Tarea 2 Calculo Integral

1: $\int \sin^5(3x)\cos(3x)\,dx$

$u = \sin(3x)$

$du = 3\cos(3x)\,dx$

$\frac{du}{3} = \cos(3x)\,dx$

$\frac{1}{3}\int u^5\,du = \frac{1}{3}\left(\frac{u^6}{6} + C\right)$

$= \frac{1}{18}u^6 + C = \boxed{\frac{1}{18}\sin(3x) + C}$

2: $\int \frac{\csc\sqrt{x}\,\cot\sqrt{x}}{\sqrt{x}}\,dx$

$u = \operatorname{sen}\sqrt{x}$

$du = \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx$

$2du = \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

$\int \frac{\left(\frac{1}{\operatorname{sen}x}\right)\left(\frac{\cos x}{\operatorname{sen}x}\right)}{\sqrt{x}}\,dx = \int \frac{\frac{\cos\sqrt{x}}{\operatorname{sen}^2\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{\cos\sqrt{x}}{\sqrt{x}\operatorname{sen}^2\sqrt{x}}\,dx$

$= 2\int \frac{du}{u^2} = 2\int u^{-2}\,du = -\frac{2}{u} + C = \boxed{-\frac{2}{\operatorname{sen}\sqrt{x}} + C}$

3: $\int \frac{e^x}{\sqrt{1-e^x}}\,dx$

$u = 1 - e^x$

$du = -e^x\,dx$

$-du = e^x\,dx$

$-\frac{du}{e^x} = dx$

$\int\left(\frac{e^x}{\sqrt{u}}\right)\left(-\frac{du}{e^x}\right) = -\int \frac{du}{\sqrt{u}}$

$= -2\sqrt{u} + C = \boxed{-2\sqrt{1-e^x} + C}$

4: $\int \sqrt[5]{(1-3x)^3}\,dx$

$u = 1 - 3x$

$du = 3\,dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\int \sqrt[5]{u^3}\,du = \frac{1}{3}\int u^{3/5}\,du$

$= \frac{5u^{3/5}}{24} + C = \boxed{\frac{5}{24}(1-3x)^{3/5} + C}$

5: $\int 5x\sqrt[2]{2x^2+3}\,dx = 5\int x\sqrt{2x^2+3}\,dx$

$u = 2x^2 + 3$

$du = 4x\,dx$

$\frac{du}{4} = x\,dx$

$= \frac{5}{4}\int \sqrt{u}\,du = \frac{5}{4}\left(\frac{2u^{3/2}}{3} + C\right) = \frac{5}{6}u^{3/2} + C$

$= \boxed{\frac{5}{6}(2x^3+3)^{3/2} + C}$

6: $\int \frac{t}{\sqrt[5]{(1+3t^2)}}\,dt = \frac{1}{6}\int \frac{du}{\sqrt[5]{u}} = \frac{1}{6}\int u^{-1/5}\,du = \frac{1}{6}\left(\frac{5u^{4/5}}{4}\right) = \frac{5}{24}u^{4/5} + C$

$= \boxed{\frac{5}{24}\sqrt[5]{(1+3t^2)^4} + C}$

$u = 1+3t^2$

$du = 6t\,dt$

$\frac{du}{6} = t\,dt$

7: $\int (4x^2-16x+7)^5\,dx = \cancel{\int (4x^2-16x+7)^2(4x^2-16x+7)^2(4x^2-16x+7)\,dx}$

$(4x^2-16x+7)^2 = 16x^4 - 64x^3 + 28x^2 - 64x^3 + 256x^2 - 112x + 28x^2 - 112x + 49$

$= 16x^4 - 128x^3 + 312x^2 - 224x + 49$

$\int (2x-7)^5(2x-7)^5\,dx = \frac{1}{2}\int u^5(u-6)^5\,du = \frac{1}{2}\left(\frac{(u-6)^5u^6}{6} - \frac{5}{6}\int u^6(u-6)^4\,du\right.$

$u = (u-6)^5 \quad dv = u^5\,du$

$du = 5(u-6)^4\,du \quad v = \int u^5\,du$

$v = \frac{u^6}{6}$

$u_1 = (u-6)^4 \quad dv_1 = u^6$

$du_1 = 4(u-6)^3\,du \quad v = \frac{u^7}{7}$

$u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \frac{1}{2}\left(\frac{(u-6)^5u^6}{6} - \frac{5}{6}\left(\frac{(u-6)^4u^7}{7} - \frac{4}{7}\int u^7(u-6)^3\,du\right.\right.$

$u_2 = (u-6)^3 \quad dv_2 = u^7$

$du_2 = 3(u-6)^2\,du \quad v_2 = \frac{u^8}{8}$

$= \frac{1}{2}\left(\frac{(u-6)^5u^6}{6} - \frac{5}{6}\left(\frac{(u-6)^4u^7}{7} - \frac{4}{7}\left(\frac{u^8(u-6)^3}{8} - \frac{3}{8}\int u^8(u-6)^2\,du\right.\right.\right.$

$u_3 = (u-6)^2$

$du_3 = 2(u-6)$

$dv_3 = u^8\,du$

$v_3 = \frac{u^9}{9}$

$= \frac{1}{2}\left(\frac{(u-6)^5u^6}{6} - \frac{5}{6}\left(\frac{(u-6)^4u^7}{7} - \frac{4}{7}\left(\frac{u^8(u-6)^3}{8} - \frac{3}{8}\left(\frac{u^9(u-6)^2}{9} - \frac{2}{9}\int u^9(u-6)\,du\right.\right.\right.\right.$

8: $\int \frac{t^2+1}{\sqrt[3]{t^3+3t-16}}\,dt \quad \frac{1}{3}\int \frac{du}{\sqrt[5]{u}} = \frac{1}{3}\int u^{-1/5}\,du = \frac{1}{3}\left(\frac{5u^{4/5}}{4}\right) = \frac{5}{12}u^{4/5} + C$

$= \boxed{\frac{5}{12}(t^3+3t-16)^{4/5} + C}$

$u = t^3+3t-16$

$du = (3t^2+3)\,dt$

$\frac{du}{3} = (t^2+1)\,dt$

9: $\int \frac{1}{\sec(5x+7)}\,dx = \int \cos(5x+7)\,dx = \frac{1}{5}\int \cos(u)\,du = \frac{1}{5}(\operatorname{sen}(u) + C) = \frac{1}{5}\operatorname{sen}(u) + C$

$u = 5x+7$

$du = 5\,dx$

$\frac{du}{5} = dx$

$= \boxed{\frac{1}{5}\operatorname{sen}(5x+7) + C}$

10: $\int \frac{2\cos x}{\sin^2 x}dx = 2\int \frac{du}{u^2} = 2\int u^{-2}du = 2\left(-\frac{1}{u}\right) = -\frac{2}{u}+C$

$\boxed{= -\frac{2}{\sin x}+C}$

$u = \sin x$

$du = \cos x\, dx$

11: $\int (z+1)\csc^2(z^2+2z)dz = \frac{1}{2}\int \csc^2(u)\,du = \frac{1}{2}(-\cot u + C)$

$\boxed{= -\frac{1}{2}\cot(z^2+2z)+C}$

$u = z^2+2z$

$du = (2z+2)du$

$\frac{du}{2} = (z+1)du$

12: $\int (z+1)e^{z^2+2z}dz = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{z^2+2z}+C}$

$u = z^2+2z$

$du = (2z+2)dz$

$\frac{du}{2} = (z+1)dz$

13: $\int \sec^2(x)\, e^{\tan(x)}dx = \int e^u du = e^u + C = \boxed{e^{\tan x}+C}$

$u = \tan x$

$du = \sec^2 x\, dx$

14: $\int \frac{\cos(\ln x)}{x}dx = \int \cos u\, du = \operatorname{sen} u + C = \boxed{\operatorname{sen}(\ln(x)) + C}$

$u = \ln(x)$

$du = \frac{1}{x}dx$

15: $\int \sin^5(3x)\cos(3x)\,dx = \frac{1}{3}\int u^5 du = \frac{1}{3}\left(\frac{u^6}{6}+C\right) = \boxed{\frac{1}{18}\sin^6(3x)+C}$

$u = \sin(3x)$

$du = 3\cos(3x)dx$

$\frac{du}{3} = \cos(3x)\,dx$

16: $\int \frac{(\ln(x+1))^2}{x+1}dx = \int u^2 du = \frac{u^3}{3}+C = \boxed{\frac{1}{3}(\ln(x+1))^3+C}$

$u = \ln(x+1)$

$du = \frac{1}{x+1}dx$

$17: \int \sin(x)\cos(x)\, e^{\cos^2 x}\, dx = -\frac{1}{2}\int e^u du = -\frac{1}{2} e^u + C$

$u = \cos^2 x$ $\qquad \boxed{= -\frac{1}{2} e^{\cos^2 x} + C}$

$du = (-2\cos x \operatorname{sen} x) dx$

$-\frac{du}{2} = \cos x \operatorname{sen} x\, dx$

$18: \int \frac{x}{1+x^2} dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + C = \boxed{\frac{1}{2}\ln(1+x^2) + C}$

$u = 1+x^2$

$du = 2x$

$\frac{du}{2} = x dx$

$19: \int \frac{x^2}{1+x^2} dx = \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} dx = \int dx - \int \frac{1}{x^2+1} dx = \boxed{x - \tan^{-1}(x) + C}$

$20: \int \frac{\cos(x)}{\sin(x)\sqrt{\sin^2(x)-1}} dx = \int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec}(u) + C$

$u = \sin(x)$ $\qquad = \boxed{\operatorname{arcsec}(\sin(x)) + C}$

$du = \cos(x) dx$

$21: \int \frac{\sec^2(x)}{\sqrt{1-\tan^2(x)}} = \int \frac{1}{\sqrt{1-u^2}} du = \operatorname{arc\,sen}(u) + C$

$\qquad = \boxed{\operatorname{arc\,sen}(\tan(x)) + C}$

$u = \tan(x)$

$du = \sec^2(x)$

$22: \int \frac{\tan(\ln x)}{x} dx = \int \tan(u) du = \ln|\sec(u)| + C$

$\qquad = \boxed{\ln|\sec(\ln x)| + C}$

$u = \ln x$

$du = \frac{1}{x} dx$

$23: \int e^x \sec(e^x+3) dx = \int \sec(u) du = \ln(\tan u + \sec u) + C$

$u = e^x + 3$ $\qquad = \boxed{\ln(\tan(e^x+3) + \sec(e^x+3)) + C}$

$du = e^x dx$

24: $\int \sin(3x-2)\cos(2x+3)\,dx = \frac{1}{2}\int \sin\left(\frac{3}{2}u - \frac{13}{2}\right)\cos(u)\,du$

$u = 2x+3$

$\frac{3}{2}u - \frac{13}{2} = 3x + \frac{9}{2} - \frac{13}{2} = 3x-2$

$du = 2dx$

$\frac{du}{2} = dx$

25: $\int \cos(2x)\sin(2x)\,dx = \frac{1}{2}\int u\,du = \frac{1}{2}\left(\frac{u^2}{2} + C\right) = \frac{1}{4}u^2 + C$

$= \boxed{\frac{1}{4}\sin^2(2x) + C}$

$u = \text{sen}\, 2x$

$du = 2\cos(2x)$

$\frac{du}{2} = \cos(2x)$

26: $\int \cos^2(2x)\sin(4x)\,dx$

27: $\int \frac{1}{(x+2)\sqrt{x^2+4x+3}}\,dx = \int \frac{1}{u\sqrt{u^2-1}}\,du = \text{arcsec}(u) + C$

$u = x+2$

$u^2 = x^2+4x+4$

$du = dx$

Ejercicios:

$V(0) = -4.9(0)^2 + 60(0) + C = 6 \therefore V(x) = -4.9x^2 + 60x + 6$

$C = 6$

1.

$V(0) = 6 \quad V(x) = -9.81\int x\,dx + 60\int dx = -4.9x^2 + 60x + C$

$V'(0) = 60 \quad V'(0) = -9.81(0) + C = 60 \quad \therefore V'(x) = -9.81x + 60$

$C = 60$

$V''(x) = -9.81 \quad \therefore V'(x) = -9.81\int dx = -9.81x + C$

$V'(x) = 0$

$-9.81x + 60 = 0$

$-9.81x = -60$

$x = 6.1162$

$V(6.1162) = -4.9(6.1162)^2 + 60(6.1162) + 6$

$= 219.85$ metros

2.

$s(t) = -4.9t^2 + V_0 t + S_0$

$s'(0) = V_0 \quad s(0) = S_0$

$\boxed{s''(t) = -9.8}$

$s'(t) = \int -9.8\,dt$

$s'(t) = -9.8t + C_1$

$s'(0) = -9.8(0) + C = V_0$

$C = V_0$

$\boxed{s'(t) = -9.8t + V_0}$

$s(t) = \int (-9.8t + V_0)\,dt$

$s(t) = -9.8\frac{t^2}{2} + V_0 t + C = -4.9t^2 + V_0 t + C$

$s(0) = -4.9(0)^2 + V_0(0) + C = S_0$

$C = S_0$

$\therefore \boxed{s(t) = -4.9t + V_0 t + S_0}$

Si esta relacionada

3:

$s(0) = 2\ m$

$s'(0) = 10\ m/s$

$s''(x) = -9.81\ m/s^2$

$s'(x) = -9.81 \int dx = -9.81x + c$

$s'(0) = -9.81(0) + c = 10 \quad \therefore s'(x) = -9.81x + 10$

$c = 10$

$s(x) = -9.81 \int x\,dx + 10\int dx = -4.9x^2 + 10x + c$

$s(0) = -4.9(0)^2 + 10(0) + C = 2 \quad \therefore s(x) = -4.9x^2 + 10x + 2$

$C = 2$

$s'(x) = 0$

$-9.81x + 10 = 0$

$-9.81x = -10$

$x = 1$

$\boxed{s(1) = -4.9(1)^2 + 10(1) + 2 = 7.1\ m}$

4:

$s(20) = 0$

$s''(x) = -1.6$

$s'(0) = 0$

$s'(x) = -1.6 \int dx = -1.6x + c$

$s'(0) = -1.6(0) + C = 0 \quad \therefore s'(x) = -1.6x$

$C = 0$

$s(x) = -1.6 \int x\,dx = -0.8x^2 + C$

$s(20) = -0.8(20)^2 + c = 0 \quad \therefore s(x) = -0.8x^2 + 320$

$-320 + c = 0$

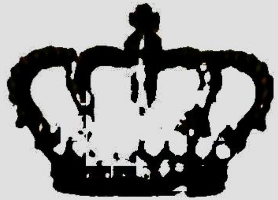
$c = 320$

$s'(20) = -1.6(20) = \boxed{-32\ m/s}$

$s(0) = -0.8x^2 + 320$

$= \boxed{320\ m}$

$\boxed{\text{cayo desde } 320\ m \text{ y impacto a } -32\ m/s}$



5:

$s(0) = 0$

$s'(0) = 25\ \text{km/h} = 6.9\bar{4}\ \text{m/s}$

$s'(13) = 80\ \text{km/h} = 22.\bar{2}\ \text{m/s}$

$$m = \frac{22.2 - 6.94}{13 - 0} = 1.1752$$

$y - y_1 = m(x - x_1)$

$y - 6.944 = 1.1752(x - 0)$

$f'(x) = 1.1752x + 6.944$

$$\boxed{f''(x) = 1.1752\ \text{m/s}^2}$$

$$f(x) = 1.1752 \int x\,dx + 6.944 \int dx = 0.5876x^2 + 6.944x + C$$

$f(0) = 0 \longrightarrow f(0) = 0.5876(0)^2 + 6.944(0) + C = 0$

$C = 0$

$\therefore f(x) = 0.5876x^2 + 6.944x$

$$f(13) = 0.5876(13)^2 + 6.944(13) = \boxed{189.5764\ \text{m}}$$