# Lasso and Feature Selection Models

## Lots of Variables

In predictive models, we frequently deal with huge numbers of variables

- When our data has lots of variables relative to the number of observations in our data set, we say that it is a **wide** data set
- When we have lots of observations relative to the number of variables, then the data set is said to be **long**

## Lots of Variables

When might we encounter **wide** data?

- New problems
- Problems that are observed with very low frequency
- Problems for which many variables are recorded

Consider the market for high-end homes. Why might we encounter wide data when trying to forecast prices for these homes?

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- Too many variables $\implies$ mathematical problems with regression analysis
- We **violate our rank and order conditions**!

**Rank**: Number of linearly-independent vectors in our matrix (# of columns)
**Order**: Number of unique observations (# of rows)

## Rank and Order Condition

If our rank is $k$ and our order is $n$, then we need to satisfy the following condition

$$n - 1 \geq k$$

In order to complete our regression, we need **at least** one observation per variable, including our dependent variable.

Sometimes, we just don't have that much data.

## Choose Your Variables

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- Use business/application understanding
- Use feature selection models
- Dimensionality reduction

## Business and Application Understanding

There are **so** many reasons it matters, but it is especially important when working with wide data.

- Do I have intuition about which variables are most important?
- Are there variables I know I cannot omit from my model to ensure that it is valid (or ensure that it is accepted by policy-makers)?

## Reducing dimensionality via algorithms

We will discuss two ways of reducing the dimensionality of our data.

1. Dimensionality Reduction - We can also try to distill the information in our model to fewer columns, thereby reducing the number of overall variables.
2. Feature Selection Models - Some models can be tuned using a **regularization** term, in order to coerce them into using fewer terms

## Likelihood Review

Recall our likelihood function for OLS:

$$L(\theta)_{OLS} = ln(\theta|y,x) = -\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}(y - x\beta)'(y - x\beta)$$

Our goal is to find values of $\beta$ and $\sigma^2$ that will maximize our likelihood function, so that our model maximizes the amount of information extracted from our data.

## Lasso Regression

To implement a Lasso regression, we introduce a **regularization** term to our likelihood function:

**Regularization**: the process of introducing additional information in order to solve an ill-posed problem (Wikipedia)

$$L(\theta)_{LASSO} = L(\theta)_{OLS} - \lambda||\theta||_1$$

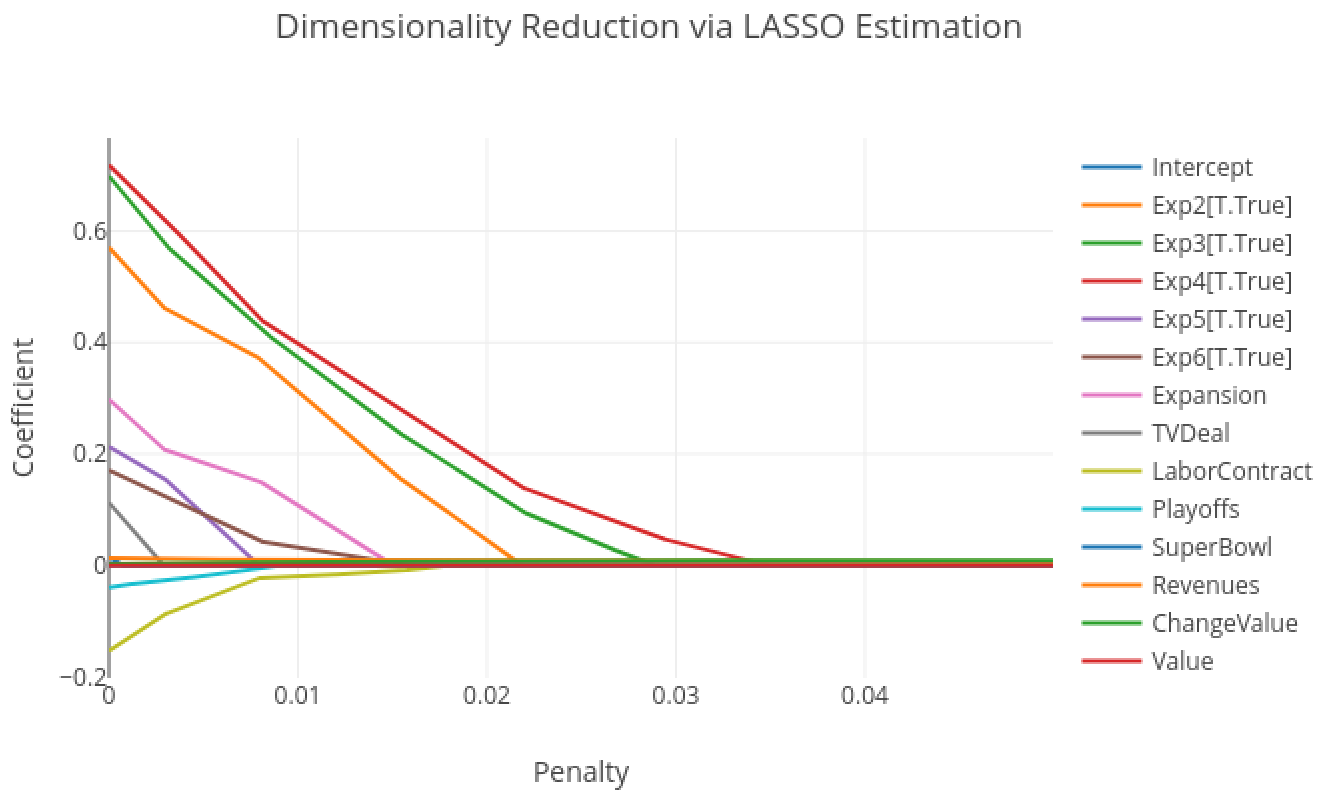
$||\theta||_1$ is the $l_1$ norm, or Manhattan distance function

## Lasso Regression

What good is our regularization term ($||\theta||_1$)?

- Larger coefficients lead to greater penalties on our likelihood function
- Reduces our model's "willingness" to use every variable
- Raising $\lambda$, our penalized likelihood function will drive some (eventually ALL) coefficients to 0!

# Lasso Regression



## Lasso Regression

1. Choose how many parameters we are willing to incorporate in our model
2. Iterate by varying $\lambda$ until our model has the preferred (or needed) number of parameters!

## Lasso Regression

- Protects us to some extent from overfitting
- Allows the data to help us shape our model
- Still interpretable!
- Can be used with least squares AND with logistic regression models

## Implementing Lasso Regression

As we continue into predictive models, it's time to introduce our next library: `sklearn`.

```
import pandas as pd
import numpy as np
from sklearn.linear_model import Lasso
import patsy as pt

data = pd.read_csv("nflValues.csv")

eqn = '''np.log(OperatingIncome) ~ Expansion + Exp2
+ Exp3 + Exp4 + Exp5 + Exp6 + TVDeal + LaborContract
+ Playoffs + SuperBowl + Revenues + ChangeValue + Value'''

y, x = pt.dmatrices(eqn, data=data)
y = np.ravel(y) # Needed to prep for sklearn models
```

## Implementing Lasso Regression

```
model = Lasso(alpha = (2/100))
reg = model.fit(x, y)

results = pd.DataFrame([reg.coef_],
            columns = x.design_info.column_names,
            index = ['Coefficients']
            ).T
```

Because `sklearn` is a *predictive* library, it does not generate results tables for us, like `statsmodels` does. We can always generate a table of results to be printed for ourselves, though.

## Classification & Lasso Regression

But wait! I wanted to use this model to predict binary outcomes!

> Great day! Regularization can also be applied to logistic regressions.

$$L(\theta)_{LOG\_LASSO} = L(\theta)_{Logistic} - \lambda||\theta||_1$$

All we have to do is apply our $l_1$ penalty term to the logistic regression likelihood function!

## Lasso Regression Classifier

```
from sklearn.linear_model import LogisticRegression \
                                 as Logit

model = Logit(penalty = 'l1', C=1/0.05)
# C is an inverse penalty weight, so that
#   smaller C = larger penalty
reg = model.fit(x, y)

results = pd.DataFrame([reg.coef_],
            columns = x.design_info.column_names,
            index = ['Coefficients']
            ).T
```

## Lasso Review

- Can be applied to various regression models
  - Linear
  - Logistic
  - ANY OTHER MAXIMUM LIKELIHOOD MODEL
- Helps us to find the most important explanatory variables using a regularization term ($\lambda \cdot ||\theta||_1$)

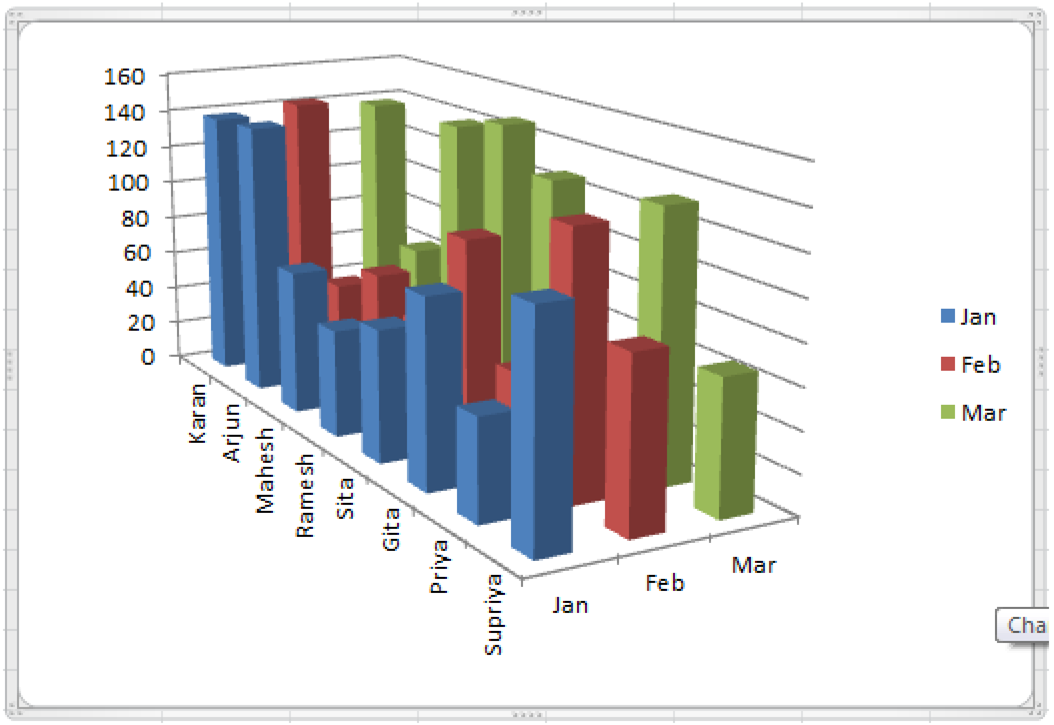
## Dimensionality Reduction

There are many dimensionality reduction tools outside of the regression paradigm. We will focus on **principal component analysis** (also called **PCA**).

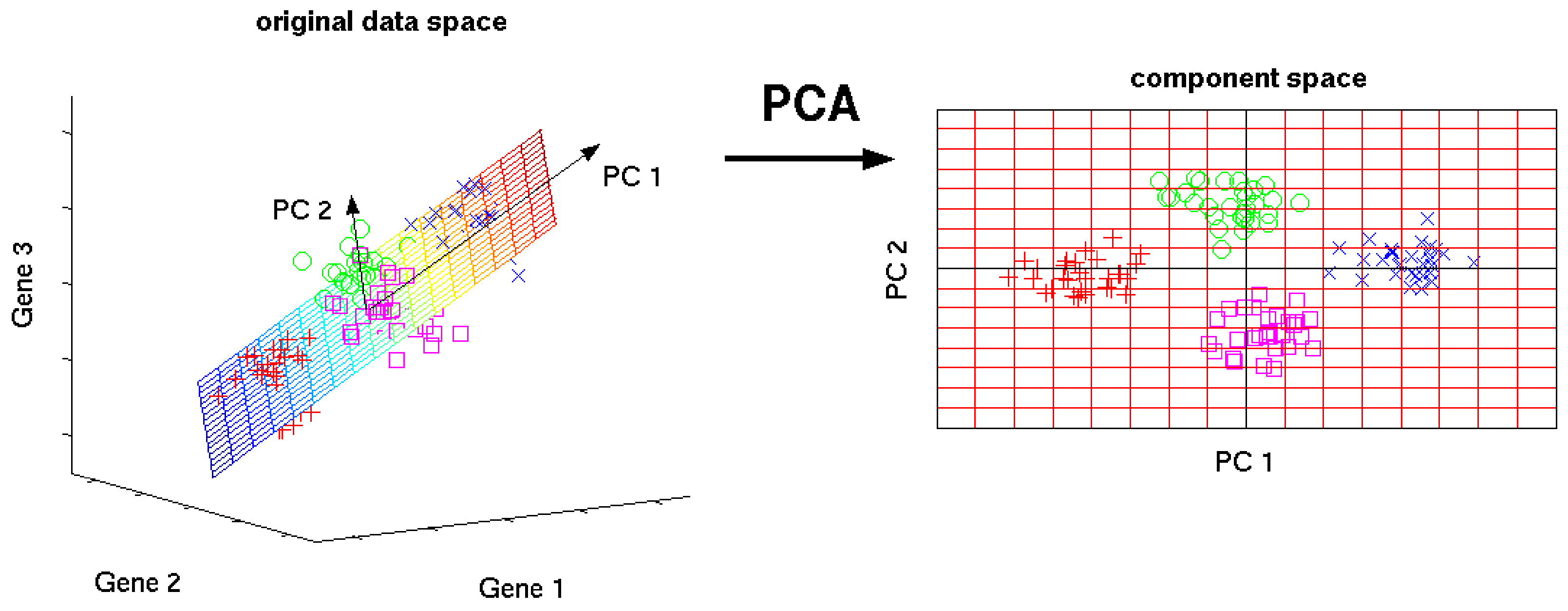
Per `sklearn`, PCA is a "Linear dimensionality reduction using Singular Value Decomposition of the data to project it to a lower dimensional space."

Let's use some figures to get the idea...

## What about this plot is unnecessary?



# Dimensions and Principal Components



## Dimensions and Principal Components

PCA condenses the information contained in our $x$ matrix into fewer columns, arranging that information in a dense structure.

- No "information" is lost$^*$ through PCA
    - But you won't know which column provided information to each new variable, so PCA cannot generally be reversed

$^*$*In theory, no information is lost, but that isn't necessarily true in practice*

## Applying PCA to Data

```
from sklearn.decomposition import PCA

pca = PCA(n_components=3)
pca.fit(x)

newX = pca.transform(x)
```

This process will find a 3-dimensional representation of our data, then transform our data to fit that new 3-dimensional space.

## PCA Summary

Using PCA, we can reduce the number of dimensions in our model, and then fit a predictor using the lower-dimension data.

- Does not lose information like Lasso regression (for the most part)
- Allows us to choose any number of dimensions for our data
- NOT interpretable! Only useful for prediction
- Can be used with any model type

## PCA Summary

I don't use PCA, because I dislike the loss of explanatory power. I find that dealing with models through regularization typically serves my goals better.

## Lab Time!