Integrales impropias:

1: $\int_{-\infty}^{\infty} x^2 e^{-x}\,dx = -\frac{x^2}{e^x} + 2\int x e^{-x}\,dx =$

$u = x^2 \quad dv = e^{-x}dx$
$du = 2x\,dx \quad v = -e^{-x}$

$u = x \quad dv = e^{-x}dx$
$du = dx \quad v = -e^{-x}$

$$= -\frac{x^2}{e^x} + 2\left[-\frac{x}{e^x} + \int e^{-x}dx\right] = -\frac{x^2}{e^x} - \frac{2x}{e^x} - \frac{2}{e^x} = -\frac{x^2+2x+2}{e^x}$$

$$\lim_{a\to-\infty}\left[-\frac{x^2+2x+2}{e^x}\right]_a^0 + \lim_{b\to\infty}\left[-\frac{x^2+2x+2}{e^x}\right]_0^b$$

$$= \lim_{a\to-\infty}\left(-2 + \frac{a^2+2a+2}{e^a}\right) + \lim_{b\to\infty}\left(-\frac{b^2+2b+2}{e^b} - (-2)\right)$$

$$= -2 + \lim_{a\to-\infty}\left(\frac{2a+2}{ae^a}\right.$$

Diverge a infinito

2: $\int_1^{\infty} \frac{1}{x(\ln x)^3}\,dx = \int_1 \frac{du}{u^3} = \left[\frac{u^4}{4}\right]_1^{\infty} = \left[\frac{(\ln x)^4}{4}\right]_1^{\infty}$

$u = \ln x$
$du = \frac{1}{x}dx$

$$\lim_{b\to\infty}\left(\frac{(\ln x)^4}{4}\right)_1^b = \lim_{b\to\infty}\left(\frac{(\ln b)^4}{4} - 0\right)$$

$$= \lim_{b\to\infty}\left(\frac{(\ln b)^4}{4}\right) = \frac{1}{4}\lim_{b\to\infty}(\ln b)^4 = \infty$$

Diverge al infinito

3: $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx = \int_{-\infty}^{\infty} \frac{e^x}{1+(e^x)^2}\,dx = \int_{-\infty}^{\infty} \frac{du}{1+u^2} = \int_{-\infty}^{\infty} \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta$

$u = e^x$
$du = e^x dx$

$u = \tan\theta$ $\quad \theta = \arctan u$
$du = \sec^2\theta\, d\theta$

$$= \int_{-\infty}^{\infty} d\theta = [\theta]_{-\infty}^{\infty} = [\arctan(u)]_{-\infty}^{\infty} = [\arctan(e^x)]_{-\infty}^{\infty}$$

$$\lim_{a\to-\infty} [\arctan(e^x)]_a^0 + \lim_{b\to\infty} [\arctan(e^x)]_0^b$$

$$= \lim_{a\to-\infty} \left(\tfrac{1}{4}\pi - \cancel{\arctan(e^a)}^{\,0}\right) + \lim_{b\to\infty} \left(\arctan(e^b) - \tfrac{1}{4}\pi\right)$$

$$\tfrac{1}{4}\pi + \left(\tfrac{1}{2}\pi - \tfrac{1}{4}\pi\right) = \boxed{\tfrac{1}{2}\pi}$$

C ~~Diverge~~ onverge a $\frac{1}{2}\pi$

4: $\int_0^{\infty} (x-1)e^{-x}\,dx = -\frac{(x-1)}{e^x} + \int e^{-x}\,dx = -\frac{(x-1)}{e^x} - \frac{1}{e^x}$

$u = (x-1)$ $\quad dv = e^{-x}dx$
$du = dx$ $\quad v = -e^{-x}$

$$= \frac{-x\cancel{+1}\cancel{-1}}{e^x} = -\frac{x}{e^x}$$

L'Hopital

$$\lim_{b\to\infty} \left[-\frac{x}{e^x}\right]_0^b = \lim_{b\to\infty} \left(-\frac{b}{e^b} - 0\right) = -\lim_{b\to\infty} \left(\frac{1}{e^b}\right) = 0$$

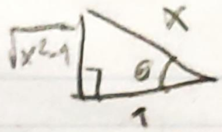
Converge a 0

5: $\int_1^{\infty} \frac{4}{x^{1/4}}\,dx = 4\int_1^{\infty} x^{-\frac{1}{4}}\,dx = 4\left[\frac{4}{3}x^{3/4}\right]_1^{\infty}$

$$\lim_{b\to\infty} \left[\frac{4}{3}x^{3/4}\right]_1^b = \lim_{b\to\infty} \left(\frac{4}{3}b^{3/4} - \frac{4}{3}\right) = \infty$$

Diverge a infinito

6: $\int_0^\infty \frac{x^3}{(x^2-1)}\,dx = \int_0^\infty \frac{(\sec^3\theta)(\sec\theta\tan\theta)}{\sec^2\theta-1}\,d\theta$



$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\theta = \operatorname{arcsec}(x)$

$= \int_0^\infty \frac{\sec^4\theta\,\cancel{\tan\theta}}{\tan^{\cancel{2}}\theta}\,d\theta = \int_0^\infty \frac{\sec^4\theta}{\tan\theta}\,d\theta$

$= \int_0^\infty \frac{\cancel{\sec}(\tan^2\theta+1)^2}{\tan\theta}\,d\theta = \int_0^\infty \frac{\tan^4\theta+2\tan\theta+1}{\tan\theta}\,d\theta$

$= \int_0^\infty (\tan^3\theta+2+\cot\theta)\,d\theta = \int_0^\infty ((\sec^2\theta-1)\tan\theta+2+\cot\theta)\,d\theta$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta$

$= \int_0^\infty (\sec^2\theta\tan\theta - \tan\theta + 2 + \cot\theta)\,d\theta$

$= \left[\frac{u^2}{2} - \ln|\sec\theta| + 2\theta + \ln|\sin\theta|\right]_0^\infty$

$= \left[\frac{\tan^2\theta}{2} - \ln|\sec\theta| + 2\theta + \ln|\sin\theta|\right]_0^\infty$

$= \left[\frac{1}{2}(\sqrt{x^2-1})^2 - \ln|x| + 2\operatorname{arcsec}(x) + \ln\left|\frac{\sqrt{x^2-1}}{x}\right|\right]_0^\infty$

$= \left[\frac{1}{2}(x^2-1) - \ln|x| + 2\operatorname{arcsec}(x) + \ln\left|\frac{\sqrt{x^2+1}}{x}\right|\right]_0^\infty$

ñ esta largo

$\int_0^\infty \frac{x^3}{(x^2-1)}\,dx = \int_0^\infty \frac{x^2}{(x^2-1)}\,x\,dx = \frac{1}{2}\int_0^\infty \frac{u+1}{u}\,du$

$u = x^2-1 \qquad x^2 = u+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\left[u + \ln|u|\right]_0^\infty$

Indefinido

$= \frac{1}{2}\lim_{b\to\infty}(u+\ln|u|)_0^b = \frac{1}{2}\lim(b+\ln|b| - 0 + \cancel{\ln|0|})$

Diverge pues no existe

7: $\int_0^\infty \sin\left(\frac{x}{2}\right) dx = 2\left[-\cos\left(\frac{x}{2}\right)\right]_0^\infty$

$u = \frac{x}{2}$

$du = \frac{dx}{2}$

$2du = dx$

$2\lim_{b\to\infty}\left(-\cos\left(\frac{x}{2}\right)\right)_0^b = 2\lim_{b\to\infty}\left(-\cos\left(\frac{b}{2}\right) - \left(-\cos\left(\frac{0}{2}\right)\right)\right)$

oscila al inf ∴ su $\lim_{x\to\infty}$ es indefinido

$= 2\lim_{b\to\infty}\left(-\cos\left(\frac{b}{2}\right) + 1\right) =$ Indefinido

Diverge pues no existe

8: $\int_0^\infty \frac{\ln(x)}{x} dx = \int_0^\infty u\, du = \left[\frac{u^2}{2}\right]_0^\infty = \left[\frac{\ln(x)^2}{2}\right]_0^\infty$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$= \lim_{b\to\infty}\left[\frac{\ln(x)^2}{2}\right]_0^b = \lim_{b\to\infty}\left(\frac{\ln(b)^2}{2} - \cancel{\frac{\ln(0)^2}{2}}\right)$ Indefinido

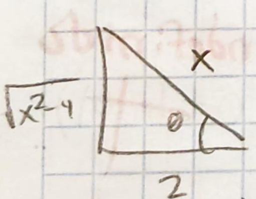
Diverge pues no existe

9: $\int_2^4 \frac{1}{\sqrt{x^2-4}} dx = \frac{2}{2}\int_2^4 \frac{\sec\theta \cancel{\tan\theta}}{\cancel{\tan\theta}} d\theta = \int_2^4 \sec\theta\, d\theta = \left[\ln(\tan\theta + \sec\theta)\right]_2^4$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

$= \left[\ln\left|\frac{\sqrt{x^2-4} + x}{2}\right|\right]_2^4 = (1.31695 - 0)$



$= 1.31695$

Converge a 1.31695

10: $\cancel{\int_0^e \frac{\ln(x)}{x} dx}$

$$\int_0^e \ln(x^2)\,dx = 2\int_0^e \ln(x)\,dx = 2\left[x\ln(x)-x\right]_0^e$$

$$2\lim_{a\to 0}\left[x\ln(x)-x\right]_a^e = 2\lim_{a\to 0}\left[\cancel{e\ln(e)-e}-(a\ln(a)-a)\right]$$

$$= 2\lim_{a\to 0}\left[-a\ln(a)+a\right] = 2\left[-\lim_{a\to 0}\frac{\ln(a)}{a^{-1}}+0\right]$$

$$= 2\left[-\lim_{a\to 0}\frac{\frac{1}{a}}{-\frac{1}{a^2}}\right] = 2\left[+\lim_{a\to 0} a\right] = 0$$

Converge a 0

11: $$\int_0^{\frac{\pi}{2}} \sec\theta\,d\theta = \left[\ln|\tan\theta+\sec\theta|\right]_0^{\frac{\pi}{2}}$$

$$\lim_{b\to\frac{\pi}{2}}\left[\ln|\tan\theta+\sec\theta|\right]_0^b = \lim_{b\to\frac{\pi}{2}}\left(\ln|\tan b+\sec b|+0\right)$$

$$= \lim_{b\to\frac{\pi}{2}}\left(\ln|\tan b+\sec b|\right) = \ln\left(\lim_{b\to\frac{\pi}{2}}\frac{\sin\theta+1}{\cos\theta}\right)$$

$$= \ln\left(\lim_{b\to\frac{\pi}{2}}\frac{\cos\theta}{-\sin\theta}\right) = \text{Indefinido}$$

Diverge pues no existe

12: $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \int_0^2 \frac{\cancel{2\cos\theta}}{\cancel{2\cos\theta}}\,d\theta = \int_0^2 d\theta = [\theta]_0^2$

$x = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$

$\theta = \arcsin\left(\frac{x}{2}\right)$

$\left[\arcsin\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{\pi}{2}$

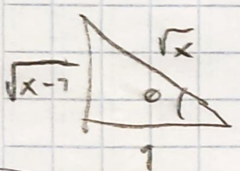
Converge a $\frac{\pi}{2}$

13: $\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx = \int_2^\infty \frac{2\sec^2\theta\,\cancel{\tan\theta}}{\cancel{\tan\theta}}\,d\theta = 2\int_2^\infty \sec^2\theta\,d\theta$

$= 2\left[\tan\theta\right]_2^\infty$

$= 2\left[\sqrt{x-1}\right]_2^\infty$

$\sqrt{x} = \sec\theta$
$x = \sec^2\theta$
$dx = 2\sec^2\theta\tan\theta\,d\theta$
$\sqrt{x-1} = \tan\theta$



$2\lim_{b\to\infty}\left[\sqrt{x-1}\right]_2^b = \infty$

Diverge a ∞.

14: $\int_1^\infty \frac{1}{x^p}\,dx = $ ¿?