Solucionario 5

lunes, febrero 15, 2021 7:35 PM

## Diagrama de Fases

Trazar las curvas que son soluciones del sistema; es decir, graficar las curvas formadas por los puntos $(x(t), y(t))$.

$\dot{x}$ : Crecimiento de X cuando crece t

$\dot{x} = f(x,y)$ : determina la trayectoria de la partícula con respecto a X

Lo mismo para $\dot{y}$.

**Pasos Generales**

1) Despejar la curva

$\dot{x}=0 \longrightarrow \boxed{f(x,y)=0}$

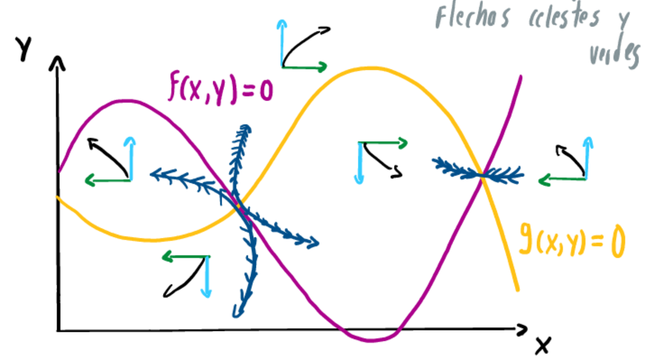
$\dot{y}=0 \longrightarrow \boxed{g(x,y)=0}$

2) Determinar las regiones:

$\frac{\partial \dot{x}}{\partial x}, \frac{\partial \dot{y}}{\partial y}, \frac{\partial \dot{y}}{\partial x}, \frac{\partial \dot{x}}{\partial y}$ $> 0$, $< 0$

3) Graficar

Acá por las preguntas las flechas celestes y verdes




Para un desarrollo matemático del diagrama de fases, revisar el desarrollo del ejercicio 2 de la PD 2

### Ejercicio 1

Pasos:

① $\dot{y} = ay + bx + h \to \dot{y}=0 \to 0 = ay + bx + h$

$Y = -\frac{b}{a}x - \frac{h}{a}$ , $\frac{\partial Y}{\partial X} = -\frac{b}{a} = m_y$

$\dot{x} = cy + dx + k \to \dot{x} = 0 \to 0 = cy + dx + k$

$Y = -\frac{d}{c}X - \frac{k}{c}$ , $\frac{\partial Y}{\partial X} = -\frac{d}{c} = m_x$

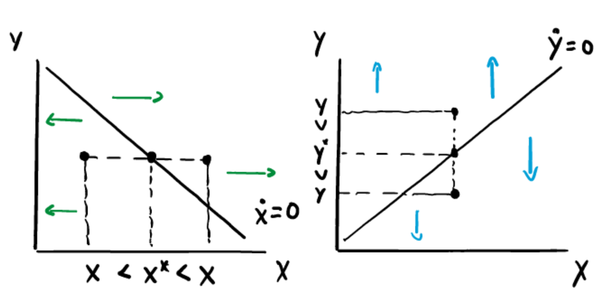
② $\frac{\partial \dot{y}}{\partial y} = a$ ; $\frac{\partial \dot{x}}{\partial x} = d$

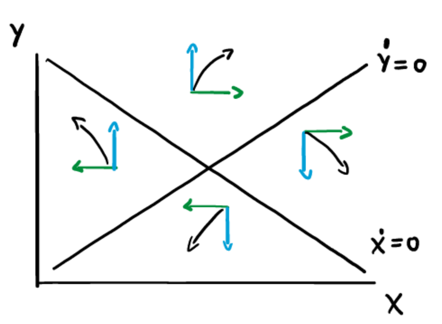
2a) a, c, d > 0 y b < 0

$\frac{\partial \dot{x}}{\partial x} > 0 \begin{cases} x > x^* \to x & \rightarrow \\ x < x^* \to x & \leftarrow \end{cases}$ $m_y > 0$

$\frac{\partial \dot{y}}{\partial y} > 0 \begin{cases} y > y^* \to y & \uparrow \\ y < y^* \to y & \downarrow \end{cases}$ $m_x < 0$

3a)



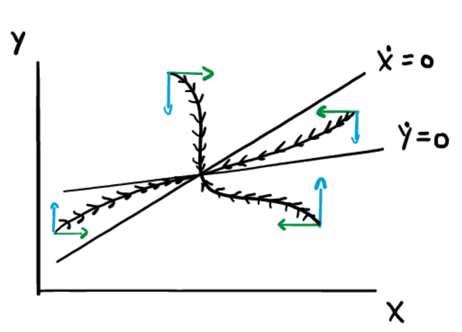




2b) b, c > 0 y a, d < 0

$\frac{\partial \dot{x}}{\partial x} < 0 \begin{cases} x > x^* \to x & \leftarrow \\ x < x^* \to x & \rightarrow \end{cases}$ $m_y > 0$

$\frac{\partial \dot{y}}{\partial y} < 0 \begin{cases} y > y^* \to y & \downarrow \\ y < y^* \to y & \uparrow \end{cases}$ $m_x > 0$

3b)

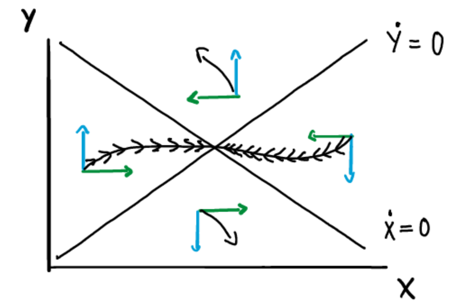



2c) a > 0 y b, c, d < 0

$\frac{\partial \dot{x}}{\partial x} < 0 \begin{cases} x > x^* \to x & \leftarrow \\ x < x^* \to x & \rightarrow \end{cases}$ $m_y > 0$

$\frac{\partial \dot{y}}{\partial y} > 0 \begin{cases} y > y^* \to y & \uparrow \\ y < y^* \to y & \downarrow \end{cases}$ $m_y < 0$

3c)

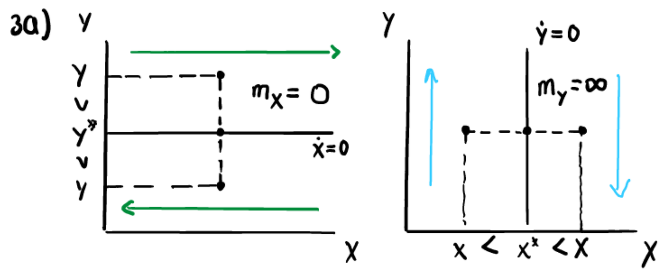


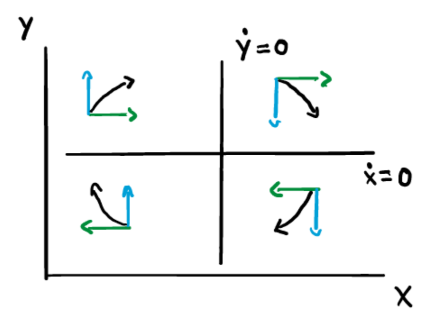

2d) c > 0, b < 0 y a, d = 0

$\frac{\partial \dot{x}}{\partial x} = 0 \to \frac{\partial \dot{x}}{\partial y} = c > 0 \begin{cases} y > y^* \to x & \rightarrow \\ y < y^* \to x & \leftarrow \end{cases}$

$\frac{\partial \dot{y}}{\partial y} = 0 \to \frac{\partial \dot{y}}{\partial x} = b < 0 \begin{cases} x > x^* \to y & \downarrow \\ x < x^* \to y & \uparrow \end{cases}$

3d)




### Ejercicio 2

a) $\dot{Y} = -3Y + Y^2 + 2$

Pasos:

1. $\dot{Y}=0 \to 0 = -3Y + Y^2 + 2 \to Y_1 = 2$ , $Y_2 = 1$
2. $\frac{\partial \dot{Y}}{\partial Y} = -3 + 2Y = 0$

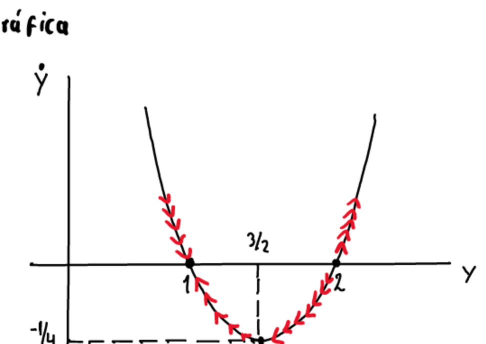
$Y = \frac{3}{2} \longrightarrow \dot{Y} \longrightarrow \dot{Y} = \frac{-1}{4}$ (Puntos críticos)

* $\frac{\partial^2 \dot{Y}}{\partial Y^2} = 2 > 0$

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=2} = 1 > 0$ : inestable

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=1} = -1 < 0$ : estable

3. Gráfica




b) $\dot{Y} = 8Y - 2Y^2$

Pasos:

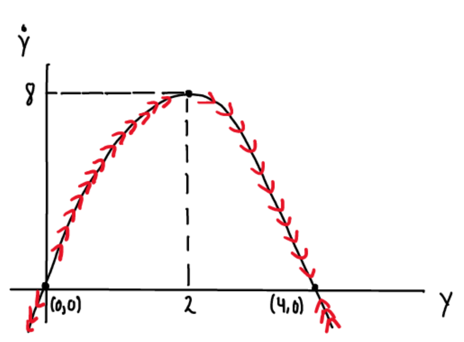
1. $\dot{Y} = 0 \to 0 = 8Y - 2Y^2 \to Y(Y-4) = 0 \to Y_1 = 0$ , $Y_2 = 4$
2. $\frac{\partial \dot{Y}}{\partial Y} = 8 - 4Y = 0 \to Y = 2 \to \dot{Y} = 8$

$\frac{\partial^2 \dot{Y}}{\partial Y^2} = -4 < 0$ max

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=0} = 8 > 0$ inestable

$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{Y=4} = -8 < 0$ estable

3.




### Ejercicio 3

b) $\dot{X} = Y - X^2 + 3$
$\dot{Y} = Y - X + 1$

Pasos:

1. $\dot{X} = 0 = Y - X^2 + 3 \longrightarrow Y = X^2 - 3$ (Parábola)

$\dot{Y} = 0 = Y - X + 1 \longrightarrow Y = X - 1$ (Recta)

intersección

$X^2 - 3 = X - 1 \to X^2 - X - 2 = 0$

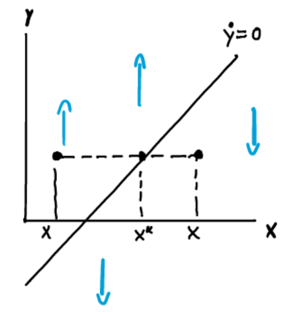
$X_1 = 2 \longrightarrow Y_1 = 1$

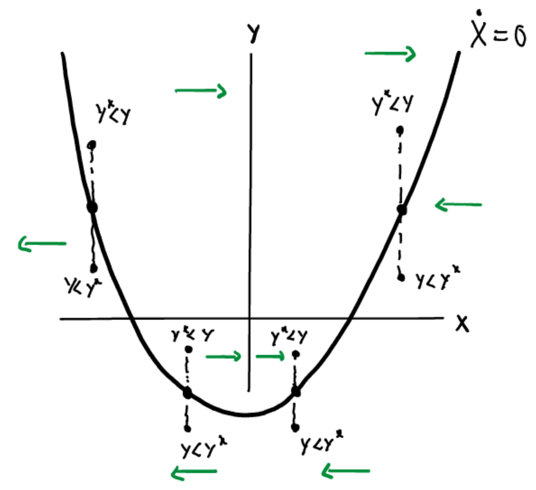
$X_2 = -1 \longrightarrow Y_2 = -2$

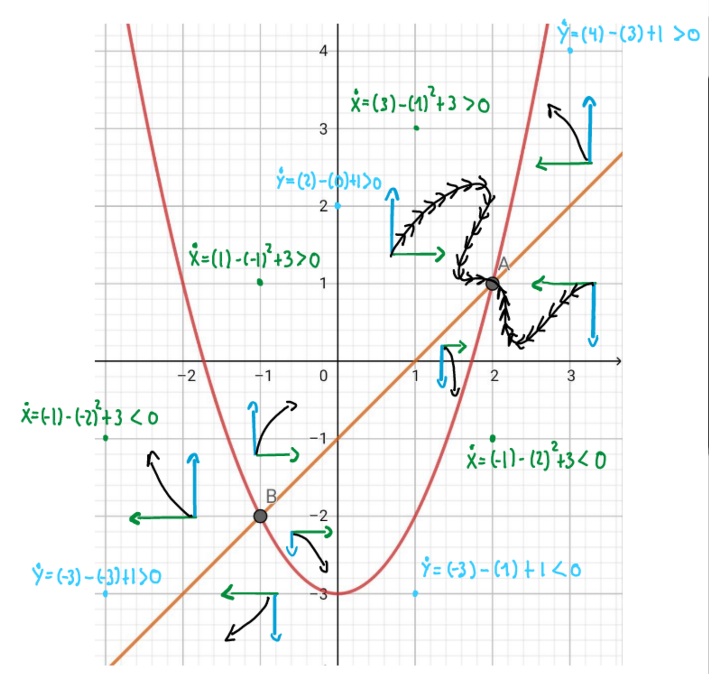
2. $\frac{\partial \dot{X}}{\partial X} = -2X \to \frac{\partial \dot{X}}{\partial Y} = 1 > 0 \begin{cases} Y > Y^* \to X & \rightarrow \\ Y < Y^* \to X & \leftarrow \end{cases}$

$\frac{\partial \dot{Y}}{\partial Y} = 1 \to \frac{\partial \dot{Y}}{\partial X} = -1 < 0 \begin{cases} X > X^* \to Y & \downarrow \\ X < X^* \to Y & \uparrow \end{cases}$

3.




c) $\dot{X} = Y - X^3$
$\dot{Y} = 1 - XY$

Pasos:

1. $\dot{X} = 0 = Y - X^3 \longrightarrow Y = X^3$

$\dot{Y} = 0 = 1 - XY \longrightarrow Y = \frac{1}{X}$

Intersección

$X^3 = \frac{1}{X} \longrightarrow X^4 = 1 \longrightarrow X^4 - 1 = 0$

$(X^2+1)(X^2-1) = 0$

$(X^2+1)(X+1)(X-1) = 0$

$X_1 = 1 \longrightarrow Y_1 = 1$

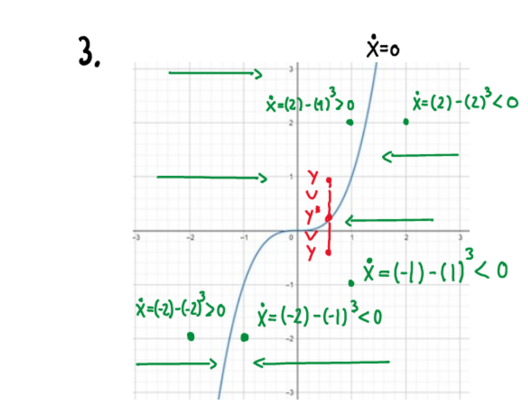
$X_2 = -1 \longrightarrow Y_2 = -1$

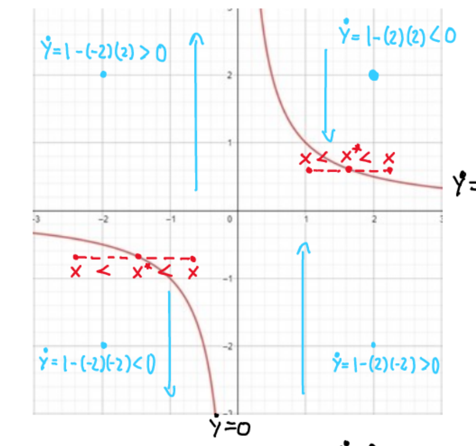
$X_3 = i$
$X_4 = -i$ ✗

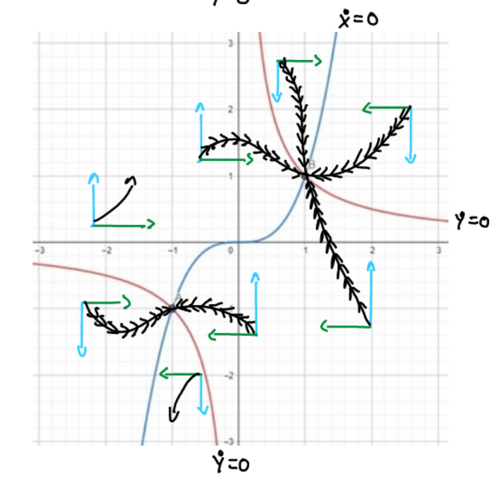
2. $\frac{\partial \dot{X}}{\partial Y} = 1 > 0 \begin{cases} Y > Y^* \to X & \rightarrow \\ Y < Y^* \to X & \leftarrow \end{cases}$

$\frac{\partial \dot{Y}}{\partial X} = -Y \begin{cases} Y > 0 \to < 0 \begin{cases} X > X^* \to Y & \downarrow \\ X < X^* \to Y & \uparrow \end{cases} \\ Y < 0 \to > 0 \begin{cases} X > X^* \to Y & \uparrow \\ X < X^* \to Y & \downarrow \end{cases} \end{cases}$

3.




### Ejercicio 4

* $f(.) = e^{-\rho t}U(c) = e^{-\rho t}U(aN - bN^2 - \dot{N})$
* $f_N = (c)(a - 2bN)e^{-\rho t}$

$f_{\dot{N}} = U'(c)(-1)e^{-\rho t}$

$\frac{\partial f_{\dot{N}}}{\partial t} = -U''(c)\dot{c}e^{-\rho t} + \rho e^{-\rho t}\dot{U}(c)$

* Euler : $U'(a)(a - 2bN)e^{-\rho t} = -U''(c)\dot{c}e^{-\rho t} + \rho e^{-\rho t}\dot{U}(c)$

$\dot{c} = \frac{U'(c)}{U''(c)}(\rho + 2bN - a)$

$\dot{N} = aN - bN^2 - C$

Pasos:

1. $\dot{c} = 0 = \frac{U'(c)}{U''(c)}(\rho + 2bN - a) \longrightarrow N = \frac{a - \rho}{2b}$

$\dot{N} = 0 = aN - bN^2 - c \longrightarrow C = aN^2 - bN$

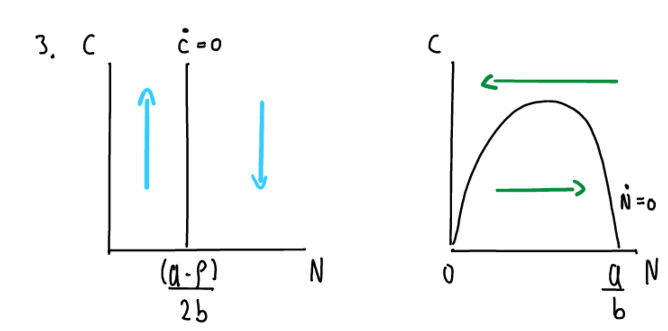
$\hookrightarrow c = 0$

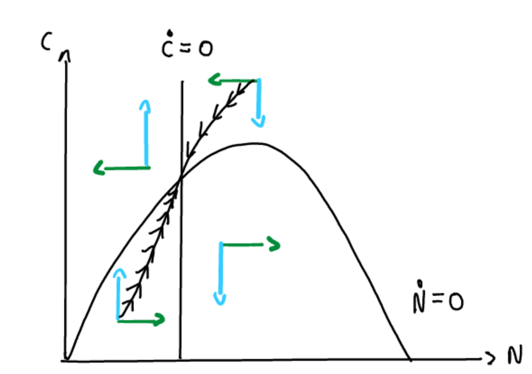
$N_1 = 0 \wedge N_2 = b/a$

2. $\frac{\partial \dot{c}}{\partial N} = \frac{U'(c)}{U''(c)}2b < 0 \begin{cases} N > N^* \to c & \downarrow \\ N < N^* \to c & \uparrow \end{cases}$

$\frac{\partial \dot{N}}{\partial c} = -1 < 0 \begin{cases} c > c^* \to N & \leftarrow \\ c < c^* \to N & \rightarrow \end{cases}$

3.




### Ejercicio 5

$\dot{P} = \alpha[D(P) - O(P^e)]$ $\dot{P}^e = \beta(P - P^e)$

Paso 1:

1. $\dot{P} = 0 = \alpha[D(P) - O(P^e)] \longrightarrow D(P) - O(P^e) = 0$

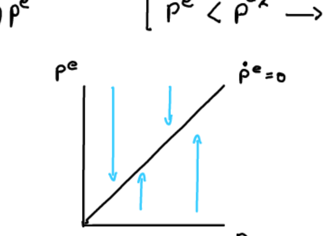
Por teorema de la función implícita

$D'(P)dP - O'(P^e)dP^e = 0$

$\frac{dP^e}{dP} = \frac{D'(P) < 0}{O'(P^e) > 0} < 0$

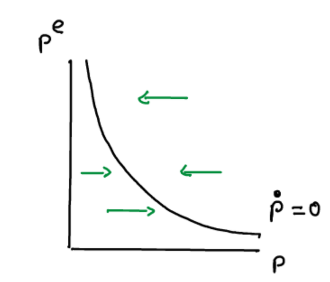
$\dot{P}^e = 0 = \beta(P - P^e) \longrightarrow P = P^e \longrightarrow \frac{dP^e}{dP} = 1$

2. * $\frac{\partial \dot{P}^e}{\partial P^e} < 0 \begin{cases} P^e > P^{e*} \longrightarrow P^e & \downarrow \\ P^e < P^{e*} \longrightarrow P^e & \uparrow \end{cases}$




* $\frac{\partial \dot{P}}{\partial P} = \frac{\partial \dot{P}}{\partial D(P)} \cdot \frac{\partial D(P)}{\partial P}$ ($\alpha > 0$, $0 <$)

$\frac{\partial \dot{P}}{\partial P} < 0 \begin{cases} P > P^* \to P & \leftarrow \\ P < P^* \to P & \rightarrow \end{cases}$

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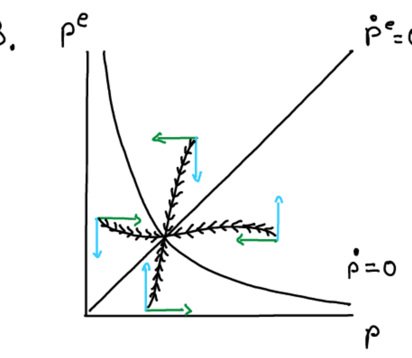



### Ejercicio 6

1. $\dot{Y} = 0 \longrightarrow Y = E(Y - T, r) + G$

$dY = \frac{\partial E}{\partial Y}dY + \frac{\partial E}{\partial r}dr$

$dY = E_Y dY + E_r dr$

$(1 - E_Y)dY = E_r dr$

$\frac{dY}{dr} = \frac{(1 - E_Y)}{E_Y} < 0$ : IS

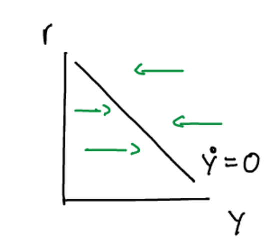
$\dot{r} = 0 \longrightarrow L(Y, r) = M/P$

$dL = \frac{\partial L}{\partial Y}dY + \frac{\partial L}{\partial r}dr = 0$

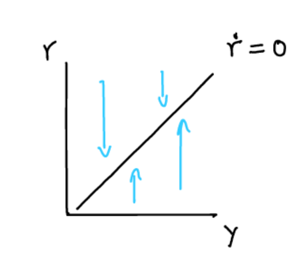
$dL = L_Y dY + L_r dr = 0$

$\frac{dY}{dr} = \frac{-L_r}{L_Y} > 0$ : LM

2. $\frac{\partial \dot{Y}}{\partial Y} = -1 < 0 \begin{cases} Y > Y^* \to Y & \leftarrow \\ Y < Y^* \to Y & \rightarrow \end{cases}$




$\frac{\partial \dot{r}}{\partial r} = L_r < 0 \begin{cases} r > r^* \to r & \downarrow \\ r < r^* \to r & \uparrow \end{cases}$

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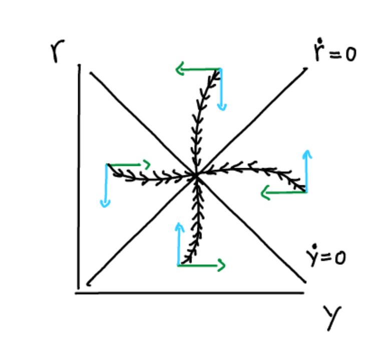


### Ejercicio 7

$f(.) = \ln(c)e^{-\rho t} = \ln(AK - \dot{K} - \delta K)e^{-\rho t}$

a) Condiciones necesarias : Condición de Euler

* $f_K = \frac{1}{C}(A - \delta)e^{-\rho t}$

$f_{\dot{K}} = \frac{1}{C}(-1)e^{-\rho t}$

$\frac{\partial f_{\dot{K}}}{\partial t} = \frac{-\dot{C}}{C^2}(-1)e^{-\rho t} + \frac{\rho}{C}e^{-\rho t}$

$f_K = \frac{\partial f_{\dot{K}}}{\partial t}$

$\frac{1}{C}(A - \delta)e^{-\rho t} = \frac{\dot{C}}{C^2}e^{-\rho t} + \frac{\rho}{C}e^{-\rho t}$

$\dot{C} = (A - \delta - \rho)C$

∴ $C_{(t)} = H_1 e^{(A - \delta - \rho)t}$

* $\dot{K} = AK - \delta K - c$

$\dot{K} - (A - \delta)K = -c$

FI: $e^{\int -(A-\delta)dt} = e^{-(A-\delta)t}$

$Ke^{-(A-\delta)t} = -\int c e^{-(A-\delta)t}dt$

$\boxed{C = H_1 e^{(A-\delta-\rho)t} \longrightarrow Ce^{-(A-\delta)t} = H_1 e^{-\rho t}}$

$Ke^{-(A-\delta)t} = -\int H_1 e^{-\rho t}$

∴ $K_{(t)} = \frac{H_1}{\rho}e^{(A-\delta-\rho)t} + H_2 e^{(A-\delta)t}$

b) Condiciones de transversalidad

$t = 0 \to K(0) = 10 = \frac{H_1}{\rho} + H_2$

$t = \infty \neq t = T$

→ $\lim_{t\to\infty}[f_{\dot{K}}] = 0$ , valor terminal desconocido

$\lim_{t\to\infty}[Y_{(t)}] = Y_\infty$ , " " conocido

El segundo es una alternativa del primero, solo cuando el horizonte de tiempo es infinito

$\lim_{t\to\infty} K = \frac{H_1}{\rho}e^{(A-\delta-\rho)t} + H_2 e^{(A-\delta)t} = 0$

$H_2 \underbrace{e^{(A-\delta)t}}_{\infty} = 0$

$H_2 = 0$

$10 = \frac{H_1}{\rho} + H_2 \longrightarrow H_1 = 10\rho$

∴ $K^*_{(t)} = 10\rho e^{(A-\delta-\rho)t}$

$C^*_{(t)} = 10\rho e^{(A-\delta-\rho)t}$

c) Condiciones suficientes

$\frac{\partial f}{\partial K} = \frac{1}{C}(A-\delta)e^{-\rho t}$ — $f_{KK} = -C^{-2}(A-\delta)^2 e^{-\rho t}$ ; $f_{K\dot{K}} = C^{-2}(A-\delta)e^{-\rho t}$

$\frac{\partial f}{\partial \dot{K}} = -\frac{1}{C}e^{-\rho t}$ — $f_{\dot{K}K} = C^{-2}(A-\delta)e^{-\rho t}$ ; $f_{\dot{K}\dot{K}} = -C^{-2}e^{-\rho t}$

$H = \begin{bmatrix} -C^{-2}e^{-\rho t} & C^{-2}(A-\delta)e^{-\rho t} \\ C^{-2}(A-\delta)e^{-\rho t} & -C^{-2}(A-\delta)^2 e^{-\rho t} \end{bmatrix}$

$H_1 = -C^{-2}e^{-\rho t} < 0$

$H_2 = C^{-4}(A-\delta)^2 e^{-2\rho t} - C^{-4}(A-\delta)^2 e^{-2\rho t} = 0$

∴ Función cóncava → Max local

d) Diagrama de Fases

1. $\dot{c} = 0 \longrightarrow (A - \delta - \rho)C = 0$
$c = 0$

$\dot{K} = 0 \longrightarrow AK - \delta K - c = 0$
$c = (A - \delta)K$

2. $\frac{\partial \dot{c}}{\partial c} = (A - \delta - \rho) < 0 \begin{cases} c > c^* \to c & \downarrow \\ c < c^* \to c & \uparrow \end{cases}$

$\frac{\partial \dot{K}}{\partial K} = (A - \delta) > 0 \begin{cases} K > K^* \to K & \rightarrow \\ K < K^* \to K & \leftarrow \end{cases}$

3.