Lecture 4: Time Series, VAR Models

What is a VAR model?

VAR models are another way that we can model time series data.

- VAR: **V**ector **A**uto**R**egressive model
- Makes use of multiple correlated time series
- Based on SUR (Seemingly Unrelated Regressions) models

Quick Overview of SUR models

Consider j regression equations:

$$Y_j = X_j \beta_j + \epsilon_j$$

where Y_j, and ϵ_j are $N \times 1$, X_j is $N \times K$, and β_j is $K \times 1$

Quick Overview of SUR models

Consider j regression equations:

$$Y_j = X_j\beta_j + \epsilon_j$$

Imagine that the outcomes Y_{ij} are correlated such that

$$Cov(\epsilon_{ij}, \epsilon_{ik}) = \sigma_{ij}$$

and

$$Cov(\epsilon_{ij}, \epsilon_{i'k}) = 0, \;\; \forall \; i \neq i'$$

Quick Overview of SUR models

We can stack our regressions to get a single system of equations:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} X_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & X_1 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & X_1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}$$

Quick Overview of SUR models

Then the FGLS estimator of the system is

$$\hat{\beta}_{FGLS} = \left(X' \left(\hat{\Sigma} \otimes I_N\right) X\right)^{-1} X' \left(\hat{\Sigma} \otimes I_N\right) Y$$

Where $\hat{\Sigma} = [\hat{\sigma}_{ij}]$, and

$$\hat{\sigma}_{ij} = \frac{1}{N} (y_i - X_i\beta_i)' (y_j - X_j\beta_j)$$

Quick Overview of SUR models

So what does all this mean?

- SUR models relax the assumption that each regression is uncorrelated with the others
- Allows us to use one dependent variable in the X matrix for another regression
  - This will in turn allow us to model simultaneous time series, where the errors across the series will certainly be correlated

VAR Models

Just an SUR model where the multiple dependent variables are time series

- We can include lags of dependent variables as part of the X matrix of covariates
- VAR models are built to capture the interactions between variables as time passes

VAR Models

We can write the VAR model

$$\mathbf{y}_t = \mu + \mathbf{\Gamma}_1 \mathbf{y}_{t-1} + \dots + \mathbf{\Gamma}_p \mathbf{y}_{t-p} + \epsilon_t$$

Representing m equations relating lagged dependent variables to the dependent variables in time t.

Implementing a VAR Model

```
# Getting started by importing modules and data

from __future__ import division , print_function
import pandas as pd, numpy as np, patsy as pt
import matplotlib.pyplot as plt
from pandas_datareader.data import DataReader
from datetime import datetime

a = DataReader('JPM',  'yahoo',
        datetime(2006,6,1), datetime(2016,6,1))

# Differencing observations to obtain stationary data

a_diff = pd.DataFrame(np.diff(a.values, axis=0),
        index=a.index.values[1:], # re-applying index
        columns=a.columns) # re-applying column names
```

Implementing a VAR Model

```
from statsmodels.tsa.api import VAR # import the model

model = VAR(a_diff) # define the model and data
model.select_order() # uses information criteria to select
                     # model order
reg = model.fit(5) # order chosen based on BIC criterion
```

- Diagnostics like those from the ARIMA(p,d,q) models are not available to determine our model order
- Use information criteria to find the optimal order of the VAR model
- Need to make our data stationary first

Forecasting with a VAR Model

```
sample = a_diff[:'2016-01-04'].values
fcast = reg.forecast(y = sample, steps = 10)
```

- When using a trained VAR model, we must include enough observations from our dataset in order to provide the expected number of lags to the model
- We have to begin our data k observations prior to our end-point, where k is the order of our model

Forecasting with a VAR Model

```
reg.plot_forecast(20) # will plot our forecast
```

- Recall that our forecast is not what we will observe in the real world
- We have **differenced** our data, and need to undo that differencing
- Apply our differenced forecasts to the most recent actual evaluation

Forecasting with a VAR Model

```
def dediff(end, forecast): # last ob, forecasts as input
    future = forecast
    for i in range(np.shape(forecast)[0]):
        if (i==0):
            future[i] = end + forecast[0]
        else:
            future[i] = future[i-1] + forecast[i]

    return future
```

- Use a function like this one to generate predicted values that can be applied to the original series

Forecasting with a VAR Model

```
nextPer = pd.DataFrame(
                dediff(a['2016-01-04':'2016-01-04'],
                fcast),
                pd.DatetimeIndex(start=datetime(2016,6,2),
                freq='D', periods=10),
                columns=a.columns)
rNext = a['2016-01-05':'2016-01-18']
```

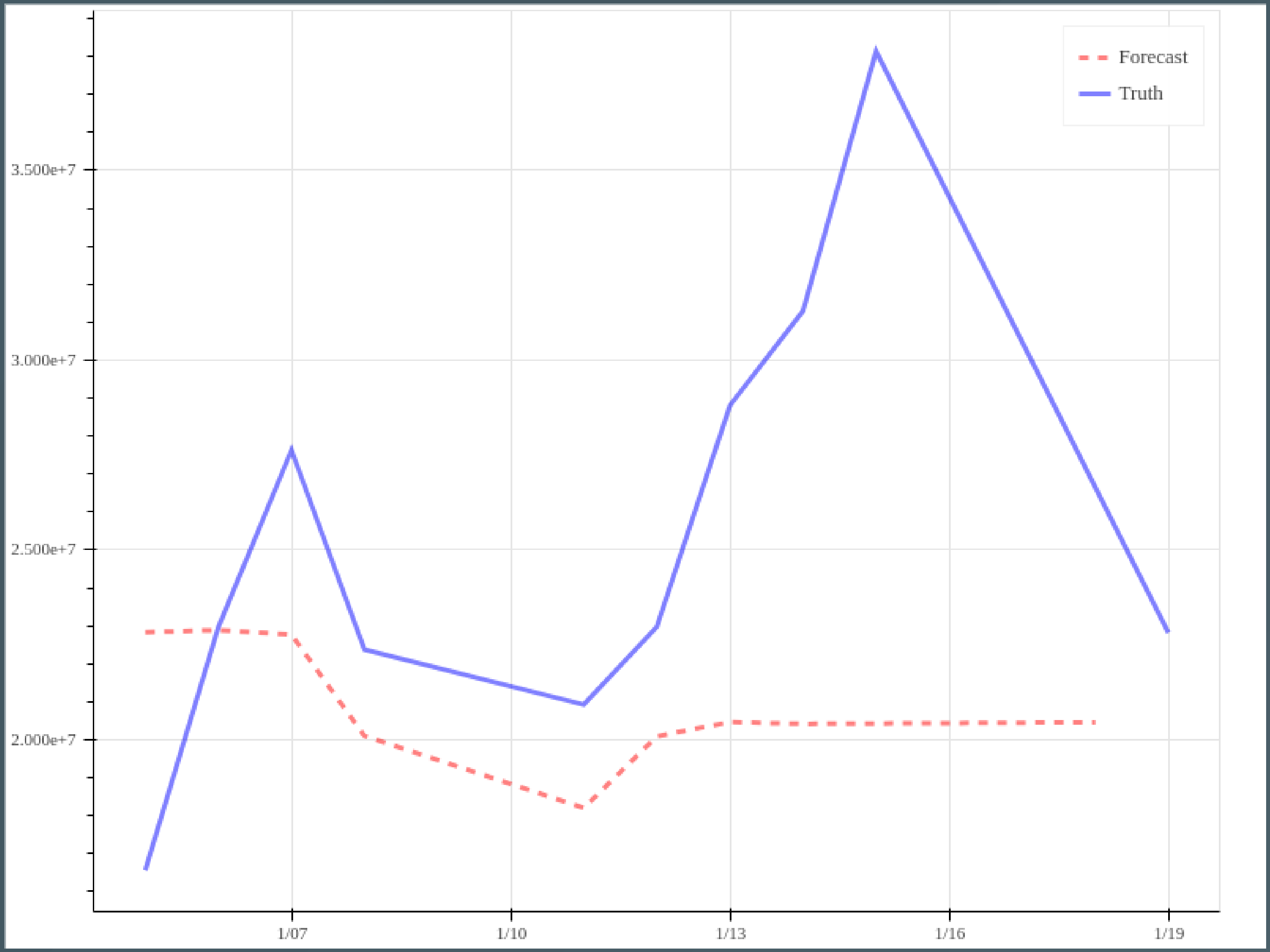
Here, we generate our predictions and isolate the truth for the predicted periods

Forecasting with a VAR Model

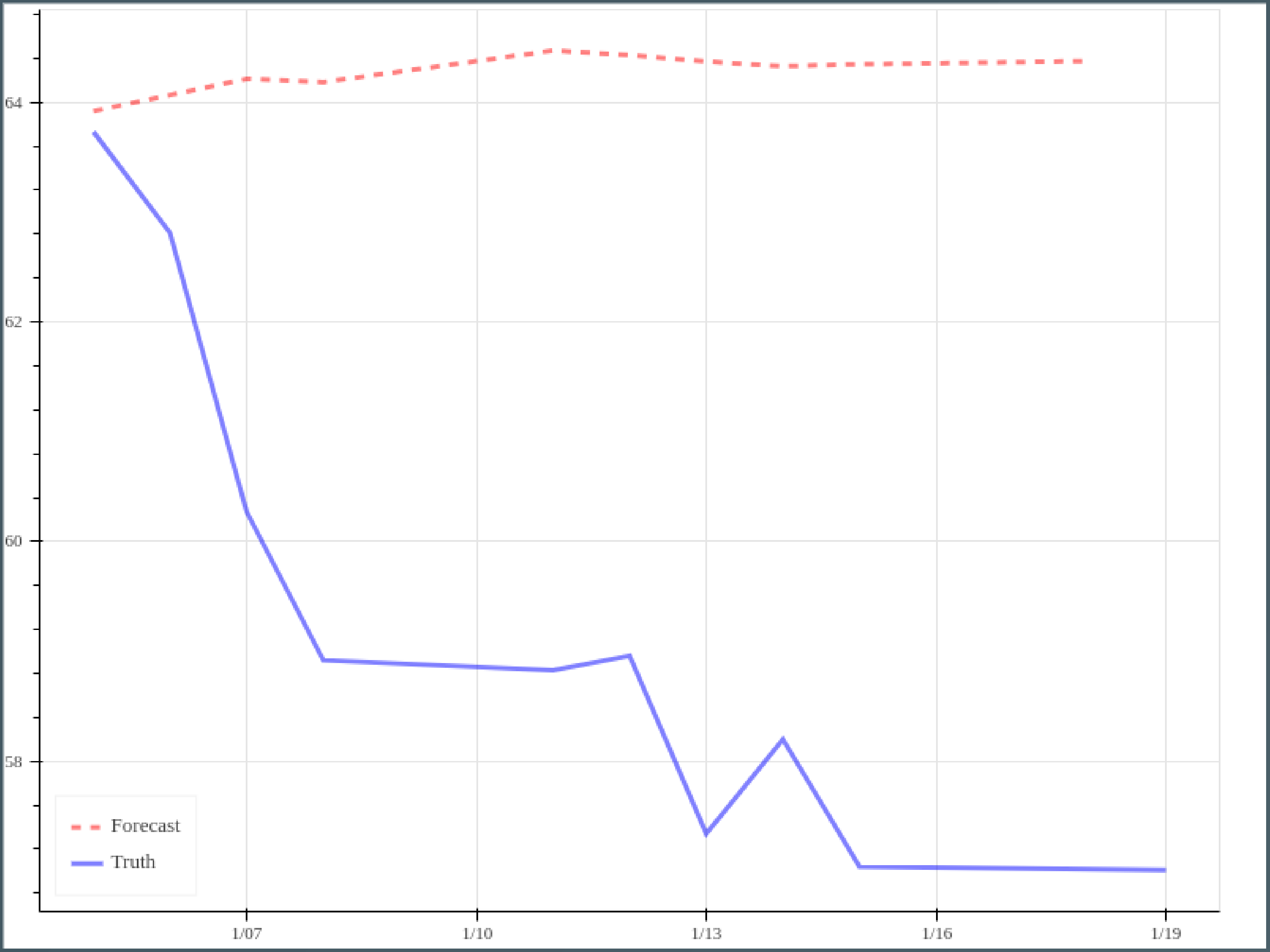
```
#Volume Plot
p = figure(plot_width=800, plot_height=600,
        x_axis_type='datetime')
p.line(nextPer.index.values, nextPer['Volume'],
        color = 'red', line_width=3,
        line_dash='dashed', alpha=0.5,
        legend='Forecast')
p.line(rNext.index.values, rNext['Volume'],
        color = 'blue', line_width=3,
        alpha=0.5, legend='Truth')
show(p)
```

Plotting prediction vs truth in Volume

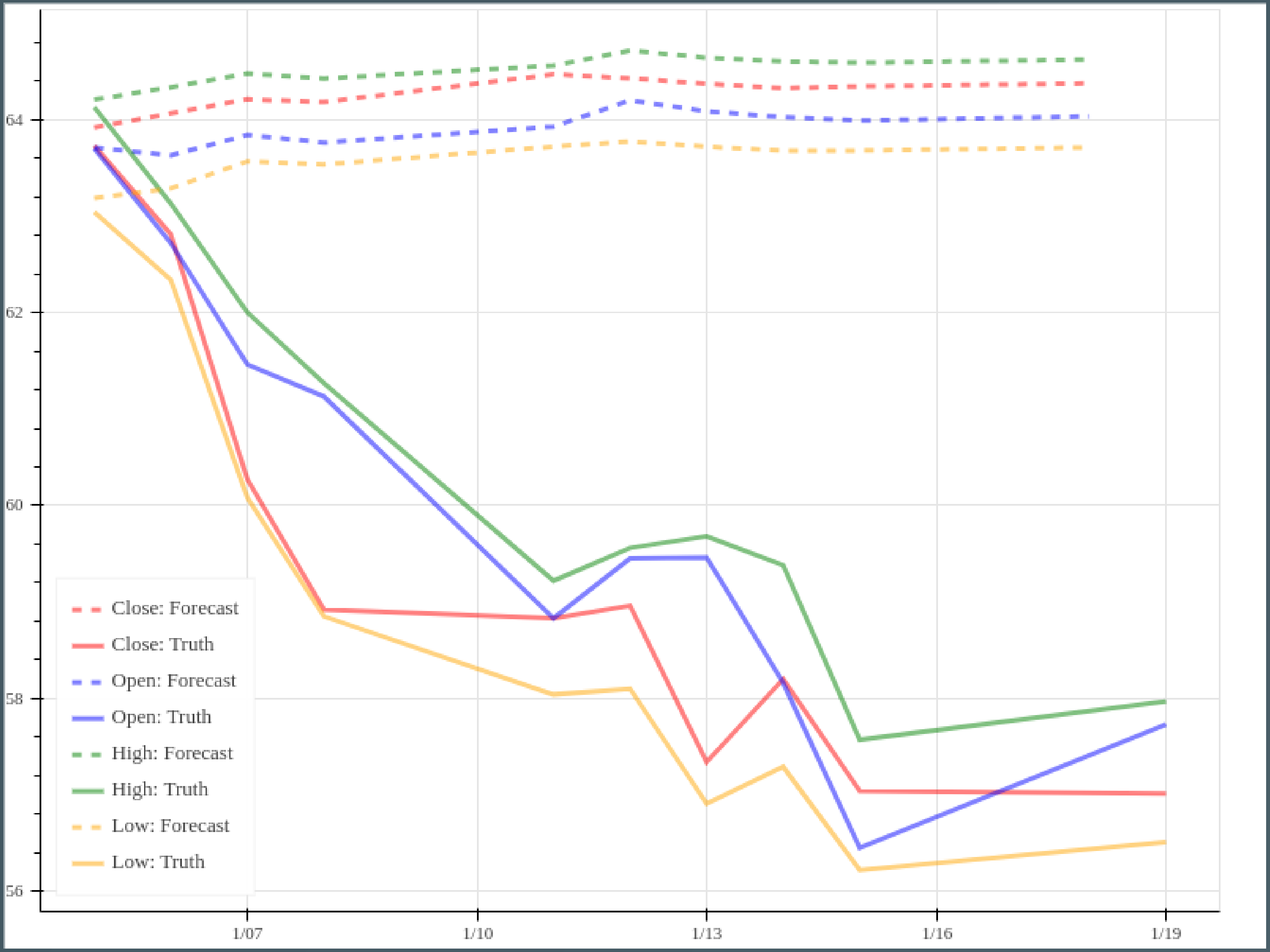
Forecasting with a VAR Model



Forecasting with a VAR Model



Forecasting with a VAR Model



Forecasting Observations

- Repeated Forecasts are needed when data is updated
- Forecasts are not accurate far into the future

Impulse Response Functions

- VAR Models can show us how each variable responds to a shock in our system
- Frequently used to determine impact of policy changes or economic shocks in Macro models
- Give us insight into how our VAR model perceives the relationship between parameters over time

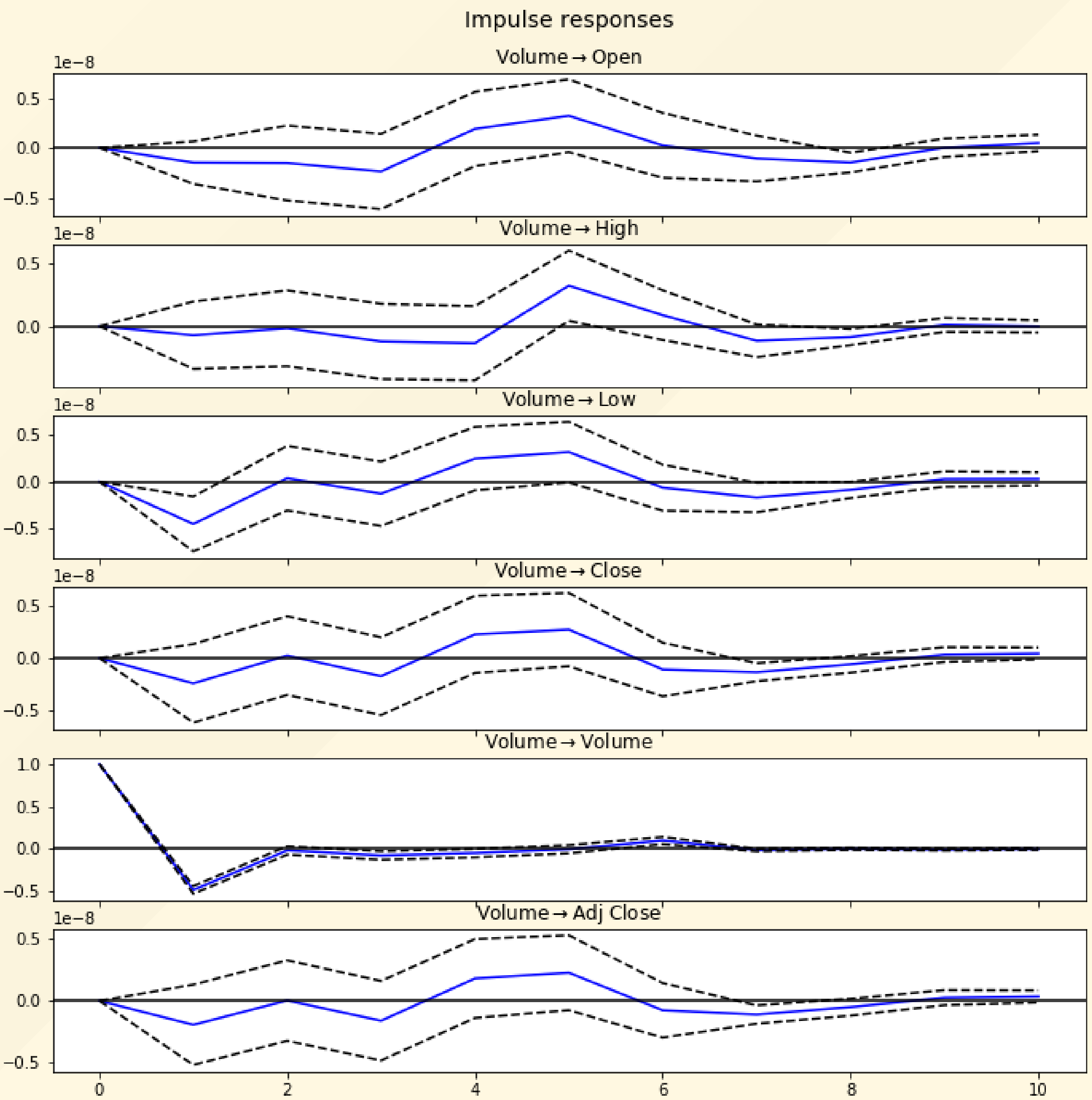
Impulse Response Functions

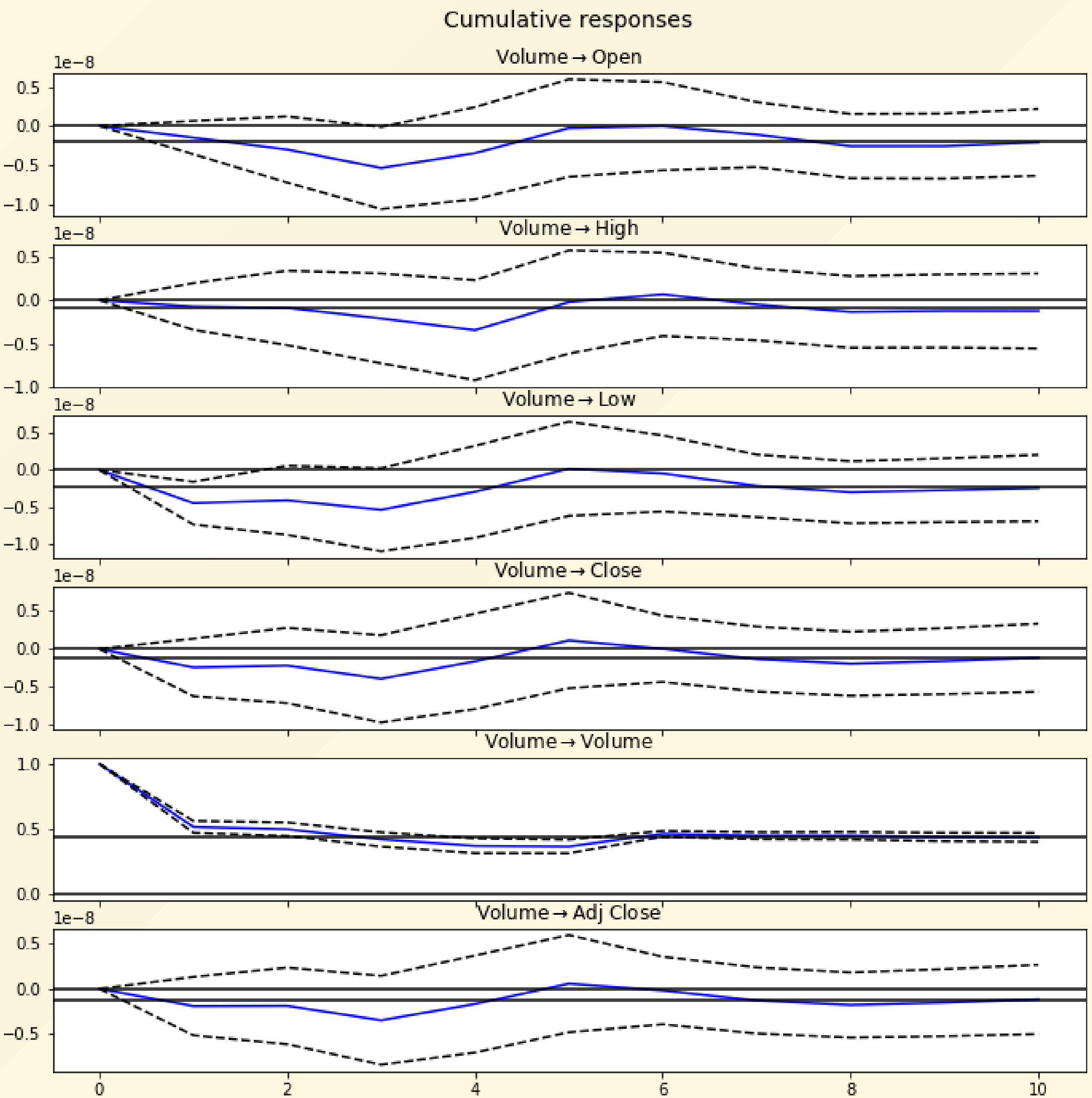
```
irf = reg.irf(10) # 10-period Impulse Response Fn

irf.plot(impulse = 'Volume') # Plot volume change impact

irf.plot_cum_effects(impulse = 'Volume') # Plot cum effect
```

- Generate a 10-period Impulse Response Function (IRF)
- Focus on plotting the effect of changes in trade volume on all variables (over 10 periods)
- Plot the cumulative effect over 10 periods


Impulse responses
Volume → Open
Volume → High
Volume → Low
Volume → Close
Volume → Volume
Volume → Adj Close
1e−8
0.5
0.0
−0.5
1.0
0
2
4
6
8
10


Cumulative responses
Volume → Open
Volume → High
Volume → Low
Volume → Close
Volume → Volume
Volume → Adj Close
1e−8

Saving Models

We can use `pickle` functions to store our models to disk, and utilize them later.

```
import cPickle as pkl

filename = '/your/directory/here' #string of file location
output = open(filename, 'wb') # allow python to write
pkl.dump(reg, output) # stores the reg object @ filename
output.close() # terminate write process
```

In this way, we can store just about any object in Python, although we have to take care with how large some objects may be.

Restoring Models

```
reg = pkl.load(open('yourfile.pkl', 'rb'))
```

When you are ready to access your model or data again, you can load your pickle back into memory.

- Forecast from same model on different days
- Share models with co-workers

For lab today:

Working with your group, use the [Occupancy Detection] data to:

- Fit a VAR model (use stationary data!)
- Forecast 10 periods into the future, and send me your forecast
- Create a plot using the last 20 periods of in-sample data, and your 10-period forecast
- Fit and Compare an ARIMAX model to your VAR model (choose a variable to be your y for the ARIMAX)