Act 1. Regresion lineal Multiple

> CO2 Emission by Vehicles

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Contenidos

- **5.1** - Temas avanzados sobre análisis regresión
- **5.1.1** - Verificación de supuestos: QQ-plots & análisis de residuales
- **5.2** - Estadística para datos multivariados

1. ¿Qué pasa con el fit del modelo y a que se lo atribuye?

Con el avance del proceso, el fit del modelo mejora. En primer lugar el modelo usó las variables regresoras ('Engine Size(L)', 'Cylinders', 'Fuel Consumption City (L/100 km)', 'Fuel Consumption Hwy (L/100 km)', 'Fuel Consumption Comb (L/100 km)', 'Fuel Consumption Comb (mpg)') y como variable de respuesta se tomó la emisón de CO2 ('CO2 Emissions(g/km)'), dicho modelo tuvo un ajuste de 0.904. Al obtener este resultado, se realizó el análisis de distribución y disperción de erroes, esto buscando definir una estrategia para mejora el ajuste general del modelo. Asím mismo se realizó el proceso de transformaciones de la variable dependiente con el fin de asimilarla a una distribución norma. Realizado este proceso, se identificó que las transformaciones asimétricas positivas aplicando la raiz cuadrada y logarítmo base 10 lograban este objetivo y al realizar nuevamente el fit del modelo, se aumentó el ajuste a 0.915 y 0.925 respectivamente.

2. ¿Qué sucede con el error y la distribución de este en los datos?

Siguinedo la respuesta de la pregunta anterior el error se reduce considerablemente conforme se aplican las transformaciones a los datos. Definitivamente el buscar una distribución lo más cercana a la normal ayuda al modelo a que las predicciones sean más acertadas.

3. Describa el impacto de las distintas variables ¿Que sucede si se omiten las variables con nulo impacto?

Finalmente, al identificar las variables que menos impactaban en modelo qe acuerdo con el P-Valor no hubo y hacer el proceso de eliminación de variables no hubo ninguna mejora en el ajuste sin embargo, identificamos que al eliminar solo una de estas las demás los P-Valores de las demás quedan en 0, denotando que impactan el modelo de regresión de manera importante.

Llamado a librerias:

In [ ]:
```
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/c
ontent/drive", force_remount=True).
```

In [ ]:
```python
import sklearn
import numpy as np
import pandas as pd
import seaborn as sns
import statsmodels.api as sm
import matplotlib.pyplot as plt
from scipy.stats import norm, uniform, skewnorm
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
```

Importamos los datos a un DataFrame

In [ ]:
```python
df = pd.read_csv('/content/drive/Shareddrives/Reto IA/Actividades/CO2_Emissions/CO2 Emissions_
df.head()
```

Out[ ]:

	Make	Model	Vehicle Class	Engine Size(L)	Cylinders	Transmission	Fuel Type	Fuel Consumption City (L/100 km)	Fuel Consumption Hwy (L/100 km)	Fu Consumptic Comb (L/1(kn
0	ACURA	ILX	COMPACT	2.0	4	AS5	Z	9.9	6.7	8
1	ACURA	ILX	COMPACT	2.4	4	M6	Z	11.2	7.7	9
2	ACURA	ILX HYBRID	COMPACT	1.5	4	AV7	Z	6.0	5.8	5
3	ACURA	MDX 4WD	SUV - SMALL	3.5	6	AS6	Z	12.7	9.1	11
4	ACURA	RDX AWD	SUV - SMALL	3.5	6	AS6	Z	12.1	8.7	10

In [ ]:
```python
df.isnull().sum()
```

Out[ ]:
```
Make                                0
Model                               0
Vehicle Class                       0
Engine Size(L)                      0
Cylinders                           0
Transmission                        0
Fuel Type                           0
Fuel Consumption City (L/100 km)    0
Fuel Consumption Hwy (L/100 km)     0
Fuel Consumption Comb (L/100 km)    0
Fuel Consumption Comb (mpg)         0
CO2 Emissions(g/km)                 0
dtype: int64
```

In [ ]:
```python
df.shape
```

Out[ ]:
```
(7385, 12)
```

In [ ]:
```python
df.columns
```

Out[ ]:
```
Index(['Make', 'Model', 'Vehicle Class', 'Engine Size(L)', 'Cylinders',
       'Transmission', 'Fuel Type', 'Fuel Consumption City (L/100 km)',
       'Fuel Consumption Hwy (L/100 km)', 'Fuel Consumption Comb (L/100 km)',
       'Fuel Consumption Comb (mpg)', 'CO2 Emissions(g/km)'],
      dtype='object')
```

In [ ]:
```python
df.describe()
```

Out[ ]:

	Engine Size(L)	Cylinders	Fuel Consumption City (L/100 km)	Fuel Consumption Hwy (L/100 km)	Fuel Consumption Comb (L/100 km)	Fuel Consumption Comb (mpg)	CO2 Emissions(g/km)
count	7385.000000	7385.000000	7385.000000	7385.000000	7385.000000	7385.000000	7385.000000
mean	3.160068	5.615030	12.556534	9.041706	10.975071	27.481652	250.584699
std	1.354170	1.828307	3.500274	2.224456	2.892506	7.231879	58.512679
min	0.900000	3.000000	4.200000	4.000000	4.100000	11.000000	96.000000
25%	2.000000	4.000000	10.100000	7.500000	8.900000	22.000000	208.000000
50%	3.000000	6.000000	12.100000	8.700000	10.600000	27.000000	246.000000
75%	3.700000	6.000000	14.600000	10.200000	12.600000	32.000000	288.000000
max	8.400000	16.000000	30.600000	20.600000	26.100000	69.000000	522.000000

Funciones generales

Funcion para graficar en base a un modelo de distribucion QQPlot:

In [ ]:
```
def QQPlot(x, y, dst):
  X = sm.add_constant(x)
  model = sm.OLS(y, X)
  result = model.fit()
  influence = result.get_influence()
  standardized_residuals = influence.resid_studentized_internal
  fig = sm.qqplot(standardized_residuals, dist = dst, line = 'q')
  plt.title('QQ Graph - Normal Distribution')
  plt.ylabel('Standarized residuals Quantiles')
  plt.grid()
  plt.show()
```

Funcion para obtener los parametros y R^2 de cada iteracion de variable predictora con el modelo OLS:

In [ ]:
```
def OLS(x, y):
  x = sm.add_constant(x)
  model = sm.OLS(y, x)
  result = model.fit()
  print('Params:', result.params)
  print('R^2:', result.rsquared)
```

Función para graficar las gráficas de asimetría

In [ ]:
```
def Asimetric_Dist(r, title):
  plt.hist(r, density = True, bins = 'auto', histtype='stepfilled', alpha=0.2)
  plt.title(title)
  plt.grid()
  plt.show()
```

Modelo de regresión

In [ ]:
```
x = np.array(df[['Engine Size(L)', 'Cylinders', 'Fuel Consumption City (L/100 km)', 'Fuel Cons
y = np.array(df['CO2 Emissions(g/km)'])
```

Obtenemos el valor de R^2 y los resultados de nuestro modelo de regresion OLS:

In [ ]:
```python
X = sm.add_constant(x)
model = sm.OLS(y, X)
results = model.fit()

print('\n', results.params)
print('\n', 'R2: ', results.rsquared)
print(results.summary())
```

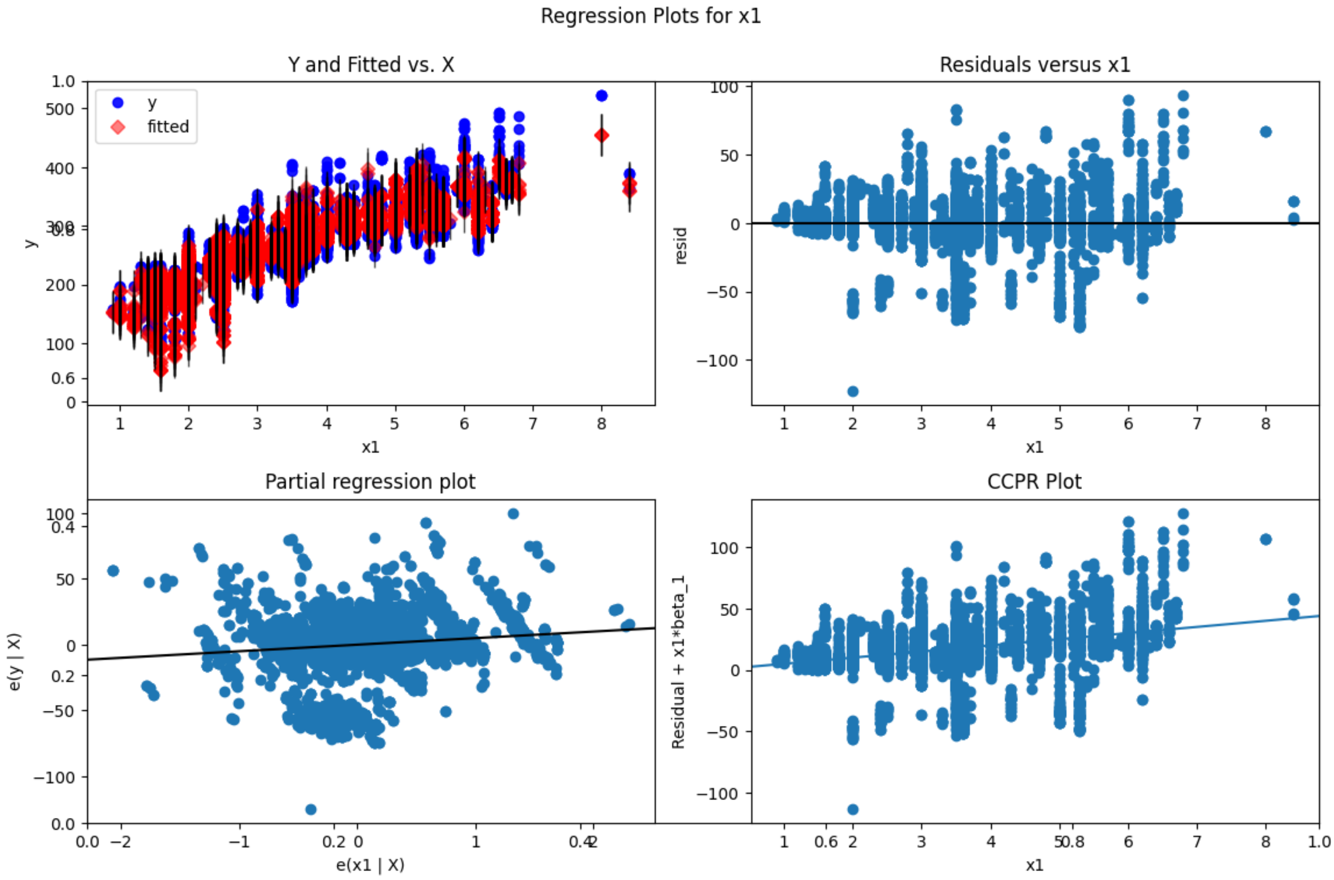
```
 [ 2.27892751e+02  4.99360380e+00  7.53852995e+00 -2.37835507e-02
  4.49061364e+00  1.67304643e+00 -3.42349241e+00]

 R2:  0.9039065926000305
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.904
Model:                            OLS   Adj. R-squared:                  0.904
Method:                 Least Squares   F-statistic:                 1.157e+04
Date:                Sat, 07 Oct 2023   Prob (F-statistic):               0.00
Time:                        04:12:07   Log-Likelihood:                -31880.
No. Observations:                7385   AIC:                         6.377e+04
Df Residuals:                    7378   BIC:                         6.382e+04
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        227.8928      4.200     54.255      0.000     219.659     236.127
x1             4.9936      0.456     10.962      0.000       4.101       5.887
x2             7.5385      0.319     23.657      0.000       6.914       8.163
x3            -0.0238      2.738     -0.009      0.993      -5.391       5.344
x4             4.4906      2.260      1.987      0.047       0.061       8.920
x5             1.6730      4.969      0.337      0.736      -8.069      11.415
x6            -3.4235      0.079    -43.545      0.000      -3.578      -3.269
==============================================================================
Omnibus:                     1193.702   Durbin-Watson:                   1.618
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7810.498
Skew:                          -0.609   Prob(JB):                         0.00
Kurtosis:                       7.889   Cond. No.                         987.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Graficamos los valores predichos vs residuos estandarizados:

In [ ]:
```python
fig, ax = plt.subplots(figsize=(12, 8))
sm.graphics.plot_regress_exog(results, 1, fig=fig)
plt.show()
```



Obtenemos los residuales estandarizados de la influencia de los resultados:

```
influence = results.get_influence()
standardized_residuals = influence.resid_studentized_internal
print(standardized_residuals)
```

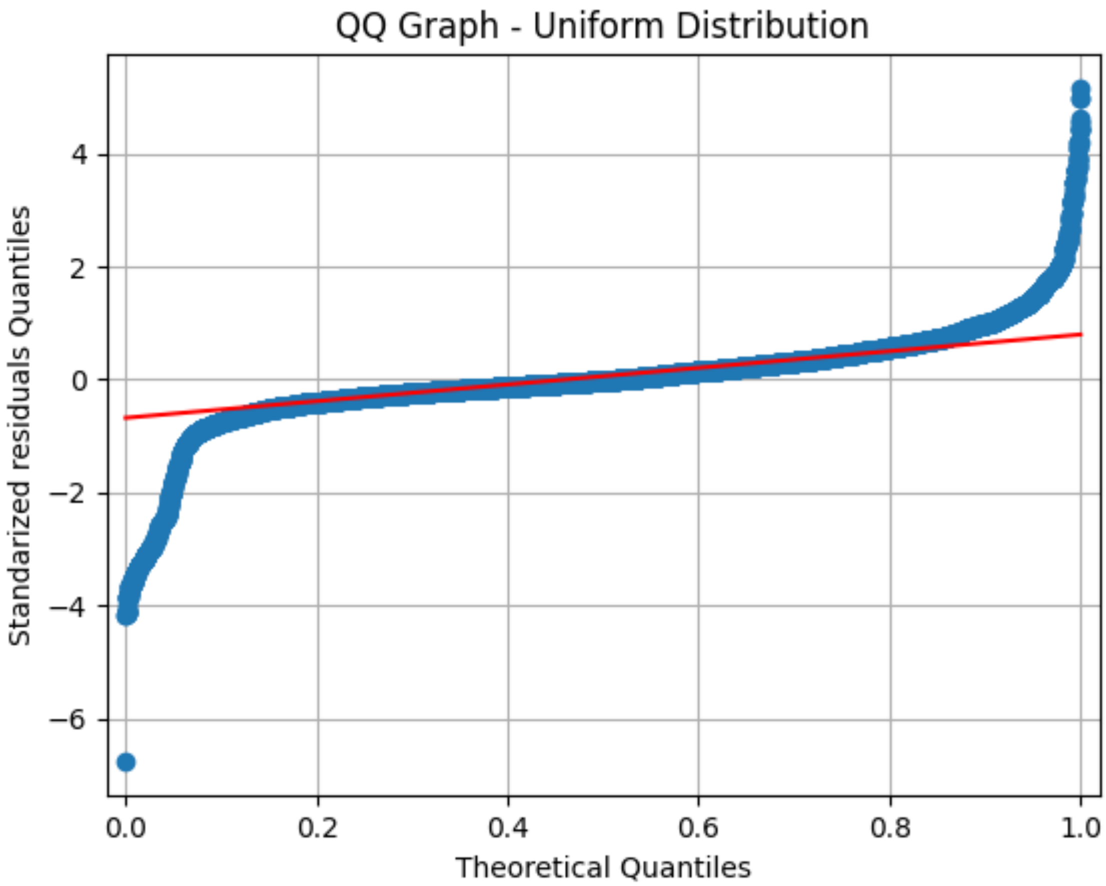
```
[-0.17261704 -0.00677114 -0.05425761 ...  0.48654435  0.53345117
  0.6779954 ]
```

Realizamos una funcion que nos permite graficar una grafica QQPlot con los datos residuales estandarizados:

```
def QQPlot_std_residual(std_res, d, l, title):
  fig = sm.qqplot(std_res, dist = d, line = l)
  plt.title(title)
  plt.ylabel('Standarized residuals Quantiles')
  plt.grid()
  plt.show()
```

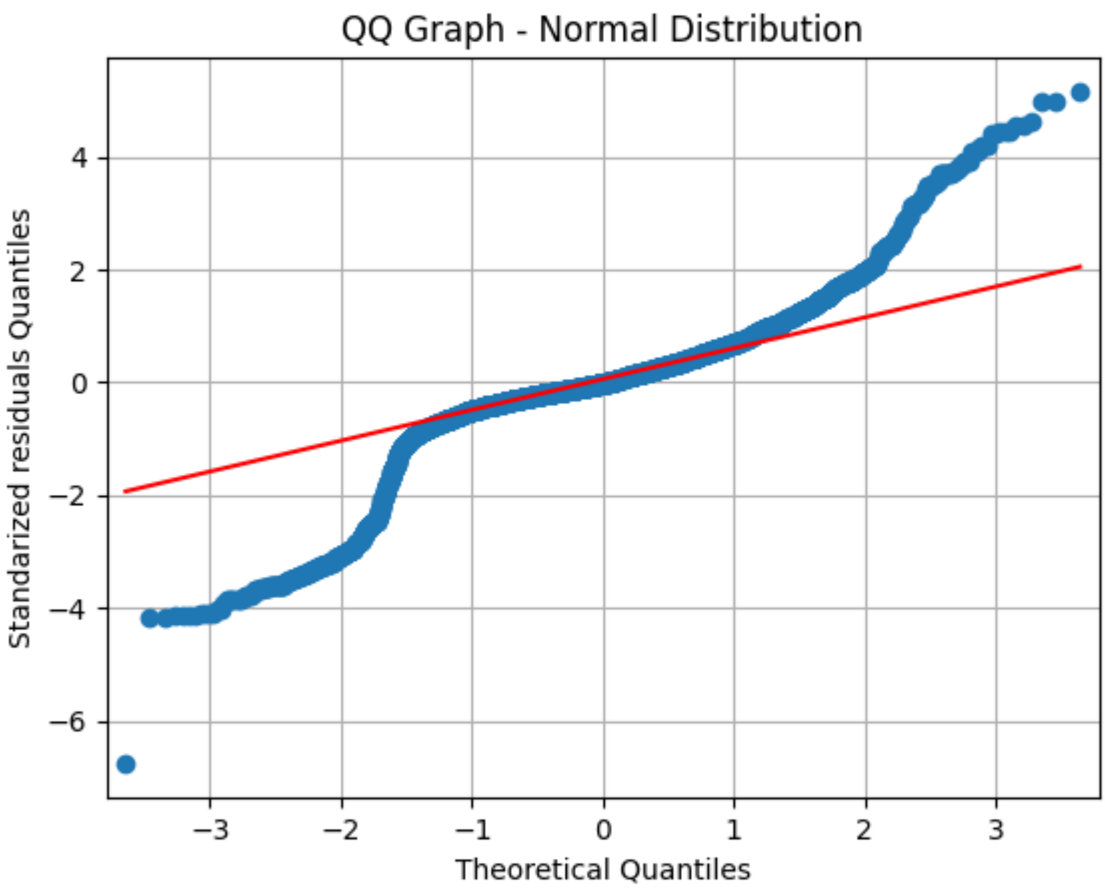
- Distribucion uniforme:

```
QQPlot_std_residual(standardized_residuals, uniform, 'q', 'QQ Graph - Uniform Distribution')
```



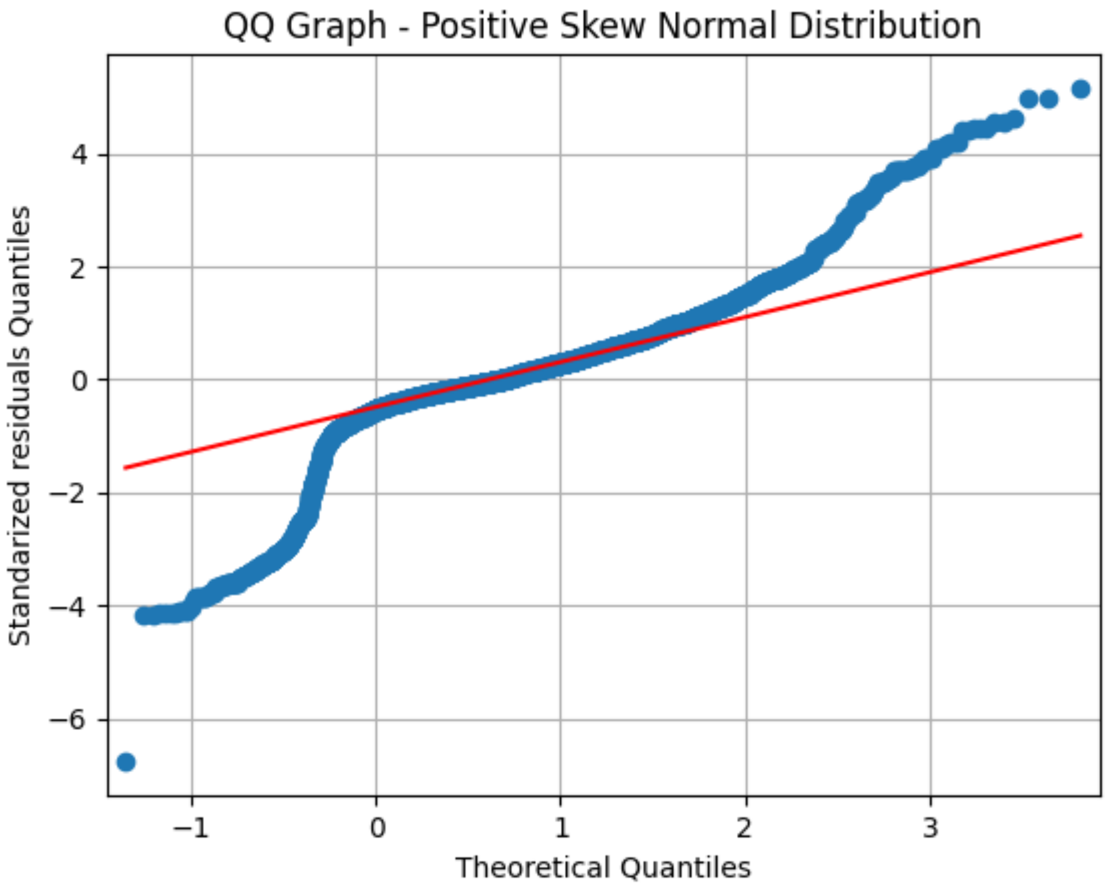
- Distribucion normal:

```
In [ ]: QQPlot_std_residual(standardized_residuals, norm, 'q', 'QQ Graph - Normal Distribution')
```



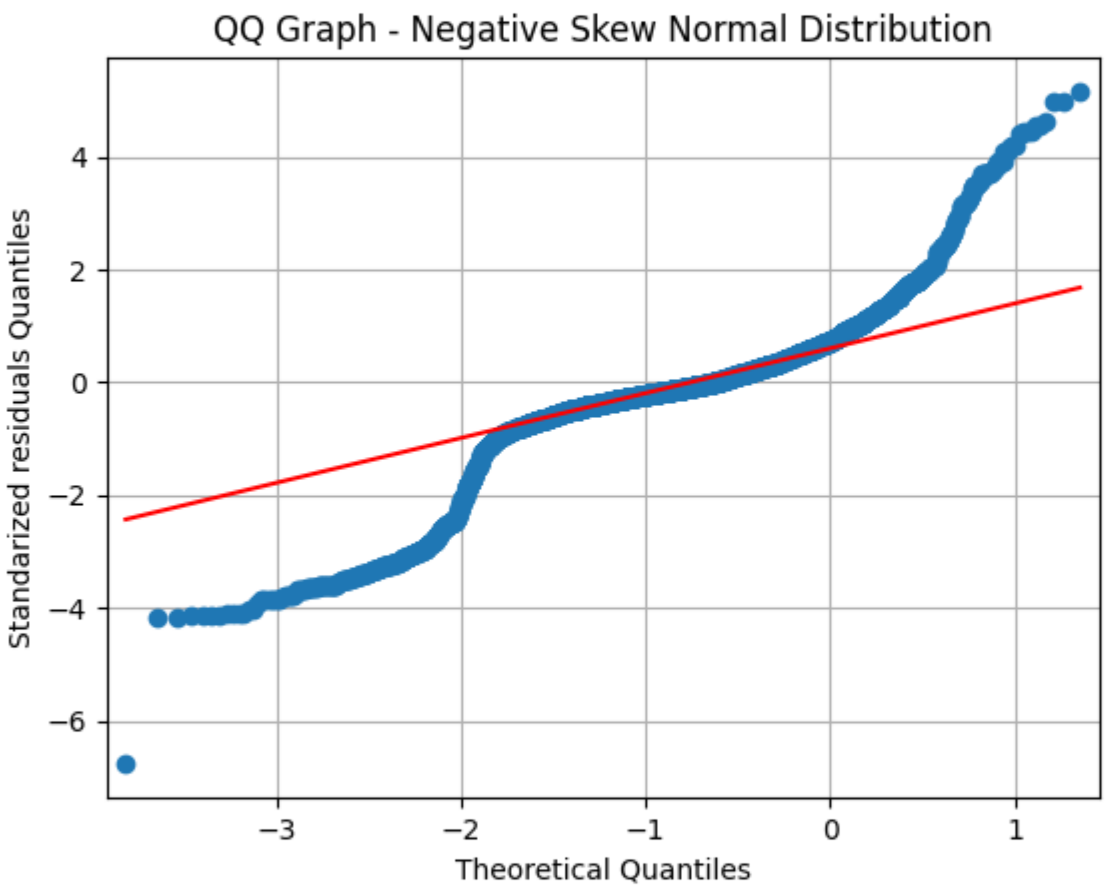
- Distribucion normal de inclinacion positiva:

```
In [ ]: QQPlot_std_residual(standardized_residuals, skewnorm(2), 'q', 'QQ Graph - Positive Skew Normal
```



- Distribucion normal de distribucion negativa:

```
In [ ]: QQPlot_std_residual(standardized_residuals, skewnorm(-2), 'q', 'QQ Graph - Negative Skew Norma
```

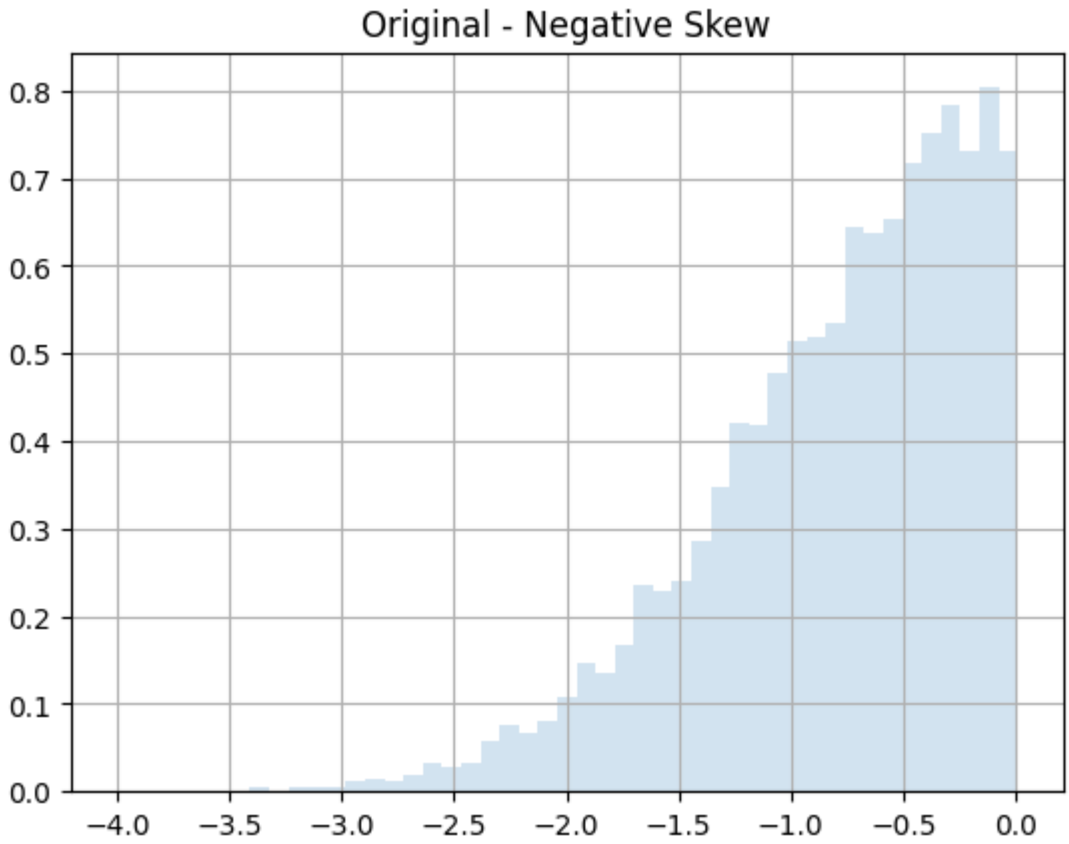


Búsqueda de la transformacion adecuada

Distribucion Asimetrica Negativa

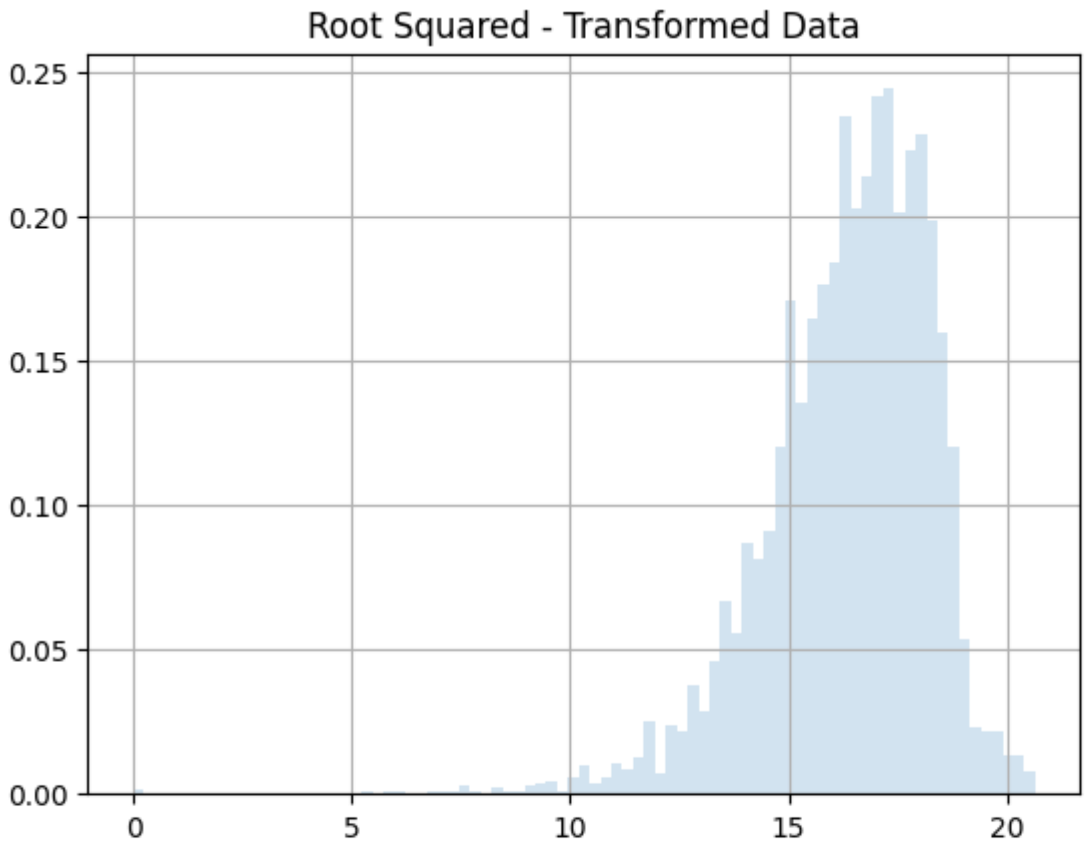
Distribucion Asimetrica Negative (1/3):

In [ ]:
```
y_skew_neg = -skewnorm.rvs(y)
Asimetric_Dist(y_skew_neg, 'Original - Negative Skew')
```



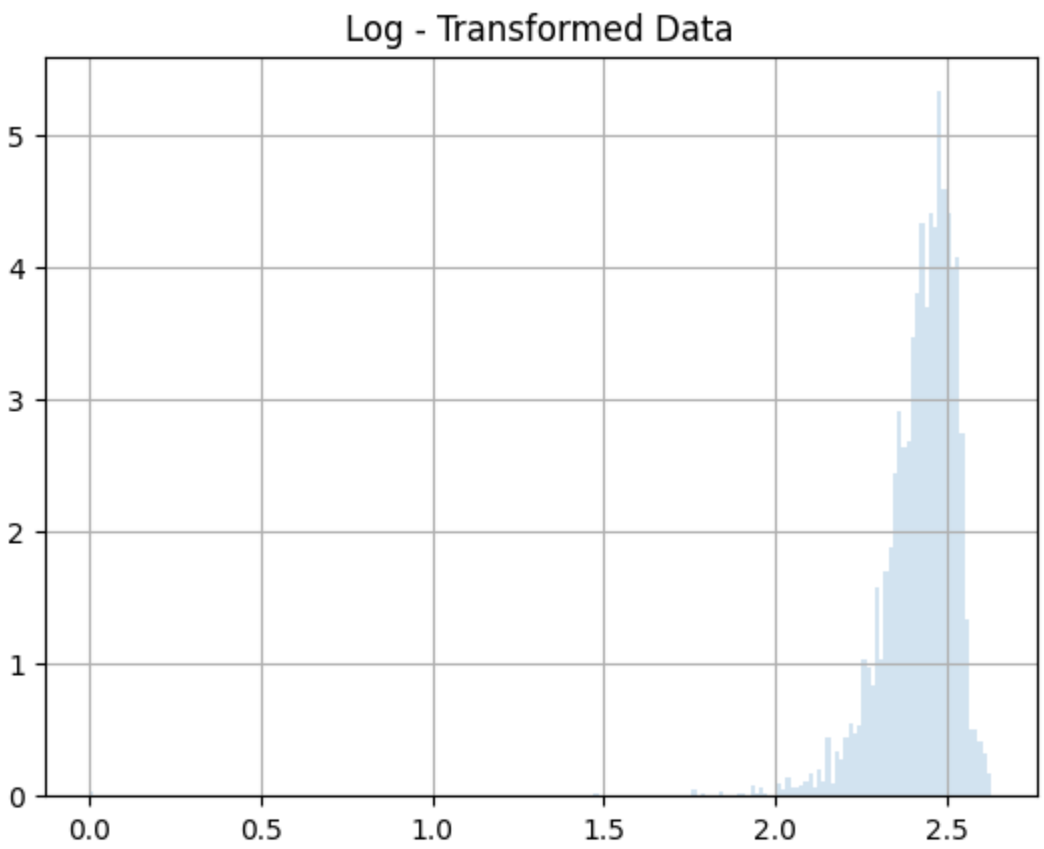
Distribucion Asimetrica Negative (2/3):

In [ ]:
```
y_pos = y + abs(min(y))
y_root_neg = np.sqrt(max(y_pos) - y_pos)
Asimetric_Dist(y_root_neg, 'Root Squared - Transformed Data')
```



Distribucion Asimetrica Negative (3/3):

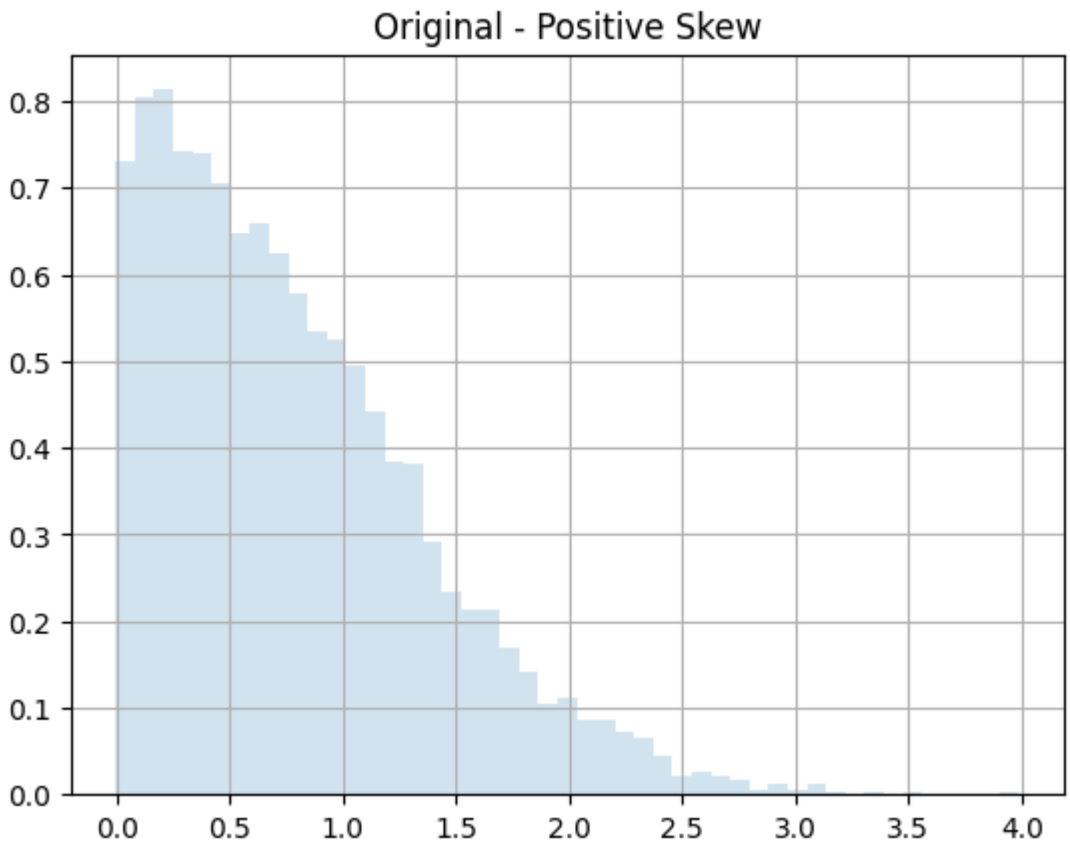
```
y_positive = y + abs(min(y))
y_log_neg = np.log10(1 + max(y_positive) - y_positive)
Asimetric_Dist(y_log_neg, 'Log - Transformed Data')
```



Distribucion Asimetrica Positiva

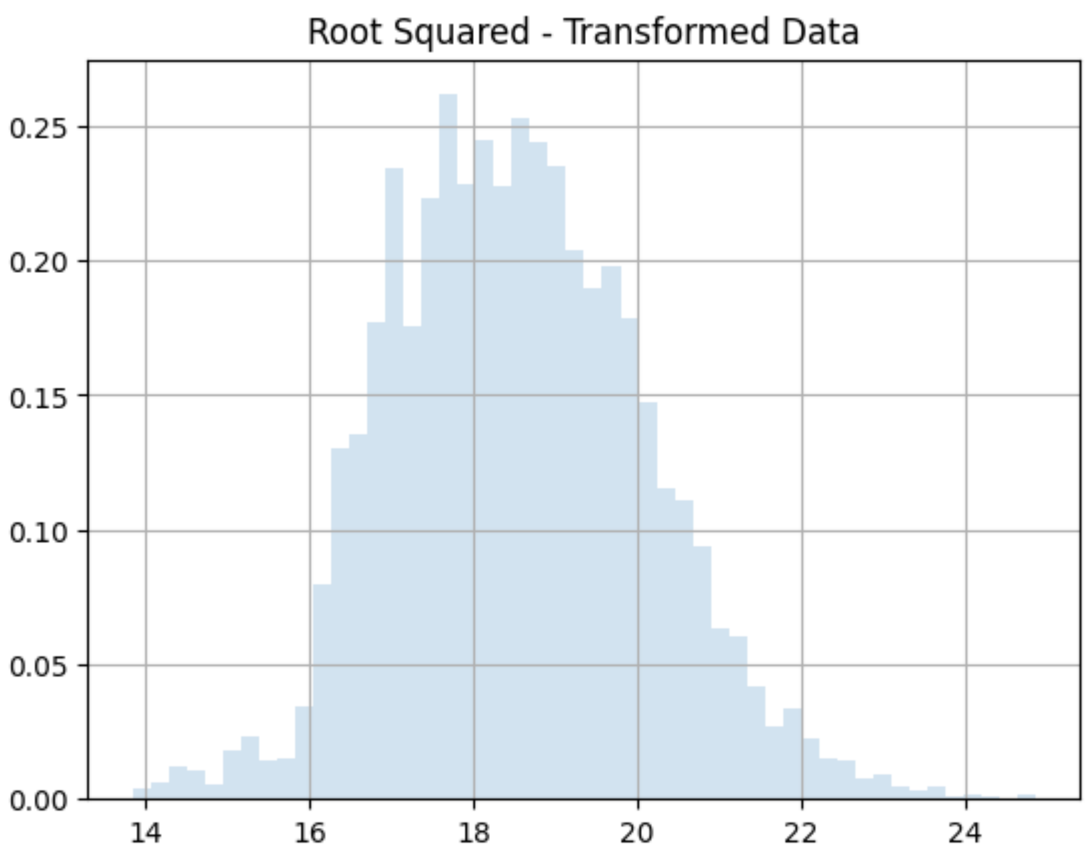
Distribucion Asimetrica Positiva:

In [ ]:
```
y_skew = skewnorm.rvs(y)
Asimetric_Dist(y_skew, 'Original - Positive Skew')
```



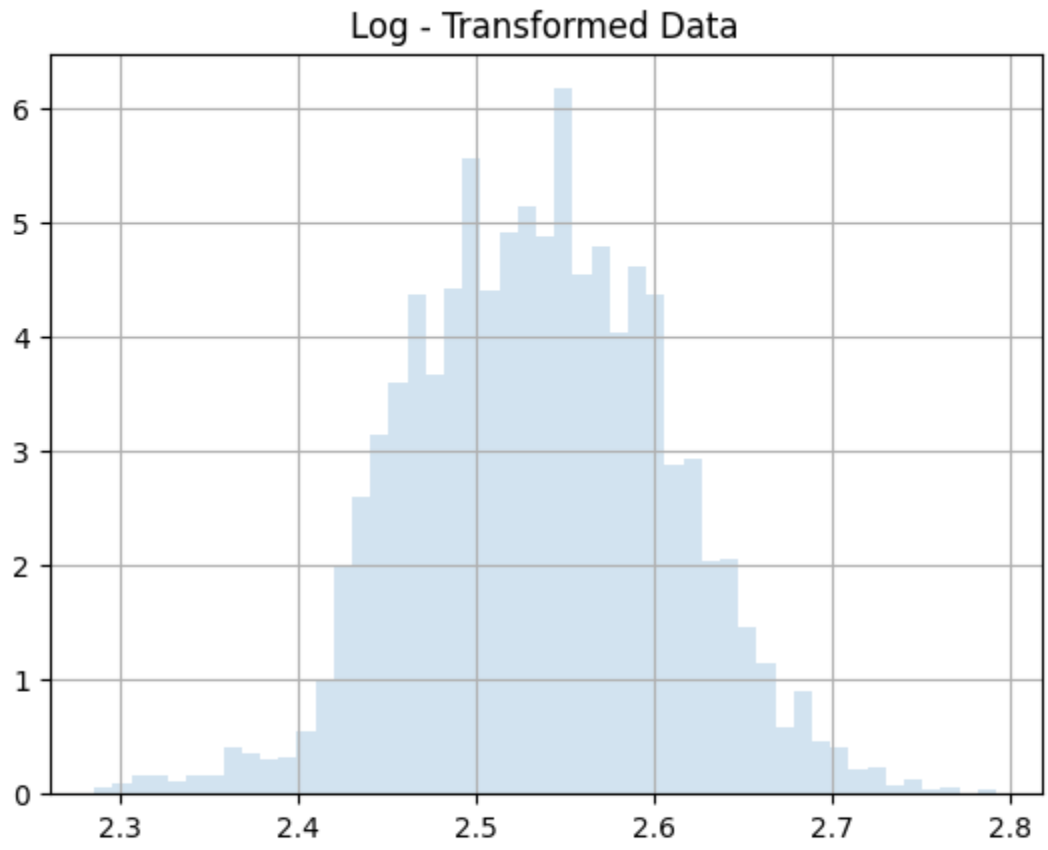
Distribucion Asimetrica Positiva:

In [ ]:
```
y_root = np.sqrt(y + abs(min(y)))
Asimetric_Dist(y_root, 'Root Squared - Transformed Data')
```



Distribucion Asimetrica Positiva:

In [ ]:
```
y_log = np.log10(1 + y + abs(min(y)))
Asimetric_Dist(y_log, 'Log - Transformed Data')
```



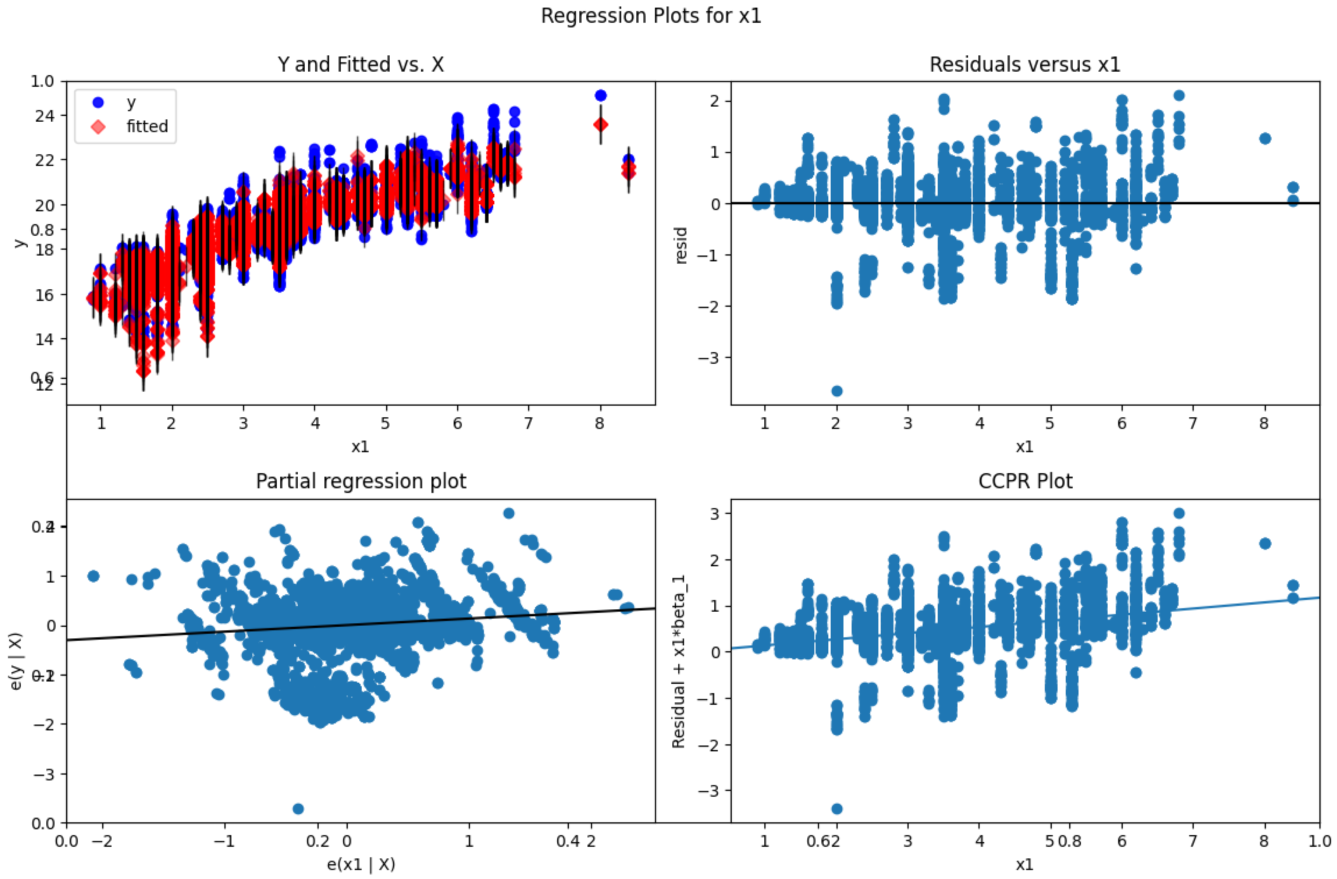
In [ ]:
```
model = sm.OLS(y_root, X)
results = model.fit()
print('\n', results.params)
print('\n', 'R2: ', results.rsquared)
print(results.summary())
```

```
 [ 1.94367810e+01  1.33053149e-01  1.85578182e-01 -5.95956234e-03
  1.18164557e-01 -6.19298157e-03 -1.19108371e-01]

 R2:  0.9152323690604602
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.915
Model:                            OLS   Adj. R-squared:                  0.915
Method:                 Least Squares   F-statistic:                 1.328e+04
Date:                Sat, 07 Oct 2023   Prob (F-statistic):               0.00
Time:                        04:12:13   Log-Likelihood:                -4635.1
No. Observations:                7385   AIC:                             9284.
Df Residuals:                    7378   BIC:                             9333.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         19.4368      0.105    185.164      0.000      19.231      19.643
x1             0.1331      0.011     11.687      0.000       0.111       0.155
x2             0.1856      0.008     23.303      0.000       0.170       0.201
x3            -0.0060      0.068     -0.087      0.931      -0.140       0.128
x4             0.1182      0.056      2.093      0.036       0.007       0.229
x5            -0.0062      0.124     -0.050      0.960      -0.250       0.237
x6            -0.1191      0.002    -60.623      0.000      -0.123      -0.115
==============================================================================
Omnibus:                     1399.064   Durbin-Watson:                   1.617
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7822.151
Skew:                          -0.794   Prob(JB):                         0.00
Kurtosis:                       7.786   Cond. No.                         987.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [ ]:
```
fig, ax = plt.subplots(figsize=(12, 8))
sm.graphics.plot_regress_exog(results, 1, fig=fig)
plt.show()
```



Evaluación del impacto de cada variable independiente

In [ ]:
```
model = sm.OLS(y_log, X)
results = model.fit()
print('\n', results.params)
print('\n', 'R2: ', results.rsquared)
print(results.summary())
```

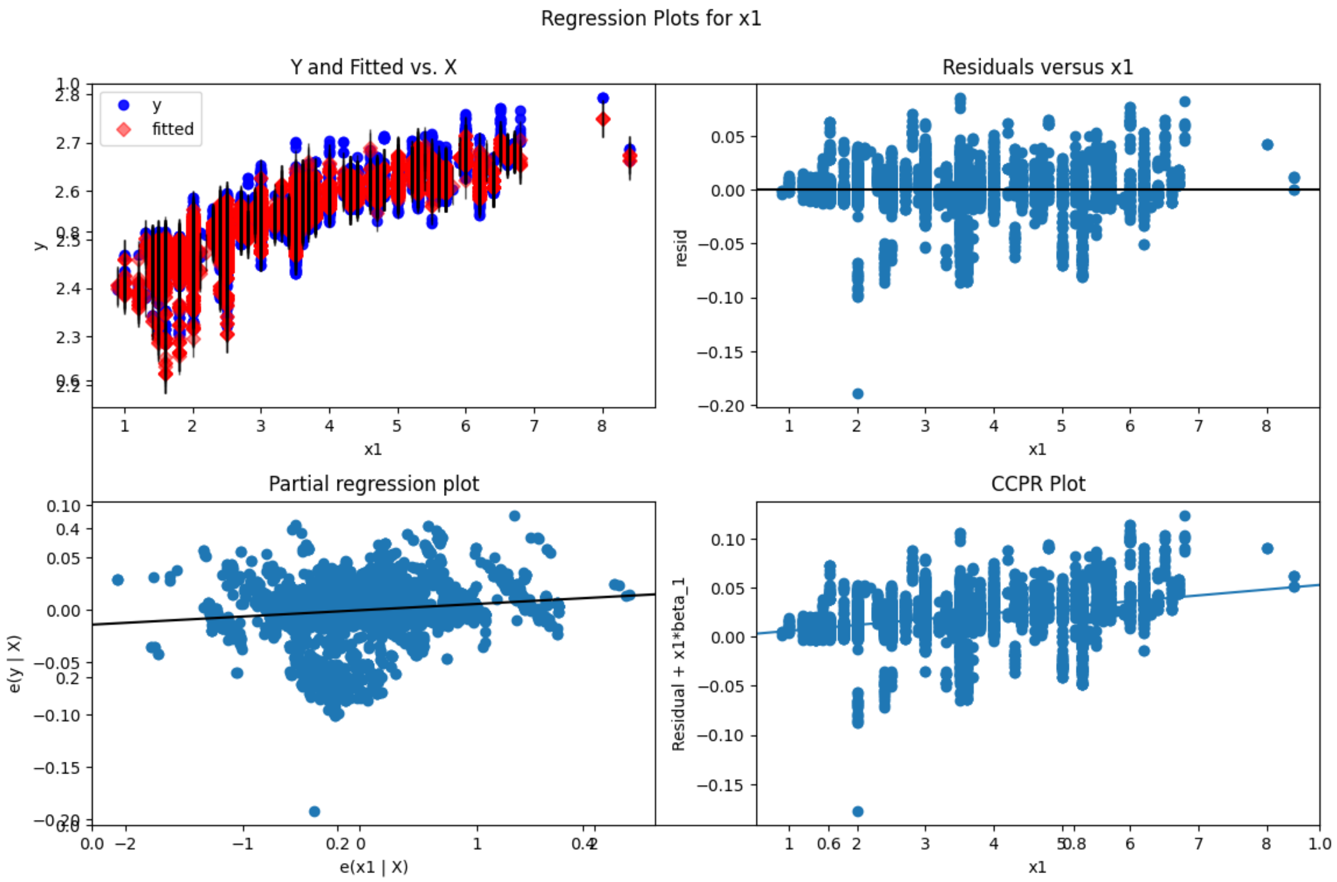
```
[ 2.64703621e+00  6.01517403e-03  7.91678618e-03 -3.12833476e-04
  5.42435642e-03 -2.91406589e-03 -6.86385862e-03]

R2:  0.9250969146244696
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.925
Model:                            OLS   Adj. R-squared:                  0.925
Method:                 Least Squares   F-statistic:                 1.519e+04
Date:                Sat, 07 Oct 2023   Prob (F-statistic):               0.00
Time:                        04:12:14   Log-Likelihood:                 18468.
No. Observations:                7385   AIC:                        -3.692e+04
Df Residuals:                    7378   BIC:                        -3.687e+04
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.6470      0.005    575.888      0.000       2.638       2.656
x1             0.0060      0.000     12.066      0.000       0.005       0.007
x2             0.0079      0.000     22.703      0.000       0.007       0.009
x3            -0.0003      0.003     -0.104      0.917      -0.006       0.006
x4             0.0054      0.002      2.194      0.028       0.001       0.010
x5            -0.0029      0.005     -0.536      0.592      -0.014       0.008
x6            -0.0069    8.6e-05    -79.783      0.000      -0.007      -0.007
==============================================================================
Omnibus:                     1698.750   Durbin-Watson:                   1.615
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            10030.801
Skew:                          -0.971   Prob(JB):                         0.00
Kurtosis:                       8.369   Cond. No.                         987.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [ ]:

```
fig, ax = plt.subplots(figsize=(12, 8))
sm.graphics.plot_regress_exog(results, 1, fig=fig)
plt.show()
```



In [ ]:

```
x = np.array(df[['Engine Size(L)', 'Cylinders', 'Fuel Consumption Hwy (L/100 km)', 'Fuel Consu
X = sm.add_constant(x)

model = sm.OLS(y_log, X)
results = model.fit()
print('\n', results.params)
print('\n', 'R2: ', results.rsquared)
print(results.summary())
```

```
 [ 2.64703287  0.00601547  0.00791563  0.00567639 -0.00347904 -0.00686376]

 R2:  0.9250968039653008
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.925
Model:                            OLS   Adj. R-squared:                  0.925
Method:                 Least Squares   F-statistic:                 1.823e+04
Date:                Sat, 07 Oct 2023   Prob (F-statistic):               0.00
Time:                        04:12:16   Log-Likelihood:                 18468.
No. Observations:                7385   AIC:                        -3.692e+04
Df Residuals:                    7379   BIC:                        -3.688e+04
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.6470      0.005    575.940      0.000       2.638       2.656
x1             0.0060      0.000     12.068      0.000       0.005       0.007
x2             0.0079      0.000     22.713      0.000       0.007       0.009
x3             0.0057      0.001     10.604      0.000       0.005       0.007
x4            -0.0035      0.001     -6.477      0.000      -0.005      -0.002
x5            -0.0069    8.6e-05    -79.791      0.000      -0.007      -0.007
==============================================================================
Omnibus:                     1698.926   Durbin-Watson:                   1.615
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            10031.533
Skew:                          -0.971   Prob(JB):                         0.00
Kurtosis:                       8.370   Cond. No.                         637.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [ ]:

```
x = np.array(df[['Engine Size(L)', 'Cylinders', 'Fuel Consumption City (L/100 km)', 'Fuel Cons
X = sm.add_constant(x)

model = sm.OLS(y_log, X)
results = model.fit()
print('\n', results.params)
print('\n', 'R2: ', results.rsquared)
print(results.summary())
```

```
 [ 2.64693737e+00  6.01076984e-03  7.91847138e-03 -1.91065345e-03
  4.11171738e-03 -6.86193779e-03]

 R2:  0.9250939993765741
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.925
Model:                            OLS   Adj. R-squared:                  0.925
Method:                 Least Squares   F-statistic:                 1.823e+04
Date:                Sat, 07 Oct 2023   Prob (F-statistic):               0.00
Time:                        04:12:16   Log-Likelihood:                 18468.
No. Observations:                7385   AIC:                        -3.692e+04
Df Residuals:                    7379   BIC:                        -3.688e+04
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.6469      0.005    576.358      0.000       2.638       2.656
x1             0.0060      0.000     12.060      0.000       0.005       0.007
x2             0.0079      0.000     22.710      0.000       0.007       0.009
x3            -0.0019      0.000     -6.455      0.000      -0.002      -0.001
x4             0.0041      0.000     12.187      0.000       0.003       0.005
x5            -0.0069    8.6e-05    -79.833      0.000      -0.007      -0.007
==============================================================================
Omnibus:                     1699.764   Durbin-Watson:                   1.615
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            10035.420
Skew:                          -0.971   Prob(JB):                         0.00
Kurtosis:                       8.370   Cond. No.                         646.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```