

IN THIS STEP WE ANALYZE THE SYMMETRY OF YOUR GRAPH

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A GRAPH CAN DISPLAY VARIOUS KINDS OF SYMMETRY. THREE MAIN SYMMETRIES ARE ESPECIALLY IMPORTANT: EVEN, ODD, AND PERIODIC SYMMETRY.

- EVEN SYMMETRY. A FUNCTION IS EVEN IF ITS GRAPH IS SYMMETRIC BY REFLECTION OVER THE Y-AXIS.
- ODD SYMMETRY. A FUNCTION IS ODD IF ITS GRAPH IS SYMMETRIC BY 180 DEGREE ROTATION AROUND THE ORIGIN.
- PERIODICITY. A FUNCTION IS PERIODIC IF AND ONLY IF ITS VALUES REPEAT REGULARLY. THAT IS, IF THERE IS A VALUE $P > 0$ SUCH THAT $f(x + P) = f(x)$ FOR ALL x IN ITS DOMAIN.

THE ALGEBRAIC TEST FOR EVEN/ODD IS TO PLUG IN $-x$ INTO THE FUNCTION.

- IF $f(-x) = f(x)$, THEN f IS EVEN.
- IF $f(-x) = -f(x)$, THEN f IS ODD.

ON THE AP CALCULUS EXAMS, PERIODICITY OCCURS ONLY IN TRIGONOMETRIC FUNCTIONS.

Symmetry:

$$f(-x) = \frac{4(-x)}{(-x)^2 + 1} = -\frac{4x}{x^2 + 1} = -f(x)$$

The function $y = f(x)$ is odd.