

20 pts

Question 1:

a. How to find the direction vector \mathbf{v} that points from \mathbf{a} toward \mathbf{b} . Subtract \mathbf{a} from \mathbf{b} . $\mathbf{v} = \mathbf{b} - \mathbf{a}$

$$\mathbf{v} = [b_x - a_x, b_y - a_y]$$

b. How the length of \mathbf{v} computed. The length of \mathbf{v} , $\|\mathbf{v}\|$ can be computed by taking the square root of the dot product of \mathbf{v} with itself.

$$\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{v_x^2 + v_y^2}$$

c. How to normalize \mathbf{v} .

To normalize \mathbf{v} , first compute $\|\mathbf{v}\|$. Multiply \mathbf{v} by the scalar $\frac{1}{\|\mathbf{v}\|}$ to get $\hat{\mathbf{v}}$.

$$\hat{\mathbf{v}} = \left[\frac{v_x}{\|\mathbf{v}\|}, \frac{v_y}{\|\mathbf{v}\|} \right]$$

Question 2:

a. The dot product $\mathbf{a} \cdot \mathbf{b}$ is computed by taking the sum of the products of the components of \mathbf{a} and \mathbf{b}

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

b. The relationship between $\mathbf{a} \cdot \mathbf{b}$ and the angle θ between \mathbf{a} and \mathbf{b} .

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta$$

c. How the cross product vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is computed.

$$c_x = a_y b_z - a_z b_y$$

$$c_y = a_z b_x - a_x b_z$$

$$c_z = a_x b_y - a_y b_x$$

d. The geometric relationship between \mathbf{a} , \mathbf{b} , \mathbf{c} is.

if \mathbf{c} is 0, \mathbf{a} and \mathbf{b} are parallel, otherwise \mathbf{c} is orthogonal to \mathbf{a} and \mathbf{b} .

e. The geometric relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

$$\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$$

f. The relationship between $\mathbf{a} \times \mathbf{b}$ and the angle θ between \mathbf{a} and \mathbf{b} .

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta$$

Question 3:

$$x^2 + 3x + 2 = 0$$

$$x = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = -1, -2$$

$$x = -1; x = -2$$

Question 4:

The distance from 2D point p to line $ax + by + c = 0$ is:

$$\frac{|ap_x + bp_y + c|}{\sqrt{a^2 + b^2}}$$

Question 5:

a. The minimum number of points to define a unique line in 3D that passes through all the points is 2. The points must not be co-linear.

b. In general it is not possible to find one line that passes through more than the minimum number of points.

c. For the 3D parametric line $\mathbf{p} = \mathbf{o} + t\mathbf{d}$ the vectors \mathbf{o} and \mathbf{d} are defined by p_1, p_2 as:

$$\mathbf{o} = \begin{bmatrix} p_{1x} \\ p_{1y} \\ p_{1z} \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} p_{2x} - p_{1x} \\ p_{2y} - p_{1y} \\ p_{2z} - p_{1z} \end{bmatrix}$$

Question 6:

Matrix multiply

$$\begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 12 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1*2 + 2*1 + 5*3 \\ 4*2 + 1*1 + 12*3 \\ 3*2 + 1*1 + 5*3 \end{bmatrix} = \begin{bmatrix} 19 \\ 45 \\ 22 \end{bmatrix}$$