Question 1:

$near = -2.\;\; far = -6.\;\; left = -2.\;\; right = 6.\; top = 4.\;\; bottom = -4$

definition:

$$\begin{bmatrix} x_{canonical} \\ y_{canonical} \\ z_{canonical} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & \frac{-(r+l)}{r-l} \\ 0 & \frac{2}{t-b} & 0 & \frac{-(t+b)}{t-b} \\ 0 & 0 & \frac{2}{n-f} & \frac{-(n+f)}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{canonical} \\ y_{canonical} \\ z_{canonical} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{6-(-2)} & 0 & 0 & \frac{-(6+(-2))}{6-(-2} \\ 0 & \frac{2}{4-(-4)} & 0 & \frac{-(4+(-4))}{4-(-4)} \\ 0 & 0 & \frac{2}{(-2)-(-6)} & \frac{-((-2)+(-6))}{(-2)-(-6)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{canonical} \\ y_{canonical} \\ z_{canonical} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & \frac{-1}{2} \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{4}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix}$$

Question 2:

a:

$P(x_v, y_v, z_v) = (100, 20, 20);\quad d = 10$

$\frac{x_s}{d} = \frac{x_v}{z_v} = \frac{100}{20} = 5$

$x_s = 5 \cdot d = 50$

$\frac{y_s}{d} = \frac{y_v}{z_v} = \frac{20}{20} = 1$

$y_s = 1 \cdot d = 10$

$P(x_s, y_s) = (50, 10)$

b:

$P(x_s, y_s) = (20, 10)$

$\frac{x_v}{z_v} = \frac{x_s}{d} = 2 \quad x_v = 2 \cdot z_v$

$\frac{y_v}{z_v} = \frac{y_s}{d} = 1 \quad y_v = 1 \cdot z_v$

There are 3 variables, but only 2 equations, we cannot uniquely determine the 3D point's coordinate because of this.

Any point on the line $(2 \cdot z_v, 1 \cdot z_v, z_v)$ could be the original coordinate.

Question 3:

(f inside s) over b

values in r,g,b,a

$$f = \begin{bmatrix} 1,1,1,1 & 1,1,1,1 & 1,1,1,1 \\ 1,1,1,1 & 1,1,1,1 & 1,1,1,1 \\ 1,1,1,1 & 1,1,1,1 & 1,1,1,1 \end{bmatrix} s = \begin{bmatrix} 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \\ 0,0,0,0 & 1,1,1,1 & 0,0,0,0 \\ 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \end{bmatrix}$$

$$\text{b} = \begin{bmatrix} 1,0,0,1 & 1,0,0,1 & 1,0,0,1 \\ 1,0,0,1 & 1,0,0,1 & 1,0,0,1 \\ 1,0,0,1 & 1,0,0,1 & 1,0,0,1 \end{bmatrix}$$

$$(f \text{ inside } s) = f_1 = \alpha_s c_f = \begin{bmatrix} 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \\ 0,0,0,0 & 1,1,1,1 & 0,0,0,0 \\ 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \end{bmatrix}$$

f_1 over $b = \alpha_s c_f = c_{f_1} + (1 - \alpha_{f_1})c_b$=

$$\begin{bmatrix} 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \\ 0,0,0,0 & 1,1,1,1 & 0,0,0,0 \\ 1,1,1,1 & 0,0,0,0 & 1,1,1,1 \end{bmatrix} + \begin{bmatrix} 0,0,0,0 & 1,0,0,1 & 0,0,0,0 \\ 1,0,0,1 & 0,0,0,0 & 1,0,0,1 \\ 0,0,0,0 & 1,0,0,1 & 0,0,0,0 \end{bmatrix} =$$

$$\begin{bmatrix} 1,1,1,1 & 1,0,0,1 & 1,1,1,1 \\ 1,0,0,1 & 1,1,1,1 & 1,0,0,1 \\ 1,1,1,1 & 1,0,0,1 & 1,1,1,1 \end{bmatrix}$$

Question 4:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$u = a \cdot x + b \cdot y + c$

$v = d \cdot x + e \cdot y + f$

$u = a \cdot 1 + b \cdot 1 + c \quad a + b + c = \frac{\sqrt{3}+1}{2}$

$v = d \cdot 1 + e \cdot 1 + f \quad d + e + f = \frac{\sqrt{3}-1}{2}$

$u = a \cdot 0 + b \cdot 0 + c \quad c = 1$

$v = d \cdot 0 + e \cdot 0 + f \quad f = -1$

$u = a \cdot 1 + b \cdot 0 + c \quad a + c = \frac{\sqrt{3}+2}{2}$

$v = d \cdot 1 + e \cdot 0 + f \quad d + f = \frac{-1}{2}$

$a = \frac{\sqrt{3}+2}{2} - c = \frac{\sqrt{3}+2}{2} - 1 = \frac{\sqrt{3}}{2}$

$d = -\frac{1}{2} - f = -\frac{1}{2} - (-1) = \frac{1}{2}$

$b = \frac{\sqrt{3}+1}{2} - a - c = \frac{\sqrt{3}+1}{2} - \frac{\sqrt{3}}{2} - 1 = -\frac{1}{2}$

$e = \frac{\sqrt{3}-1}{2} - d - f = \frac{\sqrt{3}-1}{2} - \frac{1}{2} - (-1) = \frac{\sqrt{3}}{2}$

$$a = \frac{\sqrt{3}}{2} \; b = -\frac{1}{2} \; c = 1 \; d = \frac{1}{2} \; e = \frac{\sqrt{3}}{2} \; f = -1 \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\ 0 & 0 & 1 \end{bmatrix}$$