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Well Collision Avoidance - Separation Rule

S.J. Sawaryn, Consultant and H. Wilson, Baker Hughes; J. Bang, Gyrodata Inc.,; E. Nyrnes, Statoil ASA; A. Sentance, Dynamic Graphics Inc.; B. Poedjonoand and R. Lowdon, Schlumberger; I. Mitchell and J. Codling, Halliburton; P.J. Clark, Chevron Energy Technology; W.T. Allen, BP

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Abstract

The well collision separation rule presented in this paper is a culmination of the work and consensus of industry experts from both operators and service companies in the SPE Wellbore Positioning Technical Section (WPTS). This is the second of two papers and complements the first paper SPE 184730 which described the collision avoidance management practices. These practices are fundamental in establishing the environment in which a minimum allowable separation distance (MASD) can be effectively applied. A standardised collision avoidance rule is recommended, complete with parameter values appropriate to the management of health, safety and environment (HSE) risk, and benchmarks for testing it. Together, these should help eliminate the disparate and occasionally contradictory methods in use today.

The consequences of an unplanned intersection with an existing well can range from financial loss to a catastrophic blow-out and loss of life. The process of well collision avoidance involves rules that determine the allowable separation and the management of the associated directional planning and surveying activities. The proposed separation rule is based on the pedal curve method and is expressed as a separation factor, a dimensionless number expressed as a ratio of an adjusted centre to centre distance between wells divided by a function of the relative positional uncertainty between the two. The recommended values for the rule's parameters are based on a comparison of various industry models and experience. The relationships between key concepts such as separation factor, MASD and allowable deviation from the plan (ADP) are discussed, together with their interpretation and application. The dependency on the error distributions of the survey instrument performance models used to establish the tolerance lines is also discussed.

The consequences of implementing a standardised separation rule across the industry are far reaching. It influences slot separations, trajectories, drilling practices, surveying programme and well shut-in. It is shown how the MASD can be related to a probability of crossing and being on the far side, or wrong side of an offset well. It is shown why this qualification is required for safe drilling practices to be preserved. Examples are presented to help the reader validate the calculations and software necessary to perform them. The limitations of the methods are explained and areas are highlighted for further work. The methods outlined here, taken together with SPE 184730, if implemented will improve efficiency in planning and executing wells and promote industry focus on the associated collision risks when drilling. The WPTS

is also supporting the current development of the API Recommended Practices for Wellbore Positioning (RP78). Mathematical derivations or references are shown for all the calculations presented in the paper.

Introduction

In any situation, the minimum allowable separation distance (MASD) between the reference and offset wells and the resulting allowable deviation from the plan (ADP) needs to be quantified using a separation rule that reflects the positional uncertainty between the two wells.Currently, a variety of rules are used in the industry, leading to differences in the calculated MASD. This situation leads to contention when the rules are contradictory, such as when one rule indicates that a well is drillable and the other indicates that it is not. Such situations might not be easily resolved and a benchmark or standard is required. Establishing such a benchmark has been the WPTS's goal and is the subject of this paper.

Table 1—Comparison of the proposed WPTS well separation rule with rules currently used in the industry. (1) C – casing, OH – open hole, (2) surface rules differ between the companies, (3) when $\sigma_r = \sigma_o$

Company	Uncertainty Combination Procedure	Uncertainty Scaling Factor [-]	Limit Value of *SF* [-]	Inclusion of Hole Dimensions[1]	Other Factors[2]	Equivalent WPTS *k* value [3] (Eq. 1) [-]
A	$\sqrt{\sigma_r^2+\sigma_o^2}$	2.9	1.0	C / OH	Using relatively conservative error models	2.9
Proposed WPTS Separation Rule	$\sqrt{\sigma_r^2+\sigma_o^2}$	3.5	1.0	OH / OH	Includes Surface Margin and Project-Ahead uncertainty	3.5
B	$\sigma_r+\sigma_o$	2.0	1.25	None		3.54
C	$\sqrt{\sigma_r^2+\sigma_o^2}$	2.8	1.5	C / OH	Used in combination with a separate near-surface rule	4.2
D	$\sqrt{\sigma_r^2+\sigma_o^2}$	2.8	1.0	C / OH		4.2
E	$\sigma_r+\sigma_o$	3.0	1.0	C / OH	Used in combination with a separate near-surface rule	4.24
F	$\sigma_r+\sigma_o$	3.0	1.0	C / OH		4.24
G	$\sigma_r+\sigma_o$	3.0	1.0	None	Used in combination with a separate near surface rule	4.24
H	$\sigma_r+\sigma_o$	3.0	1.0	C / OH	Includes expanding safety margin on centre to centre distance	4.5

Table 1 illustrates the range of HSE risk separation rules currently used in the industry by both operators and service companies.The final column compares them in terms of their approximate equivalent confidence interval (WPTS *k* value). The higher the value, the more conservative is the rule. The comparison is not exact because the individual rules have different formsand parameters. However, it provided a useful basis for choosing a *k* value for use in the WPTS HSE risk rule(Eq. 1).Additionally, it is noted that these treatments can lead to marked differences between the calculated MASDs in different sections of the well.

Sawaryn (2017) observed that none of the collisions recorded by the WPTSare attributable to the separation rule used. This suggests that the collision avoidance rules in common use today are effective if they are supported by rigorously applied management practices. Because of this, it is possible to position the proposed WPTS well separation rule toward the less conservative end of the existing industry scale indicated in Table 1.

Benefits of Standardisation

The standardisation of well collision avoidance management practices and separation rule offers both safety and commercial benefits. It will clarify expectation and requirements and support consistency between operators, service providers and regulators as well as increasing both planning and operational efficiencies and help reduce the burden of training. It will also simplify software content and provide focus for the automation of the associated workflows.

WPTS Separation Rule

The separationrule relates to the probability of the reference well crossing into a prohibited space associated with a specified offset well. In the terminology used in this paper, the term crossing includes a physical collision. The possibility of crossing unknowingly is undesireable because of the risk of steering into the offset well while assuming to steer away from it.

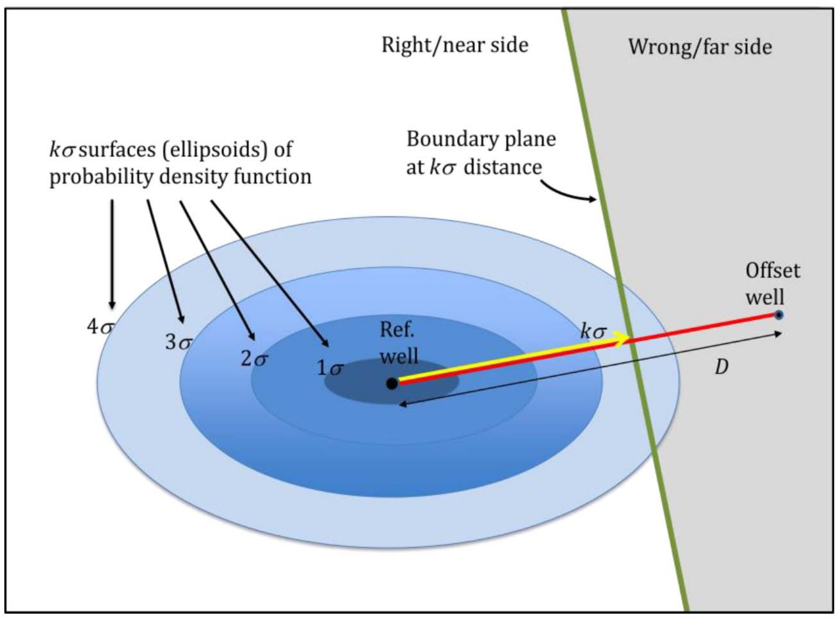


Figure 1—In this illustration of the WPTS separation rule, the relative uncertainty is assigned to the reference well and well dimensionshave been neglected. The rule defines a boundary denoted by the green linea distance $k\sigma$ from the reference well, separating space into the prohibitedregion (shaded) and the permitted region. The boundary is a plane in3D (a straight line in 2D), perpendicular to the line connecting the wells' points of interest.

The WPTS rule defines a boundary a distance $k\sigma$ from the reference well, separating space into the prohibitedregion (shaded) and the permitted region. The benefit of a planar boundary provided by the rule is that the two regions and well crossing condition are well-defined. The probabilityof unknowingly being in the wrong region can be calculated from the error distributions, (Bang, 2017).

The separation rule is expressed as a dimensionless number, a ratio of an adjusted centre to centre distance between wells divided by a function of the relative positional uncertainty between the two. This ratio is referred to as a separation factor SF and customarily the ratio is constructed so that the threshold $SF = 1$ indicates a critical condition, implying that acceptable well separations require $SF \geq 1$. Adherence to this critical value is mandatory for HSE risk wells, (Sawaryn, 2017). The SF definition adopted by the WPTS is given by Eq. 1 and its parameters are described in Table 2.

$$SF = \frac{D - R_r - R_o - S_m}{k\sqrt{\sigma_s^2 + \sigma_{pa}^2}} \tag{1}$$

The term $k\sqrt{\sigma_s^2 + \sigma_{pa}^2}$ is equivalent to the distance to the boundary plane in Fig. 1. Further *SF* thresholds can be defined as triggers for other actions. It is recommended that the value $SF = 1.25$ be used in planning as a prompt for a detailed engineering review. During execution, this threshold is used to trigger preventive measures and a Management of Change (MOC). The value $SF > 5$ is frequently used during planning to exclude offset wells from further consideration, particularly in circumstances where hundreds of wells are involved.

Table 2—Description of the parameters used in the Separation Factor equation

Parameter	Description
D	The distance between a specified point on the centreline of the reference well and the nearest point on the centreline of the offset well. The point on the reference well is specified first. The point on the offset well is identified as the point of closest approach in 3D space or in the plane normal to the reference well when travelling cylinder diagrams are being used for collision monitoring, (ISCWSA, 2013).
R_r	The open hole radius of the reference borehole.
R_o	The open hole radius of the offset borehole.
S_m	The surface margin term increases the effective radius of the offset well. It accommodates small, unidentified errors and helps overcome one of the geometric limitations of the separation rule, described in the Separation Rule Limitations section of this paper. It also defines the minimum acceptable slot separation during facility design and ensures that the separation rule will prohibit the activity before nominal contact between the reference and offset wells, even if the position uncertainty is zero.
k	The dimensionless scaling factor that determines the probability of well crossing.
σ_{pa}	Quantifies the one standard deviation uncertainty in the projection ahead of the current survey station. Its value is partially correlated with the projection distance, determined as the current survey depth to the bit plus the next survey interval. The magnitude of the actual uncertainty also depends on the planned curvature and on the BHA performance at the wellbore attitude in the formation being drilled. The project-ahead uncertainty is only an approximation and although it is predominately oriented normal to the reference well it is mathematically convenient to define σ_{pa} as being the radius of a sphere.
σ_s	The relative uncertainty at one standard deviation between the two points of interest, derived from their respective positional uncertainties σ_r and σ_o in the direction of D, (see Appendix A).

Parameter Values

The following values arerecommendedfor the surface marginS_m = 0.3m (1ft) and the project-ahead uncertainty σ_{pa} = 0.5m (1.6ft).The separation rule's assumption on project-ahead uncertainty will normally be consistent with the standard maximum survey interval recommendations (Sawaryn, 2017).However, in formations where predicting BHA steering responses is difficult, the survey interval might need to be shortened to maintain the rule's validity. ForHSE risk applications, the recommended value for k is 3.5, assuming normally distributedposition errors. Implicit in this recommendation is that the survey data are reliable with no gross errors,(Williamson, 1998). Substituting these parameter values in Eq. 1 provides the separation rule applicable to HSE risk wells, Eq. 2. Note that Eq. 2 is expressed using metric units.

$$SF = \frac{D - R_r - R_o - 0.3}{3.5\sqrt{\sigma_s^2 + 0.25}} \tag{2}$$

Crossing Probability

One WPTS requirement was to be able to quantify the crossing probabilityfor any calculated *SF*. For k= 3.5 and normally distributed position errors, the crossing probability or being on the wrong side of the boundary plane at the critical condition, $SF = 1$ is approximately 0.0002 or 1:5000. For further details see AppendixB. The probability of a well collision, with actual well-to-well contactis always less than this value,but these two probabilities should not becompared directly, (Bang, 2017).

SF Sensitivity to the Parameter Values

The significance of the fixed values S_m and σ_{pa} varies as D, R_r, R_o and σ_s vary along the well path and for different offset wells. The recommended values for S_m and σ_{pa} are small relative to typical values for D and σ_s, so in many cases S_m and σ_{pa} have a minimal effect on the *SF.* They have a much greater effect when (D-R_r-R_o) or σ_s is small, which is often the case at shallow depths when drilling from pads or platforms.

Table 3 illustrates the effect of S_m and σ_{pa} at shallow depths and the sensitivity of the *SF* to the other parameters. The odd numbered cases assume the recommended values for S_m and σ_{pa} and the even numbered cases illustrate the behavior when these parameters are absent. The pedal radialdistance (PRD) characterizes the size and relative orientation of the error ellipses. The construction of the PRD is described more fully in Appendix C. Case 9 is the same as Case 7, but with disparate PRD_r and PRD_o values which results in a larger $\sigma_s$value and a correspondingly lower *SF*. In Case 7 the two wells are surveyed with equal accuracy while in Case 9, the offset well is surveyed more accurately than the reference well. This results in a MASD difference of approximately3m, which, with 15m separation, is significant in directional drilling terms.

Table 3—Sensitivity of the Separation Factor *SF* to the parameter's values

Case	S_m [m]	σ_{pa} [m]	*Hole* Size [in]	D [m]	R_r+R_o [m]	PRD_r [m]	PRD_o [m]	σ_s [m]	*SF* Numerator [m]	*SF* Denominator [m]	*SF* [-]
1	0.3	0.5	36	2.5	0.91	0.20	0.20	0.28	1.3	2.0	0.64
2	0.0	0.0	36	2.5	0.91	0.20	0.20	0.28	1.6	1.0	1.60
3	0.3	0.5	22	2.5	0.56	0.30	0.30	0.42	1.6	2.3	0.72
4	0.0	0.0	22	2.5	0.56	0.30	0.30	0.42	1.9	1.5	1.31
5	0.3	0.5	17.5	3.5	0.44	0.50	0.50	0.71	2.8	3.0	0.91
6	0.0	0.0	17.5	3.5	0.44	0.50	0.50	0.71	3.1	2.5	1.23
7	0.3	0.5	12.25	15	0.31	3.0	3.0	4.24	14.4	15.0	0.96
8	0.0	0.0	12.25	15	0.31	3.0	3.0	4.24	14.7	14.8	0.99
9	0.3	0.5	12.25	15	0.31	5.0	1.0	5.10	14.4	17.9	0.80
10	0.0	0.0	12.25	15	0.31	5.0	1.0	5.10	14.7	17.8	0.82

MASD and ADP

Any given *SF* value represents a specific probability of the reference well crossing the offset well. The distance D at which a particular *SF* value occurs is situation specific. For any point on a reference well, the critical value $SF=1$ defines a minimum allowable separation distance (MASD) from the specified offset well along D, Eq. 3. Again, k = 3.5, S_m = 0.3m and σ_{pa} = 0.5m.

$$D_{MASD} = k\sqrt{\sigma_s^2 + \sigma_{pa}^2} + R_r + R_o + S_m \tag{3}$$

If the distance D falls below the MASD, then *SF* 1. The difference between the planned distance D_{plan} and the MASD is the allowable deviation from the plan (ADP), Eq. 4.

$$D_{ADP} = D_{plan} - D_{MASD} \tag{4}$$

If the actual wellpath deviates from the planned wellpath along D by more than the *ADP*, then $SF < 1$.

Separation Rule Limitations

The WPTS separation rule cannot be employed in situations where techniques such as magnetic/acoustic ranging or geosteering are used to position the reference well unless the positional uncertainty can be defined in terms of a covariance matrix.

The principle on which the construction of the normal plane travelling cylinder diagram is based ensures that the plane is normal to the reference well, but the offset well might penetrate the plane at any angle. Conversely, the mathematics underlying the 3D closest approach method ensures that the line joining the point of interest on the reference well and the offset well lies in a plane normal to the offset well. To add to these geometrical complexitiesit should be noted that in the general case, the principal axes of the combined ellipsoid and its pedal surface might not be aligned with either the reference or offset well's highside, rightside and along-hole wellbore axes.

Geometrical Limitations

The precision of the separation rule varies with the well to well geometry. For certain geometries the planar boundary may not define a meaningful wrong side for the offset well (limitation A), or the point selected on the offset well by the travelling cylinder method or closest approach method may not be the point of highest wrong side probability, (limitation B). These limitations may result in significant under, or over-estimation of the probability of being on the wrong side of the offset well. Both the travelling cylinder method and closest approach method will always identify the point of highest probability if the confidence regions are spherical. The severity of limitation (B) increases with the flattening of the ellipsoid.

Fig. 2 illustrates limitation (A), which may occur for any ellipsoid, including spherical confidence regions. Because of its curvature, the offset well lies closer to the reference well than does the planar boundary. Hence, the probability of being on the wrong side of the plane, inside the shaded volume, underestimates the probability of being on the wrong side of the offset well.

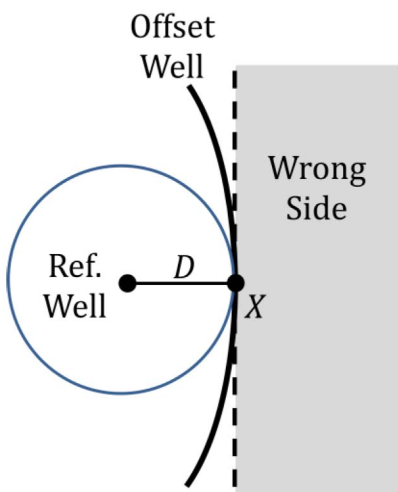


Figure 2—Example of an ill-defined wrong side region, resulting from a closest approach scan from the reference well which is perpendicular to figure's plane. The scan identifies the point X on the offset well as the point at closest distance D to the reference well, where the separation ruleassigns a planar boundary, indicated by the dashed line, perpendicular to D. However, as indicated, the offset well does not necessarily lie inside the wrong side region (shaded area). For simplicity, R_r, R_o and S_m have been neglected in this example.

Fig. 3 illustrates limitation (B). Because the curvature of the offset well is reversed compared to Fig. 2, one might expect an overestimated, conservative probability of being on the wrong side of the offset well. This will be the case for spherical confidence regions. However, for the elongated and tilted ellipsoid in the figure, the highest crossing probability is not found along the direction D that results from the closest approach. If SF=1 along the D direction, the SF will be less than one along the D' direction and so the probability of being on the wrong side of the offset well will be underestimated.Limitations (A) and (B) also apply when the clearance scan is calculated using the normal plane travelling cylinder diagram, however the corresponding SF curves may look different.

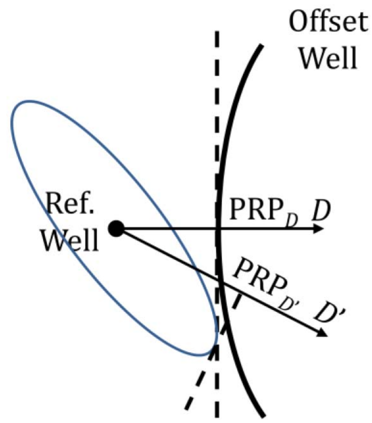
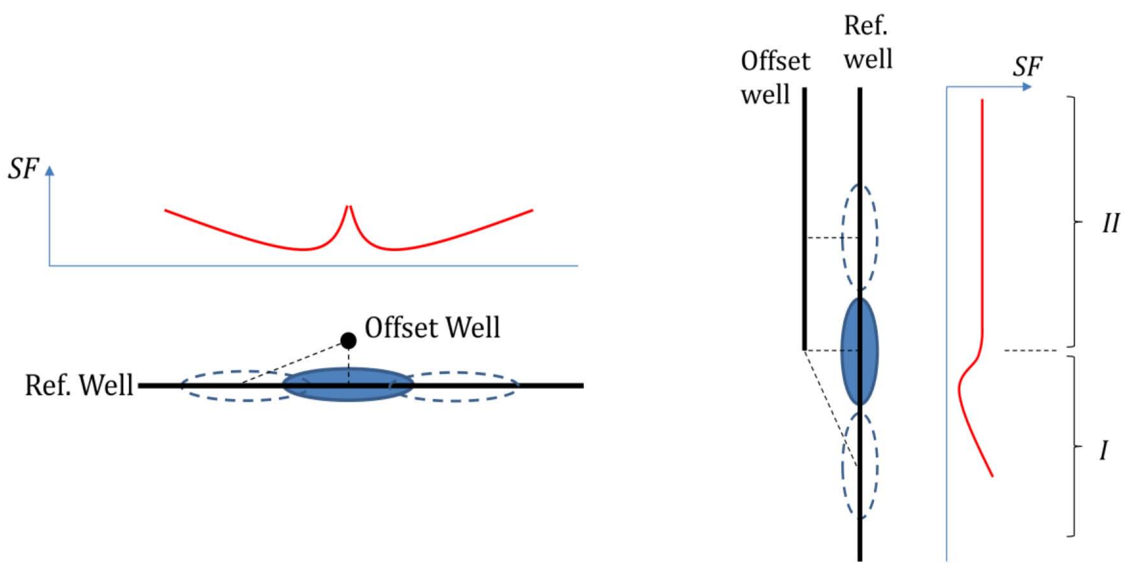


Figure. 3—Example of the scan from the reference well failing to select the offset well point with highest crossing probability. The situation is similar to Fig. 2, but with reversed curvature of the offset well, and a flatter ellipsoid. The closest approach method finds the direction D and the PRP$_D$. For another direction D', the PRP$_{D'}$ lies beyond the offset well, (Bang, 2017).

Another geometric effect is encountered for certain ellipse orientations. The direction of D can change as the point in the reference well changes and may cause non-intuitive *SF* results. This is illustrated in Fig. 4a. Moving along the reference well, D changes direction, such that both D and σ_s=|PRD$_D$| in Eq. 1 vary simultaneously (cf. Fig. C-1b). In particular, when the wells are closest, σ_s reaches a minimum that causes a peak in the *SF* curve. Although such a peak may not be intuitive, it is a valid result from Eq. 1.



4a **4b**

Figure. 4—Some scenarios that can lead to non-intuitive, but mathematically correct results for the calculated *SF* include a) the reference and offset wells crossing orthogonally and b) drilling past the end of an offset well.

The same effect is seen in region I in Fig. 4b; the similarity becomes clearer if one of the figures is rotated by 90°. In region II in Fig. 4b, limitation (B) is present. Throughout this region, D (found by the normal plane travelling cylinder or closest approach methods) is perpendicular to both wells, causing the *SF* to be calculated with the minimum σ_s. Another direction of D will give a lower *SF* value. The constant value shown in region II is therefore too optimistic, and may conceal an unacceptably low crossing probability. The non-intuitive results depicted in Fig. 4 results from particular orientations of the ellipsoids. If Fig. 4b is a section view; this scenario is typical of drilling vertically from a multi-slot facility.

These complexities can pose problems when interpreting results, particularly when comparing the separations and crossing probabilities.Despite these limitations, the recommended separation rule is considered to be the best of the currently available alternatives for defining the MASD and assessing the probability of wellbore crossing. It is conceptually simple, easy to implement in computer software and provides an unambiguous relationship between the *SF* and the crossing probability. Existing alternatives that might offer a significant improvement either lackformal documentation and adequate industry exposure,or are considered too complex or computationally intensive to be practicable at this time.

Error Distributions

Investigations have shown that the probability density functions of the error distributions are more thick-tailed than the normal distribution presentlyin common use, possibly causing the risks to be underestimated. However, various mitigating factors and practices, such as the re-running of poor directional surveys limit theadverse effects, permitting continued use of the normal distribution.Further discussion of theseis includedin Appendix B.

Presentation of Well Separation

The traveling cylinder plot is a welldocumented method for depicting drilling tolerances and supporting the effective review and approval of the drilling program, (Thorogood and Sawaryn, 1991). During execution they are also used to monitor progress, project ahead and assess closure between wells. At all stages of both planning and execution the plots are also an important communications aid for achieving common situational awareness between all the stakeholders. To satisfy all these requirements the plots should be simple, consistent and unambiguous and be the minimum set to achieve the desired goals. The north-referenced, normal plane travelling cylinder diagram is the preferred representation, although the 3D closest approach is widely usedat this time, (Sawaryn, 2017). The differences in appearance between the representations can be quite subtle and it is important to clearly identify on the plot, the basis on which it is constructed. It is recommended that the terms "travelling cylinder distance" and "centre to centre distance" are adopted for this purpose.

Note that the distances and tolerances (ADP) shown on the normal plane traveling cylinders plot will generally be greater than those reported and shown in the ladder plot (Sawaryn, 2017) for all depths except for points of closest approach of the wells, which are the same.

Tolerance Lines

The tolerance lines on a traveling cylinder plot are envelopes developed from no-go areas drawn around selected depths in offset wells. The lines represent the MASD from the offset well centre point at a specified measured depth, or range of measured depths in the reference well. They display to the directional driller the toolface dependant ADP.

At all points in the approved plan there should be space to safely drill as well as allowing the directional driller some latitude to steer the well. Because of this, re-planning is required if there is a risk of an immediate excursion when drilling starts. Re-planning is also necessaryshould the tolerances invalidate the central point of the travelling cyclinder plot at any point along the trajectory.To preserve clarity, only the minimum number of tolerance lines necessary should be used to communicate the drilling restrictions. This involves identifying the major pinch points in the plan and then adjusting the frequency of any intermediate tolerance lines to match what could reasonably be expected from the BHA performance tendencies.

Throughout, the default meaning of a tolerance line is DO NOT CROSS THIS LINE. Other uses are discouraged, but if this is deemed necessary then their purpose should be clearly stated on the diagram, differentiating the line's meaning by using a different line style.

Calculation of No-Go Areas

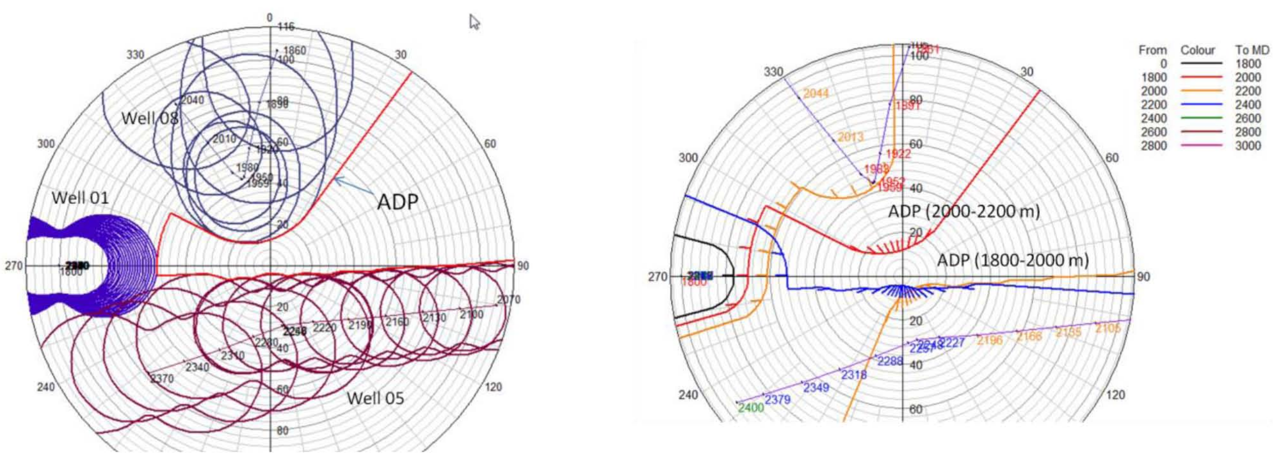
For each offset well at the depth of interest, a vector u is generated at radial points in the toolface plane, at a toolface angle τ.For the north-referenced, normal plane travelling cylinder plot set $\vartheta = \phi$. Otherwise, for highside referenced plots $\vartheta = 0$, Eq. 5.

$$\underline{u} = \begin{bmatrix} \cos\theta\cos\phi\cos(\tau-\vartheta) - \sin\phi\sin(\tau-\vartheta) \\ \cos\theta\sin\phi\cos(\tau-\vartheta) + \cos\phi\sin(\tau-\vartheta) \\ -\sin\theta\cos(\tau-\vartheta) \end{bmatrix} \tag{5}$$

If Σ_{NEV} is the covariance matrix for the relative uncertainty between the well centres, the 1σ error in the toolface direction corresponding to u is given by Eq. 6.

$$\sigma_s = \sqrt{\underline{u}^T \Sigma_{NEV}\, \underline{u}} \tag{6}$$

Substitution of this value in Eq. 3 gives the radius of the no-go line, or MASD for this direction. The resulting no-go area resembles a pedal-curve, Appendix C.



5a 5b

Figure. 5—a) The tolerance line shown in red, defines the (ADP in each toolface direction for the offset wells 01, 05 and 08 in the depth range 2000-2400m.b) To aid interpretation it is better to draw a single tolerance line on the plot that outlines the ADP.

At each depth, the distance from the plot centre to the nearest of the tolerance linesforms an envelope and represents the no-go area on the travelling cylinder plot, Fig. 5a which shows the pedal curves constructed against three offset wells for depth increments between 2,000 and 4,000 m.The figure shows how the plot can quickly become cluttered making interpretation difficult, so it is necessary tosimplify the plot showing a single tolerance line that defines the ADP. Each line is drawn with a distinctive colour or line style, keyed to the applicable depth interval so that it can be distinguished from lines for other depth zones, Fig. 5b. Automated calculation of these lines can help eliminate human error.

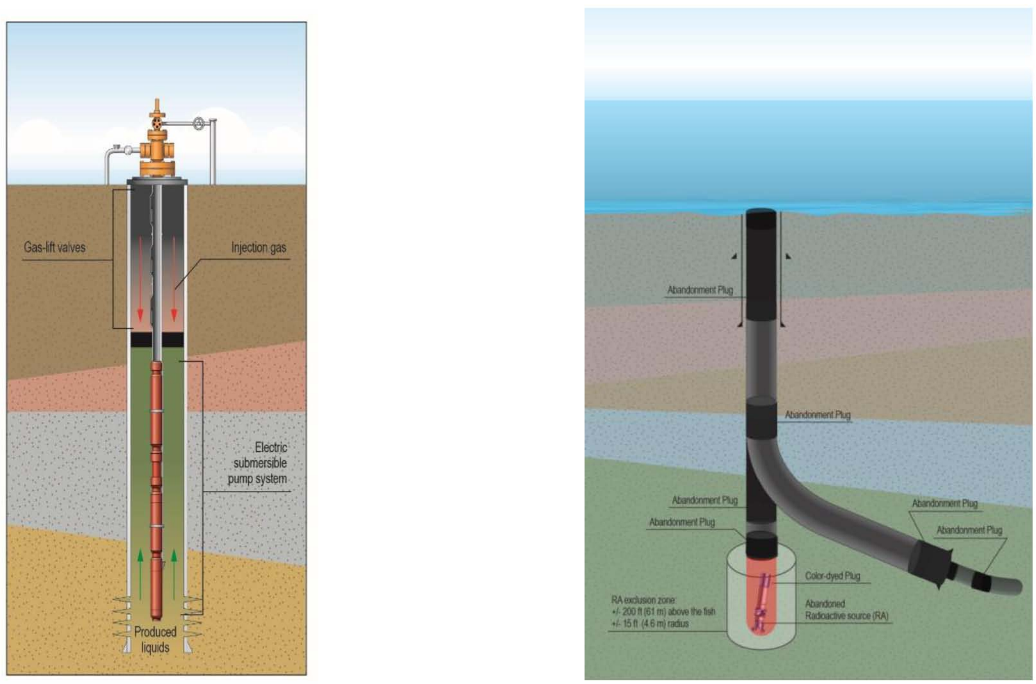
Offset Well Categorisation

After performing the MASD calculations and collision analysis the next step is to assess whether the consequence of a physicalcollision poses an HSE risk. Companies have their own preferred risk identification and assessments methods. Poedjono et al., 2009 describes one such process using hazard and risk control(HARC) and a MOC process. Another common approach is the bowtie method which visualizes the risk in just one picture, creating a clear differentiation between preventive and reactive risk management prior to drilling.

The most common HSE collision risk is a well control situation resulting froman unintended wellbore collision, but a wider range of HSE collision risks such as abandoned radioactive sources, platform piles and mines were identified, (Adel et al., 2013 and Sawaryn, 2017).

Well Pressures

To determinewhether the well control situation presents an HSE risk, it is necessary to evaluate the reference well and offset well pressures. Calculating the reference well pressure at the point where the breach might occur requires knowledge of the fluid parameters such as mud types (air or mud), weight and the TVD where the potential intersection occurs. However, evaluating the pressure in the offset wellcan be challenging because knowledge of its status: completion, production, lifting mechanism and temporal risks is required, Fig. 6. The management of collision risk is more about understanding the offset wellbores because there is greater control of the reference wellbore.



6a 6b

Figure. 6—During the risk identification process it is important to a) evaluate the reference well and offset well pressures and b) understand the offset wellbore environment and review the drilling hazards to assessthe consequences of a collision on well control.

It is also important to assess the consequences of a collision on well control. For example, during drilling if the reference well pressure is higher than the offset well pressure, it is likely that drilling fluid will flow to the offset well until the pressures are balanced. A sufficient drilling fluid volume should be available to remediate this event.

Offset Well Environment

Understanding the offset wellbore environment is paramount to successfully evaluating the risk in the event of a wellbore collision with that well. That environment can be categorised in terms of wellbore status, completion type, wellbore fluids and lifting mechanism, Table 4.

Table 4—Categorisation of the offset wellbore environment

Offset Well Environment		Description
Wellbore Status	Active	- Producer, producing hydrocarbons, steam or chemical
		- Injector, injecting fluid to maintain reservoir pressure or gas storage
		- Disposal, disposing of solids or fluids by injecting to a formation
	Inactive	- Plugged and abandoned. No chemical or radioactive source present
		- Cased but not perforated
		- Waiting on production, contains pressure or contains no pressure
		- Waiting on workover, contains pressure or contains no pressure
		- Waiting on abandonment, or temporarily abandoned
Completion Type	Open Hole	- Producing from the open hole
	Cased Hole	- The well is cased then perforated at the reservoir depth of interest or cased with pre-perforated casing or screens positioned at depth
Wellbore Fluids	Type	- Hydrocarbon, oil, gas or water
	Density	- Gas, oil or water or a mix fluid that can dynamically change along the wellbore
	Phase	- Single phase or multi-phase that could effect the fluid density
Lifting Mechanism	Natural Flow	- Flows naturally to surface (consider temporal risks – Sawaryn, 2017)
	Artificial Lift	- Increasing the reservoir pressure, or lowering the hydrostatic head in the wellbore

Non-Natural Flow

More than 75% of oil producer wells requireadditional recovery methods because the producing reservoir lacks sufficient pressure for the fluid to flow to surface naturally. Gas or water can be injected into the reservoir to maintain reservoir flowing pressure or artificial lift methods are introduced to reduce the fluid density in the borehole, (Fleshman and Lekic, 1999).

Table 5—Influence of artificial lift methods on HSE risk assessment

Artificial Lift Method	HSE Risk Assessment
Rod Pump	The consequences of collision risk with a non-naturally flowing well using a rod or sucker-rod pump will be non-HSE if collision would cause damage to the pump system only.
Progressive Cavity Pump (PCP)	The consequences of collision risk with a non-naturally flowing well using a progressive cavity pump will be non-HSE if collision would cause damage to the pump system only.
Hydraulic Pump	The consequences of collision risk with a non-naturally flowing well using a hydraulic pump will be non-HSE if collision would cause damage to the pump system only.
Gas Lift (GL)	The consequences of collision risk with a gas lifted well at shallow depth would be an HSE risk. The high-pressure gas injection would be able to displace the mud column of the reference well and therefore result in a well control situation. If it was decided to continue to operate reducing the HSE risk to non-HSE risk by shutting in the gas lift well then the high-pressure gas injector must be bled-off to a manageable low differential pressure that will not cause a well control situation. This low-pressure gas injection will moderate production until high pressure gas injection can be resumed once the HSE risk zone is passed.
Electric Submersible Pump (ESP)	The consequences of collision risk with an ESP lifted well at shallower depth could be either HSE or non-HSE. If the collision occurs above the ESP, without damaging the electrical cable, the ESP can still operate. This would displace the mud, causing a well control situation. Depending on conditions, it might be possible to reduce the well to a non HSE risk by shutting down the ESP.
Hybrid Lift System GL and ESP	The consequences of collision risk with this type of well at a shallower depth will be HSE, similar to a GL type well. At a deeper depth with the ESP it could be HSE or non-HSE. It may be possible by shut in the gas injector and bleed off the annulus. If it is not possible, the ESP must be shut in to completely maintain non-HSE conditions to drill ahead.
Hybrid Lift System ESP and Natural Flow	The consequence of collision risk with this type of well is the same as with natural flow, which could result in HSE risk. To operate under non-HSE conditions, a detailed assessment needs to be made to determine whether it is possible to shut in the ESP only. If it is not possible, non-HSE status can be achieved by setting a downhole plug.

The most common artificial lift systems are rod pumps, gas lift (GL) and electric submersible pumps (ESPs), but other systems including hybrid systems can be used with, or without natural flow to boost the production flow rate, Table 5.

Once the risk in the event of a collision with the offset well has been assessed, taking into account the factors mentioned, it should be determined whether the collision risk is an HSE risk. Drilling is not permitted unless the risk can be reduced to non-HSE by implementing preventive actions, such as the examples described above.

Assurance and Verification

Correct software implementation of the separation rule can be validated by comparison with results provided for astandard set of well paths, (ISCWSA, 2017). This document and associated workbook define a reference well path and 11 offset well paths, Table 6, each with a different spatial relationship to the reference well, Fig.7. The different spatial relationships have been carefully selected to test important geometrical features and highlight common implementation errors. The centre to centre clearance scan between the reference well and each of the offsets is included in the documentation, together with the corresponding positional uncertainties.

Table 6—Standard set of ISCWSA well separation scenarios.

Wellpath Name	Relationship to Reference Wellpath
Reference Well	Reference well. 2DJ to 85° inc, due south, then horizontal
Offset 01: East 100	Parallel to reference, offset 100m to the east
Offset 02: North 100	Parallel to reference, offset 100m to the north
Offset 03: East 10	Close top hole, then crossing reference at a low incident angle
Offset 04: East 20	Close top hole, approaching then diverging at low angle
Offset 05: Angular	Approaching at depth at an acute angle
Offset 06: Overlap Opposite	Approaching at 180° relative azimuth and overlapping
Offset 07: Short Opposite	Approaching at 180° relative azimuth and stopping short
Offset 08: Perpendicular	Approaching at 90° relative azimuth
Offset 09: Vertical	Vertical well intersecting 85° inclination tangent
Offset 10: Sidetrack from MD 900m on Reference	Sidetrack that diverges from, then approaches the reference well
Offset 11: Horizontal Approach	Approaching on horizontal plane at 90° relative azimuth

For successful assurance and verification it is important to set up the wells correctly and detailed instructions are provided in the workbook. Before using these wellpaths to evaluate more complex clearance calculations, users should first ensure that the wellpath coordinates, position uncertainties and centre to centre clearance distances agree with the definitive listings provided.

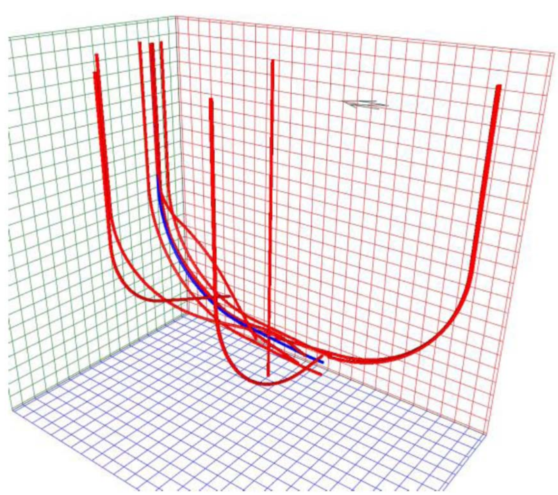


Figure. 7—3D view of thespatial relationships between the reference well and the offset wellpaths with the reference well shown in blue

The uncertainty valuesincluded in the wellpath listings are based on the ISCWSA measurement-while-drilling (MWD) model, (Williamson, 2000, Grindrod et al., 2016). The calculated results might vary slightly between implementations. However, if the wells have been set up correctly, experience shows that agreement with the uncertainties reported in the definitive listing should be repeatable to the second decimal place for values greater than 0.3m. Alternative positional uncertainty models can also be tested. Agreement with the *SF* results is expected to be within one decimal place for *SF* values less than 10.

It is suggested that a record of the results from these comparisons beretained each time a system is implemented or updated, so that it can be provided as evidence of technical compliance for internal assurance as well as assurance for partners, clients and regulatory authorities.

Further Work

At this time, the number of peer-reviewed papers dedicated to well collision avoidance is surprisingly small. Continued improvements in directional surveying tools and methods, theavailable computational power, and drilling trends leading to the closer proximity of wells all provide ample opportunities for further development of this important subject.The investigations and discussions leading to this paper have highlighted several topics that require attention. Longer step-out, complex trajectory wells from multiple drilling sites has increased pad-to-pad well interference, and has increased the variety of 3D spatial relationships and position uncertainties that may commonly occur between a reference well and its offsets. As a result, the frequency with which the geometric limitations of the existing methods may be encountered has increased. Further work is needed on the error distribution functions and crossing and collision probabilities and the manner in which these are visualized and used in both planning and operations.

Conclusions

The consequences of implementing a standardised separation rule across the industry are far reaching,but doing sooffers both safety and commercial benefits.However, to be both dependable and effective, the separation rule must be supported by rigorously applied management practices.

Practitioners should be familiar with the model's limitations, particularly with respect to complex well geometries and high well densities. Despite these limitations, the recommended separation rule is considered to be the best of the currently available alternatives for defining the MASD and assessing the probability of wellbore crossing.

When describing a collision avoidance policy the authors believe there are three key components, described in this paper that must be adhered to.

1. To establish and operate a management framework that contains all the elements outlined in the Management and Practices paper that either meets or exceeds their requirements.
2. Strict adherence to the STOP DRILLING rule for HSE risk offset wells when the separation factor reaches the threshold, *SF*=1.
3. The manual or electronic storage of all approved and executed collision avoidance plans, in amanner that clearly identifies both the author and approver.

It is acknowledged that where local regulations are more conservative they take precedence over anything stated in this paper.

SI Metric Conversion Factors

degrees × 1.745329	E-02=rad
ft × 3.048*	E-01=m
in. × 2.54*	E-02=m

*

Conversion factor is exact

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Nomenclature

COV = covariance, L^2, m^2
d = distance in the D direction, L, m
D = distance between between analysis points, L, m
k = confidence level or scaling factor, dimensionless
n,e,v = north, east and vertical ordinates in a NEV system, L,m
θ = reference well inclination at the relative depth on the travelling cylinder plot, radians
ϕ = reference well azimuth at the relative depth on the travelling cylinder plot, radians
ϑ = north reference azimuth on the travelling cylinder plot, radians
τ = Toolface angle on the travelling cylinder plot, radians
u = unit vector, dimensionless
x,y = coordinates in DXY system, L, m
R = open-hole radius, L, m
T = coordinate transformation matrix, dimensionless
S_m = surface margin, L, m
σ = standard deviation for position coordinate, L, m
σ^2 = variance for position coordinate, L^2, m^2
Σ = covariance matrix, L^2, m^2

Subscripts and Superscripts

ADP = allowable deviation from the plan
DXY = DXY reference frame
D = distance related quantity in DXY reference frame
E = east related quantity
N = north related quantity
T = transpose
X = X related quantity in DXY reference frame
Y = Y related quantity in DXY reference frame
$MASD$ = minimum allowable separation distance
NEV = north, east, vertical reference frame
V = vertical related quantity
o = offset well related quantity
p = general point
pa = project-ahead quantity
r = reference well related quantity
s = relative (combined) uncertainty

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Appendix A
Relative Uncertainty

The MASD and ADP are both functions of the relative uncertainty in the distance *D* separating the two points of interest on the reference and offset wells. Since *D* is obtained by subtracting the position coordinates of the two points of interest, each with its own position uncertainty, the relative uncertainty between the two points is a function of the two individual uncertainties. The relative uncertainty is a 3D uncertainty and can be described in terms of a covariance matrix, or illustrated by ellipsoid.It is customary to present the relative uncertainty around the offset well unless collision probabilitiesare being calculated.

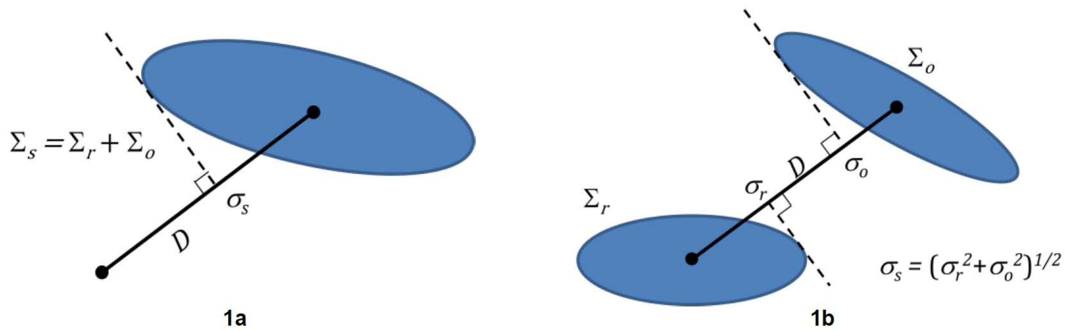


Figure. A-1—Determination of the relative uncertainty σ_s on the distance *D* between reference and offset well points.

There are two equivalent methods of determining the relative uncertainty σ_s between the reference and an offset wells, Fig. A-1. All σ values correspond toprojections of the various ellipsoids onto the direction defined by *D*. These projection points are also called pedal radius points (PRPs). For further details see Appendices C and D.In Fig. A-1a, σ_s is the projection of the combined ellipsoid onto the *D* direction. In Fig. A-1b, σ_s is formed from the projections of the individual ellipsoids. If the positional errors are statistically independent (uncorrelated)between the wells, as is assumed at this time, the two approaches are equivalent and give the same σ_s value. If the errors are correlated between the wells, both sums (Σ_r+Σ_o) and (σ_r^2+σ_o^2) must be modified.However,a rigorous methodology for this has yet to be developed and this limitation must therefore be regarded as a potential unknown simplification inherent in the present practice.

Appendix B

Probability Distributions

At this time the normal distribution is the most widely used distribution for both statistical analysis and quality assessment of well bore directional survey data. However, studies have shown that heavy-tailed distributions might be more appropriatebecause of the inherent nature of particular error terms (Williamson 1998, Gjerde 2008, Macmillan and Grindrod 2010, Torkildsen 2013), or because of gross errors relating to poor survey practice, (Ekseth et al., 2010).

Student's *t*-Distribution

An alternative to the normal distribution is the Student's *t*-distribution. The tails of this distribution can be tuned by the parameter degrees of freedom. Torkildsen, (2013, 2014a, 2014b) showed that a Student's *t*-distribution with sixdegrees of freedom (DOF) fits well withempirical survey data. This particular choice gives a significantly higher crossing probability at *SF*=1 than does the normal distribution, Fig. B-1.

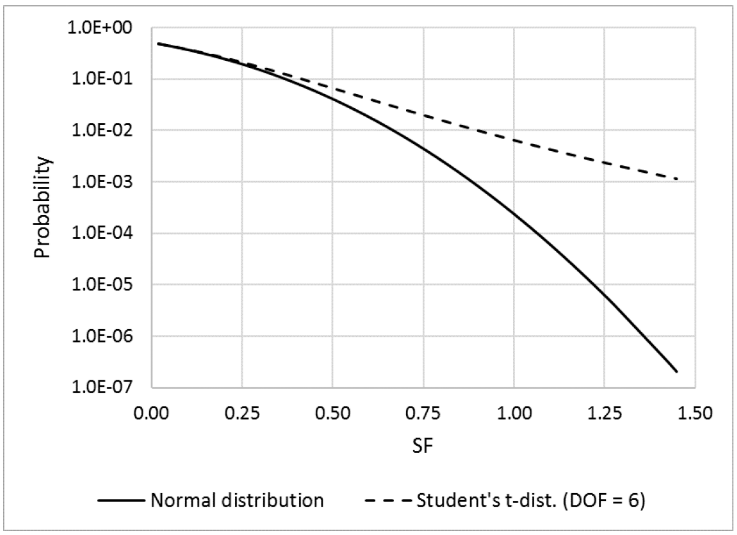


Figure. B–1—Probability of wellbore crossingfor a normal distribution and for a Student's *t*-distribution,plotted on a logarithmic scale as a function of *SF*. At *SF*=1.0 the probabilities are approximately 2.3E-4 and 6.4E-3respectively.

Using the Student's *t*-distribution with six DOF in place of the normal distribution, the *k*-value in the separation rulewould have to be increased to approximately 6.9to matchthe crossing probability of 2.3E-4 at *SF*=1. An adjustment is therefore required if the normal Distribution is to be safely employed.

Continued Use of the Normal Distribution

A procedure recommended by the ISCWSA Collision Avoidance Work Group to reduce the discrepancy between the normal and the actual error distributions is to define one standard deviation as the 95.4% confidence interval divided by 2, (ISCWSA 2013). This providesa better agreement between the normal and the actual distributions in the critical region between twoandthree standard deviations.

Other factors also come into play. Data quality control tests have pass/fail limits that partially crop the tails of the distribution, often at 3σ (Ekseth et al. 2010). This reinforces the importance of applying effective quality control procedures to exclude corrupted measurements. The standard deviation σ_s in Eq. 1 represents position uncertainty, which is the statistical summation of several independent measurement uncertainties. It is possible that this results in a distribution which is closer to a Normal distribution than the distributions of each of the individual errors, an effect similar to that of the central limit theorem, but this is yet to be confirmed.

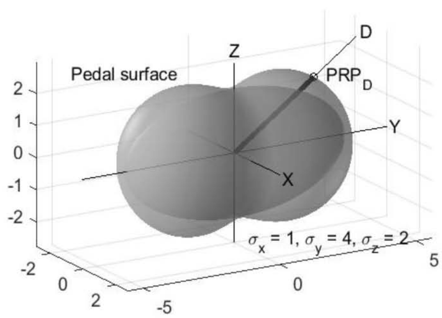
The combined effect of the previous procedures and factors reduces the calculation's sensitivity to the choice of distribution and improves the validity of the separation rule based on the assumption of normally distributed errors.

Where the previous procedures are not applied, it is necessary to acknowledge the normal distribution's possible underestimation of actual probabilities at confidence intervals beyond two to three standard deviations. This can be achieved by increasing the standard deviations of error terms within the error model or by increasing the separation rule's k factor. The recommended k value of 3.5 was chosen based on evaluation of collision frequency resulting from the application of various separation rules currently in use. It therefore includes some allowance that the procedure mentioned previously might not have been rigorously applied in all cases.

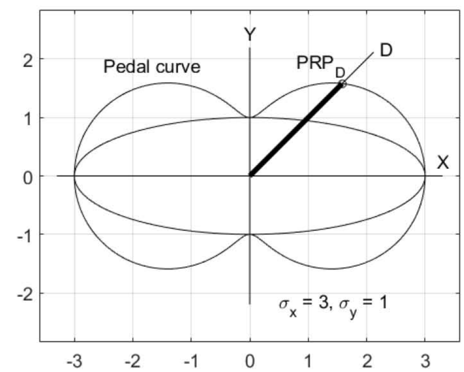
Appendix C

Pedal Surface

The projection procedure indicated in Fig. 1 provides a straight-forward graphical representation of σ_s. The 1σ ellipsoid is projected onto the *D* direction, and σ_s is then the extreme point of the projected length, or equivalently the projection onto *D* of the extreme extension of the ellipsoid in the *D* direction. Such a projection point will trace out a surface in 3D space when the direction *D* is varied, Fig. C-1. This surface is known as a pedal surface. A characteristic feature is that it lies outside the ellipsoid, except in the directions of the principal axes where the two surfaces coincide. A planar section taken through the origin normal to one of the three principal axes results in a two-dimensional (2D) curve termed the pedal curve, (Weisstein, 2017).



1a



1b

Figure. C-1–a)—The projection of an ellipsoid onto a given vector $\underline{D}$ gives the PRPD in the direction $\underline{D}$. As the direction of $\underline{D}$ varies in 3D, PRP_D traces out a pedal surface, b) a pedal curve is obtained by projecting the 2D ellipse onto a vector $\underline{D}$ that changes direction in the plane.

A point on the pedal surface or curve will be termed the Pedal Radius Point (PRP_D), where the subscript *D* indicates the direction *D*. Similarly, the distance of PRP_D from the origin will be termed Pedal Radius Distance (PRD_D). The PRP and PRD can be defined for any *kσ* ellipsoid. When $k = 1$, $PRD_D = \sigma_D$. The standard deviation σ_D, in the direction of *D* can be derived from the covariance matrix, Appendix D.

Appendix D
Derivation of Directional Uncertainty

The uncertainties in the relative positions between the two wells are represented mathematically by 3×3 covariance matrices σ, and graphically by ellipsoids, also referred to as error ellipsoids which represent confidence regions of the positions.The directional uncertainty along an arbitrary set of orthogonal axes can be obtained from the covariance matrix. Let *D* be the distance between two points connecting the reference and offset wellbores, possibly modified by wellbore radii and additional margins, σ_D the standard deviation of the position uncertainty in the direction along *D* and *k* a scaling factor to provide an acceptable, pre-defined crossing probability when *SF*=1.The *SF* that is used in the separation rule is of the form Eq. D-1.

$$SF = \frac{D}{k\sigma_D} \tag{D-1}$$

The concept of wellbore crossing poses two questions. Firstly, what is the shape of the spatial boundary that the reference well should not cross and secondly, what is the probability of crossing this boundary? The proposed separation rule, with the *SF* of the form Eq. (D-1), implies that the boundary is the plane perpendicular to the *D* direction at a distance $k\sigma_D$ from the origin. This boundary divides space into an allowed region defined by $d < k\sigma_D$, and a prohibited or wrong side region defined by $d \geq k\sigma_D$. Hence, the crossing probability can be determined as a one-dimensional (1D) integral along the *D* axis from $k\sigma_D$ to infinity, (Bang, 2017). The integral is over the 1D projection onto *D* of the initial 3D probability density function (PDF). Compared to the general 3D integral forarbitrary boundary geometry this represents a significant computational simplification, which is one major benefit of basing the separation rule on Eq. (D-1).

The 1D PDF that results from projecting the initial 3D PDFonto an arbitrary direction *D*, inherits its basic properties, shape and confidence level from the 3D PDF. In particular, σ_D is the projection point of the 1σ uncertainty ellipsoid. As illustrated in Fig. 1, σ_D can be determined by two equivalent procedures: either by projecting the appropriately combined ellipsoid (covariance matrix), or as a combination of projections of individual ellipsoids (covariance matrices).

The mathematical procedure of projecting the corresponding covariance matrix Σ onto a direction *D* is described in the following. This projection becomes particularly simple when *D* aligns with any of the three axes of the coordinate system in which Σ is described. For example, the usual north, east and vertical (*NEV*) representation of the covariance matrix is given by Eq. D-2.

$$\Sigma_{NEV} = \begin{bmatrix} \sigma_N^2 & \mathrm{cov}(N,E) & \mathrm{cov}(N,V) \\ \mathrm{cov}(E,N) & \sigma_E^2 & \mathrm{cov}(E,V) \\ \mathrm{cov}(V,N) & \mathrm{cov}(V,E) & \sigma_V^2 \end{bmatrix} \tag{D-2}$$

The variance along,for example the *N* axis is obtained by eliminating all elements involving either of the other two directions, *E* and *V* from the covariance matrix; hence, only remains. The variance along an arbitrary direction *D*can therefore bedetermined by first expressing the covariance matrix in another coordinate system *DXY* (*D*defines one of the system axes), followed by the elimination of all elements involving *X* and *Y*. In the present context, the directions of the *X* and *Y* axes can be chosen freely, as long as the *DXY* system is Cartesian and right-handed, as with the *NEV* system.The covariance matrix Σ_{NEV} is transformed tothe *DXY* coordinate system by application of the coordinate transformation matrix $T_{NEVtoDXY}$, Eq. D-3.

$$\Sigma_{DXY} = T_{NEVtoDXY} \Sigma_{NEV} T_{NEVtoDXY}^T \tag{D-3}$$

The superscript T denotes the transpose of the matrix. The NEV and DXY systems must have the same origin. $T_{NEVtoDXY}$ is constructed from the unit length axis vectors u_D, u_X and u_Y as these are expressed in the NEV system, Eq. D-4.

$$T_{NEVtoDXY} = \begin{bmatrix} \underline{u}_D^T \\ \underline{u}_X^T \\ \underline{u}_Y^T \end{bmatrix} = \begin{bmatrix} n_D & e_D & v_D \\ n_X & e_X & v_X \\ n_Y & e_Y & v_Y \end{bmatrix} \quad \text{D-4}$$

A point expressed in NEV coordinates as $\underline{p}_{NEV} = \begin{bmatrix} n_p & e_p & v_p \end{bmatrix}^T$ will therefore have the following coordinates in the DXY system:

$$\underline{p}_{DXY} = \begin{bmatrix} d_p \\ x_p \\ y_p \end{bmatrix} = T_{NEVtoDXY}\,\underline{p}_{NEV} \quad \text{D-5}$$

Because T is orthonormal, the inverse coordinate transformation T^{-1} equals its transpose T^T, Eq. D-6.

$$T_{DXYtoNEV} = T_{NEVtoDXY}^{-1} = T_{NEVtoDXY}^{T} \quad \text{D-6}$$

Writing out Eq. (D-3) with Eqs. (D-4) and (D-2) followed by elimination of the X and Y directions shows that the variance along the D direction can be found from Eq. D-7.

$$\sigma_D = \sqrt{\underline{u}_D^T \Sigma_{NEV} \underline{u}_D} \quad \text{D-7}$$

The 2D equivalent to the formulas can be obtained by eliminating all V and Y coordinates and axes from the equations.

Example Calculation

Consider the ellipse and pedal curve in Fig. B-1. The covariance matrix (in units of m^2) is given by Eq. D-8.

$$\Sigma = \begin{bmatrix} 3^2 & 0 \\ 0 & 1^2 \end{bmatrix} \quad \text{D-8}$$

The unit direction vector parallel to D at 45° to the X axis is u_D=[cos(45°) sin(45°)]^T. Using Eq. D-7 the distance σ_D to the PRP$_D$ is given by Eq. D-9.

$$\sigma_D = \sqrt{\underline{u}_D^T \Sigma \underline{u}_D} = \sqrt{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = 2.24m \quad \text{D-9}$$

Assuming that £ is the combined covariance matrix and D is the required direction, such that σ_s=σ_D, using Eq. 3 yields a MASD of:

$$D_{MASD}(45^\circ) = 3.5\sqrt{2.24^2 + 0.5^2} + R_r + R_o + 0.3 = 8.3m + R_r + R_o \quad \text{D-10}$$

For comparison, the MASD in the directions 0° (σ_D=3m) and 90° (σ_D=1m) become D_{MASD}(0°)=10.9m+(R_r+R_o) and D_{MASD}(90°)=4.7m+(R_r+R_o) respectively.