# Day 5: Generalized Additive Models

# Linear and Nonlinear Models

## Linear Models (ARIMA and VAR)

- Make strong assumptions about the relationships between dependent and independent variables
- But they are easily interpretable

## Non-linear Models

- Reduce (or eliminate) these assumptions
- But this is often done at the cost of interpretability

# Non-linear Modeling

Non-linear models can be written generally as

$$y = g(x) + \epsilon$$

where $g(\cdot)$ can be **any** function.

- Tremendous flexibility
- Low likelihood of interpretability

# Non-linear Modeling

If $g(\cdot)$ is a function of more than one parameter, interpretation may quickly become difficult.

$$y = x_1^2 x_2^2 + \epsilon$$

In this case, the marginal effect of $x_1$ on $y$ is

$$\frac{\partial y}{\partial x_1} = 2x_1x_2^2$$

and depends on the values of both $x_1$ and $x_2$.

# Generalized Additive Models

GAMs allow us much of the flexibility of non-linear models, without the difficulty of interpretation.

- Each parameter's effect on the dependent variable is modeled as its own function
- Since the model is additive, interpretation is straightforward, and parameter effects can be isolated

# Generalized Additive Models

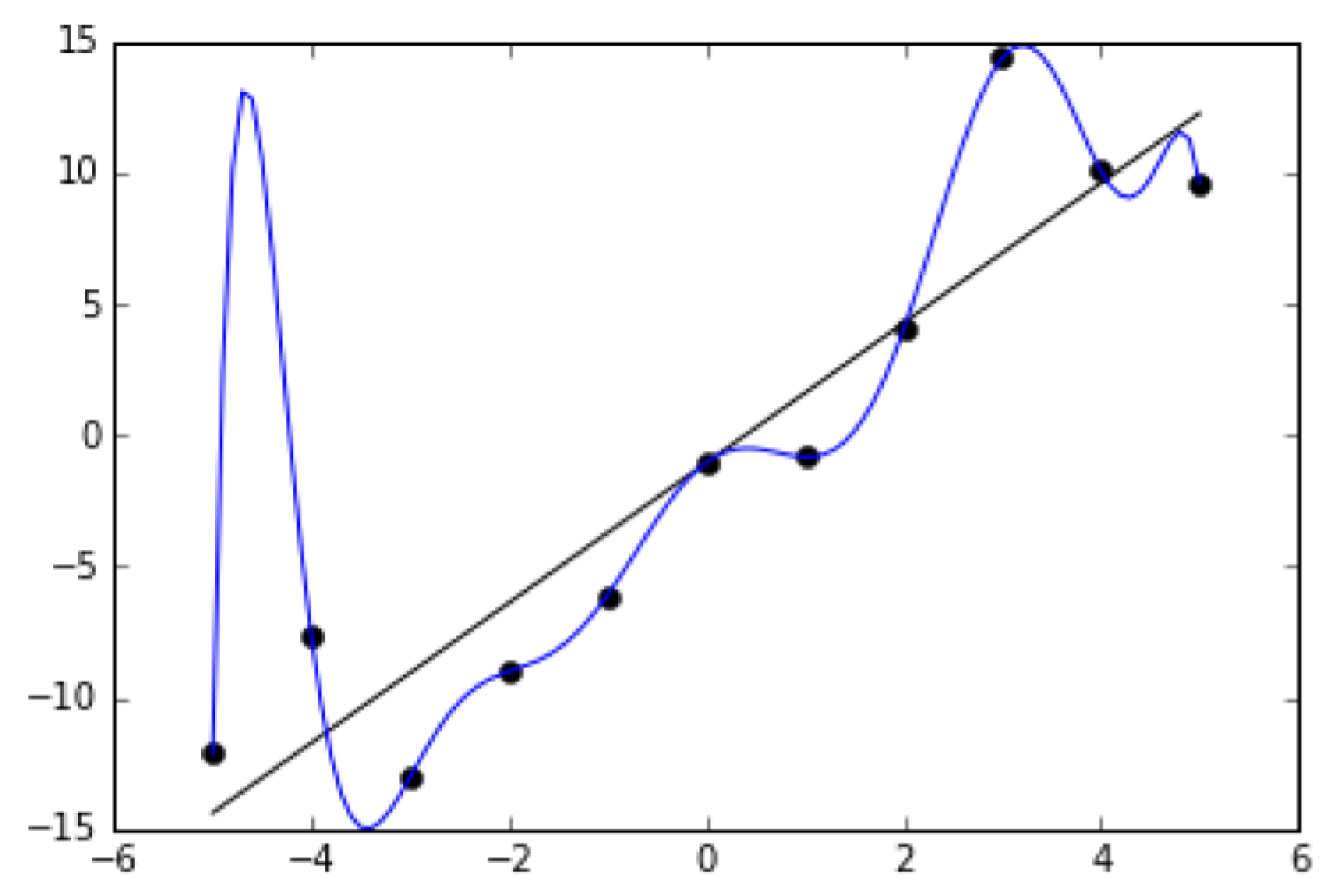
GAMs allow us much of the flexibility of non-linear models, without the difficulty of interpretation.

$$y = \sum_{i=1}^{N} f_i(x_i) + \epsilon$$

For two parameters, this could be expressed as

$$y = f_1(x_1) + f_2(x_2) + \epsilon$$

# Non-linearity and Smoothness

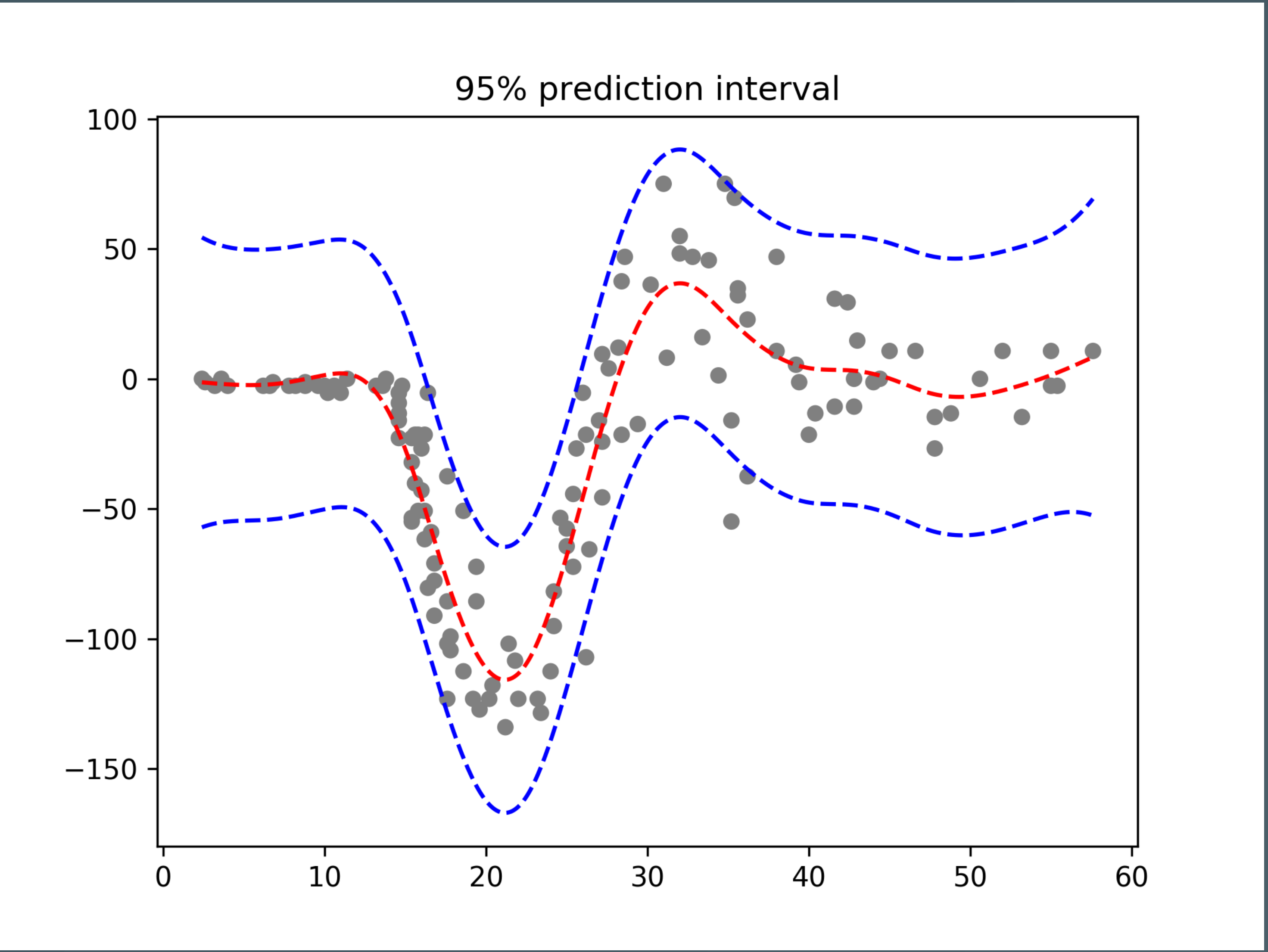


# Non-linearity and Smoothness

On the previous slide, a high-order polynomial was fitted to a parameter.

- Was the fit perfect? **Yes**
- Was it likely to fit the true data-generating process? **No**

# Non-linearity and Smoothness



# Non-linearity and Smoothness

This time, our high-order polynomial actually seems to represent the true relationship between the input and the output.

- Take care not to overfit your model
- Our true test will be when we fit a model, and use it to make predictions out-of-sample
- In sample, we can never do worse by applying a more complex functional form
- Out of sample, excess complexity can ruin our predictions

# GAM Fitting Procedure

If we want to fit an additive model, we need to create a loss function that we can optimize. For one parameter, we need to optimize

$$y = a + f(x) + \epsilon$$

Sum of squared errors for this function is

$$SSE = \sum_{i=1}^{n} (y_i - a - f(x_i))^2$$

# Choosing GAM Smoothness

In addition to minimizing the SSE term, we need to include a term that will regulate how smooth our function is, penalizing our model for "less smooth" functional forms.

Our *Penalized* Sum of Squared Errors (PSSE) is

$$\sum_{i=1}^{n}(y_i - a - f(x_i))^2 + \lambda \int_0^1 (f''(x))^2 dx$$

# Choosing GAM Smoothness

$\lambda$ is the parameter that we can adjust in order to choose how much we want to penalize our function for increased complexity.

$$\int_0^1 (f''(x))^2 dx$$

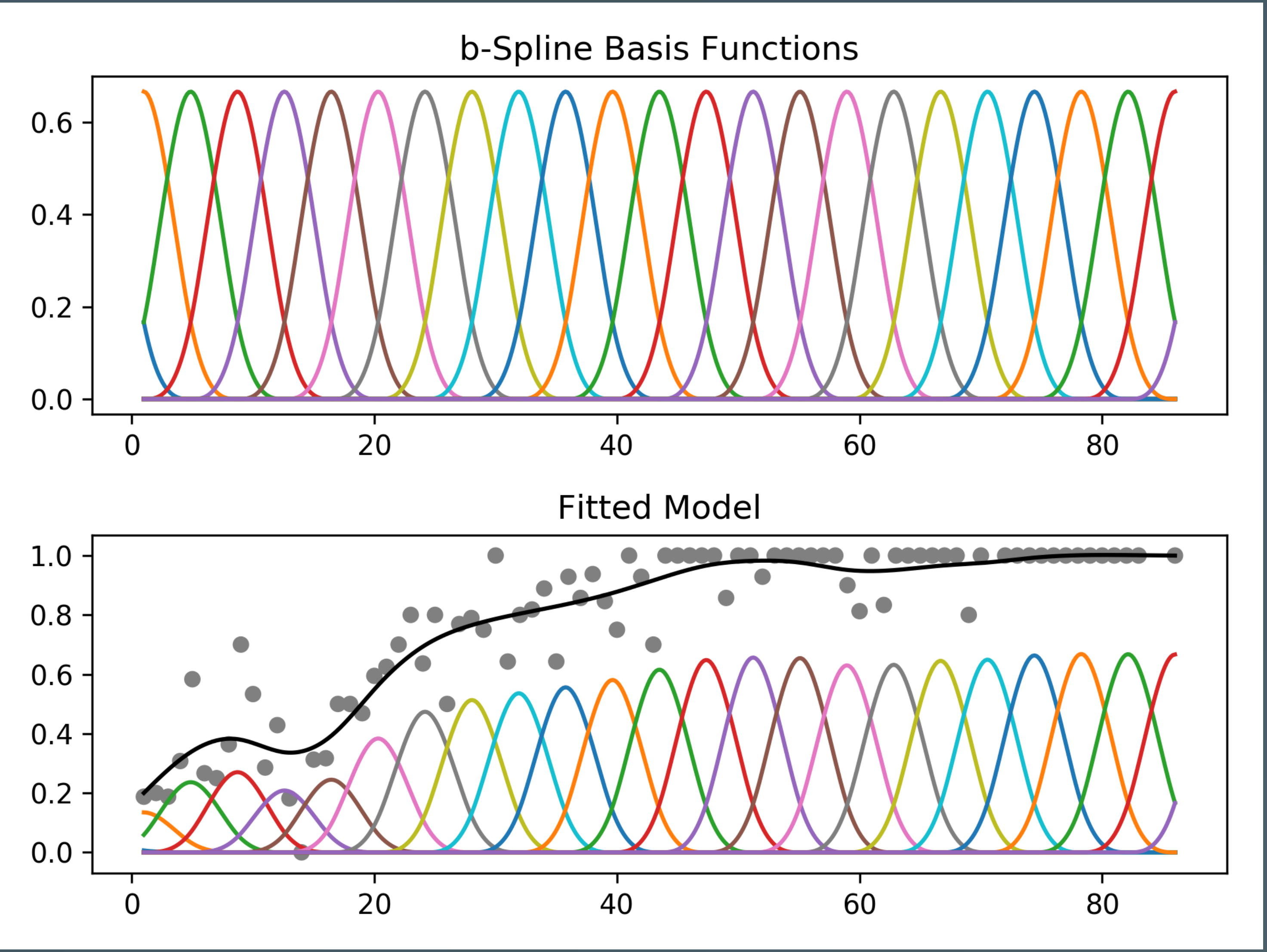
The integral term takes into account how quickly the slope of our function is changing over the interval [0,1], and penalizes our SSE when this value is high.

# Fitting Functional Forms

In order to fit a GAM to the data, we need to be able to choose an arbitrary function from among nearly infinite options.

**Splines** are a way for us to generate these functions without having to use computationally expensive searches through the function space (the group of possible function matches to the true function)

# Using Splines



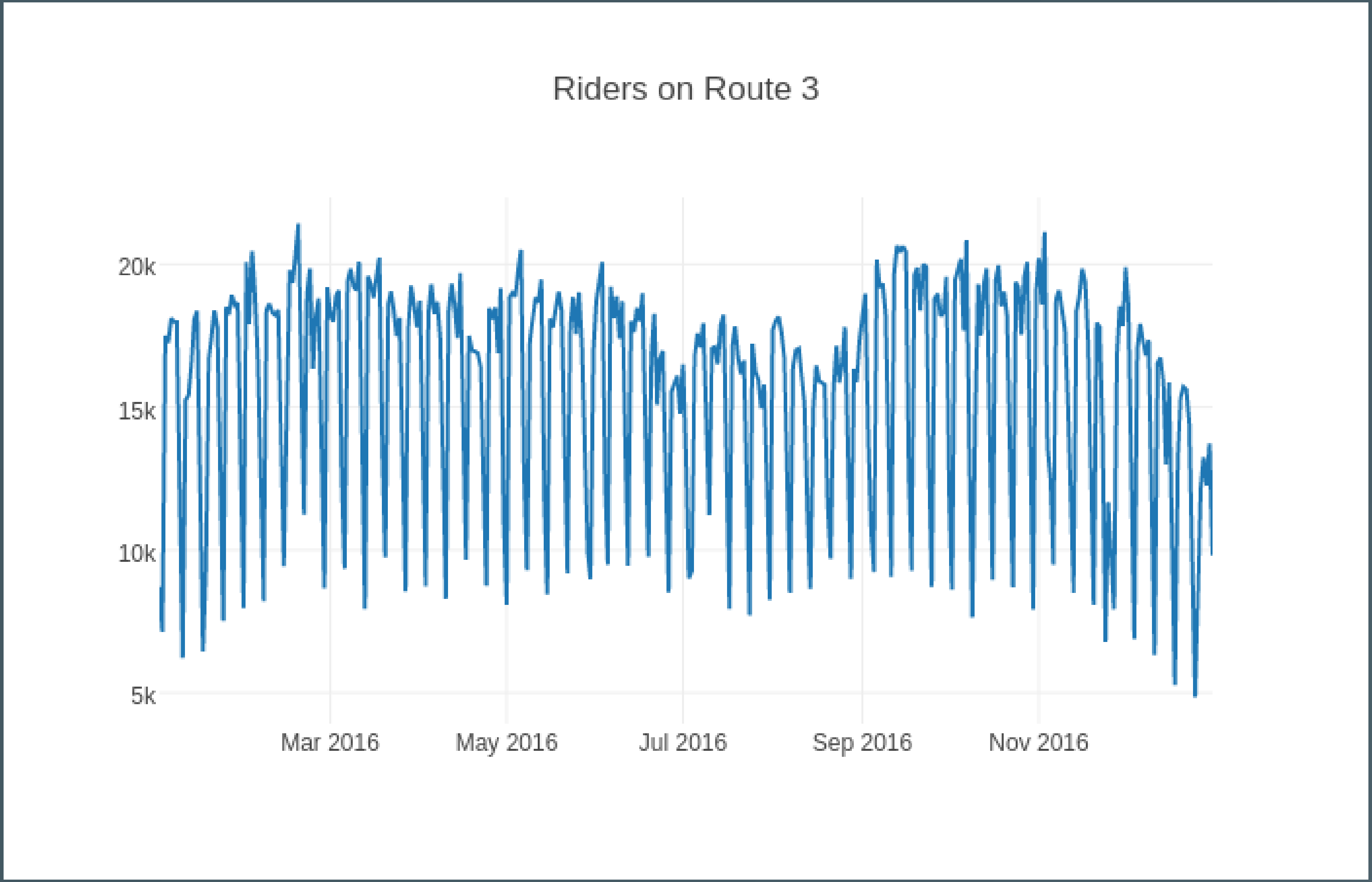
# Implementing a GAM

```
#Import statements
import pandas as pd
import numpy as np
from fbprophet import Prophet

# Prep the dataset

data = pd.read_csv(
    "/home/dusty/Econ8310/DataSets/chicagoBusRiders.csv")
route3 = data[data.route=='3'][['date','rides']]
route3.date = pd.to_datetime(route3.date,
    infer_datetime_format=True)
route3.columns = [['ds', 'y']]
```

# Implementing a GAM



# Implementing a GAM

```
# Initialize Prophet instance and fit to data

m = Prophet(changepoint_prior_scale=0.5)
# Higher prior values will tend toward overfitting
#     Lower values will tend toward underfitting

m.fit(route3)
```

In order to adapt the flexibility of our model, we are able to change the value of `changepoint_prior_scale`. We can use this to make a more flexible or rigid model, depending on our needs.

# Implementing a GAM

```
# Create timeline for 1 year in future,
#   then generate predictions based on that timeline

future = m.make_future_dataframe(periods=365)
forecast = m.predict(future)
```

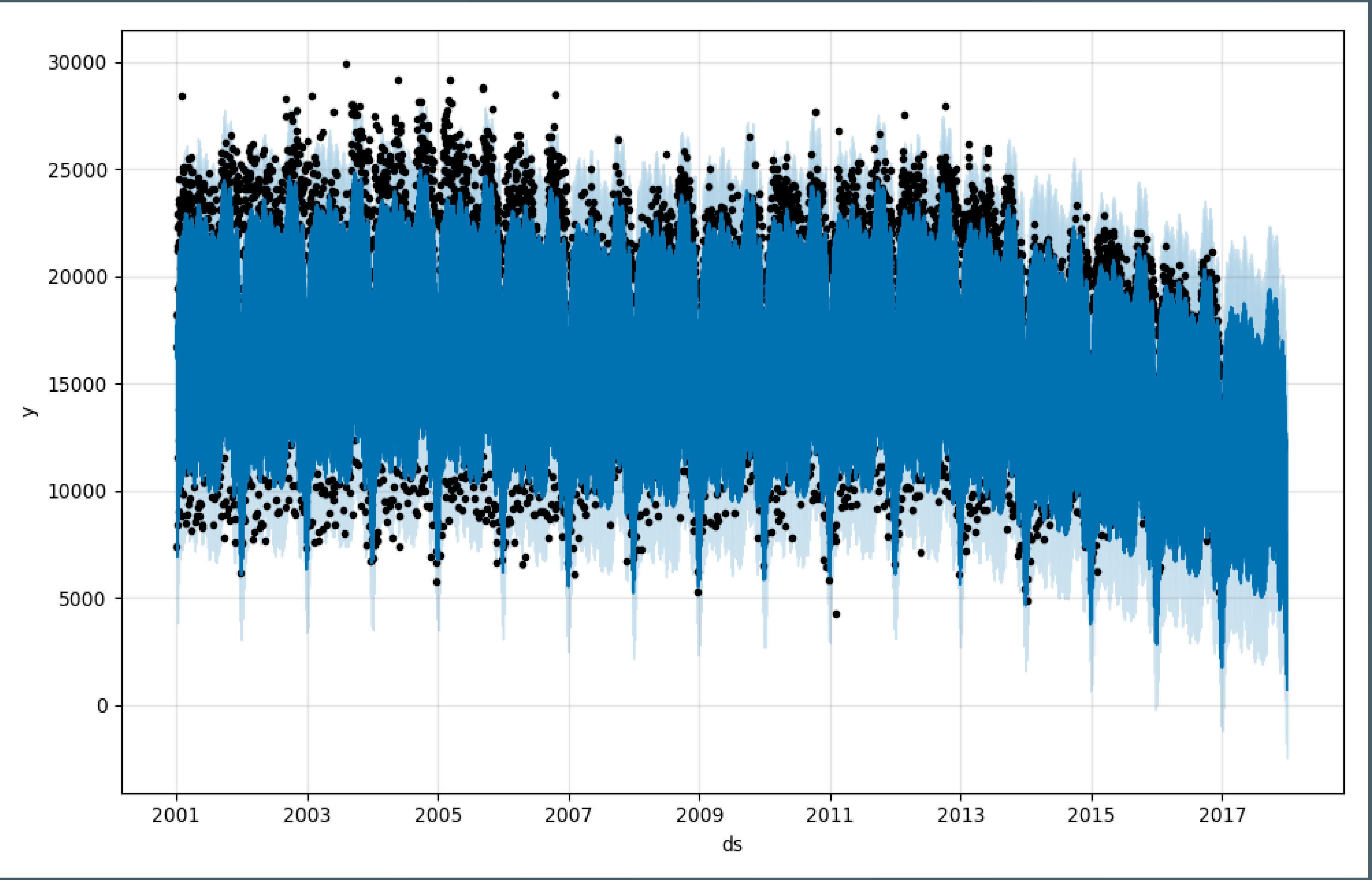
# Implementing a GAM

```
# Create plots of forecast and truth,
#   as well as component breakdowns of the trends

plt = m.plot(forecast)
plt.show()

comp = m.plot_components(forecast)
comp.show()
```

# Implementing a GAM



# Implementing a GAM

# For Lab Today

In your teams, work to model future rider counts on bus route 111, using the data in `chicagoBusRiders.csv`. You should attempt to generate a model with low SSE. In order to determine the SSE for a model, you will need to write a function to calculate the sum of squared errors.

Does changing the smoothness parameter `changepoint_prior_scale` allow you to improve your model accuracy?