IN THIS STEP WE FIND THE ASYMPTOTES FOR OUR GRAPH

A VERTICAL ASYMPTOTE FOR A FUNCTION IS A VERTICAL LINE $X = K$ SHOWING WHERE THE FUNCTION BECOMES UNBOUNDED.

IF YOUR FUNCTION IS RATIONAL, THAT IS, IF F(X) HAS THE FORM OF A FRACTION, F(X) = P(X) / Q(X), IN WHICH BOTH P(X)AND Q(X) ARE POLYNOMIALS, THEN WE FOLLOW THESE TWO STEPS:

1. FACTOR BOTH THE NUMERATOR (TOP) AND DENOMINATOR (BOTTOM). THIS IS VERY IMPORTANT BECAUSE IF ANY FACTORS END UP CANCELING, THEN THEY WOULD NOT CONTRIBUTE ANY VERTICAL ASYMPTOTES.

2. ONCE YOUR RATIONAL FUNCTION IS COMPLETELY REDUCED, LOOK AT THE FACTORS IN THE DENOMINATOR. IF THERE IS A FACTOR INVOLVING $(X - A)$, THEN $X = A$ IS A POSSIBLE ASYMPTOTE. IF THERE IS A FACTOR INVOLVING $(X + A)$, THEN $X = -A$ IS A POSSIBLE ASYMPTOTE. NOTE HOW THE SIGN SEEMS TO BE OPPOSITE BOTH TIMES (JUST LIKE SOLVING A FACTORED POLYNOMIAL THAT HAS BEEN SET EQUAL TO ZERO).

3. TAKE EACH “POSSIBLE ASYMPTOTE” AND CHECK IF A LIMIT EXISTS AT THAT POINT IN THE GRAPH, THIS MEANS A CANNOT OBEY $\lim\limits_{\infty^- \to a}$ AND $\lim\limits_{\infty^+ \to a}$

4. TO FIND THE HORIZONTAL ASYMPTOTE SET $\lim\limits_{x \to \infty}$ FOR YOUR FUNCTION

Asymptote

$$\frac{x^2}{x-1} = y$$

domain restrictions $x \neq 1$

$\lim\limits_{\infty^+ \to 1} \frac{x}{x-1}$

$\frac{1.001}{1.001-1} = +y$

$\boxed{+\infty}$

$\lim\limits_{\infty^+ \to x} = \frac{x}{x-1}$

$\frac{0.9999}{0.9999-1} = -y$

$\boxed{-\infty}$

because $\lim\limits_{\infty^+ \to 1} f(x) \neq \lim\limits_{\infty^- \to 1} f(x)$

$\therefore$ Vertical asymptote is at $x = 1$