

CS 447/547: Computer Graphics

Homework 2

This homework must be done individually. Submission date is October 23, 2013, in class.

Total points: 31

Question 1: This question concerns human's intensity perception. Humans are tuned to the *ratio* of intensities, not their absolute difference. If we want to make a perceptually uniform intensity system with intensities $l_1 = 1$, l_2 , l_3 , l_4 , and $l_5 = 256$. What are the values of l_2 , l_3 , and l_4 ? (3 points)

Answer:

$$\begin{cases} \frac{l_2}{l_1} = \frac{l_3}{l_2} = \frac{l_4}{l_3} = \frac{l_5}{l_4} \\ l_5 = 256 \\ l_1 = 1 \end{cases} \rightarrow l_5 = l_2^4 = 256 \rightarrow l_2 = 4, l_3 = 16, \text{ and } l_4 = 64.$$

Question 2: CIE L*a*b* color space is often considered approximately perceptually uniform. We can convert RGB into L*a*b* in two steps:

Step 1: Convert RGB to XYZ using the formula in our lecture 2.

Step 2: Convert XYZ to L*a*b*.

L*a*b* is not a linear color space, so converting XYZ to L*a*b* is more complicated than RGB to XYZ. We will use the following formulas to the conversion.

$$L^* = \begin{cases} 116 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 & \frac{Y}{Y_n} > 0.008856 \\ 903.3 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} & \text{else} \end{cases}$$
$$a^* = 500 * \left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right)$$
$$b^* = 200 * \left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right)$$

where

$$f(t) = \begin{cases} t^{\frac{1}{3}} & t > 0.008856 \\ 7.787 * t + \frac{16}{116} & \text{else} \end{cases}$$

Here $Y_n = 1.0$ is the luminance, and $X_n = 0.950455$, $Z_n = 1.088753$.

Suppose we have two colors in RGB color space (0.5, 0, 0) to (1, 1, 1).

a. What are the coordinates for these two colors in L*a*b* color space? (2 points)

Answer:

(38.9502, 63.5844, 53.3516) and (99.9923, 0.0412, -0.0285)

In computer graphics, we often need to perform linear interpolation between two colors. The linear interpolation from (r_1, g_1, b_1) and (r_2, g_2, b_2) can be implemented as follows.

$$r(u) = (1 - u)r_1 + ur_2$$

$$g(u) = (1 - u)g_1 + ug_2$$

$$b(u) = (1 - u)b_1 + ub_2$$

b. We want to interpolate from $(0.5, 0, 0)$ to $(1, 1, 1)$ in 5 steps, which can be achieved by using $u=0, u=0.25, u=0.5, u=0.75, u=1$, respectively. Compute the 5 RGB colors. (5 points)

Answer:

Let C_i ($1 \leq i \leq 5$) be the resulting color by interpolating from $R_1(0.5, 0, 0)$ to $R_2(1, 1, 1)$,

$$C_i = uR_2 + (1 - u)R_1$$

$$C_1 = (0.5, 0, 0) ; C_2 = (0.625, 0.125, 0.125) ; C_3 = (0.75, 0.25, 0.25) ; C_4 = (0.875, 0.375, 0.375) ; C_5 = (1, 1, 1)$$

c. Compute the corresponding coordinates in $L^*a^*b^*$ color space of the above 5 RGB colors. (5 points)

Answer:

Let L_i ($1 \leq i \leq 5$) be (L^*_i, a^*_i, b^*_i) corresponding to the RGB colors in (c),

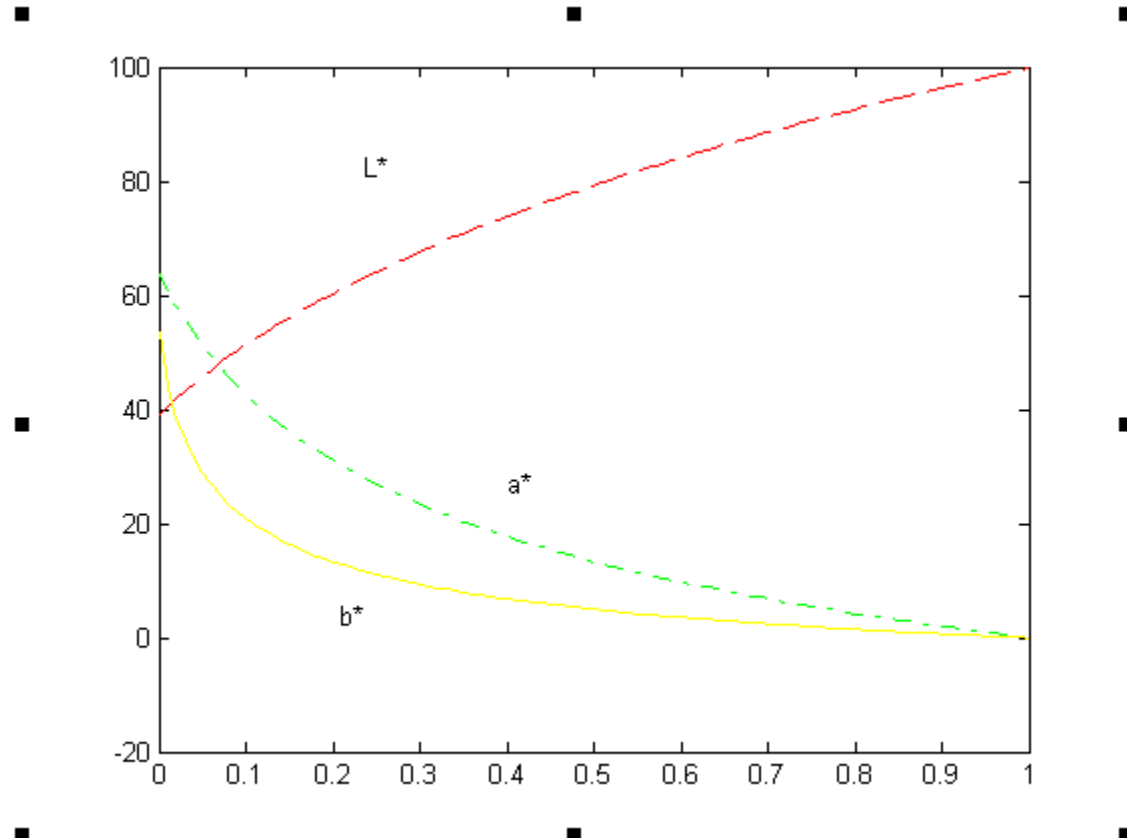
$$L_1 = (38.9502, 63.5844, 53.3516) ; L_2 = (64.1346, 26.8609, 11.0547)$$

$$L_3 = (79.2166, 13.2815, 4.9469) ; L_4 = (90.6165, 5.4298, 1.9170)$$

$$L_5 = (99.9923, 0.0412, -0.0285)$$

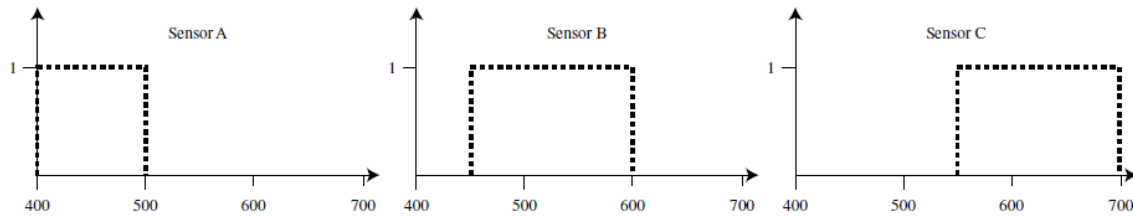
d. Plot two graphs: one showing L^* as a function of u and the other showing a^* . (2 points)

Answer:

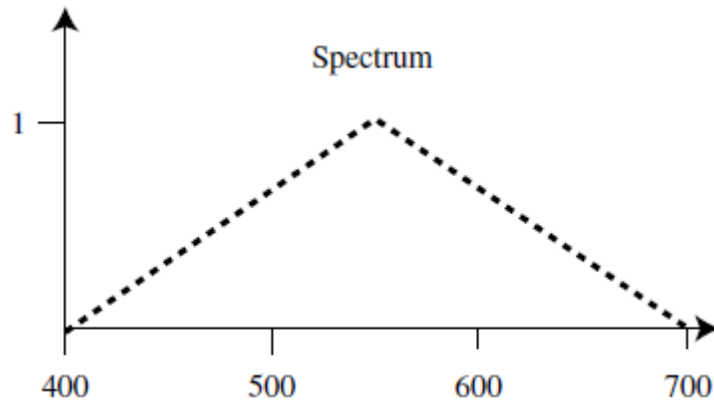


Here we can see that L^* and a^* are not a linear function of u .

Question 3: Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.



What is the response of each of these three sensors to the following spectrum? You need to give actual values. (6 points)



Answer:

$$r_A = 100/3 = 33.3; r_B = 325/3 = 108.3; r_C = 75$$

Question 4: The **Sobel operator** is used in image processing for edge detection. The following shows a 3×3 Sobel filter mask for a horizontal edge detector.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

a. What is the response of this filter to the following 6×6 image? Ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image. (2 points)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ -4 & -4 & -4 & -4 \\ -4 & -4 & -4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b. What is the response of this filter to the following 6×6 image? Again, ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a 4×4 image. (2 points)

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c. Can you design a 3×3 filter that can detect the vertical edge in the image shown in (b)? (1 point)

Answer:

There is no unique answer. Here is an example.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Any design that will have a similar response to the above one is ok.

Question 5: Gaussian is one of the most popular filters in computer graphics. What is the 9×9 2D Gaussian filter mask? (Use the method described in Lecture 4).

Answer: Our lecture gave a 1×7 1D Gaussian filter mask. We can then first construct the following 1×9 1D Gaussian filter mask.

$$\mathbf{G}_9 = \frac{1}{256} [1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1]^T \quad (2 \text{ points})$$

We can use obtain the corresponding 9×9 2D Gaussian filter mask by vector outer product.

(1 point)

$$\mathbf{G}_{9 \times 9} = \mathbf{G}_9 \mathbf{G}_9^T$$