# Lecture 4: Time Series, VAR Models

# What is a VAR model?

VAR models are another way that we can model time series data.

- VAR: **V**ector **A**uto**R**egressive model
- Makes use of multiple correlated time series
- Based on SUR (Seemingly Unrelated Regressions) models

# Quick Overview of SUR models

Consider $j$ regression equations:

$$Y_j = X_j\beta_j + \epsilon_j$$

where $Y_j$, and $\epsilon_j$ are $N \times 1$, $X_j$ is $N \times K$, and $\beta_j$ is $K \times 1$

# Quick Overview of SUR models

Consider $j$ regression equations:

$$Y_j = X_j\beta_j + \epsilon_j$$

Imagine that the outcomes $Y_{ij}$ are correlated such that

$$Cov(\epsilon_{ij}, \epsilon_{ik}) = \sigma_{ij}$$

and

$$Cov(\epsilon_{ij}, \epsilon_{i'k}) = 0, \ \ \forall \ i \neq i'$$

# Quick Overview of SUR models

We can stack our $J$ regressions to get a single system of equations:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_J \end{bmatrix} = \begin{bmatrix} X & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & X & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & X \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{J \times K} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{J \times K} \end{bmatrix}$$

Where $Y_j$ is a vector of length $N$, and $X$ is an $N \times K$ matrix

# Quick Overview of SUR models

Then the FGLS estimator of the system is

$$\hat{\beta}_{FGLS} = \left(X'\left(\hat{\Sigma} \otimes I_J\right)X\right)^{-1} X'\left(\hat{\Sigma} \otimes I_J\right)Y$$

Where $\hat{\Sigma} = [\hat{\sigma}_{ij}]$, and

$$\hat{\sigma}_{ij} = \frac{1}{N}\left(y_i - X_i\beta_i\right)'\left(y_j - X_j\beta_j\right)$$

# Quick Overview of SUR models

So what does all this mean?

- SUR models relax the assumption that each regression is uncorrelated with the others
- Allows us to use some of our dependent variables in the $X$ matrices for other regressions
    - This will in turn allow us to model simultaneous time series, where the errors across the series will certainly be correlated

# VAR Models

Just a SUR model where the multiple dependent variables are time series

- We can include lags of dependent variables as part of the $X$ matrix of covariates
- VAR models are built to capture the interactions between variables as time passes

# VAR Models

We can write the VAR model

$$\mathbf{y}_{i,t} = \mu_{\mathbf{i}} + \mathbf{\Gamma}_{i,1}\mathbf{y}_{i,t-1} + \sum_{j=1}^{J} \mathbf{\Gamma}_{j,1}\mathbf{y}_{i,t-1} + \epsilon_{i,t}$$

where $i \neq j$

Representing the $i$th equation relating lagged dependent variables to the dependent variable in time $t$.

# Implementing a VAR Model

```
# Getting started by importing modules and data
import pandas as pd, numpy as np
from statsmodels.tsa.api import VAR
import statsmodels.tsa.stattools as st
import plotly.express as px
from datetime import datetime

# Collect data, set index
# Be sure to put the URL back onto a single line!!
data = pd.read_csv("https://github.com/dustywhite7/Econ8310/
raw/master/DataSets/pollutionBeijing.csv")
# Difference and log dep. var.
format = '%Y-%m-%d %H:%M:%S'
data['datetime'] = pd.to_datetime(data['datetime'],
        format=format)
data.set_index(pd.DatetimeIndex(data['datetime']),
        inplace=True)
```

# Implementing a VAR Model

```
# Select variables for VAR model
varData = data[['pm2.5','TEMP','PRES',
                        'Iws']].dropna()[:-50]
```

- **REMEMBER: We need ALL stationary variables**
  - `st.adfuller` is the test to use on each variable
- We also need the terminal (or starting) values of each variable PRIOR to differencing to reconstruct the original time series

# Implementing a VAR Model

```
model = VAR(varData) # define the model and data
model.select_order() # uses information criteria to
                     # select the model order
modelFit = model.fit(30)
                # order chosen based on BIC criterion
```

- Diagnostics like those from the ARIMA(p,d,q) models are not available to determine our model order
- Use information criteria to find the optimal order of the VAR model
  - You could also do this with your ARIMA models if you so choose

# Forecasting with a VAR Model

```
# Forecasting
pred = modelFit.forecast(varData['2013-01-04':].values,
                         steps = 50)
```

- When using a trained VAR model, we must include enough observations from our dataset in order to provide the expected number of lags to the model
- We have to begin our data **at least** $k$ observations prior to our end-point, where $k$ is the order of our model

# Forecasting with a VAR Model

Recall that our raw forecast is not always what we will observe in the real world

- If we have **differenced** our data, we need to undo that differencing
- THEN we apply our transformed forecasts to the most recent actual evaluation

# Forecasting with a VAR Model

```
# Make predictions a DataFrame
nextPer = pd.DataFrame(pred,
                # Use the index from the last 50 observations
                #   as the index for the predictions
                pd.DatetimeIndex(
                varData.iloc[-50:].index),
                columns=varData.columns)

# Isolate the testing data, last 50 periods
test = data.iloc[-50:]
```

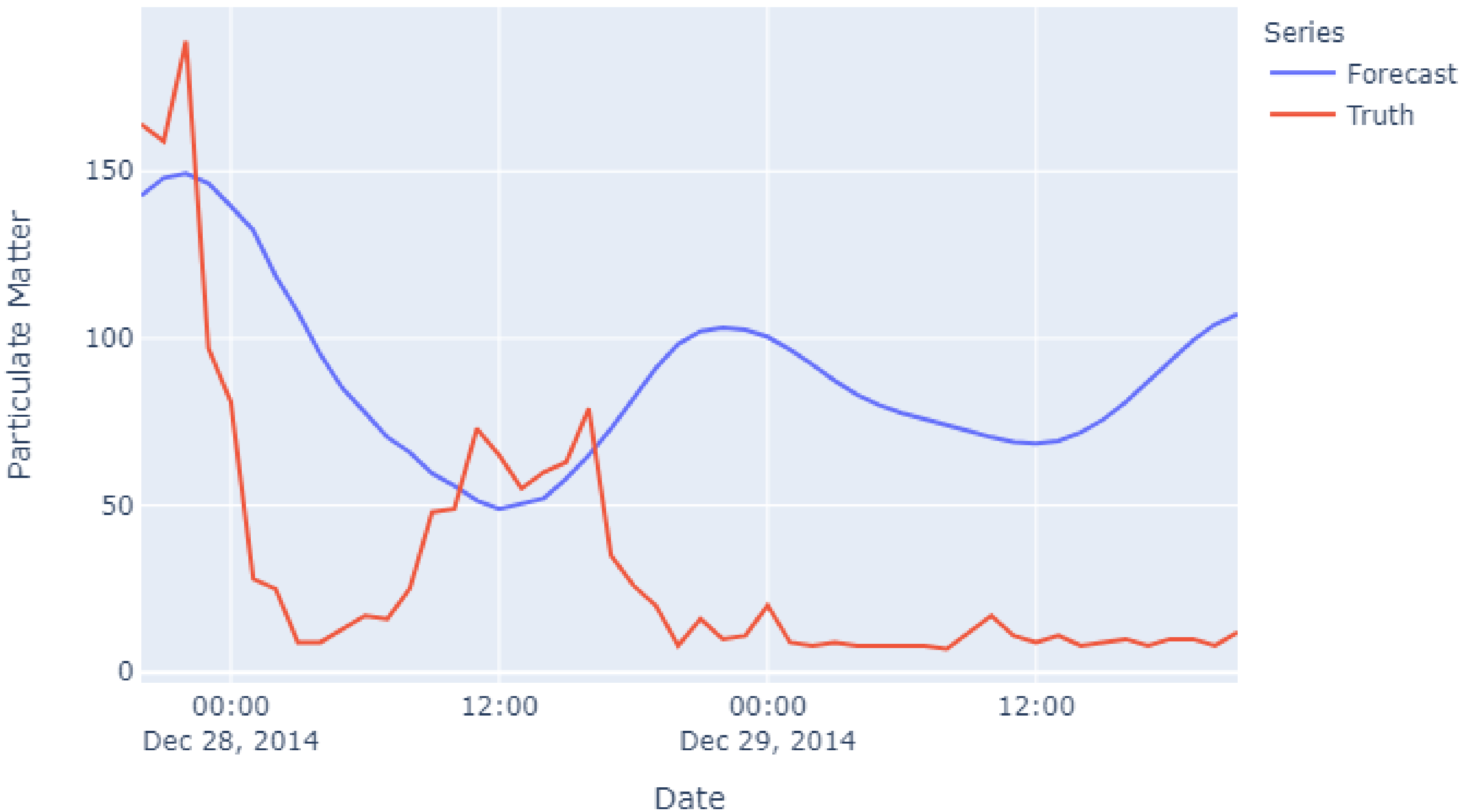
Here, we transform our predictions (and truth) into datetime formatted values, so that we can more easily plot them.

# Forecasting with a VAR Model

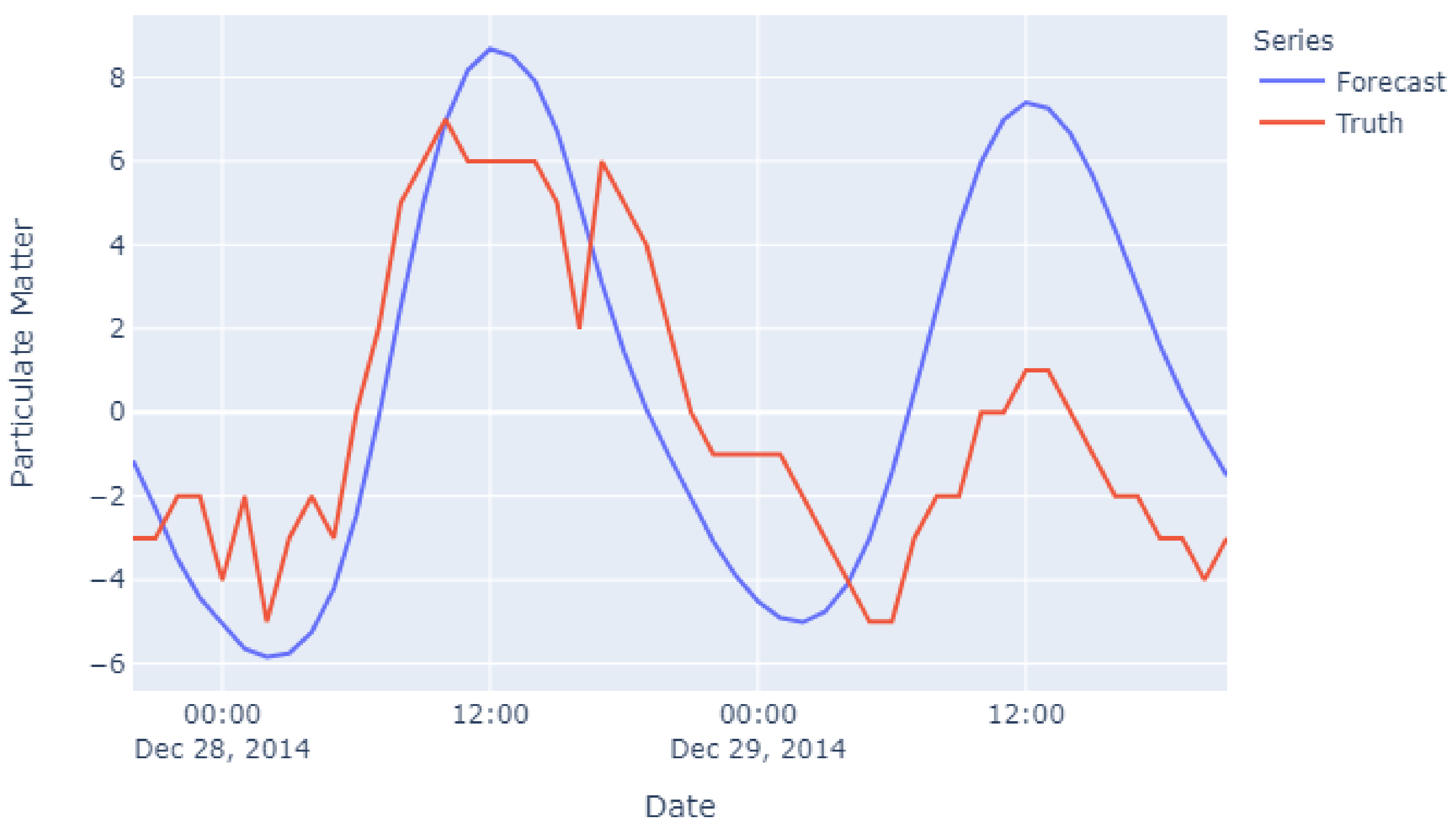
```
# Create and format the figure
fig = px.line(x = nextPer.index,
                y=[nextPer['pm2.5'], test['pm2.5']],
                labels = {
                        'value' : 'Particulate Matter',
                        'x' : 'Date',
                        'variable' : 'Series'
                })

# Renaming the series
fig.data[0].name = "Forecast"
fig.data[1].name = "Truth"

# Render the plot
fig.show()
```

Plotting prediction vs truth

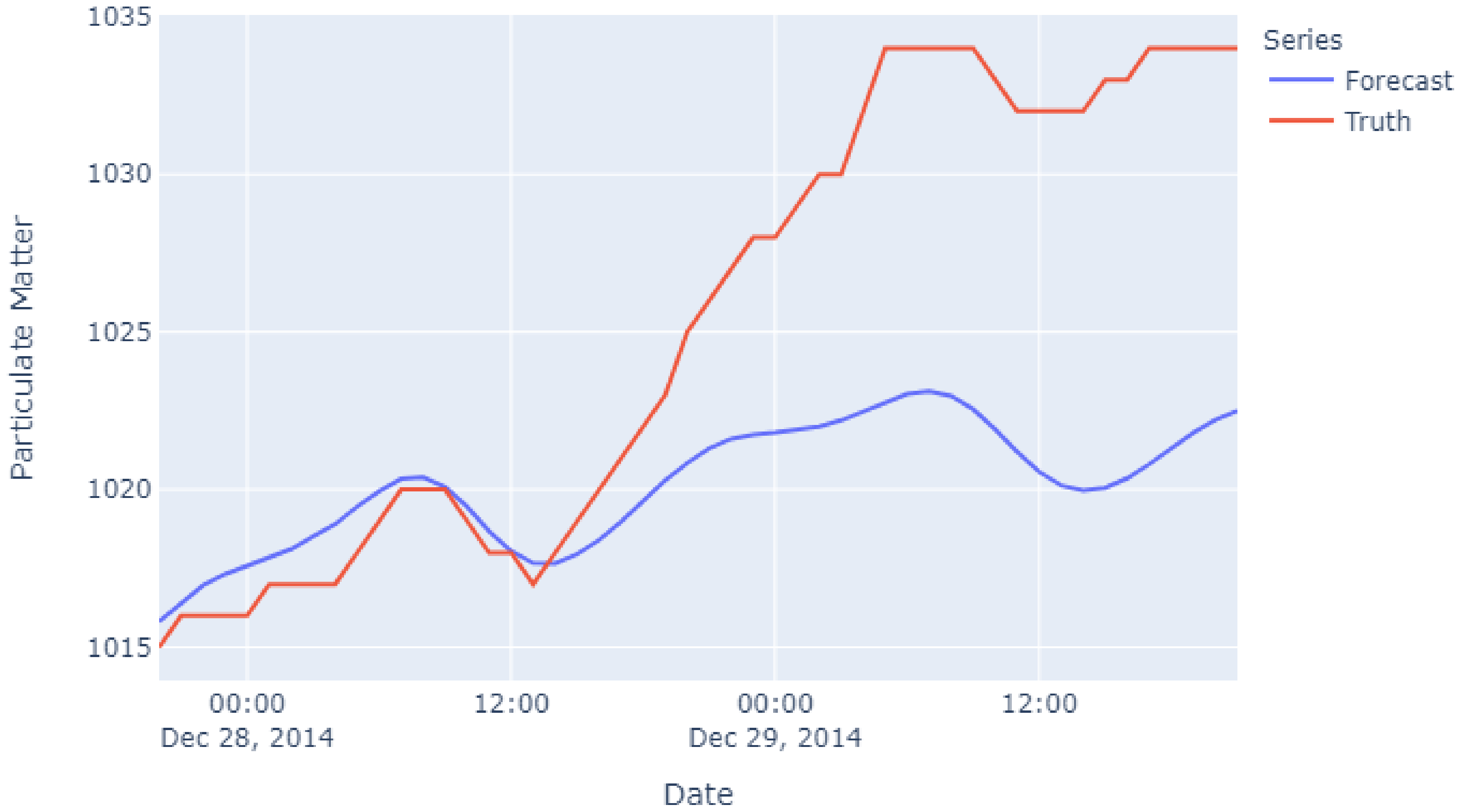
# Particulate Matter



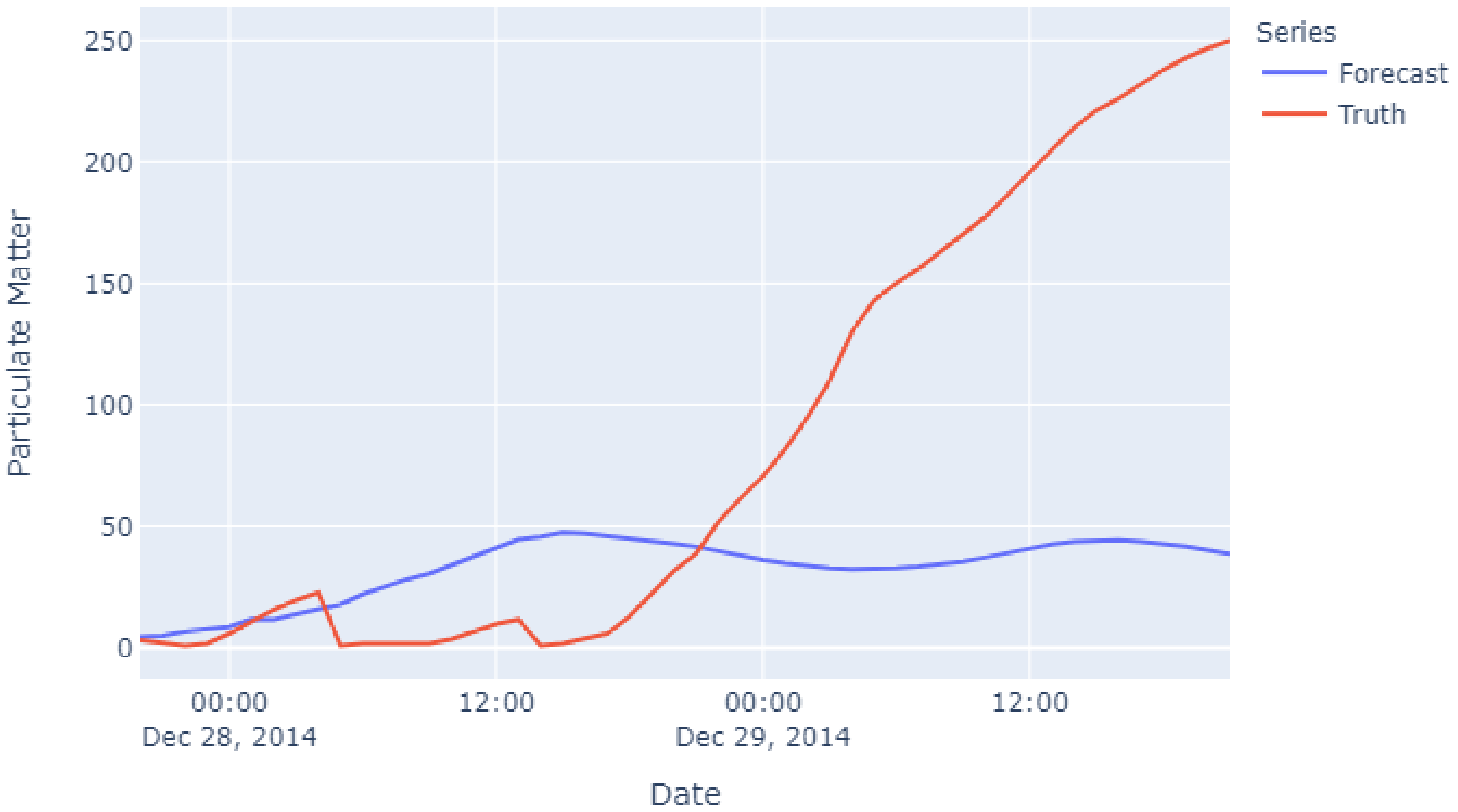
# Temperature



# Air Pressure



# Wind Speed


Series
Forecast
Truth
Particulate Matter
250
200
150
100
50
0
00:00
Dec 28, 2014
12:00
00:00
Dec 29, 2014
12:00
Date

# Forecasting Observations

- Repeated Forecasts are needed when data is updated
- Forecasts are not accurate far into the future

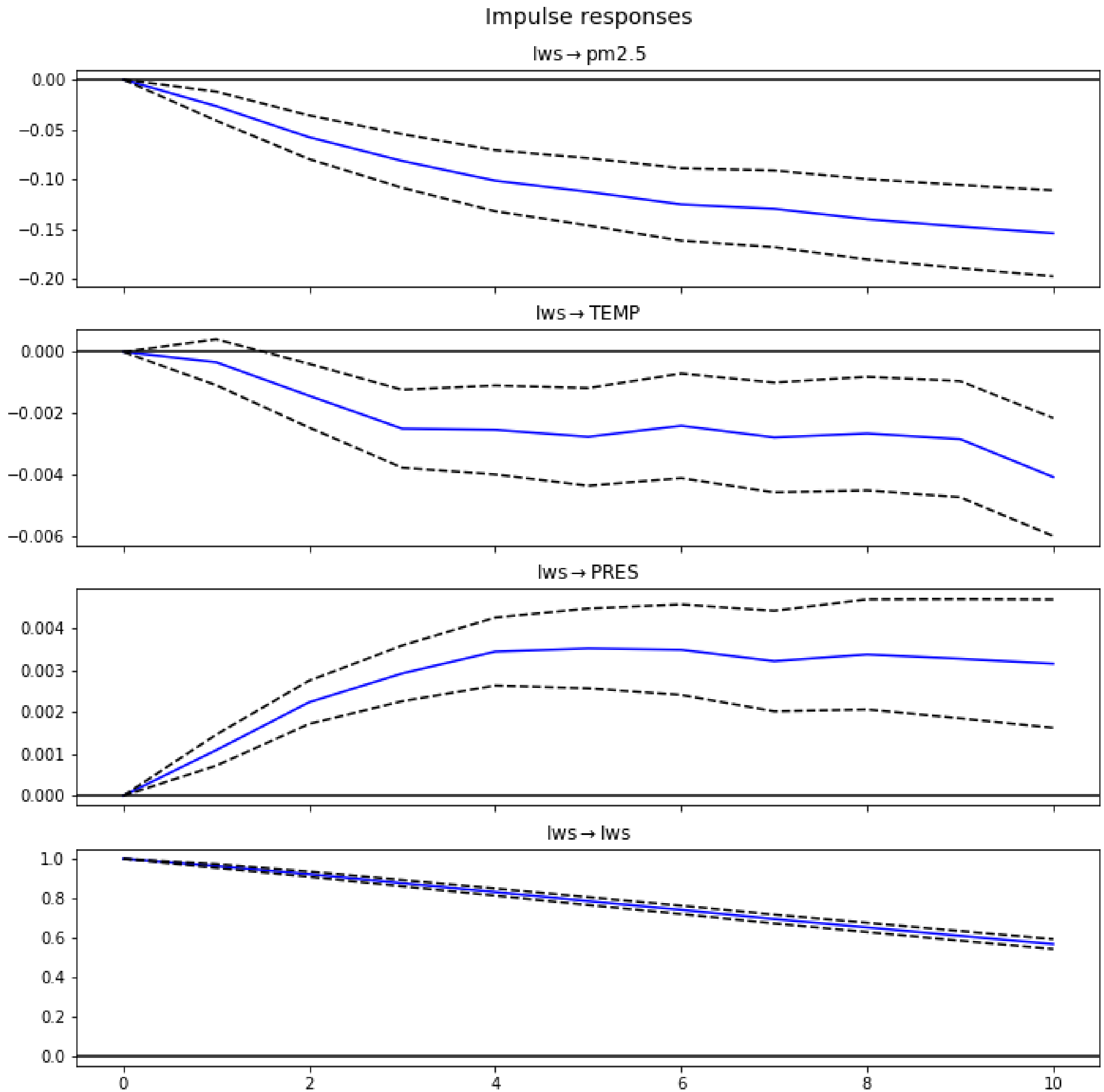
# Impulse Response Functions

- VAR Models can show us how each variable responds to a shock in our system
- Frequently used to determine impact of policy changes or economic shocks in Macro models
- Give us insight into how our VAR model perceives the relationship between parameters over time

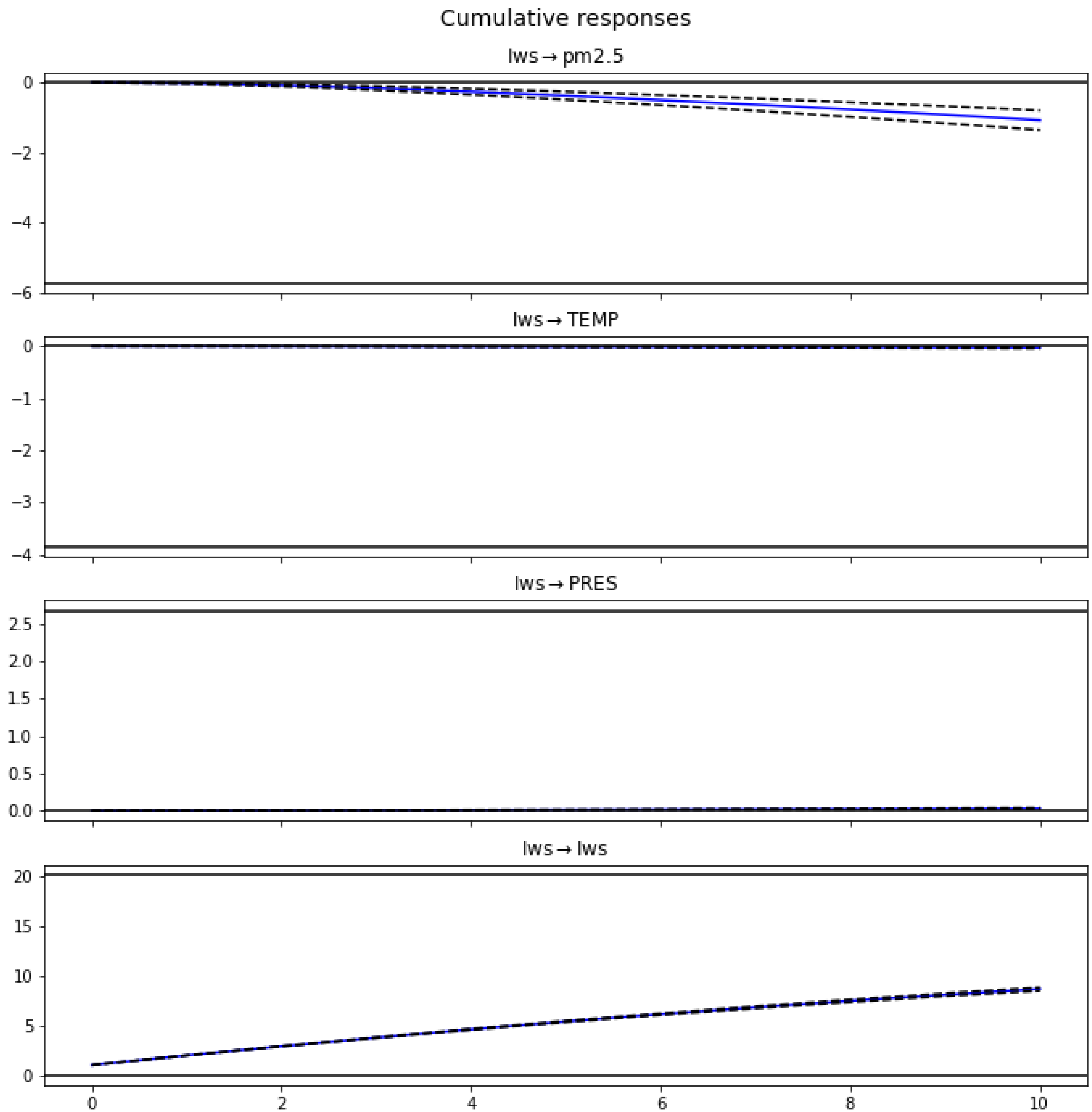
# Impulse Response Functions

```
irf = modelFit.irf(10) # 10-period Impulse Response Fn
irf.plot(impulse = 'Iws') # Plot volume change impact
irf.plot_cum_effects(impulse = 'Iws') # Plot effects
```

- Generate a 10-period Impulse Response Function (IRF)
- Focus on plotting the effect of changes in trade volume on all variables (over 10 periods)
- Plot the cumulative effect over 10 periods



Impulse responses
lws → pm2.5
0.00
−0.05
−0.10
−0.15
−0.20
lws → TEMP
0.000
−0.002
−0.004
−0.006
lws → PRES
0.004
0.003
0.002
0.001
0.000
lws → lws
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


Cumulative responses
lws → pm2.5
0
−2
−4
−6
lws → TEMP
0
−1
−2
−3
−4
lws → PRES
2.5
2.0
1.5
1.0
0.5
0.0
lws → lws
20
15
10
5
0
0
2
4
6
8
10

# Lab Time!