Question 1:

a. How to find the direction vector $\mathbf{v}$ that points from $\mathbf{a}$ toward $\mathbf{b}$. Subtract $\mathbf{a}$ from $\mathbf{b}$. $v = b - a$

$\mathbf{v} = [b_x - a_x,\ b_y - a_y]$

b. How the length of $\mathbf{v}$ computed. The length of $\mathbf{v}$,$\|\mathbf{v}\|$ can be computed by taking the square root of the dot product of $\mathbf{v}$ with itself.

$\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{{v_x}^2 + {v_y}^2}$

c. How to normalize $\mathbf{v}$.

To normalize $\mathbf{v}$, first compute $\|\mathbf{v}\|$. Multiply $\mathbf{v}$ by the scalar $\frac{1}{\|\mathbf{v}\|}$ to get $\hat{\mathbf{v}}$.

$\hat{\mathbf{v}} = [\frac{v_x}{\|\mathbf{v}\|}, \frac{v_y}{\|\mathbf{v}\|}]$

Question 2:

a. The dot product $\mathbf{a} \cdot \mathbf{b}$ is computed by taking the sum of the products of the components of $\mathbf{a}$ and $\mathbf{b}$

$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$

b. The relationship between $\mathbf{a} \cdot \mathbf{b}$ and the angle θ between $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$

c. How the cross product vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is computed.

$c_x = a_y b_z - a_z b_y$

$c_y = a_z b_x - a_x b_z$

$c_z = a_x b_y - a_y b_x$

d. The geometric relationship between a,b,c is.

if c is 0, a and b are parallel, otherwise c is orthogonal to a and b.

e. The geometric relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

$\mathbf{a} \times \mathbf{b}$ = -($\mathbf{b} \times \mathbf{a}$)

f. The relationship between $\mathbf{a} \times \mathbf{b}$ and the angle θ between a and b.

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\|\sin\theta$

Question 3:
$x^2 + 3x + 2 = 0$
$x = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = -1, -2$
$x = -1;\ x = -2$

Question 4:
The distance from 2D point p to line $ax + by + c = 0$ is:
$\frac{|ap_x + bp_y + c|}{\sqrt{a^2+b^2}}$

Question 5:
a. The minimum number of points to define a unique line in 3D that passes through all the points is 2. The points must not be co-linear.

b. In general it is not possible to find one line that passes through more than the minimum number of points.

c. For the 3D parametric line $\mathbf{p} = o + t\mathbf{d}$ the vectors $\mathbf{o}$ and $\mathbf{d}$ are defined by p_1, p_2 as:

$$\mathbf{o} = \begin{bmatrix} p_{1x} \\ p_{1y} \\ p_{1z} \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} p_{2x} - p_{1x} \\ p_{2y} - p_{1y} \\ p_{2z} - p_{1z} \end{bmatrix}$$

Question 6:
Matrix multiply

$$\begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 12 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1*2 + 2*1 + 5*3 \\ 4*2 + 1*1 + 12*3 \\ 3*2 + 1*1 + 5*3 \end{bmatrix} = \begin{bmatrix} 19 \\ 45 \\ 22 \end{bmatrix}$$