# IN THIS STEP WE FIND THE CONCAVITY OF THE GRAPH.

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HERE WE FIND THE INFLECTION POINTS AND WHERE THE GRAPH CONCAVES UP AND DOWN.

-TO FIND THE INFLECTION POINTS OF THE GRAPH WE NEED TO SOLVE FOR

F(X)'' = 0;

-NEXT WE FIND WHERE THE GRAPH IS CONCAVE UP AND DOWN ---->

-IF THE F(X)'' > 0 THEN THERE IS A CONCAVE UP

-IF THE F(X)'' < 0 THEN THERE IS A CONCAVE DOWN

*Second Derivative:*

$$f''(x)=\frac{-8x(x^2+1)^2-4(1-x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$=\frac{-8x^3-8x-16x+16x^3}{(x^2+1)^3}=\frac{8x^3-24x}{(x^2+1)^3}=\frac{8x(x^2-3)}{(x^2+1)^3}$$

$$f''(x)=0 \Rightarrow x=0 \text{ or } x=\pm\sqrt{3}$$

$$f(0)=0,\quad f(\pm\sqrt{3})=\frac{\pm 4\sqrt{3}}{(\pm\sqrt{3})^2+1}=\pm\sqrt{3}$$

| $x$ | | $-\sqrt{3}$ | | $0$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $\cap$ | $-\sqrt{3}$ | $\cup$ | $0$ | $\cap$ | $\sqrt{3}$ | $\cup$ |
| $f''(x)$ | - | $0$ | + | $0$ | - | $0$ | + |

$(-\sqrt{3},-\sqrt{3})$, $(0,0)$, and $(\sqrt{3},\sqrt{3})$ are points of inflection.