Task 1:

1. You survey households in your area to find the average rent they are paying. Find the standard deviation from the following data:

\$1550, \$1700, \$900, \$850, \$1000, \$950.

Solution:

$\mathbf{X_1 = 1550},\ \mathbf{X_2 = 1700}, \mathbf{X_3 = 900}, \mathbf{X_4 = 850}, \mathbf{X_5 = 1000}, \mathbf{X_6 = 950}$

There are total 6 sample data. $\mathbf{n = 6}$

To calculate the average or the mean of the above data, we use the formula:

$$\mathbf{\overline{X} = \frac{\sum_{i=1}^{n}(X_i)}{n}}$$

$$\overline{X} = \frac{1550+1700+900+850+1000+950}{6}$$

$$\mathbf{\overline{X} = 1158.33}$$

The Mean of the above data is **1158.33**

To calculate the variance we use the formula:

$$\mathbf{S^2 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}}$$

$$S^2 = \frac{(1550-1158.33)^2+(1700-1158.33)^2+(900-1158.33)^2+(850-1158.33)^2+(1000-1158.33)^2+(950-1158.33)^2}{6-1}$$

$$S^2 = \frac{153402.78 + 293402.78 + 66736.11 + 95069.44 + 25069.44 + 43402.78}{5}$$

$$\mathbf{S^2 = 135416.67}$$

The Variance is **135416.67**

To calculate the Standard Deviation we use the formula:

$$\mathbf{S = \sqrt{\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}}\ \ Or\ \ S = \sqrt{S^2}}$$

$$S = \sqrt{135416.67}$$

S =367.99

The standard Deviation of the above data is **367.99**

	Data (X)	X - $\overline{X}$	(X - $\overline{X}$)²
X1	1550	391.67	153402.78
X2	1700	541.67	293402.78
X3	900	-258.33	66736.11
X4	850	-308.33	95069.44
X5	1000	-158.33	25069.44
X6	950	-208.33	43402.78
Sum	**6950**		**677083.33**
n	**6**		
Mean ($\overline{X}$)	**1158.33**		
Variance (S^2)	**135416.67**		
Standard Deviation (S)	**367.99**		

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2. Find the variance for the following set of data representing trees in California (heights in feet):

3, 21, 98, 203, 17, 9

Solution:

$\mathbf{X_1 = 3,\ X_2 = 21, X_3 = 98, X_4 = 203, X_5 = 17, X_6 = 9}$

There are total 6 data. Therefore

$\mathbf{n = 6}$

To calculate the average or the mean of the above data, we use the formula:

$$\mathbf{\overline{X} = \frac{\sum_{i=1}^{n}(X_i)}{n}}$$

$$\overline{X} = \frac{3+21+98+203+17+9}{6}$$

$$\mathbf{\overline{X} = 58.50}$$

The Mean of the above data is **58.50**

To calculate the variance we use the formula:

$$\mathbf{S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}$$

$$S^2 = \frac{(3-58.50)^2+(21-58.50)^2+(98-58.50)^2+(203-58.50)^2+(17-58.50)^2+(9-58.50)^2}{6-1}$$

$$S^2 = \frac{3080.25 + 1406.25 + 1560.25 + 20880.25 + 1722.25 + 2450.25}{5}$$

$$\mathbf{S^2 = 6219.90}$$

The Variance is **6219.90**

	Data (X)	X - $\bar{X}$	(X - $\bar{X}$)²
X1	3	-55.50	3080.25
X2	21	-37.50	1406.25
X3	98	39.50	1560.25
X4	203	144.50	20880.25
X5	17	-41.50	1722.25
X6	9	-49.50	2450.25
Sum	**351**		**31099.50**
n	**6**		
Mean ($\bar{X}$)	**58.50**		
Variance (S^2)	**6219.90**		

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3. In a class on 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in two subjects and 3 failed in three subjects. Find the probability distribution of the variable for number of subjects a student from the given class has failed in.

Solution:

The probability $\mathrm{P(X)}$ of failing in 0 subjects$(\mathrm{X} = 0)$, $\mathrm{P}(0) = 80/100 = \mathbf{0.8}$

The probability $\mathrm{P(X)}$ of failing in 1 subject$(\mathrm{X} = 1)$, $\mathrm{P}(1) = 10/100 = \mathbf{0.1}$

The probability $\mathrm{P(X)}$ of failing in 2 subjects$(\mathrm{X} = 2)$, $\mathrm{P}(1) = 7/100 = \mathbf{0.07}$

The probability $\mathrm{P(X)}$ of failing in 3 subjects$(\mathrm{X} = 3)$, $\mathrm{P}(1) = 3/100 = \mathbf{0.03}$

Task 2:

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1. A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution:

Total number of questions $\mathbf{n = 20}$

Number of wrong answers $\mathbf{k = 5}$

Number of correct answers $\mathbf{n - k = 20 - 5 = 15}$

Probability of wrong answer $\mathbf{P = 3/4}$

Probability of correct answer $\mathbf{Q = 1 - (3/4) = 1/4}$

The binomial distribution formula is:

$$\mathbf{P(k) = \binom{n}{k} P^k Q^{n-k}}$$

$$P(k) = \frac{n!}{(n-k)!k!} P^k Q^{n-k}$$

On substituting the values to the formula

$$P(k) = \frac{20!}{(20-5)!5!} (3/4)^5 (1/4)^{20-5}$$

$$P(k) = \frac{20!}{15!5!} (3/4)^5 (1/4)^{15}$$

$$P(k) = \frac{20*19*18*17*16}{5*4*3*2*1} (3/4)^5 (1/4)^{15}$$

$$P(k) = 0.000003426495$$

The probability is $\mathbf{0.000003426495}$

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2. A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

Solution:

Total number of trials $\mathbf{n = 50}$

Count of success $\mathbf{k = 5}$

Count of failure $\mathbf{n - k = 50 - 5 = 45}$

Probability of getting a D: $\mathbf{P = 1/5}$

Probability of not getting a D: $\mathbf{Q = 1 - (1/5) = 4/5}$

The binomial distribution formula is:

$$\mathbf{P}(\boldsymbol{k}) = \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{P}^{\boldsymbol{k}} \boldsymbol{Q}^{\boldsymbol{n-k}}$$

$$\mathrm{P}(k) = \frac{n!}{(\mathrm{n-k})!\mathrm{k}!} P^k Q^{n-k}$$

On substituting the values to the formula

$$\mathrm{P(k)} = \frac{50!}{(50-5)!5!} (1/5)^5 (4/5)^{50-5}$$

$$\mathrm{P(k)} = \frac{50!}{45!5!} (1/5)^5 (4/5)^{45}$$

$$\mathrm{P(k)} = \frac{50*49*48*47*46}{5*4*3*2*1} (1/5)^5 (4/5)^{45}$$

$$\mathrm{P(k)} = 0.029531$$

The probability is $\mathbf{0.029531}$

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3. Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes.

Solution:

Red balls: $\mathbf{R} = \mathbf{4}$

Black balls: $\mathbf{B} = \mathbf{6}$

Total balls: $\mathbf{T} = \mathbf{10}$

The probabilities of all possible outcomes are:

Probability of drawing both red balls: $\mathbf{P(RR)} = \left(\frac{\mathbf{4}}{\mathbf{10}}\right) * \left(\frac{\mathbf{3}}{\mathbf{9}}\right) = \frac{\mathbf{12}}{\mathbf{90}} = \mathbf{.1333} = \mathbf{13.33}\%$

Probability of drawing 1 red ball & 1 black ball: $\mathbf{P(RB)} = \left(\frac{\mathbf{4}}{\mathbf{10}}\right) * \left(\frac{\mathbf{6}}{\mathbf{9}}\right) = \frac{\mathbf{24}}{\mathbf{90}} = \mathbf{.2666} = \mathbf{26.66}\%$

Probability of drawing 1 black ball & 1 red ball: $\mathbf{P(BR)} = \left(\frac{\mathbf{6}}{\mathbf{10}}\right) * \left(\frac{\mathbf{4}}{\mathbf{9}}\right) = \frac{\mathbf{24}}{\mathbf{90}} = \mathbf{.2666} = \mathbf{26.66}\%$

Probability of drawing both black balls: $\mathbf{P(BB)} = \left(\frac{\mathbf{6}}{\mathbf{10}}\right) * \left(\frac{\mathbf{5}}{\mathbf{9}}\right) = \frac{\mathbf{30}}{\mathbf{90}} = \mathbf{.3333} = \mathbf{33.33}\%$
