8

Frequency Response Analysis

Background

Our primary objective in studying control systems is to understand how they behave. Then through understanding their behavior, hopefully we can exert some influence to cause them to perform in a manner that will be beneficial.

Just observing behavior is not quite enough, however. It is necessary to have definite criteria of behavior, and the criteria must be measurable so that comparisons can be made. Frequency response analysis is one method of meeting these needs.

A frequency response test of a control system component (or even of a whole control system) is conducted by forcing a test signal which varies in sine wave fashion into the input of the component. At the same time, the output of the component is tracked so that the input and the output can be compared. The unique property of the sine wave input is that it is the only type of input that produces an output of the identical form. The output will also have the sine wave shape, and its frequency of oscillation will be the same as that of the input. Hence the name, *frequency response.* This is where the similarity ends, however.

Figure 8-1 is a graph of the frequency response input and output of a component under test. Comparing the output wave with the input, two factors are significant. First, the inherent gain of the component has modified the magnitude of the output wave. In this case it emerges smaller in magnitude than that of the input wave. Second, on the time scale, the output wave is out of phase with the input wave. It actually lags behind the input

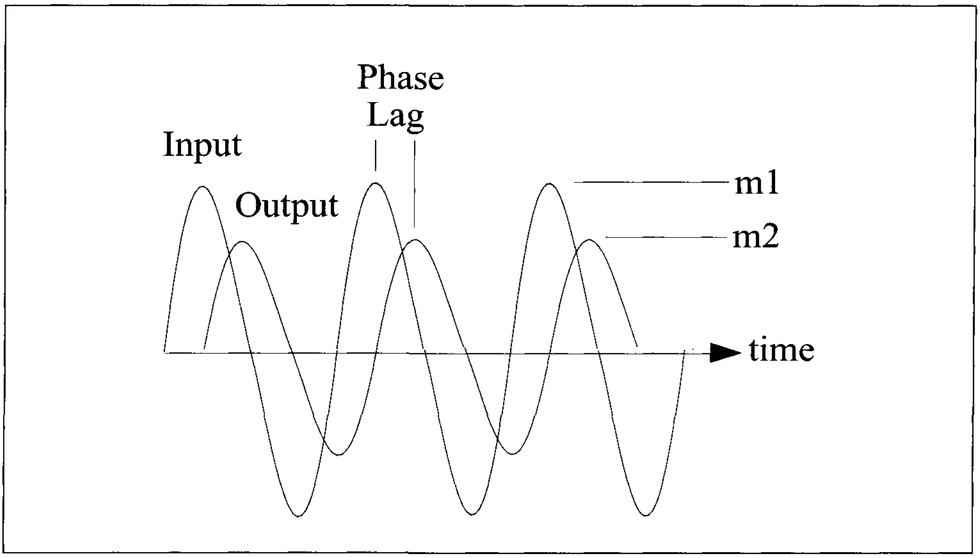

Figure 8-1. Input and output waves in a frequency response test.

wave. This is caused by the component's inherent reaction time. The slower the reaction of the component, the greater will be the time lag.

The ratio of the magnitude of the output wave to that of the input wave is called the *magnitude ratio*. It is measured by the ratio of m_2/m_1 in Figure 8-1. The phase (time) lag of the output wave is measured in degrees. If the output wave trailed the input by one whole cycle, then the phase lag would be 360°. The phase lag shown would be about 1/4 cycle or 90°.

The graph shows the magnitude and the phase lag at only one frequency. In a complete analysis, the component would be subjected to a sine wave input over a whole range of frequencies, with the magnitude ratio and phase lag being measured at each one. The variation of the magnitude ratio and the phase lag over the relevant range of frequencies are the two performance criteria that frequency response analysis yields.

The Bode Diagram

Once the magnitude ratio and phase lag data have been accumulated for the range of frequencies of interest, it is customary to plot both of these data on a base of frequency. The graph in Figure 8-2 is an example. Graphs with the frequency response data made visible in this form are often called Bode diagramsgain, after H.W. Bode, who was a noted pioneer in the development of the theory of feedback amplifiers. While process control systems function at considerably lower frequencies than those with which Bode would have been dealing, his manner of presenting the data is nevertheless applicable.

In the Bode diagram, the magnitude ratio of the output to the input is generally abbreviated as *gain*. The frequency and gain scales in the diagram are logarithmic, so they can cover a range of two or more decades in a reasonable space. Setting out the scales linearly would spread the diagram out to the point where it would be unwieldy.

Frequencies are measured in cycles per minute (cpm); the practical unit considering the slow rate of the oscillations. A commonly used scale is from 0.01 to 1 cpm, as shown in Figure 8-2. In process control, frequency rates are so low that control theorists usually talk in terms of the period of oscillation, which is the time required to make one cycle, rather than frequency. The period of oscillation will be 1/frequency.

Frequency Response of a Time Constant Element

The frequency response data in Figure 8-2 is for an element that occurs in all process control systems, namely, a time constant. The value of the time constant in this case is 2 minutes. This element also has a steady state gain of 5.0. In frequency response analysis, steady state is equivalent to zero frequency. Logarithmic scales, by their nature, cannot go down to zero, but a value of 0.01 cpm is usually low enough to reveal the steady state gain of the component under test.

The steady state gain of 5.0 means that at very low frequencies the amplitude of the output wave will be 5 times that of the amplitude of the input wave. The gain graph shows that even with a time constant of 2 minutes, which implies a relatively slow reaction to any input, at the very low frequencies the output of the element still tracks the input, and the gain of 5 is maintained. As the frequency increases, however, a point is reached where the output wave is not completed before a new wave arrives at the input of the element. From then on, the amplitude of the output wave decreases until at a frequency of 1 cpm, the gain has dropped to 0.4.

The phase lag graph shows that even at the minimum frequency of 0.01 cpm, the output wave lags slightly behind the input wave. From then on, the phase lag increases with the frequency. Interestingly, even at a theoretical infinite frequency, the phase lag never exceeds –90°. This is a unique property of a time constant element.

Frequency Response of a Dead Time Element

A frequency response plot for a dead time element is shown in Figure 8-3. Graphs are shown for three different levels of dead time: 0.2, 0.5, and 1 minute. In process control studies, dead time elements are considered to contribute phase lag to the system but no gain. Thus the gain graph for

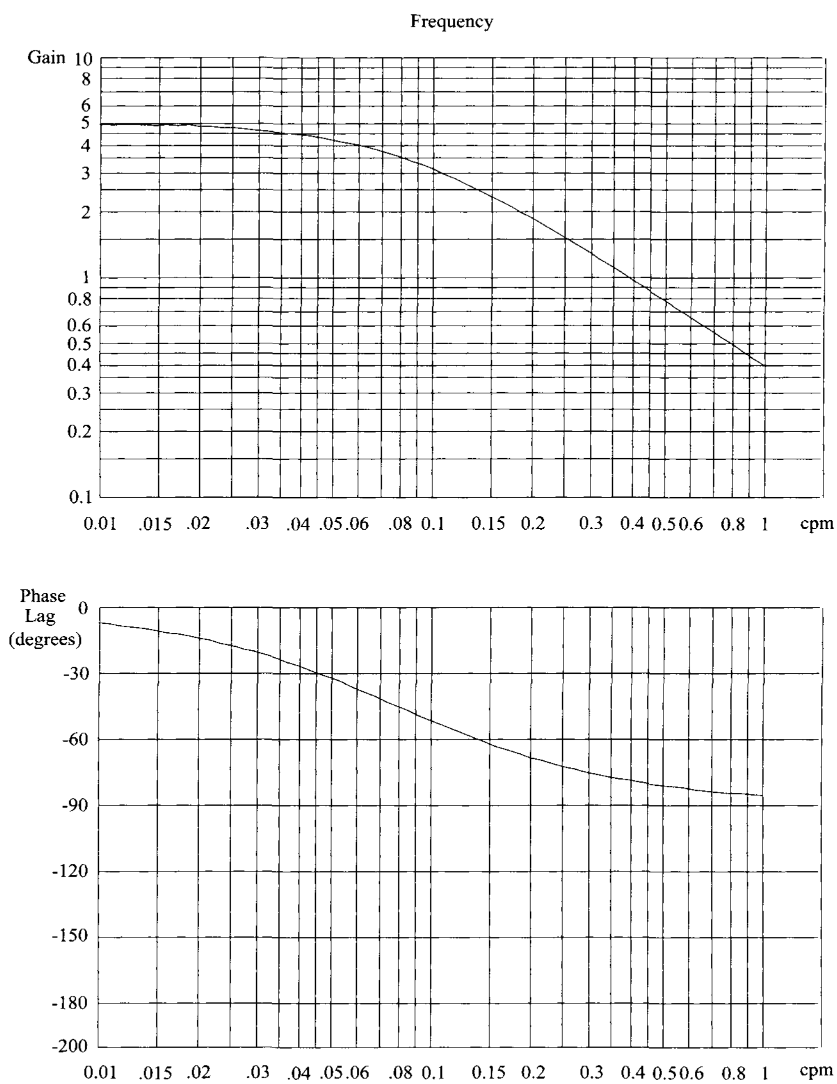

Figure 8-2. Frequency response data for a single time constant.

each of the three dead time elements is 1.0 across the whole range of frequencies.

The damage that dead time creates in feedback control systems is shown in the rapid way in which dead time increases phase lag as the frequency increases. This can have a harmful effect on the time required for the control system to recover after it is disturbed. Further discussion of this important point will follow later.

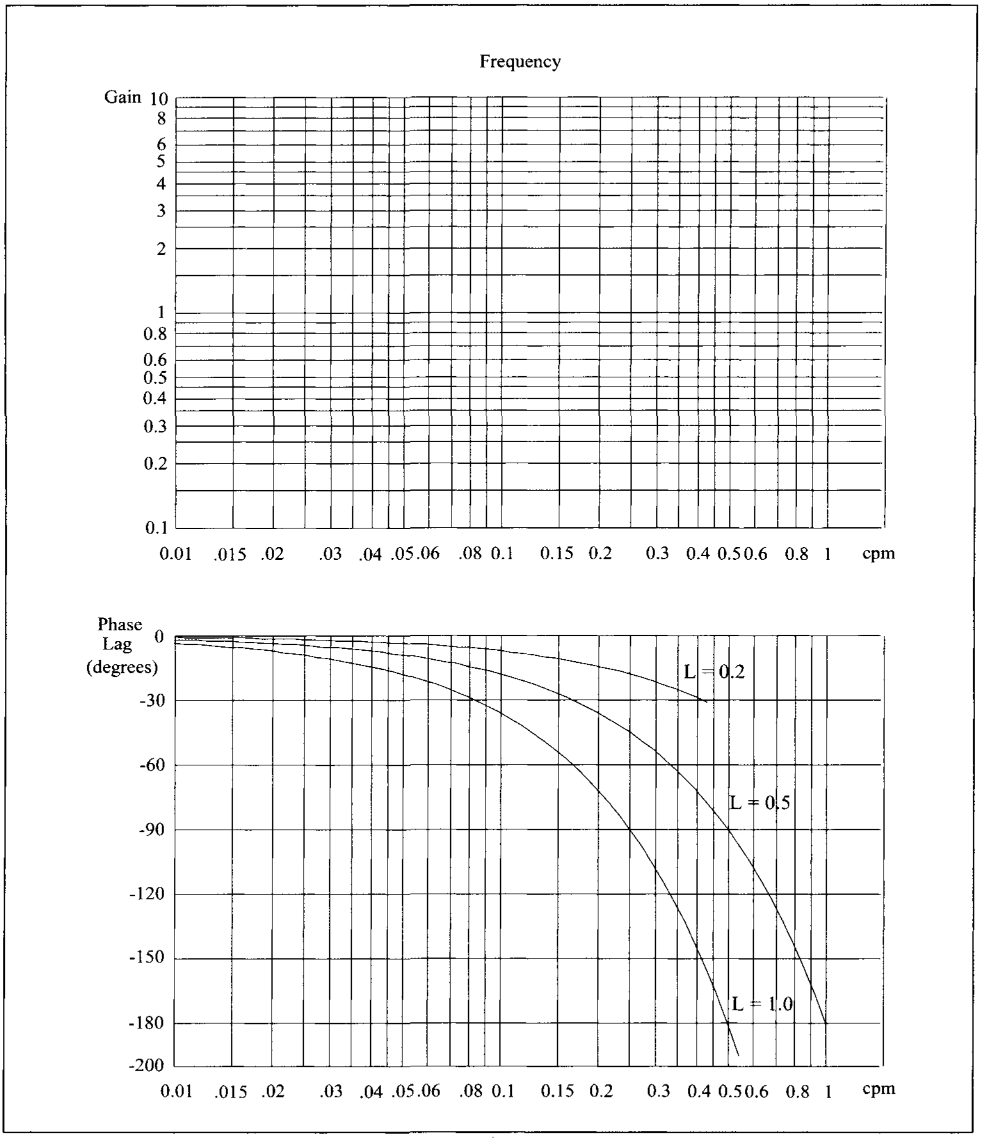

Figure 8-3. Frequency response data for assorted dead time elements.

Notice that at a frequency of 0.25 cpm, the phase lag contributed by a 1 minute dead time element is –90°. For the 0.2 and 0.5 minute dead time elements it is –18° and –45°, respectively, at the same frequency. This shows clearly that the phase lag created is in direct proportion to the dead time that is present.

Combinations of Components

If the frequency response data for the individual components are available, then the frequency response characteristics that two or more components in tandem would have can be easily computed. It is only necessary to *multiply* the individual *gain* values, and *add* the individual *phase lag* values, taken at the selected frequency. The rule: gains multiply, phase lags add.

Suppose that a system consisted of a 0.5 minute dead time, followed by a 2 minute time constant. The gain and phase lag of the combination can be determined at any frequency from the graphs in Figure 8-2 and 8-3. From Figure 8-2, at a frequency of 0.5 cpm, the gain for the time constant is 0.78, and the phase lag is –81°. From Figure 8-3, at the same frequency, the gain for the 0.5 minute dead time element is 1.0, and the phase lag is –90°. Accordingly, at a frequency of 0.5 cpm, the gain of the time constant and dead time together will be 0.78 x 1.0 = 0.78, and the phase lag will be –81° + (–90°) = –171°.

It follows that if the frequency response characteristics of the automatic controller, the control valve, the process, and the measurement sensor, which are the essential components of the control system, were available, then the frequency response characteristics of the whole system could be computed, and from that, the ultimate performance of the control system on control predicted. During the 1960s, there was a definite impetus to predict control system performance in this way. Unfortunately, the procedure requires that the characteristics of all of the components be known, and while it was not difficult to obtain this data for the controller, the control valve, and the sensor, the data for the dominant component—the process that was to be controlled—was always lacking.

Since there was no telling theoretically what mixture of dynamic elements the process might have in it, the only alternative was to obtain the data by making an actual field test. This would involve using a special signal generator to disturb the process in sinusoidal fashion, over a whole range of frequencies, and recording what resulted at the process's output. No process operator in his right mind who was in charge of a boiler, a fractionation column, or a reactor, would allow such a test to be made.

This obstacle would have brought about the demise of frequency response analysis as it applies to process control systems, had it not been for the contribution of J.G. Ziegler, who developed another more realistic method of determining the characteristics of a process. This is discussed in Chapter 10.

Period of Oscillation

When part of the output of any system is fed back into its input, then a closed loop is created, which effectively sets the stage for oscillations to occur. Anyone who has pointed a microphone at a loudspeaker knows this. Of the various performance criteria that apply to control systems, the time required to make one oscillation has the greatest impact. This time value is called the *period of oscillation.* It is actually the inverse of the frequency of oscillation that is the basis for plotting gain and phase lag in frequency response diagrams.

The period of oscillation of a control system is an inherent property of the system. It is created by the dynamic characteristics of all of the components in the system in combination. As such, in real life process control systems the period of oscillation cannot be appreciably altered. We have to live with it.

If a feedback control system is disturbed, the automatic controller usually does not recognize the upset in the process and its effect on the controlled variable until the measurement sensor has actually measured a change in the controlled variable and has fed this information back to the controller. Owing to the response times of the process and the sensor, corrective action by the controller does not take place until some time after the disturbance has occurred; in other words, it happens too late. This means that after each disturbance, the control system has to go through an interval of upset and recovery before it can get back on control.

When the controller is correctly tuned to provide the right amount of corrective action and to apply it no faster than the process can absorb it, the pattern of the recovery will be a sine wave with each peak smaller than the peak that preceded it, until the oscillations die out altogether. Figure 8-4 illustrates this.

Adjusting the controller for a good recovery in a feedback control system is a compromise between minimizing the height of the first peak, as that is when the controlled variable deviates farthest from the desired value, and minimizing the recovery time, which is the time required to get back on control. Most control experts agree that this compromise is best achieved if the recovery curve exhibits oscillations of decreasing amplitude, with the amplitude of each peak being about 1/4 of the amplitude of the peak that preceded it, as shown in Figure 8-4. The important feature that Figure 8-4 illustrates is that if the control system recovers in the optimum manner, then following a disturbance it usually takes two or three oscillations to get back on control. This being so, then the time required to make one oscillation, that is, the period of oscillation, becomes all important.

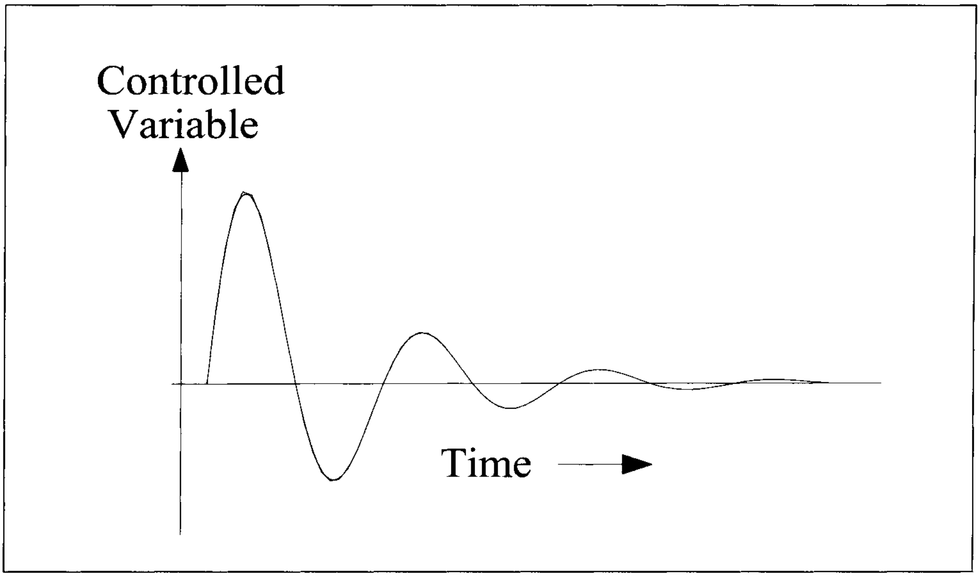

Figure 8-4. Control system recovery at an amplitude ratio of ¼.

Summary

It may well be, therefore, that the most valuable item of information that the frequency response diagram can give us is the value of the period of oscillation created by the composite of components which comprise the control system.

If conditions are favorable, or in another sense, unfavorable, a feedback control system can go into a state of continuous oscillations. Chapter 9 describes the conditions that must prevail to cause this. Chapter 9 also explains the fact that when a control system oscillates, it will do so at the frequency at which the cumulative phase lag of all the components of the system becomes –180°.

Thus, if the frequency response gain and phase lag can be plotted for the complete system of components, then the frequency at which the phase lag curve crosses –180° will be the frequency at which the control system is going to oscillate. The period of oscillation will be the inverse of this frequency, and the recovery time of the control system following a disturbance will be two or three times the period of oscillation. It is necessary to settle for a ballpark factor of two or three times, rather than a definite number, since the recovery time will also depend on the size of the disturbance. A more severe disturbance may result in a greater number of oscillations before the control system settles down.

Frequency response analysis proves, among other things, that the bad control system components are those that contribute excessive phase lag and, consequently, cause the phase lag curve to cross the –180° line at lower frequencies. Lower frequencies mean longer periods of oscillation and longer control system recovery times.