Day 5: Generalized Additive Models

Linear and Nonlinear Models

Linear Models (ARIMA and VAR)

- Make strong assumptions about the relationships between dependent and independent variables
- But they are easily interpretable

Non-linear Models

- Reduce (or eliminate) these assumptions
- But this is often done at the cost of interpretability

Non-linear Modeling

Non-linear models can be written generally as

$$y = g(x) + \epsilon$$

where $g(\cdot)$ can be **any** function.

- Tremendous flexibility
- Low likelihood of interpretability

Non-linear Modeling

If $g(\cdot)$ is a function of more than one parameter, interpretation may quickly become difficult.

$$y = x_1^2 x_2^2 + \epsilon$$

In this case, the marginal effect of x_1 on y is

$$\frac{\partial y}{\partial x_1} = 2x_1 x_2^2$$

and depends on the values of both x_1 and x_2.

Generalized Additive Models

GAMs allow us much of the flexibility of non-linear models, without the difficulty of interpretation.

- Each parameter's effect on the dependent variable is modeled as its own function
- Since the model is additive, interpretation is straightforward, and parameter effects can be isolated

Generalized Additive Models

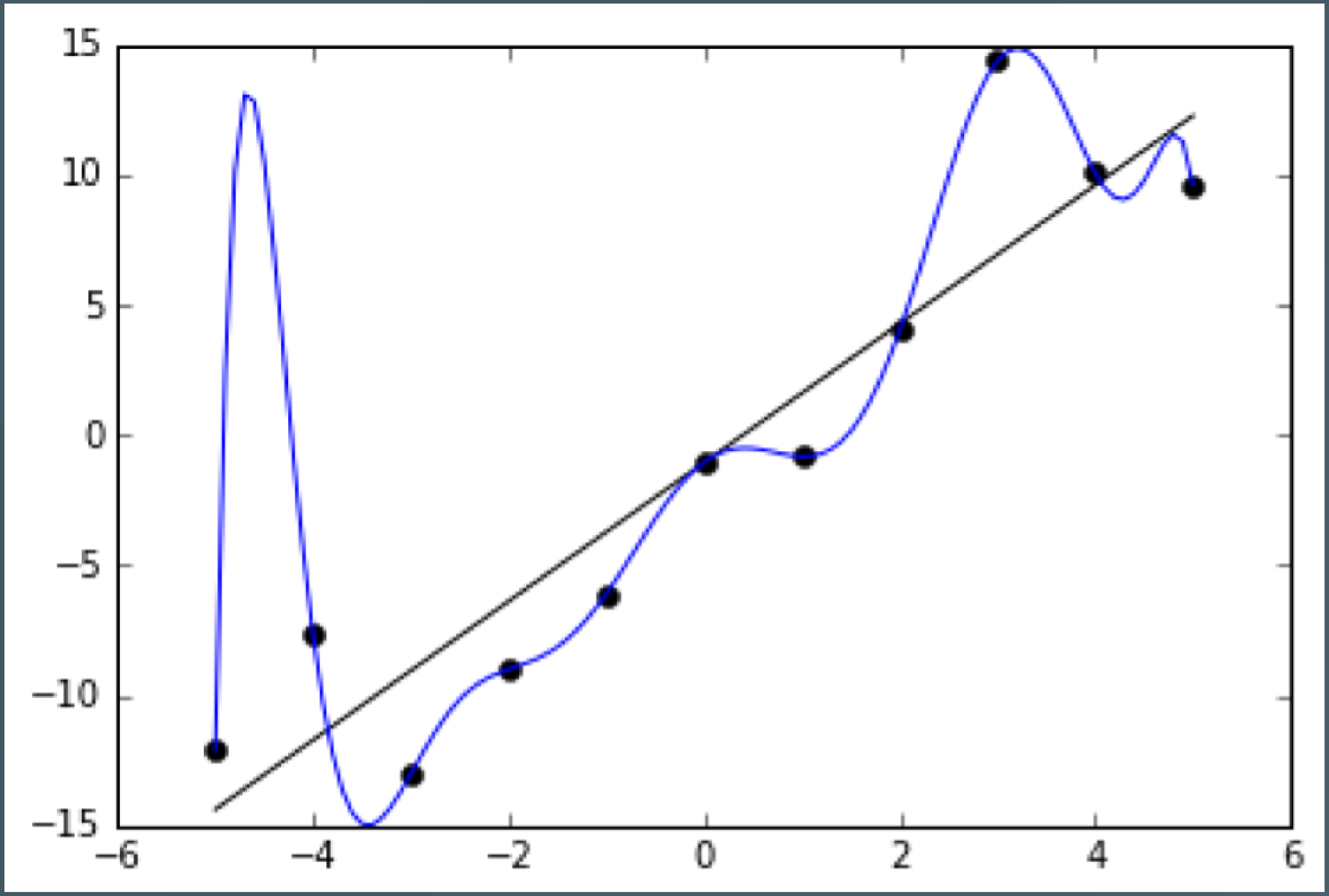
GAMs allow us much of the flexibility of non-linear models, without the difficulty of interpretation.

$$y = \sum_{i=1}^{N} f_i(x_i) + \epsilon$$

For two parameters, this could be expressed as

$$y = f_1(x_1) + f_2(x_2) + \epsilon$$

Non-linearity and Smoothness

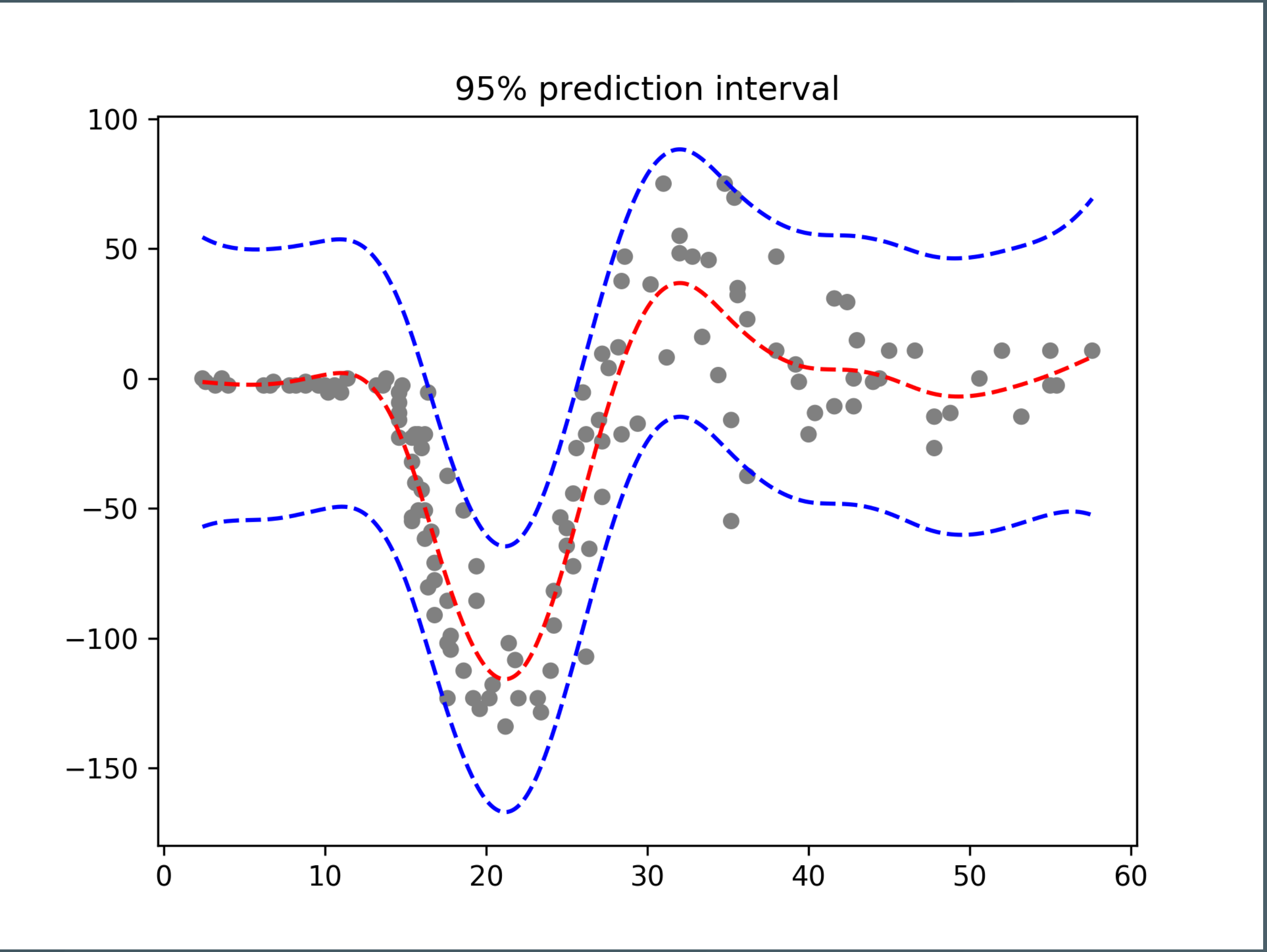


Non-linearity and Smoothness

On the previous slide, a high-order polynomial was fitted to a parameter.

- Was the fit perfect? **Yes**
- Was it likely to fit the true data-generating process? **No**

Non-linearity and Smoothness



Non-linearity and Smoothness

This time, our high-order polynomial actually seems to represent the true relationship between the input and the output.

- Take care not to overfit your model
- Our true test will be when we fit a model, and use it to make predictions out-of-sample
- In sample, we can never do worse by applying a more complex functional form
- Out of sample, excess complexity can ruin our predictions

GAM Fitting Procedure

If we want to fit an additive model, we need to create a loss function that we can optimize. For one parameter, we need to optimize

$$y = a + f(x) + \epsilon$$

Sum of squared errors for this function is

$$SSE = \sum_{i=1}^{n} (y_i - a - f(x_i))^2$$

Choosing GAM Smoothness

In addition to minimizing the SSE term, we need to include a term that will regulate how smooth our function is, penalizing our model for "less smooth" functional forms.

Our *Penalized* Sum of Squared Errors (PSSE) is

$$\sum_{i=1}^{n}(y_i - a - f(x_i))^2 + \lambda \int_0^1 (f''(x))^2 dx$$

Choosing GAM Smoothness

λ is the parameter that we can adjust in order to choose how much we want to penalize our function for increased complexity.

$$\int_0^1 (f''(x))^2 dx$$

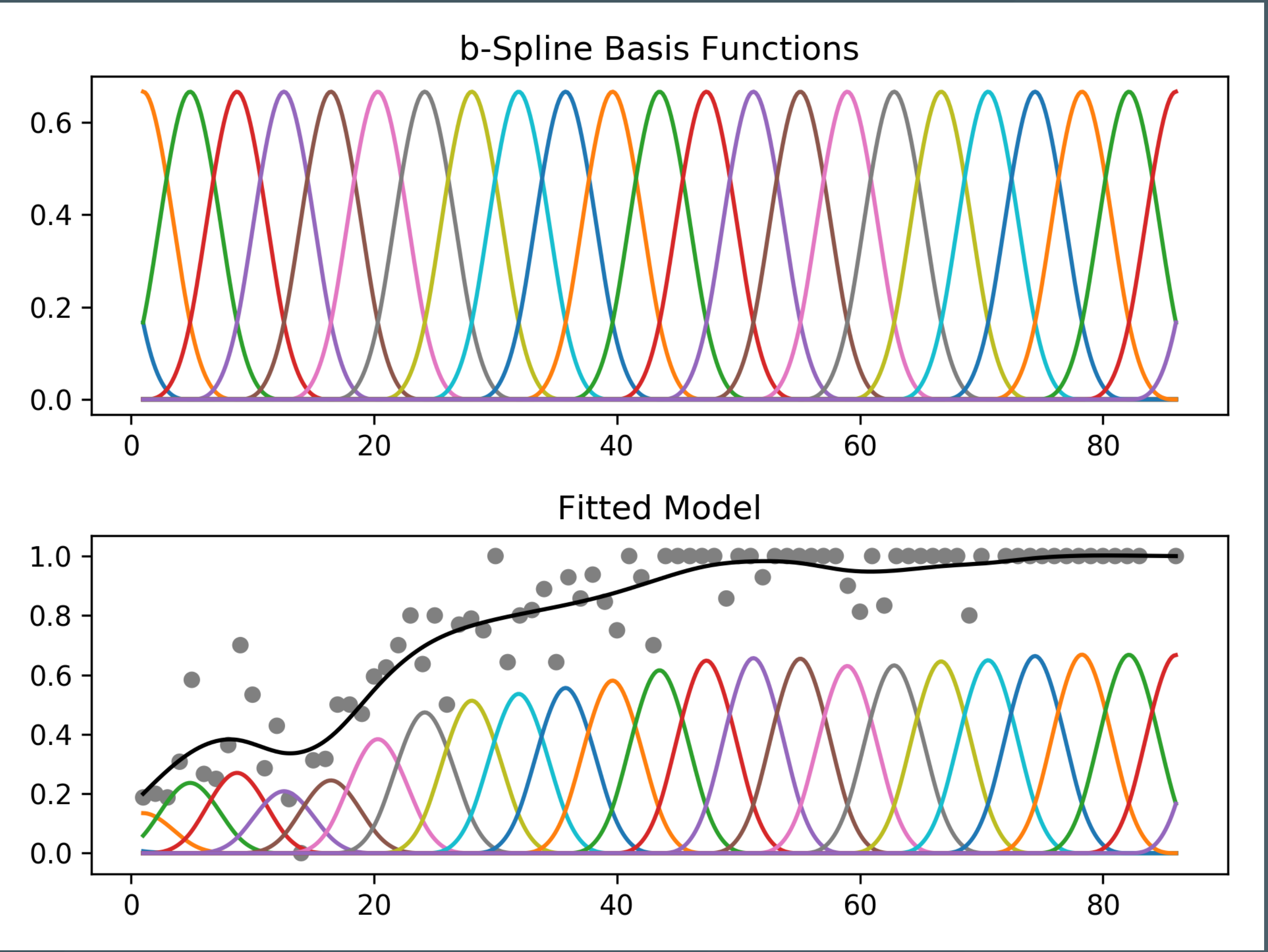
The integral term takes into account how quickly the slope of our function is changing over the interval [0,1], and penalizes our SSE when this value is high.

Fitting Functional Forms

In order to fit a GAM to the data, we need to be able to choose an arbitrary function from among nearly infinite options.

Splines are a way for us to generate these functions without having to use computationally expensive searches through the function space (the group of possible function matches to the true function)

Using Splines



Implementing a GAM

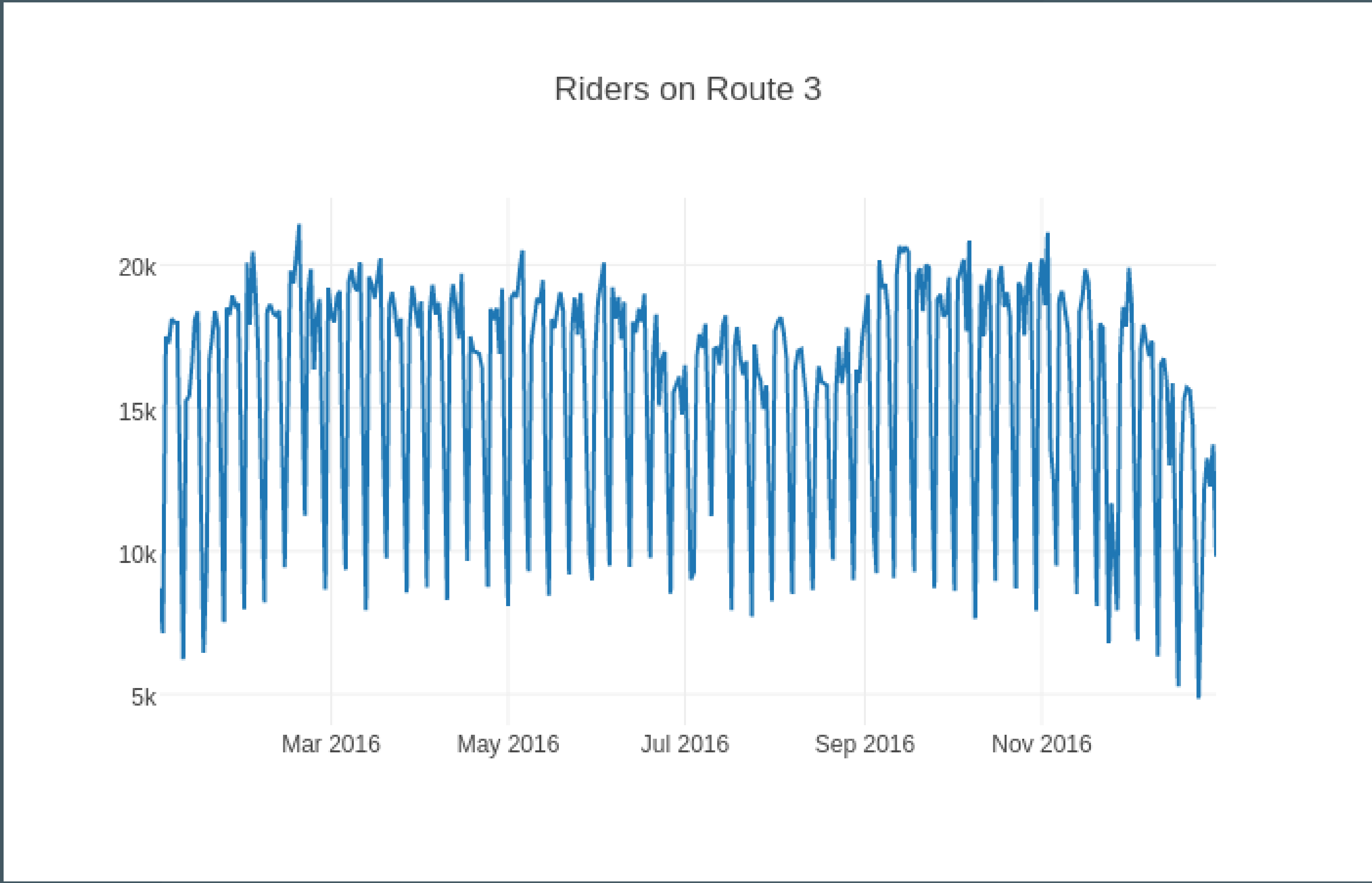
We can use either of two libraries to implement our GAM models, depending on our specific needs:

- Time series models: prefer Facebook's `Prophet` library
- General predictive analytics: prefer `pyGAM` library

Implementing a GAM in Prophet

```
#Import statements
import pandas as pd
import numpy as np
from fbprophet import Prophet

# Prep the dataset

data = pd.read_csv(
    "/home/dusty/Econ8310/DataSets/chicagoBusRiders.csv")
route3 = data[data.route=='3'][['date','rides']]
route3.date = pd.to_datetime(route3.date,
    infer_datetime_format=True)
route3.columns = [['ds', 'y']]
```

Implementing a GAM in Prophet



Implementing a GAM in Prophet

```
# Initialize Prophet instance and fit to data

m = Prophet(changepoint_prior_scale=0.5)
# Higher prior values will tend toward overfitting
#     Lower values will tend toward underfitting

m.fit(route3)
```

In order to adapt the flexibility of our model, we are able to change the value of `changepoint_prior_scale`. We can use this to make a more flexible or rigid model, depending on our needs.

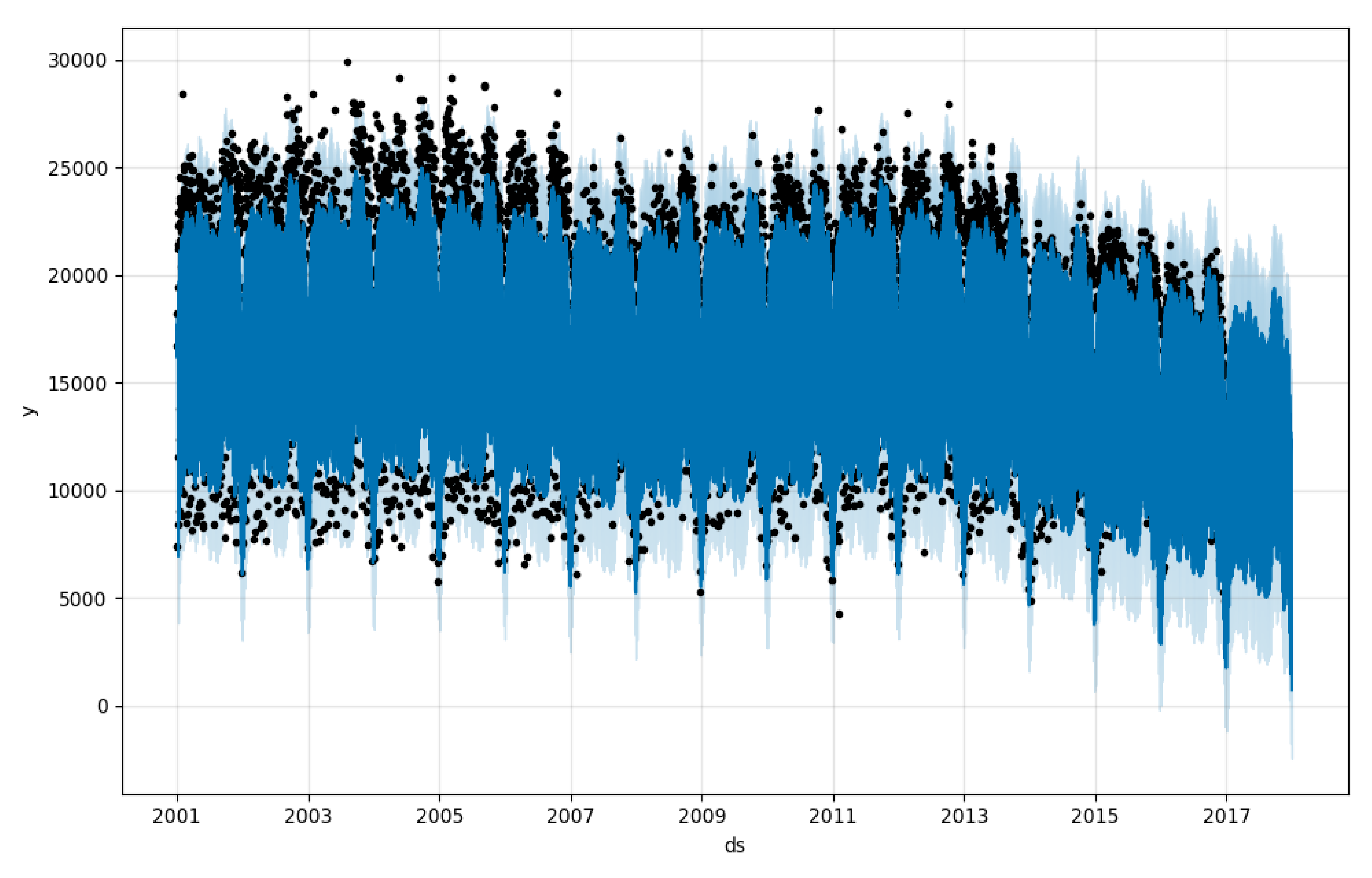
Implementing a GAM in Prophet

```
# Create timeline for 1 year in future,
#   then generate predictions based on that timeline

future = m.make_future_dataframe(periods=365)
forecast = m.predict(future)
```

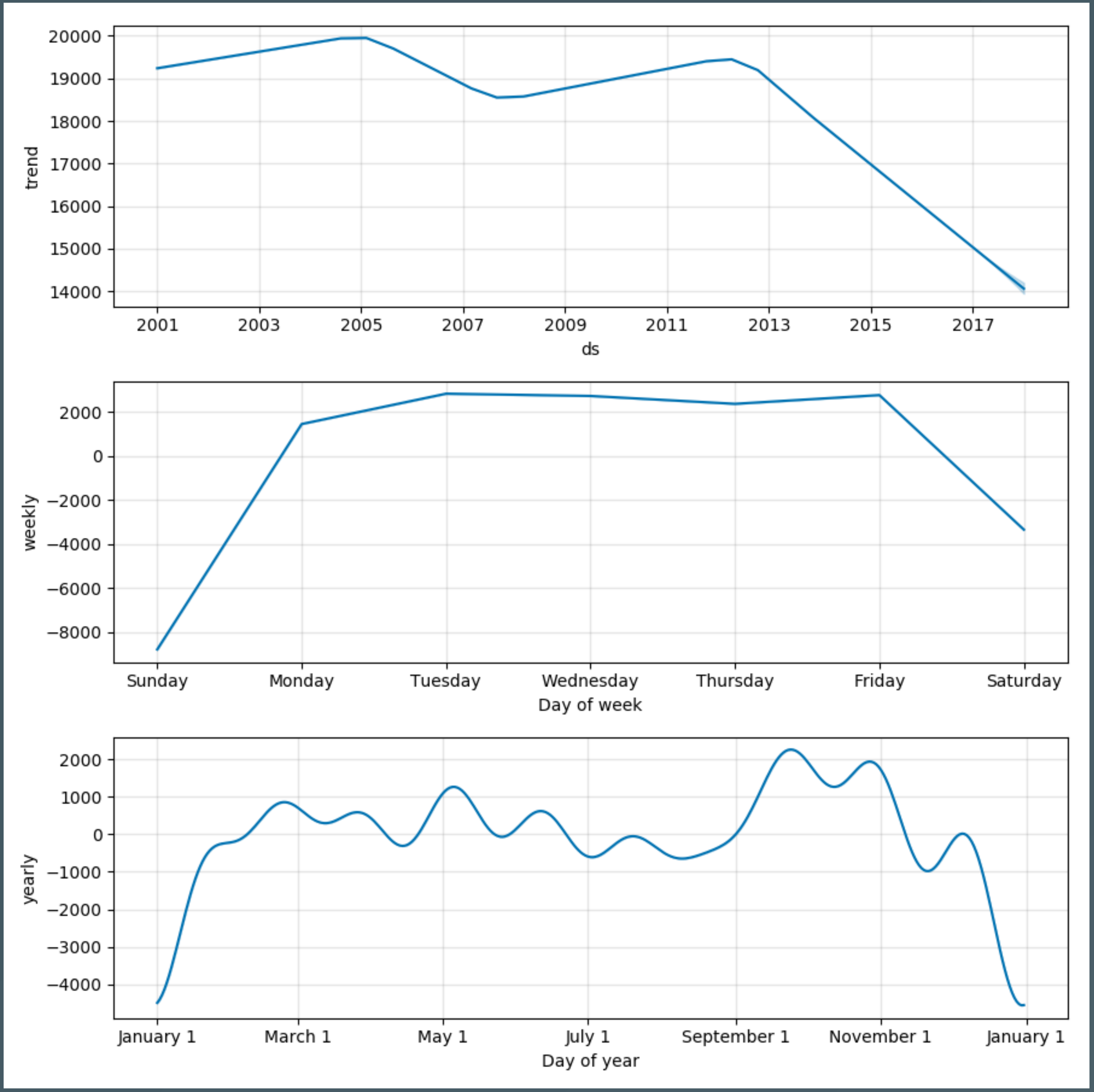
Implementing a GAM in Prophet

```
# Create plots of forecast and truth,
#   as well as component breakdowns of the trends

plt = m.plot(forecast)
plt.show()

comp = m.plot_components(forecast)
comp.show()
```

Implementing a GAM in Prophet



Implementing a GAM in Prophet



Implementing a GAM in pyGAM

```
from pygam import LinearGAM, s, f
import pandas as pd
import patsy as pt
import numpy as np
from plotly import tools
import plotly.offline as py
import plotly.graph_objs as go

# Prep the dataset
data = pd.read_csv(
    "/home/dusty/Econ8310/DataSets/HappinessWorld.csv")
```

Implementing a GAM in pyGAM

```
# Generate x and y matrices
eqn = """happiness ~ -1 + freedom + family +
      year + economy + health + trust"""
y,x = pt.dmatrices(eqn, data=data)

# Initialize and fit the model
gam = LinearGAM(s(0) + s(1) + s(2) + s(3) + s(4) + s(5))
gam = gam.gridsearch(np.asarray(x), y)
```

We create our `x` and `y` matrices using `patsy`, then we construct our GAM model.

Implementing a GAM in pyGAM

```python
# Generate x and y matrices
eqn = """happiness ~ -1 + freedom + family +
      year + economy + health + trust"""
y,x = pt.dmatrices(eqn, data=data)

# Initialize and fit the model
gam = LinearGAM(s(0) + s(1) + s(2) + s(3) + s(4) + s(5))
gam = gam.gridsearch(np.asarray(x), y)
```

The `LinearGAM` object requires a functional argument (`l`, `s` or `f`) for each variable in our data

Use `l` for linear functions, `s` for "smooth" functions of a variable, and `f` for "factor"-type variables (binary or step variables)

Implementing a GAM in pyGAM

```
# Specify plot titles and shape
titles = ['freedom', 'family', 'year', 'economy',
          'health', 'trust']

fig = tools.make_subplots(rows=2, cols=3,
        subplot_titles=titles)
fig['layout'].update(height=800, width=1200,
        title='pyGAM', showlegend=False)
```

To plot our marginal effects jointly, we need to create subplots, and assign a subplot to each of our variables.

Here, we prepare the canvas by creating the grid and shape of the overall figure.

Implementing a GAM in pyGAM

```
for i, title in enumerate(titles):
  XX = gam.generate_X_grid(term=i)
  pdep, confi = gam.partial_dependence(term=i, width=.95)
  trace = go.Scatter(x=XX[:,i], y=pdep, mode='lines',
        name='Effect')
  ci1 = go.Scatter(x = XX[:,i], y=confi[:,0],
        line=dict(dash='dash', color='grey'),
        name='95% CI')
  ci2 = go.Scatter(x = XX[:,i], y=confi[:,1],
        line=dict(dash='dash', color='grey'),
    name='95% CI')
  ...
```

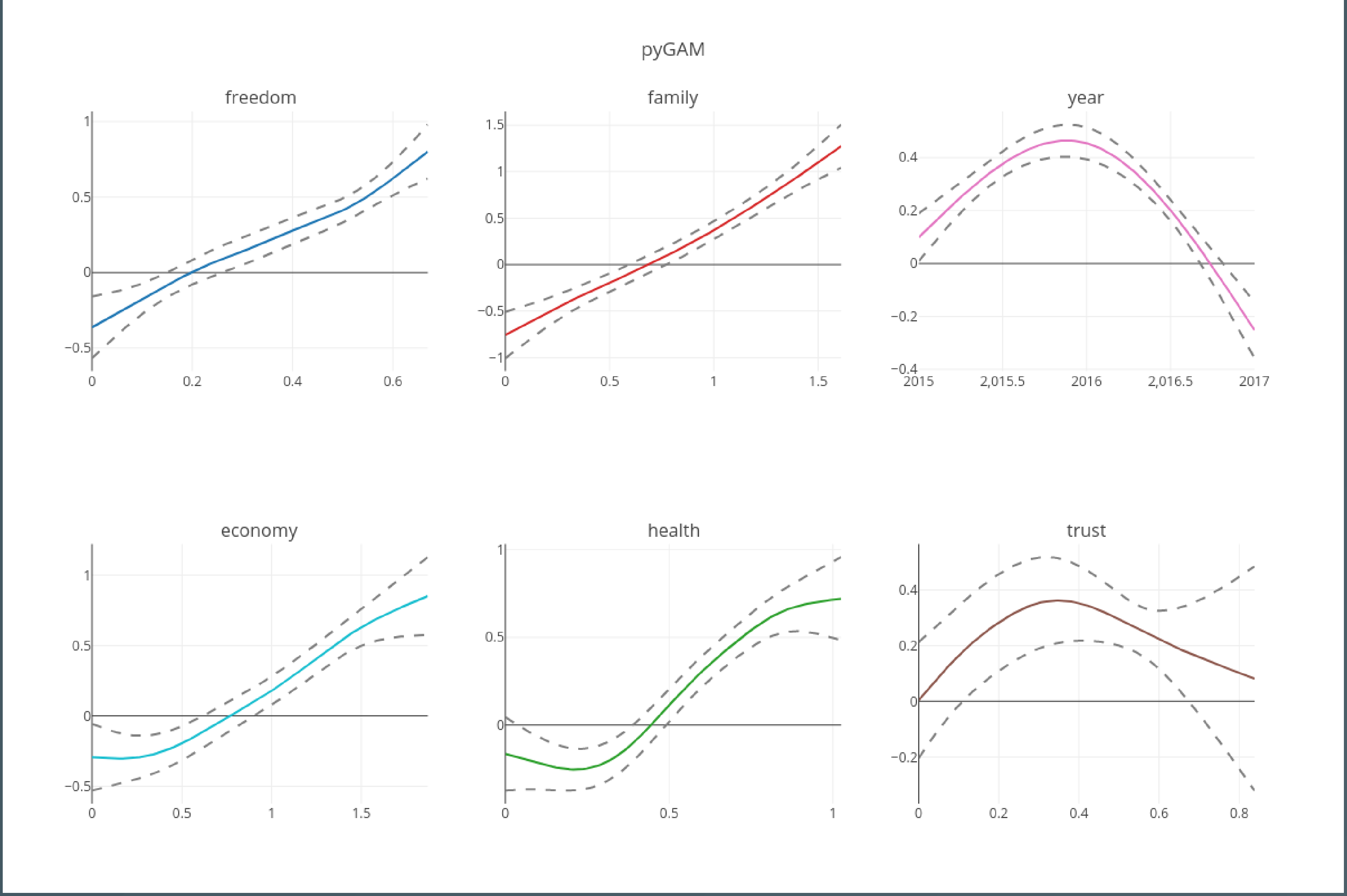
First, we iterate over each variable, creating the traces of the marginal effect and confidence interval

Implementing a GAM in pyGAM

```
for i, title in enumerate(titles):
  ...

  if i<3:
    fig.append_trace(trace, 1, i+1)
    fig.append_trace(ci1, 1, i+1)
    fig.append_trace(ci2, 1, i+1)
  else:
    fig.append_trace(trace, 2, i-2)
    fig.append_trace(ci1, 2, i-2)
    fig.append_trace(ci2, 2, i-2)

py.plot(fig)
```

Then we put the traces in their place on our grid, and plot the figure.

Implementing a GAM in pyGAM



Forecasting with pyGAM Models

```
# Making a Forecast

# predicting the outcome of the UAE in 2015
gam.predict([[0.64, 1.13, 2015, 1.47, 0.81, 0.38]])
```

We need to provide a 2-dimensional array of parameters for generating forecasts (this is why there are double brackets `[`)

For Lab Today

Using either the happiness data (in the GitHub repository), or the weather data from Lab 2, try out models in both `fbprophet` and `pygam`.

- How does each perform?
- Do GAM models provide advantages over VAR models for the data we are focused on?
- What kinds of information might still be missing from our models that could prove helpful?