Basics of the system calculus.
(Grundzüge des Systemenkalküls.)

Part 1*).

by

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[...]

1 Basic concepts and axioms

[...]

2 The system calculus

[...]

Postulate VII. If $x \in B$, then $^{a})$ $\overline{x} \in B$, $^{b})$ $x \cdot \overline{x} = 0$, and $^{c})$ $x + \overline{x} = 1$.

The system of the algebra of logic can be extended, by introducing two infinite operations: "$\sum_{y \in X} y$" — "the sum of all things in the set X" and "$\prod_{y \in X} y$" — "the product of all things in the set X". It is then necessary to add the following postulates:

Postulate VIII. If $X \subset B$, then $^{a})$ $\sum_{y \in X} y \in B$ and $^{b})$ $x < \sum_{y \in X} y$ for all $x \in X$; $^{c})$ if furthermore $z \in B$ and $x < z$ for all $x \in X$, then $\sum_{y \in X} y < z$.

Postulate IX. If $X \subset B$, then $^{a})$ $\prod_{y \in X} y \in B$ and $^{b})$ $\sum_{y \in X} y < x$ for all $x \in X$; $^{c})$ if furthermore $z \in B$ and $z < x$ for all $x \in X$, then $z < \prod_{y \in X} y$.

Postulate X. If $x \in B$ and $X \subset B$, then $^{a})$ $x \cdot \sum_{y \in X} y = \sum_{y \in X} (x \cdot y)$ and $^{b})$ $x + \prod_{y \in X} y = \prod_{y \in X} (x + y)$.

The postulates I–VII, together with all the theorems that follow from them, form the *common* and the postulates I–X the *extended system of the algebra of logic* [1]).

*Note from the editor: The second part of this treatment arrived at the same time as the first one and will — in agreement with the author — be published in the next volume of "Fundamenta Mathematicae".

[1]) Compare with T_5, p. 177–180. One should beware that the postulates I–X are not mutually independent; in particular one can show that any three out of the four postulates V^{a}, V^{b}, X^{a} and X^{b} can be omitted from the rest of the postulates of the system considered (this sharpens a remark made in T_5, p. 180).

We proceed to the calculus of propositions, which we will mention only briefly; in order to avoid misunderstandings, we will not simply say propositional calculus, but rather *algorithm of propositions*: the jargon “propositional calculus” already has a definite meaning, which different from what we are intending.

The range of considerations of the algorithm of propositions is formed by the set S. We will define two relations between the elements of this set: “$x \supset y$” (“x implies y”) and “$x = y$” (“x is equivalent to y”).

Definition 4. [a]) $x \supset y$ holds precisely when $x, y \in S$ and $x \to y \in L$; [b]) $x = y$ holds precisely when both $x \supset y$ and $y \supset x$ hold.

Then one can prove the following:

Theorem 4. We replace the the symbols “B”, “$<$” and “$=$” by “S”, “$\supset$” and “$\equiv$” in all postulates of the common systems of the algebra of logic; furthermore we replace “0” by the variable “u” and “1” by the variable “v”, and we make the assumptions $u \in S$ and $\overline{u} \in L$ as well as $v \in L$ everywhere in the postulates; finally we leave the remaining symbols unchanged. Then the postulates I–VII are satisfied.

[...]