Tarea 2:

1: $\int x\sin x\,dx = -x\cos x - \int \cos x\,dx = x\cos x - (-\operatorname{sen} x)$

$u = x \qquad dv = \sin x\,dx \qquad = -x\cos x + \operatorname{sen} x + C$

$du = dx \qquad v = \int \sin x$

$V = -\cos x$

2: $\int e^{4x}\sin(3x)\,dx = e^{4x}\sin(3x) - \int e^{4x}\cos(3x)\,dx$

$u = \sin(3x) \qquad dv = e^{4x}dx \qquad u_1 = \cos(3x) \qquad dv_1 = e^{4x}dx$

$du = \cos(3x)dx \qquad V = \int e^{4x}dx \qquad du_1 = -\sin(3x)dx \qquad v = \int e^{4x}dx$

$V = e^{4x} \qquad v = e^{4x}$

$= e^{4x}\sin(3x) - \left(e^{4x}\cos(3x) + \int e^{4x}\sin(3x)\,dx\right)$

$= e^{4x}\sin(3x) - e^{4x}\cos(3x) - \int e^{4x}\sin(3x)\,dx$

$\int e^{4x}\sin(3x)\,dx = e^{4x}\sin(3x) - e^{4x}\cos(3x) - \int e^{4x}\sin(3x)\,dx$

$\int e^{4x}\sin(3x)\,dx = \boxed{\dfrac{e^{4x}\sin(3x) - e^{4x}\cos(3x)}{2} + C}$

3: $x^2\sin(4x)\,dx = -\dfrac{x^2\sin(4x)}{4} + \dfrac{1}{2}\int x\cos(4x)\,dx$

$u = x^2 \qquad dv = \sin(4x)dx \qquad u_1 = x \qquad dv = \cos(4x)dx$

$du = 2x\,dx \qquad V = \int \sin(4x)\,dx \qquad j = 4x \qquad du_1 = dx \qquad V = \int\cos(4x)dx$

$dj = 4dx \qquad = \frac{1}{4}\sin(4x)$

$V = \frac{1}{4}(-\cos(4x)) \qquad \frac{dj}{4} = dx$

$= -\frac{1}{4}\cos(4x)$

$= -\dfrac{x^2\sin(4x)}{4} - \dfrac{1}{2}\left(\dfrac{x\sin(4x)}{4} - \dfrac{1}{4}\int \sin(4x)\,dx = -\dfrac{x^2\sin(4x)}{4} - \dfrac{1}{2}\left(\dfrac{x\sin(4x)}{4} + \dfrac{1}{8}\cos(4x)\right)\right.$

$= \boxed{\frac{1}{4}x^2\sin(4x) - \frac{1}{8}x\sin(4x) + \frac{1}{16}\cos(4x) + C}$

4: $x^2e^{3x}dx = x^2e^{3x} - 2\int xe^{3x}dx = x^2e^{3x} - 2\left(xe^{3x} - \int e^{3x}dx\right.$

$u = x^2 \qquad du = e^{3x}dx \qquad u_1 = x \qquad dv = e^{3x}dx \qquad = x^2e^{3x} - 2(xe^{3x} - e^{3x}) + C$

$du = 2x\,dx \qquad V = \int e^{3x}dx \qquad du_1 = dx \qquad v = \int e^{3x}dx \qquad = \boxed{x^2e^{3x} - 2xe^{3x} + 2e^{3x} + C}$

$= e^{3x} \qquad V = e^{3x}$

5: $\int x e^{-2x} dx = -\frac{e^{-2x} x}{2} + \frac{1}{2}\int e^{-2x} dx = \boxed{-\frac{e^{2x} x}{2} - \frac{1}{4} e^{-2x} + C}$

$= -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + C$

$u = x$ $\quad dv = e^{-2x} dx$ $\quad u = -2x$

$du = dx$ $\quad v = \int e^{-2x} dx$ $\quad du = -2\,dx$

$v = -\frac{1}{2}\int e^{u} du$ $\quad -\frac{du}{2} = dx$

$v = -\frac{1}{2} e^{u} = -\frac{e^{-2x}}{2}$

6: $\int x\cos(5x)\,dx = \frac{x\sin(5x)}{5} - \frac{1}{5}\int \sin(5x)\,dx = \boxed{\frac{x\cos(5x)}{5} - \frac{\sin(5x)}{25} + C}$

$u = x$ $\quad dv = \cos(5x)\,dx$ $\quad y = 5x$

$du = dx$ $\quad v = \int \cos(5x)\,dx$ $\quad dy = 5\,dx$

$v = \frac{1}{5}\int \cos(y)\,dy$ $\quad \frac{dy}{5} = dx$

$v = \frac{\sin(5x)}{5}$

7: $\int x\csc^2(x)\,dx = -x\cot(x) + \int \cot(x)\,dx = \boxed{-x\cot(x) + \ln|\sin(x)| + C}$

$u = x$ $\quad dv = \csc^2(x)\,dx$

$du = dx$ $\quad v = \int \csc^2(x)\,dx$

$v = -\cot(x)$

8: $\int x\sec(x)\tan(x)\,dx = x\sec(x) - \int \sec(x)\,dx = \boxed{x\sec(x) - \ln|\tan(x) + \sec(x)| + C}$

$u = x$ $\quad dv = \sec(x)\tan(x)\,dx$

$du = dx$ $\quad v = \int \sec(x)\tan(x)\,dx$

$v = \sec(x)$

9: $\int x^3 e^{-x} dx = -\frac{x^3}{e^x} + 3\int x^2 e^{-x} dx = -\frac{x^3}{e^x} + 3\left(-\frac{x^2}{e^x} + 2\int x e^{-x} dx\right)$

$u = x^3$ $\quad dv = e^{-x} dx$ $\quad y = -x$ $\quad u_1 = x^2$ $\quad dv_1 = e^{-x} dx$

$du = 3x^2 dx$ $\quad v = \int e^{-x} dx$ $\quad dy = -dx$ $\quad du_1 = 2x\,dx$ $\quad v_1 = \int e^{-x} dx$

$-dy = dx$ $\quad v_1 = -e^{-x}$

$v = -\int e^{y} dy$

$= -e^{-x}$

$= -\frac{x^3}{e^x} - \frac{3x^2}{e^x} + 6\int x e^{-x} dx = -\frac{x^3}{e^x} - \frac{3x^2}{e^x} + 6\left(-\frac{x}{e^x} + \int e^{-x} dx\right)$

$u_2 = x$ $\quad dv_2 = e^{-x} dx$

$du_2 = dx$ $\quad v_2 = \int e^{-x} dx$

$v_2 = -e^{-x}$

$= -\frac{x^3}{e^x} - \frac{3x^2}{e^x} + 6\left(-\frac{x}{e^x} - \frac{1}{e^x}\right) + C$

$= -\frac{x^3}{e^x} - \frac{3x^2}{e^x} - \frac{6x}{e^x} - \frac{6}{e^x} + C$

$\boxed{= -\frac{x^3 + 3x^2 + 6x + 6}{e^x} + C}$

10: $\int x^2\cos(x)\,dx = x^2\sin(x) - 2\int x\sin(x)\,dx$

$u = x^2$ $\quad dv = \cos(x)\,dx$ $\quad u_1 = x$ $\quad dv_1 = \sin(x)\,dx$

$du = 2x\,dx$ $\quad v = \int\cos(x)\,dx$ $\quad du_1 = dx$ $\quad v_1 = \int\sin(x)\,dx$

$v = \sin(x)$ $\quad v_1 = -\cos(x)$

$$= x^2\sin(x) - 2\left(-x\cos(x) + \int\cos(x)\,dx\right) = \boxed{x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C}$$

11: $\int \sin^{-1}x\,dx = \int\csc(x)\,dx = \boxed{-\ln|\csc(x) + \cot(x)| + C}$

12: $\int \tan^{-1}(x)\,dx = \int\cot(x)\,dx = \boxed{\ln|\sin(x)| + C}$

13: $\int x^2\ln(x)\,dx = \frac{\ln(x)x^3}{3} - \frac{1}{3}\int x^2\,dx = \boxed{\frac{\ln(x)x^3}{3} - \frac{x^3}{9} + C}$

$u = \ln(x)$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \int x^2\,dx$

$v = \frac{x^3}{3}$

14: $\int \sqrt{x}\,\ln x\,dx = \frac{2x^{3/2}\ln(x)}{3} - \frac{2}{3}\int x^{1/2}\,dx = \boxed{\frac{2x^{3/2}\ln(x)}{3} - \frac{4x^{1/2}}{9} + C}$

$u = \ln(x)$ $\quad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx$ $\quad v = \int x^{1/2}\,dx$

$v = \frac{2x^{3/2}}{3}$

15: $\int x\tan^{-1}(x)\,dx = \frac{\cot(x)x^2}{2} - \frac{1}{2}\int x^2(1+x^2)^{-1}\,dx$

$u = \cot(x)$ $\quad dv = x\,dx$ $\quad u_1 = (1+x^2)^{-1}$ $\quad dv = x^2\,dx$

$du = -\csc^2(x)$ $\quad v = \int x\,dx$ $\quad du_1 = -\frac{2x}{(1+x^2)^2}$ $\quad v = \int x^2\,dx$

$v = \frac{x^2}{2}$ $\quad v = \frac{x^3}{3}\cot(x)$

$du = \frac{1}{(1+x^2)}$

$$= \frac{\cot(x)x^2}{2} - \frac{1}{2}\left(\frac{x^3}{3(1+x^2)} - \frac{2}{3}\int\frac{2x^4}{x^4+2x^2+1}\right.$$

16: $\int x\csc^2(x)\,dx = -x\cot(x) + \int\cot(x)\,dx = \boxed{-x\cot(x) + \ln|\sin(x)| + C}$

$u = x$ $\quad dv = \csc^2(x)\,dx$

$du = dx$ $\quad v = \int\csc^2(x)\,dx$

$v = -\cot(x)$

17: $\int e^{3x}\cos(3x)\,dx = \frac{\cos(3x)e^{3x}}{3} + \int e^{3x}\sin(3x)\,dx$

$u = \cos(3x)$ $\quad dv = e^{3x}\,dx$ $\quad y = 3x$ $\quad u_1 = \sin(3x)$ $\quad dv_1 = e^{3x}\,dx$

$du = -3\sin(3x)$ $\quad v = \int e^{3x}\,dx$ $\quad dy = 3\,dx$ $\quad du_1 = 3\cos(3x)$ $\quad v_1 = \int e^{3x}\,dx$

$v = \frac{1}{3}\int e^y\,dy$ $\quad \frac{dy}{3} = dx$ $\quad v_1 = \frac{e^{3x}}{3}$

$v = \frac{e^{3x}}{3}$

$$= \frac{\cos(3x)e^{3x}}{3} + \frac{\sin(3x)e^{3x}}{3} - \int e^{3x}\cos(3x)\,dx$$

$$2\int e^{3x}\cos(3x)dx = \frac{\cos(3x)e^{3x} + \sin(3x)e^{3x}}{3} = \boxed{\frac{\cos(3x)e^{3x} + \sin(3x)e^{3x}}{6} + C}$$

18: $\int e^{-x}\sin(x)\,dx = -e^{-x}\sin(x) + \int e^{-x}\cos(x)dx$

$u = \sin(x)$ $\quad dv = e^{-x}dx$ $\quad y = -x$, $dy = -dx$, $-dy = dx$

$du = \cos(x)dx$ $\quad v = -\int e^{y}dy$

$v = -e^{-x}$

$u_1 = \cos(x)$ $\quad dv = e^{-x}dx$

$du_1 = -\sin(x)$ $\quad v = \int e^{-x}dx$

$v = -e^{-x}$

$$= -e^{-x}\sin(x) - e^{-x}\cos(x) - \int e^{-x}\sin(x)dx$$

$$\therefore\ 2\int e^{-x}\sin(x)\,dx = -e^{-x}\sin(x) - e^{-x}\cos(x) = -\frac{(\sin(x) + \cos(x))}{e^{x}}$$

$$\int e^{-x}\sin(x)\,dx = \boxed{-\frac{\sin(x) + \cos(x)}{2e^{x}} + C}$$

19: $\int \sec^3(x)\,dx = \int \sec^2(x)\sec(x)\,dx = \sec(x)\tan(x) - \int \sec(x)\tan^2(x)\,dx$

$u = \sec(x)dx$ $\quad dv = \sec^2(x)\,dx$

$du = \sec(x)\tan(x)dx$ $\quad v = \int \sec^2(x)\,dx$

$v = \tan(x)$

$= \sec(x)\tan(x) - \int \sec(x)(\sec^2(x) - 1)\,dx$

$= \sec(x)\tan(x) - \int \sec^3(x) - \sec(x)\,dx$

$$= \sec(x)\tan(x) - \int \sec^3(x)dx + \int \sec(x)dx)$$

$$2\int \sec^3(x)\,dx = \sec(x)\tan(x) + \ln|\tan(x) + \sec(x)|$$

$$\int \sec^3(x)dx = \boxed{\frac{\sec(x)\tan(x) + \ln|\tan(x) + \sec(x)|}{2} + C}$$

20: $\int \sin(x)\ln|\cos(x)|\,dx = -\ln|\cos(x)|\cos(x) - \int \sin(x)\,dx$

$u = \ln|\cos(x)|$ $\quad dv = \sin(x)\,dx$

$du = -\frac{\sin(x)}{\cos(x)}dx$ $\quad v = \int \sin(x)\,dx$

$v = -\cos(x)$

$= \boxed{-\ln|\cos(x)|\cos(x) + \cos(x) + C}$

21: $\int \sin(\ln(x))dx = \int \sin(u)\,x\,du = \int \sin(u)e^{u}\,du = e^{u}\sin(u) - \int e^{u}\cos(u)du$

$u = \ln|x|$

$du = \frac{dx}{x}$

$x\,du = dx$

$u_1 = \sin(u)$ $\quad dv = e^{u}du$

$du_1 = \cos(u)du$ $\quad v = \int e^{u}du$

$v = e^{u}$

$u_2 = \cos(u)$ $\quad dv_1 = e^{u}du$

$du_2 = -\sin(u)du$ $\quad V = e^{u}$

$$= e^{u}\sin(u) - e^{u}\cos(u) - \int \sin(u)e^{u}du$$

$$2\int \sin(u)e^{u}du = e^{u}\sin(u) - e^{u}\cos(u)$$

$$\int \sin(u)e^{u}du = \frac{e^{u}\sin(u) - e^{u}\cos(u)}{2} = \boxed{\frac{x\sin(\ln(x)) - x\cos(\ln(x))}{2} + C}$$

22: $\int x\sec^2(5x)\,dx = \frac{x\tan(5x)}{5} - \frac{1}{5}\int \tan(5x)\,dx = \frac{x\tan(x)}{5} - \frac{1}{5}\left(\frac{\ln|\tan(5x)+\sec(5x)|}{5}\right)$

$u = x$ $\quad dv = \sec^2(5x)\,dx$ $\quad y = 5x$ $\quad u_2 = 5x$

$du = dx$ $\quad v = \frac{1}{5}\int \sec^2(y)\,dy$ $\quad dy = 5dx$ $\quad du_2 = 5dx$

$\frac{dy}{5} = dx$ $\quad \frac{du_2}{5} = dx$

$v = \frac{\tan(5x)}{5}$

$= \boxed{\frac{x\tan(5x)}{5} - \frac{\ln|\tan(5x)+\sec(5x)|}{25} + C}$

23: $\int \frac{x}{\sqrt{x-1}}\,dx = \int \frac{u+1}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}}du + \int \frac{1}{\sqrt{u}}\,du = \int u^{1/2}du + \int u^{-1/2}du$

$u = x-1$

$du = dx$

$= \frac{2u^{3/2}}{3} - 2u^{1/2} = \frac{2}{3}u^{3/2} - 2u^{1/2} + C = \boxed{\frac{2}{3}(x-1)^{3/2} - 2(x-1)^{1/2} + C}$

24: $\int (2x+1)\sqrt{x-5}\,dx = \int (2u+11)\sqrt{u}\,du = \frac{2(2u+11)u^{3/2}}{3} - \frac{4}{3}\int u^{3/2}du$

$u_1 = 2u+11$ $\quad dv = \sqrt{u}\,du$

$u = x-5$ $\quad du_1 = 2du$ $\quad v = \int u^{1/2}du$

$du = dx$ $\quad v = \frac{2u^{3/2}}{3}$

$= \frac{2(2u+11)u^{3/2}}{3} - \frac{4}{3}\left(\frac{2u^{5/2}}{5}\right) = \frac{2}{3}(2u+11)u^{3/2} - \frac{8}{15}u^{5/2} + C$

$= \boxed{\frac{2}{3}(2x+1)(x-5)^{3/2} - \frac{8}{15}(x-5)^{5/2} + C}$

25: $\int \frac{x^2}{(x-1)^{-1}}\,dx = \int x^2(x-1)\,dx = \frac{(x-1)x^3}{3} - \frac{1}{3}\int x^3 dx$

$u = x-1$ $\quad dv = x^2\,dx$

$du = dx$ $\quad v = \frac{x^3}{3}$

$= \boxed{\frac{(x-1)x^3}{3} - \frac{x^4}{12} + C}$

$\int (x^3 - x^2)dx = \int x^3dx - \int x^2dx$

$= \boxed{\frac{x^4}{4} - \frac{x^3}{3} + C}$

Mas facil por multiplicación