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The Z–N Approximation

In Chapter 8, which deals with frequency response analysis, the point was made that attempting to obtain frequency response data on actual operating equipment was impractical. Process operators would not allow their equipment to be upset with sine wave disturbances over a lengthy time period, when their job was to ensure production and safe operations.

Nonetheless, the dynamic behavior of most operating equipment cannot be predicted by sitting in front of one's computer and applying theory. Real life operating equipment often consists of multiple time constants, with some dead time thrown in. The only way to get at the dynamic characteristics is with a field test. The question then becomes: Can a field test be devised that would be acceptable to plant operating people and would also yield the required information?

J.G. Ziegler, during his career as a control systems engineer with the Taylor Instrument Companies, answered this question and made a significant contribution to control systems analysis. His method has come to be called the *Ziegler–Nichols approximation*, or for short, the Z–N approximation.

Historical

The events that led up to the Z–N approximation are interesting and deserve some space. During the late 1930s and early 1940s, there was no ISA—The Instrumentation, Systems, and Automation Society. In the United States, any technical work on control systems that was felt to be noteworthy was made public through accredited professional societies such as the American Society of Mechanical Engineers or the American Institute of Chemical Engineers, at their national or regional conferences or through their publications. Some work on control theory was being

done independently in the United States and England by individuals working in universities or manufacturing facilities, but the only organized group that had been officially formed up until that time was a Control Systems group, made up of several ASME members. As the one formal group that was recognized as such, the group members felt that they were in the forefront in the understanding of control systems theory.

To broaden their studies, the ASME group felt they needed an analogous system that exhibited typical process control characteristics. An obvious feature of feedback control systems is that they oscillate, and on this basis, the ASME group selected a mass oscillating at the end of a spring, with damping added to cause the oscillations to eventually die out. Thus, any conclusions that the ASME group reached with regard to control systems behavior were based on their analysis of their damped spring mass system.

It was at that time that Ziegler came to the Taylor Instruments headquarters at Rochester, N.Y., hoping to benefit from the knowledge of the resident control systems experts there. Much to his disappointment, he found that control theory, in their minds, was apparently limited to “capacity is good, and lags are bad.”

Since this hardly satisfied his inquiring mind, Ziegler spent much of his time in the Taylor research lab working with a rudimentary process simulator. His technique consisted for the most part of sending a step change input into the simulator and watching what happened at its output. He collected numerous chart records of simulator outputs, and to these he gave the name *process reaction curves*. For our purposes, the term process reaction curve will mean the output response of a process, component, or whatever, to a *step change* in input.

One thing Ziegler noticed almost immediately was when the dynamics in the simulator were any more complex than those of a single time constant, the process reaction curve assumed an S shape. This happened because the output response did not start out from $t = 0$ at its maximum rate of change, as it would do if there were only a single time constant involved. The S shape showed that the response would begin slowly and then would build up to its maximum rate of change only after a period of time. Therefore, if a step change input is applied to any process or control system component, and the reaction curve that results shows the S shape, then the presence of something more than a single time constant, possibly multiple time constants or a time constant plus dead time, is indicated.

Figure 10-1A is a process reaction curve made by a single time constant. In this case, the output starts out from zero at its maximum rate of change.

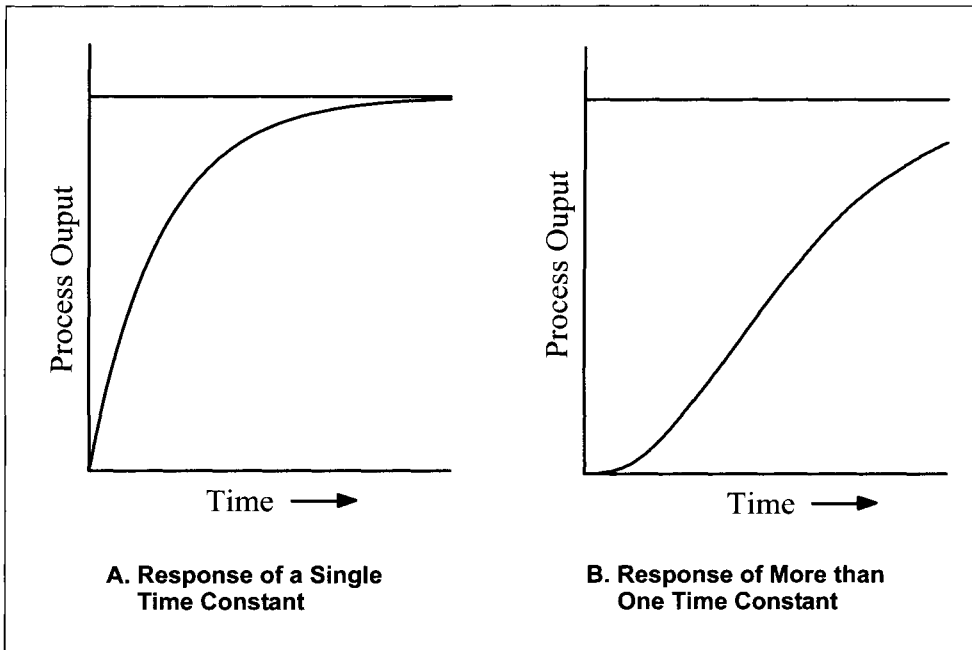


Figure 10-1. Response of a Single Time Constant (A) and Response of More than One Time Constant (B).

Figure 10-1B is a reaction curve made by two time constants in series. The reaction curve now starts out relatively slowly and builds up to its maximum rate of change later, thus creating the S shape.

Describing mathematically the reaction curve created by a single time constant is not a problem. This was done in Chapter 6, which dealt with differential equations. Describing the S shaped reaction curve in mathematical terms, however, requires a knowledge of mathematics, which most engineering graduates do not possess.

One person who did have this capability was N.B. Nichols, the Director of Research at Taylor Instruments while Ziegler was working in the research lab. Ziegler had a great admiration for Nichols' knowledge of mathematics and considered himself fortunate that he could call on Nichols when he needed help with the mathematics of control systems. In fact, Ziegler felt that the assistance he received from Nichols was valuable enough that he included Nichols' name as a co-author on some of the technical articles that Ziegler himself wrote.

Real life processes, when subjected to a step change disturbance, will inevitably exhibit the S shaped reaction curve. Since analysis using mathematics was out of the question, Ziegler was led to investigate the possibility of using a graphical method. As part of his ongoing research, Ziegler not

only tested simulated processes by running process reaction curves, he also connected the processes to an automatic controller in an attempt to understand the pattern of controller settings which were required to have the simulated control system recover from a disturbance with the desired 1/4 amplitude ratio.

One point that particularly interested Ziegler was the controller gain setting which was required for optimum control, and whether anything could be learned from the process reaction curve that would give him a clue to the correct controller gain value. As his work progressed, he became convinced that two properties of the reaction curve had a significant effect on the correct gain setting. The first was the time interval at the start of the reaction curve during which the value of the curve hardly changed from the value at which it started out. The second was the maximum rate of change that the reaction curve eventually attained before it began to taper off to a new steady state value. To put numbers on these two properties, Ziegler drew a tangent to the reaction curve at the point of its steepest slope and extended the tangent downward until it reached the vertical $t = 0$ axis. This is illustrated in Figure 10-2.

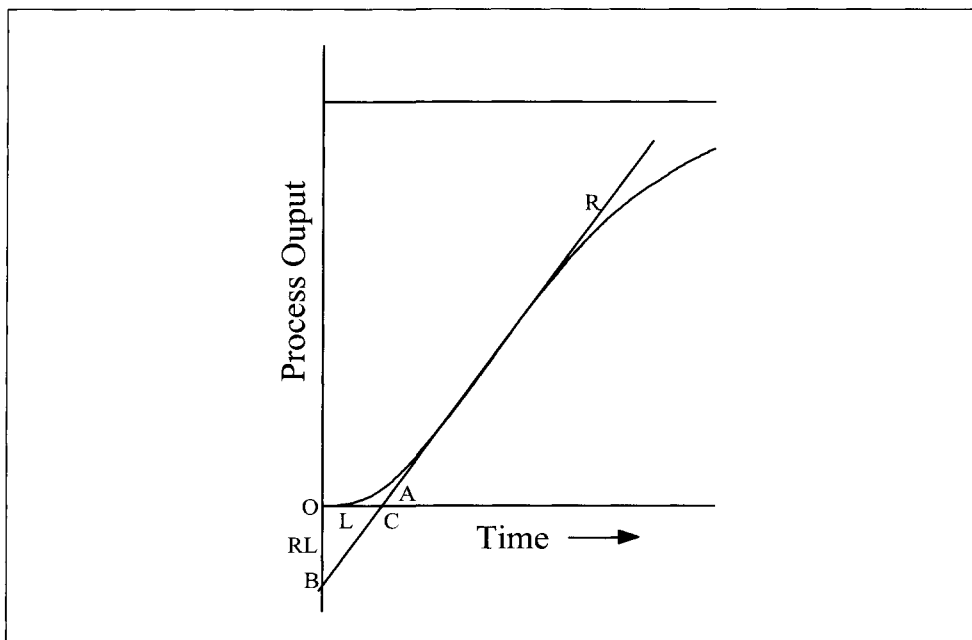


Figure 10-2. A Process Reaction Curve.

In the diagram, R is the slope of the tangent. Its units will be units of the process output per minute. The tangent intersects the horizontal line that represents the initial ($t = 0$) value of the process output at point C. The distance OC along this line is a time quantity and is designated L minutes.

The tangent then extends downward to intersect the vertical axis at point B. In this way, the mathematically complicated S curve has been replaced by the line segment OC, which is L time units in length, and the tangent with slope R.

If the controller gain setting depends on both R and L, then it is a reasonable bet that it depends on the product of them both, or RL. Since R is the slope of the tangent, then R is equal to the tangent of the angle A and also to the tangent of the angle OCB, which is equal to angle A. The tangent of angle OCB is equal to

$$\frac{OB}{OC} = \frac{OB}{L} = R, \text{ from which OB must be equal to RL.}$$

Ziegler's tests showed that if the quantity L, which is essentially dead time, increased, then a lower controller gain was required. The same applied if the slope R became greater. Could it be possible, therefore, that the required controller gain setting for a recovery with optimum stability depended on 1/RL? Numerous test runs that Ziegler conducted after developing this theory, proved that he was on the right track.

During the course of his work, Ziegler had occasion to talk informally with other engineers who were interested in what he was doing, and at one point the suggestion was made that he should prepare a paper for presentation at the ASME national conference in 1941. While this was encouraging, Ziegler knew that a paper that dealt with gain settings alone would not satisfy the audience. The following comment would obviously be made: That is all well and good, but how do we set the other controller settings, automatic reset, and derivative?

Further investigation by Ziegler, along with frequent consultations with Nichols, led him to the conclusion that the optimum reset and derivative settings had to be dependent on the apparent dead time L alone. As a result of some work by Nichols, the optimum values for reset and derivative were arrived at as simple functions of L. Thus, all of the ingredients for the complete paper were present.

The famous paper, *Optimum Settings for Automatic Controllers* (American Society of Mechanical Engineers, 1941) was then put together. Ziegler included their formulas for determining the optimum gain, reset, and derivative settings, based on first obtaining a process reaction curve. He also included a second approach, which involved finding the period of oscillation of the control system in the automatic mode, by turning up the gain of the controller until, in response to a disturbance, the system oscillated continuously on its own. The paper was then submitted.

To ensure that some questions would be asked in the question period following the presentation of a paper, it was customary to send out a copy of each paper to selected individuals prior to the conference. It was at this point that the storm broke. When the ASME Control Systems group received their preprint of Ziegler's paper, they hit the roof. What shook them was Ziegler's assertion that a process control system could be made to oscillate continuously. On the basis of their observations of their spring mass model, they claimed continuous oscillations were impossible.

As far as their model was concerned, they were right. A spring mass system will act the same as a pure two time constant system. Figure 10-3 shows the frequency response data for a system that contains two time constants and nothing more. The time constants are 2 minutes and 1 minute. The diagram shows that the phase lag curve, even with a second time constant in place, is not going to reach -180° , except at a theoretical infinite frequency. Therefore, a two time constant system cannot be made to oscillate on its own. The result was that the Control Systems group assailed the conference program committee and demanded that Ziegler's paper be withdrawn.

The fact that eluded these poor souls was that their model did not accurately represent a real life process control system. There will always be a time constant in the control valve, one or more time constants in the process, and at least one time constant in the measurement sensor, without counting any dead time that may have crept in as well. Any dynamic delay, no matter how small, that is present over and above the two time constants, will push the phase lag curve down below -180° , and make it possible for the control system to oscillate continuously on its own, given the correct amount of gain.

In the end, over the objections of the ASME Control Systems group, Ziegler was allowed to present the paper. Today, *Optimum Settings for Automatic Controllers* by Ziegler and Nichols would likely get most people's vote as the all time most significant article ever prepared on the subject of process control. Ziegler and Nichols both died in 1997, knowing that their article had received world wide acclaim.

The Z–N Approximation

Since most real life process reaction curves have the S shape and are complicated mathematically as a result, a practical question would be: Could it be possible to replace the S shaped curve with another reaction curve, which had dynamic properties equivalent to those of the S curve, but which was manageable mathematically through the use of transfer functions?

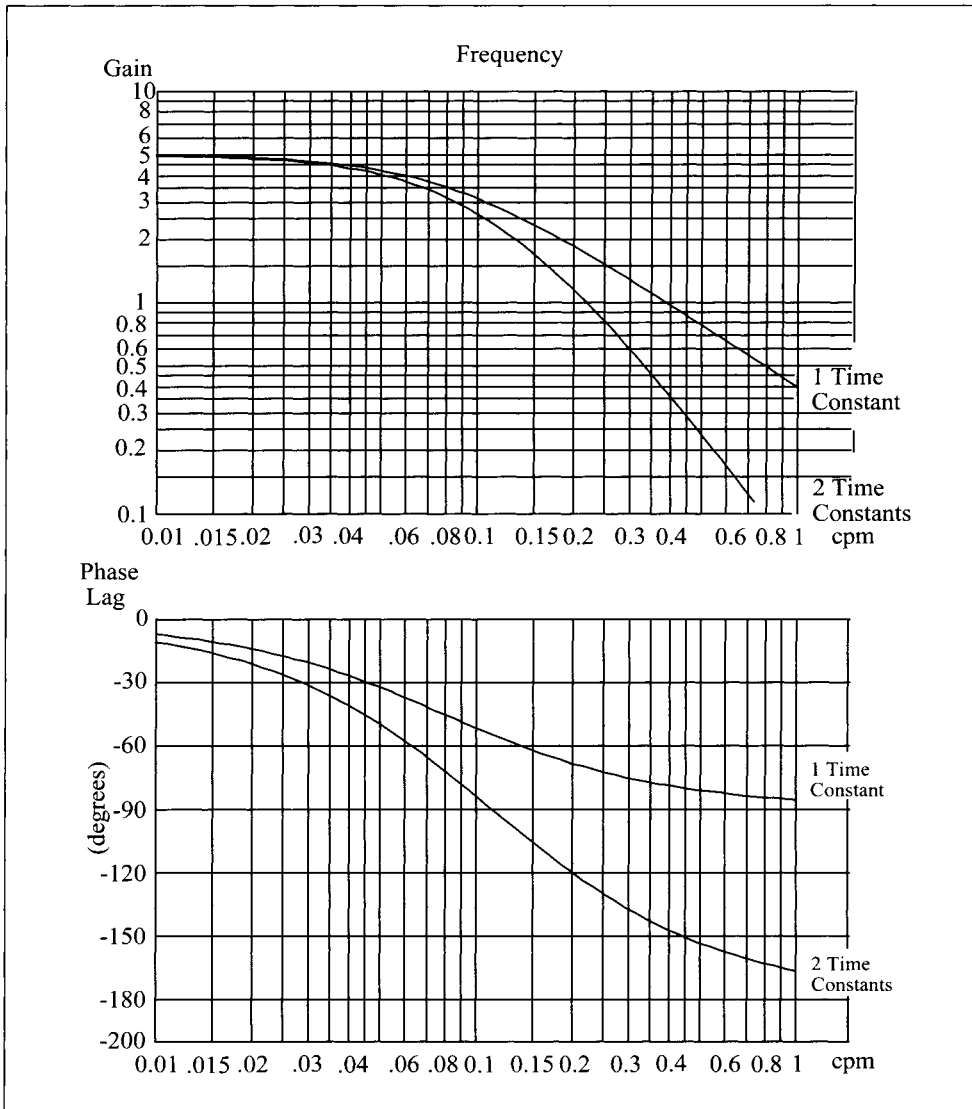


Figure 10-3. Frequency Response Data for Single and Two Time Constant Systems.

Ziegler's response to this question was the proposition that a synthetic process reaction curve that consisted of a dead time interval followed by a single time constant could be substituted for the S curve, thus rendering it possible to analyze the behavior of the complete control system. The frequency response data for dead time plus a single time constant in tandem can be easily derived. Unfortunately, in his paper, *Optimum Settings for Automatic Controllers*, Ziegler did not propose a method for determining the actual values that should be assigned to the dead time and time constant. Since we can no longer call upon Ziegler for assistance, we have to improvise.

Whatever method is contrived, the prime requirement will be that the replacement curve must generate a phase lag of -180° at a frequency that is at least close to the frequency at which the original S curve produces a phase lag of -180° . With this stipulation, the plan will be to start with a model process reaction curve that is sufficiently complex to generate an S shaped reaction curve, but for which the components are accurately known, so that the overall frequency response data can be accurately determined. Then a second curve consisting of dead time and a single time constant can be fitted to it and tested to see if it meets the important criterion.

The model process reaction curve will be one created by three time constants in series. The time constant values are 2.0, 1.5, and 0.5 minutes, respectively. Figure 10-4 shows the S shaped process reaction curve created by this combination. The phase lag curve for this synthetic process will cross the -180° line at a frequency of 0.26 cpm. The verification for this is not difficult. In general, the phase lag contributed by a time constant is equal to the angle whose tangent is $-\omega T$, where ω is the frequency in radians per minute, and T is the time constant in minutes.

$$\omega = 2 \pi \times f = 2 \pi \times 0.26 = 1.634 \text{ rad/min}$$

For time constant 1, phase lag = $\tan^{-1} (-1.634 \times 2.0) = \tan^{-1} (-3.27) = -73.0^\circ$

For time constant 2, phase lag = $\tan^{-1} (-1.634 \times 1.5) = \tan^{-1} (-2.45) = -67.8^\circ$

For time constant 3, phase lag = $\tan^{-1} (-1.634 \times 0.5) = \tan^{-1} (-0.82) = -39.2^\circ$

Total phase lag from all three time constants = -180.0° .

In Figure 10-4, the scale for the process variable has been selected arbitrarily on a 0 to 4 basis to simplify the analysis. It could be in temperature, pressure, or other process units, with an appropriate factor. The process reaction curve for the three time constant system starts out at 0 time and 0 on the vertical scale, proceeds through its S shape, and eventually reaches its ultimate value of 4 after a long time.

The line AB is the tangent to the reaction curve at its steepest point on the curve. The tangent meets the horizontal (time) axis at 0.85 minutes. This is the time value of the quantity L , in this case.

At first glance, it would appear that the search for the values for the dead time and the time constant of the replacement reaction curve has been successful. The dead time should be 0.85 minutes. Therefore, it only remains to find the value for a time constant that, in combination with the 0.85

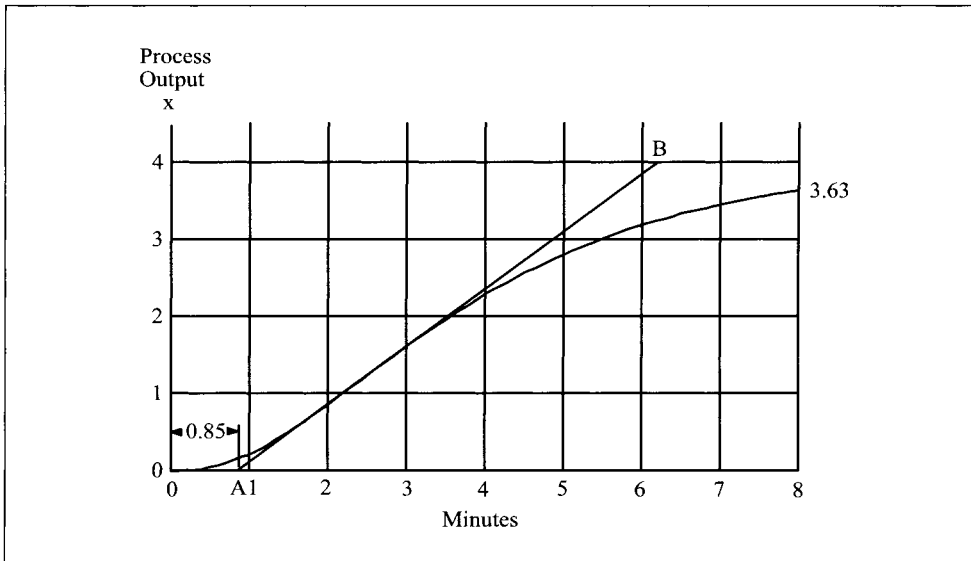


Figure 10-4. A Process Reaction Curve Created by Three Time Constants in Series.

minute dead time, will produce a phase lag of -180° at a frequency of 0.26 cpm.

Alas, it won't be that easy. How much phase lag does 0.85 minutes of dead time create at a frequency of 0.26 cpm?

$$0.26 \text{ cpm} = 1.634 \text{ rad/min, as before.}$$

$$\text{Phase lag} = -\omega L = -1.634 \times 0.85 = -1.39 \text{ rad} = -79.6^\circ$$

The graph in Figure 10-3 shows that the maximum phase lag that one time constant can produce is -90° . Therefore, no single time constant, in combination with a dead time of 0.85 minutes, can produce -180° of phase lag at 0.26 cpm. $L = 0.85$ minute does not work.

There is another problem. It is known that the three time constant process will oscillate at a frequency of 0.26 cpm because it is known that the time constants are 2.0, 1.5, and 0.5 minutes. In the case of a real life process reaction curve, the dynamic elements involved are not known, so the frequency at which the control system will oscillate can't be determined by that route. Conclusion? Back to the drafting board.

The starting point in the search for the appropriate time constant is the three things that are known about it.

- In Figure 10-4, the time constant would start out from a point on the horizontal (time) axis, proceed upward in standard time constant fashion, and eventually reach a final value of $x = 4$.
- The point from which it starts must be a time value > 0.85 minutes.
- The process variable y should track the path of the original three time constant reaction curve as closely as possible. One way that this could be achieved would be to have the single time constant curve intersect with the three time constant curve at its end point, at which $t = 8$, and the process variable $x = 3.63$.
- The values of the dead time L and the time constant T must create a phase lag of -180° at some frequency that is reasonably close to 0.26 cpm.

The function $f(t)$, which is consistent with these conditions, is

$$x = 4 \left(1 - e^{-\frac{1}{T}(t-L)} \right).$$

The math will be easier to handle if $1/T$ is made equal to z . Then

$$x = 4 \left(1 - e^{-z(t-L)} \right).$$

If $t = 8$ and $x = 3.63$ satisfy this expression, then $3.63 = 4(1 - e^{-z(8-L)})$.

Reducing this,

$$\frac{3.63}{4} = 1 - e^{-z(8-L)}, \text{ and } e^{-z(8-L)} = 1 - \frac{3.63}{4} = 0.0925$$

$$-z(8-L) = \log_e 0.0925 = -2.38$$

$$z = \frac{2.38}{8-L}, \text{ and } T = \frac{1}{z} = \frac{8-L}{2.38}.$$

By using this relationship, it is possible to enter trial values for the dead time L and calculate the associated value of the time constant T , so that the time constant graph will begin at $t = L$ and pass through the point $t = 8$, $x = 3.63$. This does not, however, yield the rest of the information that is really required, namely, the phase lags, which will be contributed by the dead time and the time constant, and their sum. The phase lag calculation cannot be done without knowing the frequency.

Estimating the Frequency of Oscillation

Optimum Settings for Automatic Controllers contains a clue as to how the frequency of oscillation can be estimated. Ziegler and Nichols concluded that the optimum controller reset rate and derivative settings depended on the dead time L alone. Accordingly, they included in the article the formulas for calculating the optimum settings for both procedures: finding the period of oscillation (P_u) or making a process reaction curve and finding the dead time L by drawing the tangent to the curve. The formulas they published were:

$$\text{Optimum derivative} = \frac{P_u}{8} = 0.5L$$

$$\text{Optimum reset rate} = \frac{2.0}{P_u} = \frac{0.5}{L}$$

The constants that Ziegler and Nichols used in these relationships were based to some extent on the pneumatic controllers that Taylor Instruments were marketing in the 1940s, and on the internal pneumatic circuitry that these controllers used. Consequently, these formulas will not necessarily give the best values for the reset rate and derivative settings for the analog and digital electronic controllers in use today. However, the point here is not to be able to calculate reset and derivative settings. Rearranging either of these formulas, it is apparent that the period of oscillation P_u is equal to four times the dead time L and this does not depend on hardware. This is the information that is needed to make it possible to estimate the frequency of oscillation from any process reaction curve.

The tangent to the steepest point on the three time constant reaction curve meets the horizontal axis at 0.85 min. The period of oscillation is therefore estimated to be four times this, or 3.4 minutes, and the frequency of oscillation will be the inverse of the period, or 0.294 cpm. The true value is 0.26 cpm, so the value obtained by the estimation procedure is reasonably close. In the absence of any better method, this procedure will have to suffice.

Values for the Dead Time and Time Constant

Completing the example will best illustrate the method.

Starting with the relation

$$T = \frac{8 - L}{2.38}$$

select a trial value for L .

A previous calculation showed that $L = 0.85$ is too small a value, so the first trial value will be $L = 1.0$.

$$\text{If } L = 1.0, \text{ then } T = \frac{8 - 1}{2.38} = 2.94 \text{ min.}$$

Frequency of oscillation is estimated to be $0.294 \text{ cpm} = 1.847 \text{ rad/min}$.

Phase lag for $L = 1.0 \text{ min.}$ is $-1.847 \times 1 = -1.847 \text{ rad.} = -105.8^\circ$.

Phase lag for $T = 2.94 \text{ min.}$ is $\tan^{-1}(-1.847 \times 2.94) = \tan^{-1}(-5.43) = -79.6^\circ$.

Total phase lag = $-105.8 + (-79.6) = -185.4^\circ$, which is 5.4° too much.

At this point it is apparent that it would be easy to design a computer program that would quickly calculate the values for the time constant T , the phase lags for the dead time and for the time constant, and the total phase lag, from trial values of L . By using such a program, the values sought turn out to be:

Dead Time $L = 0.95 \text{ min.}$ Time Constant $T = 2.96 \text{ min.}$

Phase lag for $L = 0.95$ will be $-1.847 \times 0.95 = -1.755 \text{ rad.} = -100.5^\circ$.

Phase lag for $T = 2.94$ will be $\tan^{-1}(-1.847 \times 2.94) = \tan^{-1}(-5.47) = -79.6^\circ$.

Total phase lag = $-100.5 + (-79.6) = -180.2^\circ$.

Just How Good Is the Approximation?

It is now appropriate to investigate how well the dead time plus one time constant approximation matches the original three time constant process reaction curve. This should be done in two ways:

- By overlaying the dead time plus one time constant reaction curve on the three time constant reaction curve (Figure 10-4).
- By comparing the frequency response gain and phase data for both reaction curves.

Figure 10-5 shows how the two reaction curves compare on a time basis. As planned, the curves intersect at $t = 8$, $x = 3.63$, although the dead time (L) values are 0.85 and 0.95 minutes, respectively.

What is most significant, however, is the comparison of the two phase lag curves. Figure 10-6 shows that they track fairly well, including at the point where they reach -180° . The original process reaction curve consisting of

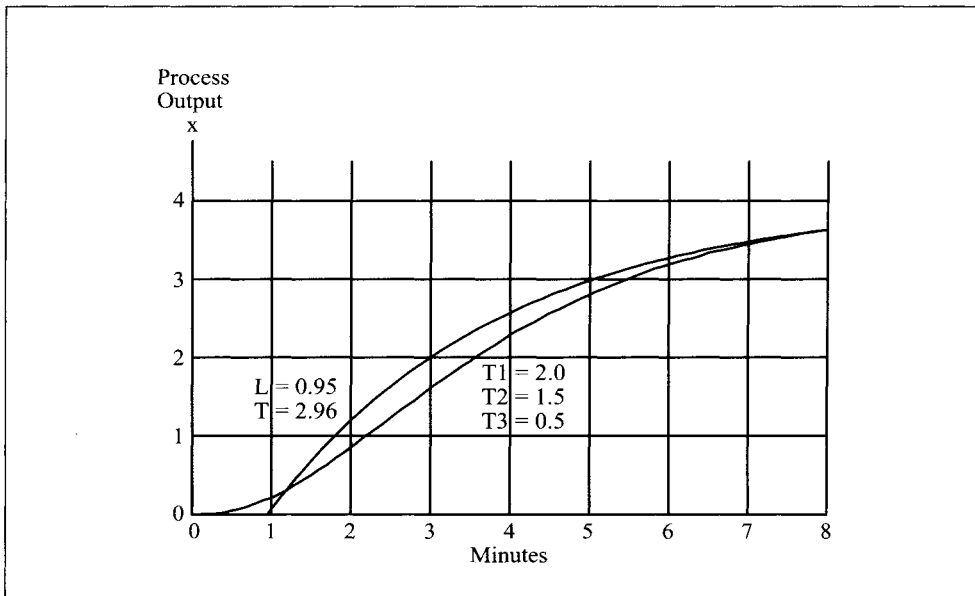


Figure 10-5. Process Reaction Curves for the Three Time Constant System and the Z-N Approximation.

three time constants shows a frequency of oscillation of 0.26 cpm, while the approximation reaction curve, with dead time and a single time constant, crosses -180° at a frequency of 0.29 cpm. The equivalent periods of oscillation are therefore 3.8 and 3.4 minutes. This is about the level of accuracy that is to be expected from the approximation.

At the frequency of oscillation, the gain values are considerably different, of the order of 0.3 and 0.75. This is not a problem, however, because the adjustable gain in the automatic controller makes it possible to set up the overall control system gain, which includes the gain of the process, to the value that produces a recovery with a 1/4 amplitude ratio.

Making a Process Reaction Curve

The process reaction curve was given its name in Ziegler and Nichols original 1941 article. Constructing a process reaction curve is really a dynamic test in which the input test signal is a step change. A test using this particular input pattern is a very useful one, in that it is easy to generate and it yields most, if not all, of the dynamic data needed to make a practical analysis of a process control system.

A process reaction curve is made with the automatic controller on Manual control. The procedure:

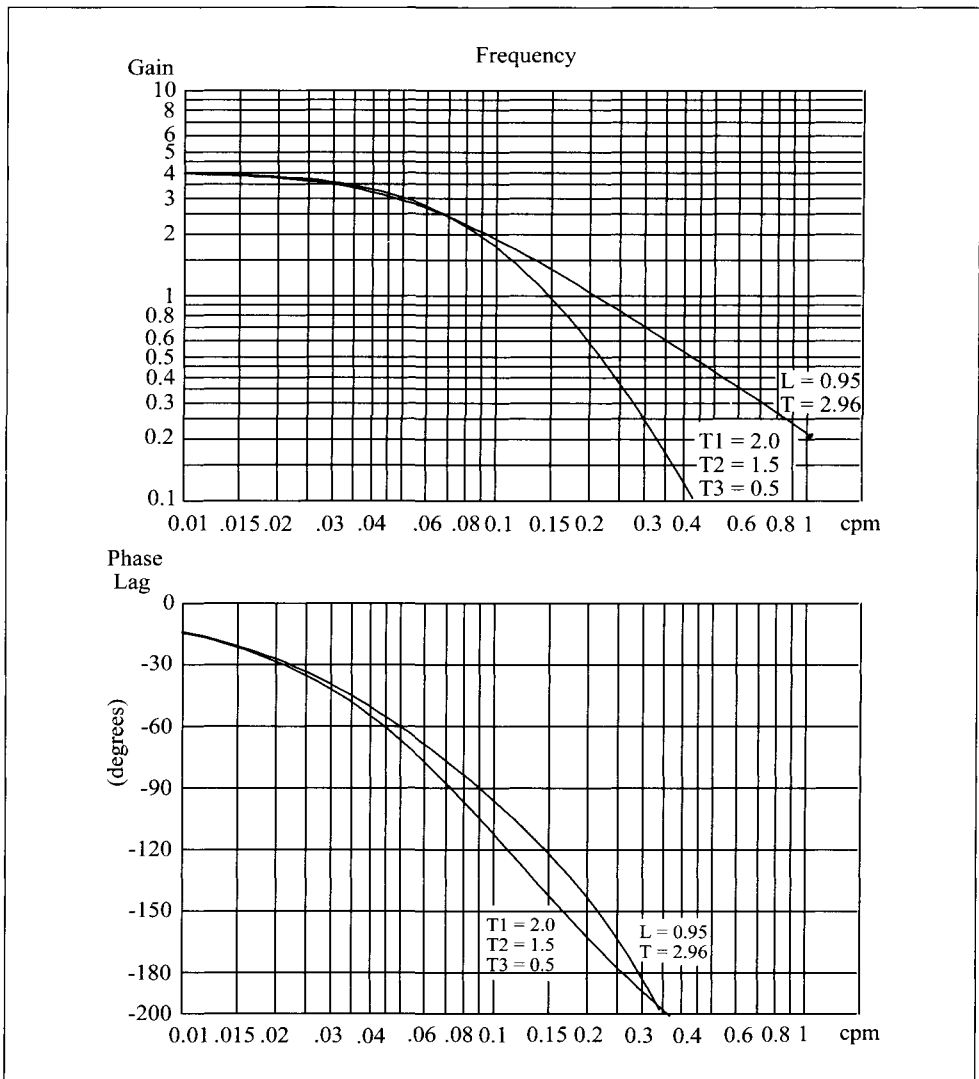


Figure 10-6. Frequency Response Graphs for the Three Time Constant System and the Z-N Approximation.

1. Set the controller on Manual and allow the control system and the controlled variable to stabilize. Record the manual output to the control valve.
2. Step the manual output to the control valve quickly to a new value. The size of the step change must be great enough that the controlled variable, when plotted either manually or automatically, exhibits the familiar S shaped curve, as shown in Figure 10-7. If no better information is available, then a 5% change in output to the valve is a reasonable number to start with. Record the size of the step change, Δm , in percent output signal to the control valve.

Once again, bear in mind that this test is being done on an operating plant. Consequently, it can be done only with the concurrence of the plant operator. One usually finds that if the operator can be convinced that running the test will ultimately lead to better control, he or she will usually agree to the test. On most processes the equivalent data will be obtained whether the step change drives the controlled variable up or down. Ask the operator which he or she would prefer.

3. The test is begun when the step change in output to the control valve is made. Starting from this moment, record corresponding readings of time and the controlled variable; in Figure 10-7, it is the process temperature. Continue taking readings until it is obvious that the rate of change of the process variable is approaching its new steady state value. The readings should be spaced closely enough that they can be plotted later to produce a reasonably representative process reaction curve.
4. When Step 3 is completed, set the output signal to the control valve, still on manual control, back to the original value recorded in Step 1. If this procedure is followed, the only disturbance that the process will see is a hump, upward or downward, in the controlled variable, as shown in Figure 10-7.

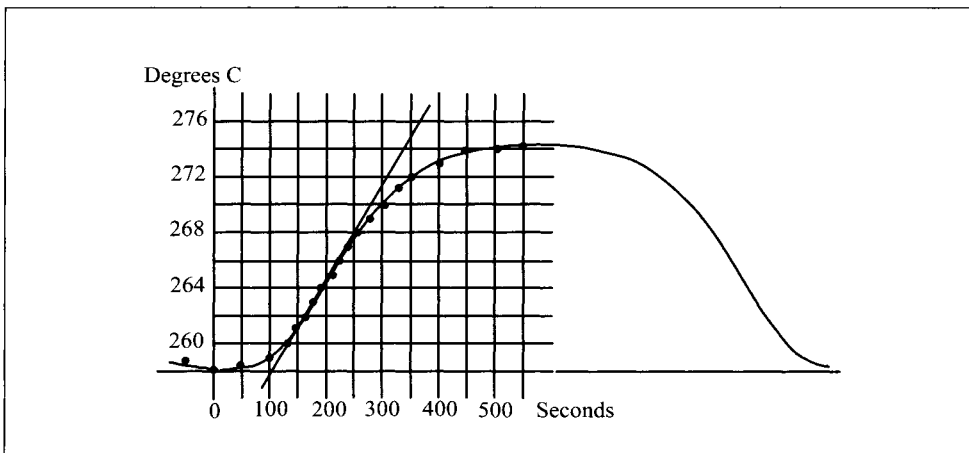


Figure 10-7. A Process Reaction Curve Test from an Actual Process.

Before describing what should be done with the data obtained, the following points on technique for taking the data should be noted. Ordinarily, a team of two observers will be required.

The documentation is made easier if the data sheet is set up beforehand. This is where another problem can be inadvertently created. It would

seem that a logical approach would be to concentrate on the independent variable, that is, the time values. When the data sheet is laid out, the times, usually at about 15 second intervals, are written in. Then when the test begins, it is up to the team member who is tracking the process variable to call out its values as his partner calls out the times.

We now have the potential for a real confrontation. If the process variable is displayed on an analog indicator or recorder, and the value is in between two of the divisions, it may be difficult to decide quickly enough what the value of the controlled variable really is. In fact, it may happen that before the team member who has this job has figured out the value at one time point, his partner is calling for the next value. At this stage, there may be some pointed criticism about the capabilities of the harassed team member, with the result that he quits and goes back to the office or shop.

This problem should not occur if it is recognized that the time device, stop watch or otherwise, is the easier instrument to read. Some timers even have a digital readout. Consequently, a better procedure is to set down even division and mid division values of the controlled variable, and as the test proceeds, record the time values at which the controlled variable attains these values.

The ultimate arrangement, of course, would be to have a high speed recorder to record the values of the controlled variable on a time base over the course of the process reaction curve run. Unfortunately, not many engineering departments will have this equipment. However, one person with a cassette recorder might be able to do the job adequately.

The measuring system of an automatic controller contributes its share to the overall dynamics of the control loop. Consequently, the readings of the process variable which are taken must come from the measuring system of the automatic controller, not from some other meter which may be measuring the same process variable, and which may be easier to read.

At this point, a word of caution is appropriate. The procedure just described works only for processes that exhibit self regulation. Self regulation means that for each position of the control valve, as set on manual control, the controlled process variable will settle out at some reasonable value. Unfortunately, there are processes which are not self regulating. If a process lacking in self regulation is holding steady on manual control at $t = 0$, an unbalance which is subsequently caused by a step movement of the control valve will be integrated over time. The controlled variable will not settle at some new steady state value, but will drift up to maximum or down to zero.

The good news, however, is that most of the non self regulating processes have reasonably good dynamic behavior, are easier to control, and thus have no real need for obtaining dynamic data by making a process reaction curve, or by any other test. Process reaction curves are generally reserved for the hard jobs.

Example 1: Reaction of a Real Process

Inasmuch as the process reaction curve shown in Figure 10-7 is from an actual process, it would be meaningful to determine the values of the dead time and the time constant, and complete the approximation. Figure 10-8 shows the same process reaction curve with the trailing end, after the cut-off point, deleted. The temperature scale has been replaced with an arbitrary scale that has its zero point at the level from which the reaction curve starts out. Since the two values being sought—the dead time and the time constant—are both time values, changing the scale of the dependent variable will not affect the outcome.

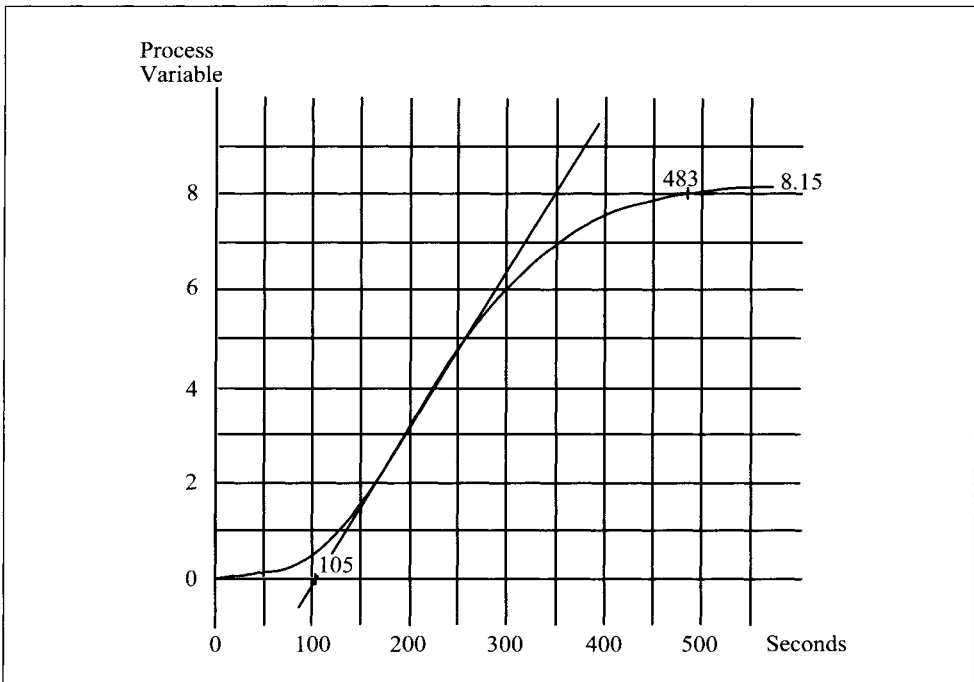


Figure 10-8. Process Reaction Curve from Figure 10-7.

The equation for the time constant, as before, is,

$$\text{PV (Process Variable)} = X \left[1 - e^{-\frac{1}{T}(t-L)} \right].$$

In this equation, X is the difference between the start of test and end of test steady state values of the controlled variable, T is the time constant, and L is the dead time.

The curve appears to show that the ultimate value X is 8.15 on the PV scale. The tangent to the curve at its steepest point crosses the base line at about 105 seconds. Thus, the estimated period of oscillation will be four times this value or 420 seconds. The frequency of oscillation will be $1/420$ Hz, and in radians per second, the frequency will be

$$\omega = 2\pi \times \frac{1}{420} = 0.0150 \text{ rad/s}.$$

A point near the end of the reaction curve is also needed. A logical question would be: Why not use the point at the end of the curve where the PV is 8.15 (its ultimate value) and $t = 550$ s? This does not compute, however, because an inherent characteristic of any time constant is that it does not attain its ultimate value until t equals infinity. Therefore, some other point, near, but not at, the end of the curve, must be chosen. The curve appears to cross $PV = 8$ at $t = 483$ s, so this point will be used.

Substituting these values in the equation for the time constant gives,

$$8.0 = 8.15 \left[1 - e^{-\frac{1}{T}(483-L)} \right].$$

Simplifying this and rearranging yields,

$$T = \frac{483-L}{4.00}.$$

The procedure now calls for selecting values of the dead time L , determining the value of the time constant T from the relation above, and then calculating the phase lag that the dead time and time constant in combination will create at a frequency of 0.0150 rad/s. This trial and error routine is to be repeated until the values of L and T that create a phase lag of -180° have been found.

For this process reaction curve the values turn out to be $L = 150$ s and $T = 83.3$ s. The phase lag values are:

$$\text{Dead time phase lag} = -\omega L = -0.0150 \times 150 = -2.24 \text{ rad} = -128.6^\circ.$$

$$\text{Time constant phase lag} = \tan^{-1}(-\omega T) = \tan^{-1}(-0.0150 \times 83.3) = -0.895 \text{ rad} = -51.3 \text{ deg}.$$

Total phase lag = $-128.6 + (-51.3) = -179.9^\circ$.

Figure 10-9 shows how closely the Z-N approximation matches the original process reaction curve.

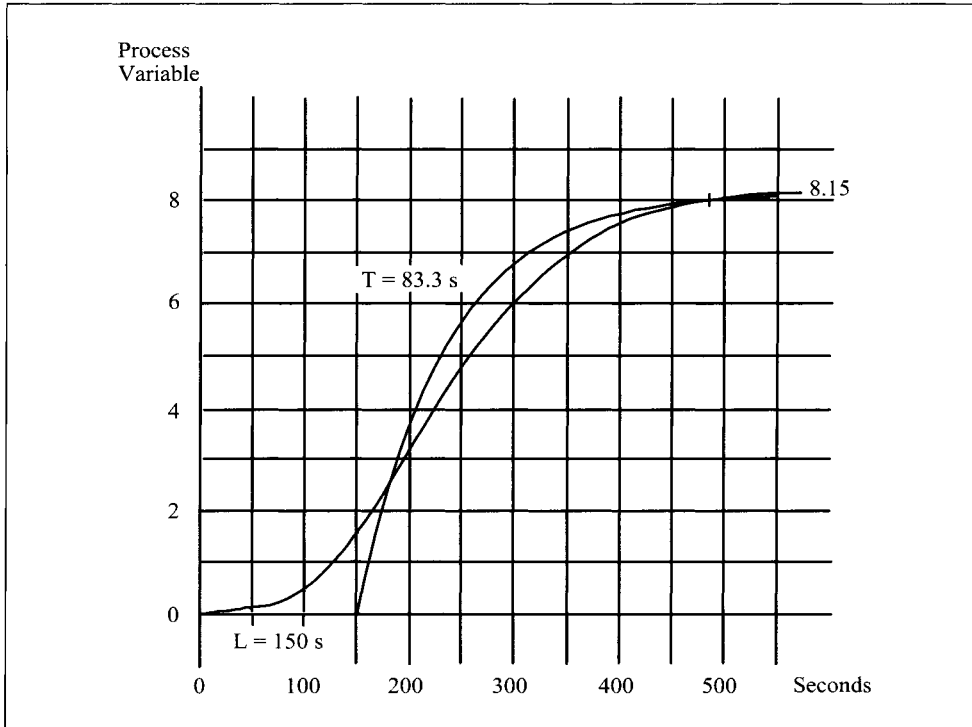


Figure 10-9. The Original Process Reaction Curve and the Z-N Approximation.