Day 12: Random Forests, Other Ensembles

When we use single learning algorithms, we are very vulnerable to overfitting our model to in-sample variations, which reduces our ability to accurately model out-of-sample.

Who chose the next Roman emperor? Who chose the next king of France or England?

Strange women lying in ponds, distributing swords, is no basis for a system of government. Supreme executive power derives from a mandate from the masses, not from some farcical aquatic ceremony!

Who chose the next Roman emperor? Who chose the next king of France or England?

The current emperor or king (mostly)

Why is this a bad idea?

- A single person (algorithm?) making a decision can easily make an error of judgement.
- If we choose a Nero, we end up with Rome in flames

How do most developed countries now choose leaders?

They vote!

Why?

- NOT because we care if everyone has their opinion heard (see history of voting eligibility)
- Because large groups of people, when their opinions are averaged, make good better choices

I had a class of undergraduates who averaged 55% on their final exam.

On the other hand, a student who had chosen the most popular response to each question based on the responses of their classmates would have scored ~85%.

In aggregate, poor students can select good answers

This principle also applies to statistical learning algorithms. Aggregating "poor" algorithms can lead to a "good" algorithm.

Collections of learning algorithms are called **ensembles**.

Bagging

Bagging (**B**ootstrap **Agg**regation) is a simple way to start creating an ensemble model.

Standard Model:

$$\hat{f}\left(x
ight)=f^{st}(x)$$

All training data is used to generate our best estimate of the true functional form, f(x).

Bagging

$$\hat{f}_{bag}(x) = rac{1}{B} \sum_{b=1}^{B} f_b^*(x)$$

In bagging, each estimate utilizes a bootstrap (random) sample of the training data with each observation receiving probability $\frac{1}{N}$.

The bagged estimate is then based on the weighted average of all of the models.

Boosting

If we boost an algorithm using M stages, then we need to define $f_m(x)$ at each stage.

$$f_0(x) = 0$$

At each subsequent stage, we solve for

$$f_m(x) = f_{m-1}(x) + \hat{f}_m(x)$$

So that each stage adds more information to our model.

Boosting vs Bagging

Bagging:

 An averaged model utilizing bootstrapped samples of the complete dataset

Boosting:

 An additive model, where the predictions are incrementally improved

Boosting vs Bagging

Bagging:

- Much easier to implement
- Less Overfitting

Boosting:

- Better Performance (generally)
- Can be vulnerable to overfitting still

Tree Problems

One drawback to bagging can be illustrated by thinking about how decision trees are generated.

- 1. Find the biggest information gain
- 2. Split the tree
- 3. On each branch, find the next best information gain
- 4. Split again
- 5. Repeat 3 and 4 until stopping rule is reached (depth, purity, number of observations, etc.)

Random Forests

Using bagging on decision trees in a situation where one variable is clearly superior to other inputs, the data sample will almost never allow a model to explore other inputs.

- The most informative input will mask the other options (always be chosen)
- Each tree in the bagging algorithm is highly correlated with the other trees
 - Permits overfitting, and reduces predictive power

Random Forests

How can we alleviate this tendency?

- Restrict the inputs that the tree is allowed to choose from
- Bootstrap the sample
- Aggregate the forecasts to make a single, more accurate, prediction.

Restricting Inputs

When a classification tree looks for maximum information gain, it searches across **all** available inputs.

Trees in a random forest are restricted to a random subset of inputs at each branching:

- Typically, \sqrt{p} inputs are provided at each branch
- When a new branch occurs, a new random subset of inputs is provided
- This is repeated for all branches on all trees

Making a Classification

Once each tree in a random forest has been grown, we can use the trees to create a decision rule based on a vote by the classifiers:

- Each tree classifies an observation
- Whichever class receives the most votes (has highest predicted probability of being the true observed class) "wins," and is assigned as the predicted class for the observation

MNIST Dataset

MNIST Handwriting Recognition Data

- Contains digits 0-9
- Full Dataset is 60,000 training observations, 10,000 testing observations
- 28 x 28 pixel images, (784 factors)
- Numbers are centered in the image
- Goal is to predict the written digit based on the image
- Great learning dataset

Implementing Ensembles

We will implement Random Forests, Bagging, and Boosting using the scikit-learn module in Python, as we did for Decision Trees

Prepare the Data

Baseline Decision Tree Classifier

Resulting in:

The decision tree has an accuracy of: 0.651555555556

Random Forest Classifier

Resulting in:

The random forest has an accuracy of: 0.938481481481

Boosting Ensemble (of decision trees)

Resulting in:

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The boosting algorithm has an accuracy of: 0.934888888889
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Bagging Ensemble (of decision trees)

Resulting in:

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The bagging algorithm has an accuracy of: 0.918166666667
```

Summary of Results

- 1. Decision Tree: 65.2%
- 2. Random Forest: 93.8%
- 3. Boosting Algorithm: 93.5%
- 4. Bagging Algorithm 91.8%

Note: In this case, the Random Forest performs essentially as well as the Boosting Algorithm (all four of these are based on decision trees). It also takes less time to execute, since trees can be fit in parallel.

For Lab Today:

With your group, use the ensemble methods that we discussed today to our new Lab Assignment 4. We want to predict if an NFL passing play is going to result in a first down or touchdown (combined to be considered 'success')

Which ensemble method performed best?

How much of a difference did it make to increase the number of models in the ensemble?

How much better did you perform than a decision tree classifier?