Day 9: Classification Algorithms - Logistic Regression

Why Classification?

To date, we have focused on *regression* algorithms. While useful, there is a critical feature of regression tools worth noting:

- Do not understand different "groups" of data when presented as a dependent variable, just one sliding scale in $\mathbb{R}$

When to Choose Classification

When we have **discrete** dependent variables

- Binary variables
- Categorical Data
- When there is no clear dependent variable, or when we don't exactly know what we are looking for

Regression vs Classification

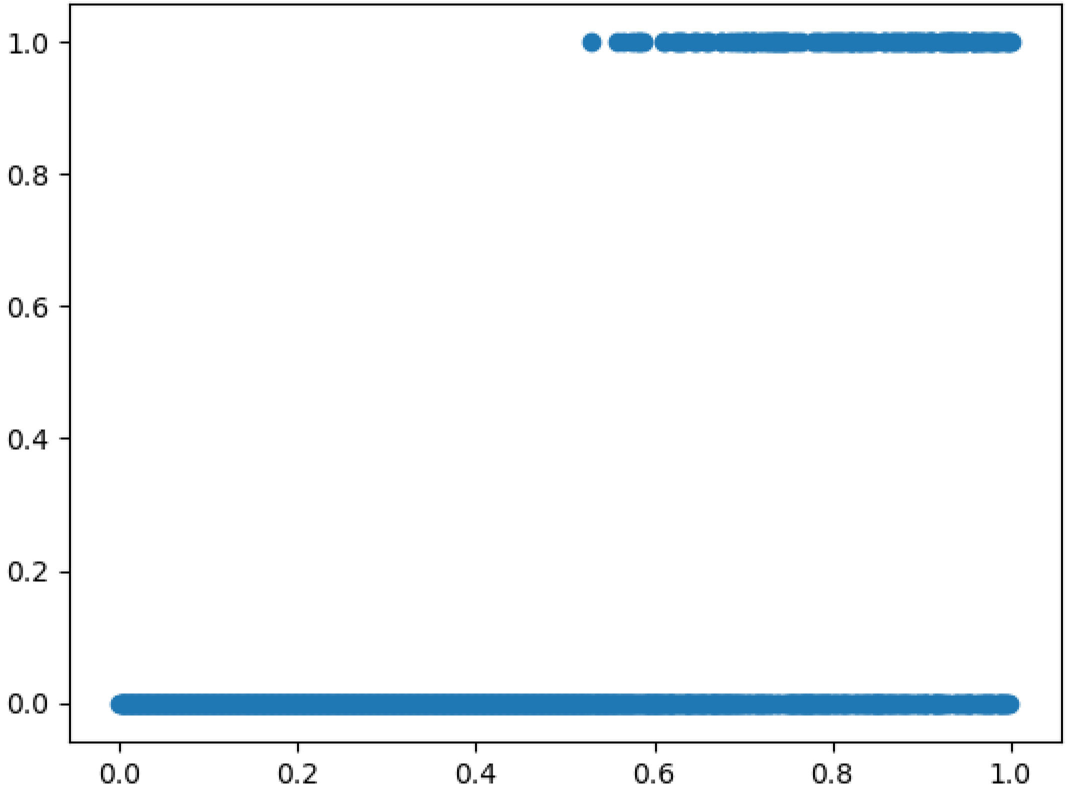
Regression asks:

- What is the predicted price of commodity x in the next period?

Classification asks

- Does the price of commodity x rise or fall in the next period?

Classification - Logistic Regression



What about Linear Probability Models?

Good:

- Just use OLS to estimate likelihood of outcome
- Has the advantage of simplicity

Bad:

- Assumes continuity of outcomes (which is not true in a classification problem)

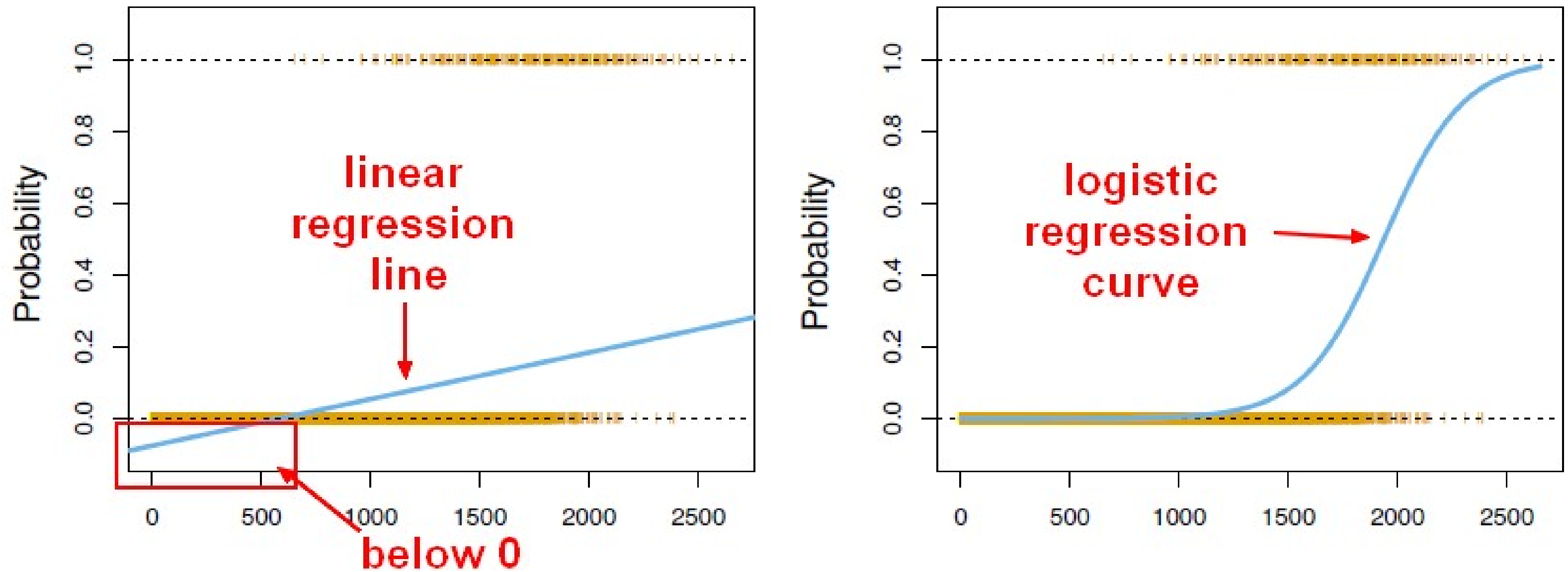
What about Linear Probability Models?

Ugly:

- Is not restricted to the $[0, 1]$ interval!
- Can have meaningless probabilities (greater than 1, and less than 0)

Logistic Regression

What if we transform our regression model in a way that requires it to remain within the $[0, 1]$ interval?



Logistic Regression

- We no longer have a linear function (linear functions are not bounded to our unit interval)
- We no longer assume that treatments have constant effect
- **But** our output can now be interpreted as

$$p(y = 1)$$

Logistic Transformation

The transformation that is required in order to coerce our output to remain between 0 and 1 is

$$p(y=1|x) = \frac{exp(x'\beta)}{1+exp(x'\beta)} = \Lambda(x'\beta)$$

and is called the **logistic transformation**.

Maximum Likelihood Estimation

OLS Log-Likelihood function:

$$ln\ \mathcal{L}(\theta|y,x) = -\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}(y - x\beta)'(y - x\beta)$$

Logistic Log-Likelihood function:

$$ln\ \mathcal{L}(\theta|y,x) = \sum_{i=1}^{n}(y_i\ ln(\Lambda(x_i'\beta)) + (1 - y_i)\ ln(1 - \Lambda(x_i'\beta)))$$

Marginal Effects in a Logit Model

In order to obtain a point estimate of the marginal effect of a given input on y, we must use the function

$$\frac{\partial E(y|x)}{\partial x} = \Lambda(x'\beta) \cdot (1 - \Lambda(x'\beta)) \cdot \beta$$

Thus, our marginal effects will depend on the values of our inputs.

Note: the Lambda (Λ) function is defined on the previous slide

Marginal Effects in Regressions

OLS:

$$\frac{\partial E(y|x)}{\partial x} = \beta$$

Logit:

$$\frac{\partial E(y|x)}{\partial x} = \Lambda(x'\beta) \cdot (1 - \Lambda(x'\beta)) \cdot \beta$$

Implementing Logistic Regression

```
import numpy as np
import patsy as pt
import statsmodels.api as sm

data = pd.read_csv('passFailTrain.csv')

y, x = pt.dmatrices('G3 ~ G1 + age + goout', data = data)

model = sm.Logit(y, x)

reg = model.fit()

print(reg.summary())
```

Implementing Logistic Regression

```
import numpy as np
import patsy as pt
import statsmodels.api as sm
```

We need to import our libraries, and particularly, import the `statsmodels` library.

Implementing Logistic Regression

```
data = pd.read_csv('passFailTrain.csv')

y, x = pt.dmatrices('G3 ~ G1 + age + goout', data = data)
```

Recall that we generate our y and x matrices in order to use them in our model. Output goes on the left of the "~", inputs on the right, separated by "+"

Note: this is also the formula that R uses when performing regressions.

Implementing Logistic Regression

```
model = sm.Logit(y, x)

reg = model.fit()

print(reg.summary())
```

First, we create our Logit model, then we store the fitted model as `reg`. Afterward, we can print out our summary table.

Implementing Logistic Regression

It should look something like this:

```
                           Logit Regression Results
==============================================================================
Dep. Variable:                     G3   No. Observations:                  296
Model:                          Logit   Df Residuals:                      292
Method:                           MLE   Df Model:                            3
Date:                Thu, 23 Mar 2017   Pseudo R-squ.:                  0.3567
Time:                        16:21:12   Log-Likelihood:                -119.01
converged:                       True   LL-Null:                       -185.01
                                        LLR p-value:                 2.010e-28
==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept      6.9131      2.282      3.030      0.002         2.441    11.386
G1             3.1671      0.344      9.218      0.000         2.494     3.840
age           -0.4124      0.136     -3.043      0.002        -0.678    -0.147
goout         -0.3163      0.147     -2.150      0.032        -0.605    -0.028
==============================================================================
```

Predictions from Logit Model

Now, we may want to use our logit model to make predictions about new observations.

All we need are new values:

`New Observation: [Term 1 Grade: Pass, Age: 16, Frequency of Going Out: 4]`

Predictions from Logit Model

```
reg.predict((1,1,16,4))
# OR
xpred = pt.build_design_matrices([x.design_info],
    testData)[0] # Recycle patsy model w/ test data
reg.predict(xpred) # Use test data to generate predictions
```

Note that we have to include values for all necessary variables, as well as a 1 for the intercept term.

Marginal Effects from Logit Model

```
reg = model.fit() # We need to start with a fitted model

mEff = reg.get_margeff(
  at='overall', # Where the ME is estimated
  method='eydx', # Calculates d(ln y)/dx, or % effect
  dummy=True, # Caclulates effects on dummies as 0 to 1
  count=True) # Calculates effects on count as value + 1

mEff.summary()
```

Using the `get_mareff` method, we can easily estimate the marginal effects of our regressors on the dependent variable. (No ugly home-made functions needed!)

Notes on R^2

While R^2 values are not always helpful in a regression setting, they are very valuable when forecasting using regressions.

- Tell us how much of the variance our model is capable of explaining
- If our R^2 is 0.3567 (like it was for the regression earlier), then the model explains 35.67% of the variation in pass/fail outcomes among students in our sample.

Notes on R^2

Even **more** useful in a forecasting setting is the out-of-sample R^2

- Tell us how much of the variance our model is capable of explaining with respect to **new** observations
- Basically, it tells us if we are doing a good job creating accurate forecasts

Generating a Tjur R^2

Since we cannot use the standard R^2 measure for Logit models, we need to calculate a pseudo-R^2, and `statsmodels` does not calculate out-of-sample R^2 automatically.

Generating a Tjur R^2

Tjur (2009) suggested an R^2 measure for Logit models calculated as the difference between the mean value of predictions for "failures" and "successes" in a binary model.

$$Tjur\ R^2 = \bar{\hat{y}}_{successes} - \bar{\hat{y}}_{failures}$$

$$Tjur\ R^2 = \text{Mean prediction for successes}$$
$$-\text{Mean prediction for failure}$$

Generating a Tjur R^2

Tjur (2009) suggested an R^2 measure for Logit models calculated as the difference between the mean value of predictions for "failures" and "successes" in a binary model.

$$Tjur\ R^2 = \bar{\hat{y}}_{successes} - \bar{\hat{y}}_{failures}$$

The measure is bounded by 1 and 0, and gives us a measure of how well we separate our two outcomes

Lab for Today

1. Fit a Logit model predicting whether or not a marketing campaign initiated by a bank results in successful sales of a financial product (CD's in this case)

2. Create a function that will take a fitted logit model, and y and x matrices, and return the Tjur R^2 value for that sample

3. Do your best to find a model with the **highest** Tjur R^2 value given the data that was provided to you (always feel free to compare code and models with others!)