}{1-a}$$

Determinación del movimiento

Converge $|a| \leq 1$; Monótono $a \geq 0$

Diverge $|a| > 1$; Oscilante $a < 0$

Esta Estacionario

$$\lim_{t \to \infty} = \bar{X}$$

a) Monótono y Divergente

$$X_t = 7X_{t-1} + 16$$

$$\bar{X} = \frac{16}{1-7} = -\frac{8}{3}$$

$$X_t = A(7)^t - \frac{8}{3}$$

$t=0 \rightarrow 5 = A - \frac{8}{3}$

$$A = \frac{7}{3}$$

$$\therefore X_t = \frac{7}{3}(7)^t - \frac{8}{3}$$

$7 \geq 0$ y $|7| > 1$

b) Monótono y Converge

$\therefore X_t = -8(1/3)^t + 9$; $1/3 \geq 0$ y $|1/3| \leq 1$

c) Oscilante y Divergente

$\therefore X_t = 2/3(-2)^t + 1/3$; $-2 < 0$ y $|-2| > 1$

d) Oscilante y Convergente

$\therefore X_t = -2(-1/4)^t + 4$; $-1/4 < 0$ y $|-1/4| \leq 1$

Ejercicio 2

$$\left| \frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}} \right| < 1, \ \bar{Y} \text{ estable}. \quad \left| \frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}} \right| \geq 1, \ \bar{Y} \text{ inestable}$$

$$\frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}} \geq 0, \text{ monótona} \ ; \quad \frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}} < 0, \text{ oscilante}$$

a) $Y_t = \frac{Y_{t-1}}{2} + 5$

$\bar{Y} = 0.5\bar{Y} + 5$

$0.5\bar{Y} = 5$

$\bar{Y} = 10$

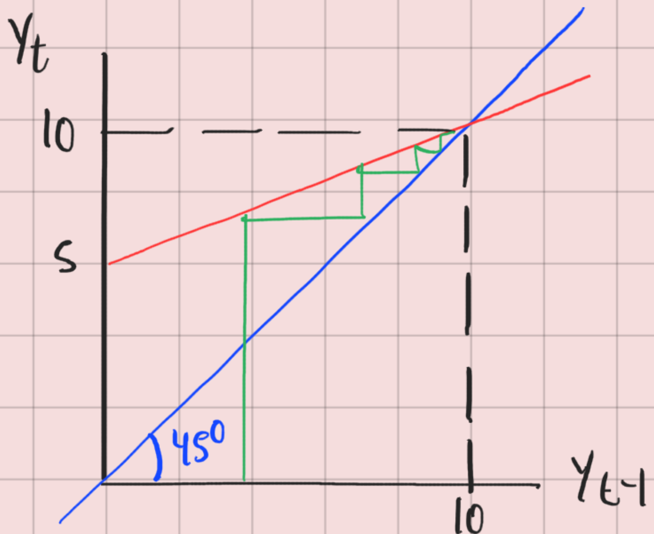
$\frac{dY_t}{dY_{t-1}} = 0.5 \quad > 0$

$\frac{dY}{dY_{t-1}} = 0$

linea recta de pendiente +

$\rightarrow \left| \frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}=10} \right| = 0.5 < 1$ estable

$\frac{dY_t}{dY_{t-1}} \Big|_{\bar{Y}=10} = 0.5 \geq 0$ monótona



c) $\bar{Y} = \bar{Y}^{0.5}$

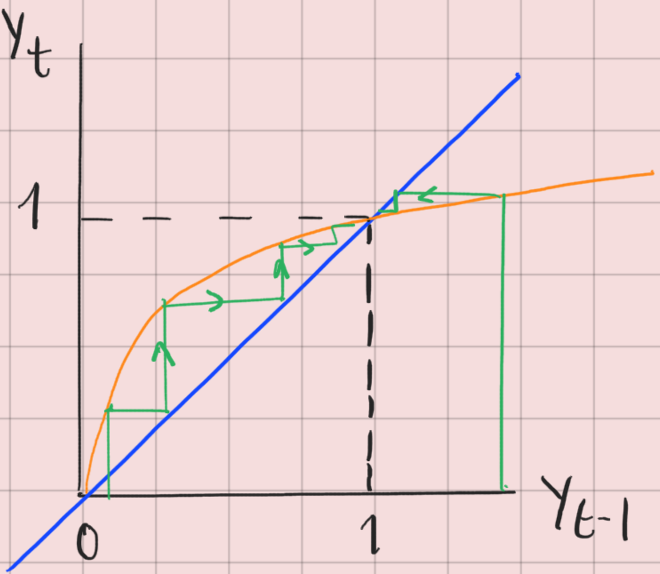
$\bar{Y}(\bar{Y}^{-0.5} - 1) = 0$

$\bar{Y}_1 = 0 \wedge \bar{Y}_2 = 1$

$$\left.\begin{array}{l} \dfrac{dY_t}{dY_{t-1}} = 0.5 Y_t^{-0.5} > 0 \\[2ex] \dfrac{d^2 Y_t}{dY_{t-1}^2} = -0.25\, Y^{-1.5} < 0 \end{array}\right\} \text{Cóncavo}$$

$|f'(\bar{Y}_1)| = \infty > 1$, y $f'(\bar{Y}_1) = \infty \gtrless 0$ oscilante inestable

$|f'(\bar{Y}_2)| = 0.5 < 1$, y $f'(\bar{Y}_2) = 0.5 > 0$ monótono estable



e) $Y_t = Y_{t-1}^{-0.25}$

$\bar{y}(1 - \bar{y}^{-1.25}) = 0$

$\bar{Y}_1 = 0 \wedge \bar{Y}_2 = 1$

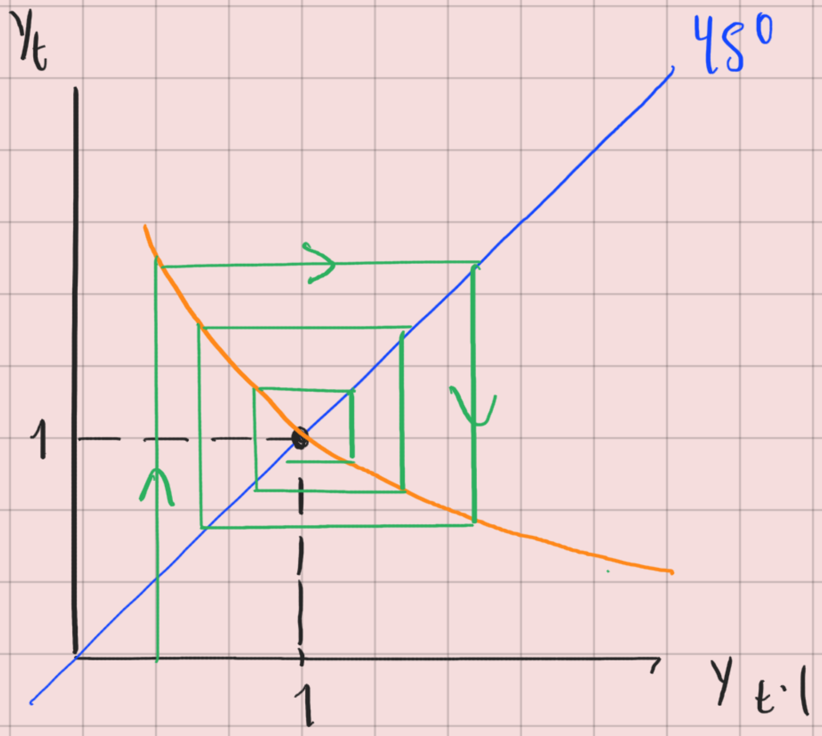
$f' = -0.25\, Y_{t-1}^{-1.25} < 0$

$f'' = 0.325\, Y_{t-1}^{-2.25} > 0$

convexa

$|f'(0)| = \infty \geq 1$ y $f'(0) = \infty \geq 0$ oscilante inestable

$|f'(1)| = 0.25 < 1$ y $f'(1) = -0.25 < 0$ oscilante estable



E en D de Segundo orden

$$Y_t + a_1 Y_{t-1} + a_2 Y_{t-2} = b$$

$$Y_t = Y_c + \bar{Y}$$

$$\rightarrow Y_c = A_1 r_1^t + A_2 r_2^t$$

$$r^2 + a_1 r + a_2 = 0$$

$$\rightarrow \bar{Y} = \begin{cases} \dfrac{b}{1+a_1+a_2} & \leftrightarrow \; a_1 + a_2 \neq -1 \\ \dfrac{b}{2+a_1} t & \leftrightarrow \; a_1 + a_2 = -1 \wedge a_1 \neq -2 \\ \dfrac{b}{2} t^2 & \leftrightarrow \; a_1 + a_2 = -1 \wedge a_1 = -2 \end{cases}$$

Nota

Si $r_1 = r_2 = r$

$$Y_c = A_1 r^t + A_2 t r^t$$

$$\therefore Y = A_1 r_1^t + A_2 r_2^t + \bar{Y}$$

Si $|r| > 1$ divergencia

$|r| < 1$ Convergencia

$|r_1|$ y $|r_2|$ el dominante dirige la trayectoria

Ejercicio 3

a) $Y_{t+2} - 11Y_{t+1} + 10Y_t = 27 \quad Y(0) = 2 \quad , \quad Y(1) = 53$

$Y_c: \; r^2 - 11r + 10 = 0$

$r \qquad -10$

$r \qquad -1$

$r = 10 \, , 1$

$Y_p: \; a_1 + a_2 = -1 \; \wedge \; a \neq -2$

$\bar{Y} = \frac{27}{1+(-11)} t = -3t$

$SG: \; Y = A_1 10^t + A_2 - 3t$

$CI: \; t=0 \rightarrow Y(0) = 2 = A_1 + A_2$

$t=1 \rightarrow Y(1) = 53 = 10A_1 + A_2 - 3$

$A_1 = 6$

$A_2 = -4$

$SE: \; Y_t = 6(10)^t - 4 - 3t$ #

b) $Y_t = \frac{1}{2} 5^t + \frac{2}{3} t (5)^t + \frac{1}{2}$

Usar la Nota

Ejercicio 4

Modelo de Crecimiento de Harrod

en el equilibrio

$$2.66(Y_t - Y_{t-1}) = 0.16\,Y_t$$

$$2.5\,Y_t = 2.66\,Y_{t-1}$$

$$Y_t = 1.064\,Y_{t-1}$$

$$Y_t = \left(9000 - \frac{0}{1-1.064}\right)(1.064)^t + \frac{0}{1-1.064} = 9000(1.064)^t$$

tasa de Crecimiento Garantizado

$$G_w = \frac{s}{a-s}$$

s : Propensión marginal a Ahorrar
a : " " " Invertir

$$G_w = \frac{0.16}{2.66-0.16} = 0.064$$