Tarea 4

1: $\int \cos^7(x)\,dx = \int \cos^2(x)\cos^2(x)\cos^2(x)\cos(x)\,dx$

$$= \int (1-\sin^2(x))^3\cos(x)\,dx = (1 - 3\sin^2(x) + 3\sin^4(x) - \sin^6(x))\cos(x)\,dx$$

$$= \int \cos(x)\,dx - 3\int \cos(x)\sin^2(x)\,dx + 3\int \cos(x)\sin^4(x)\,dx - \int \cos(x)\sin^6(x)\,dx$$

$$= \sin(x) - 3\int u^2\,du + 3\int u^4\,du - \int u^6\,du$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \sin(x) - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7 + C$$

$$\boxed{= \sin(x) - \sin^3(x) + \frac{3}{5}\sin^5(x) - \frac{1}{7}\sin^7(x) + C}$$

2: $\int \sin^4(x)\cos^2(x)\,dx = \int \sin^2(x)\sin^2(x)\cos^2(x)\,dx$

$$= \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx$$

$$\frac{1}{8}\int (1-\cos(2x))(1-\cos^2(2x))\,dx = \frac{1}{8}\int (1 - \cos^2(2x) - \cos(2x) + \cos^3(2x))\,dx$$

$y = 2x$

$dy = 2dx$

$\frac{dy}{2} = dx$

$$= \frac{1}{8}\left(\int dx - \int \cos^2(2x)\,dx - \int \cos(2x)\,dx + \int \cos^3(2x)\,dx\right)$$

$$= \frac{1}{8}x - \frac{1}{2}\sin(2x) - \frac{1}{8}\left(\int \cos^2(2x)\,dx - \int \cos^3(2x)\,dx\right)$$

$m = 4x$

$dm = 4dx$

$\frac{dm}{4} = dx$

$$= \frac{1}{8}x - \frac{1}{2}\sin(2x) - \frac{1}{8}\left(\frac{1}{2}\int (1+\cos(4x))\,dx - \int (1-\sin^2(2x))\cos(2x)\,dx\right)$$

$$= \frac{1}{8}x - \frac{1}{2}\sin(2x) - \frac{1}{8}\left[\frac{1}{2}\left(\int dx + \int \cos(4x)\,dx\right) - \left(\int \cos(2x) - \int \sin^2(2x)\cos(2x)\,dx\right)\right]$$

$$= \frac{1}{8}x - \frac{1}{2}\sin(2x) - \frac{1}{8}\left[\frac{1}{2}x + \frac{1}{8}\sin(4x) - \frac{1}{2}\sin(2x) + \boxed{\int \sin^2(2x)\cos(2x)\,dx}\right]$$

$$2\int u_1^2\,du_1 = \frac{2}{3}u_1^3 = \frac{2}{3}\sin^3(2x)$$

$u_1 = \sin(2x)$

$du_1 = 2\cos(2x)\,dx$

$$= \frac{1}{8}x - \frac{1}{2}\sin(2x) - \frac{1}{16}x - \frac{1}{64}\sin(4x) + \frac{1}{16}\sin(2x) - \frac{1}{12}\sin^3(2x) + C$$

$$\boxed{= \frac{1}{16}x - \frac{7}{16}\sin(2x) - \frac{1}{64}\sin(4x) - \frac{1}{12}\sin^3(2x) + C}$$

3: $\int \sin^2(x)\cos^2(x)\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx$

$= \frac{1}{4}\int (1-\cos^2(2x))\,dx = \frac{1}{4}\int \sin^2(2x)\,dx = \frac{1}{4}\int \frac{1-\cos(4x)}{2}$

$= \frac{1}{8}\left[\int dx - \int \cos(4x)\,dx\right]$ $\quad u = 4x,\ du = 4dx,\ \frac{du}{4} = dx \quad = \boxed{\frac{1}{8}x - \frac{1}{32}\sin(4x) + C}$

4: $\int \sin^5(x)\sqrt[3]{\cos x}\,dx = \int \sin^3(x)\sin^2(x)\cos^{1/3}(x)\,dx$

$= \int \sin^3(x)(1-\cos^2(x))\cos^{1/3}(x)\,dx = \int \sin^3(x)(\cos^{1/3}x - \cos^{7/3}x)\,dx$

$= \int \sin^3(x)\cos^{1/3}(x)\,dx - \int \sin^3(x)\cos^{7/3}(x)\,dx$

$= \int \sin(x)(1-\cos^2(x))\cos^{1/3}(x)\,dx - \int \sin(x)(1-\cos^2(x))\cos^{7/3}(x)\,dx$

$= \int \sin(x)(\cos^{1/3}(x) - \cos^{7/3}(x))\,dx - \int \sin(x)(\cos^{7/3}(x) - \cos^{13/3}(x))\,dx$

$= \int \sin(x)\cos^{1/3}(x)\,dx - \int \sin(x)\cos^{7/3}(x)\,dx - \int \sin(x)\cos^{7/3}(x)\,dx + \int \sin(x)\cos^{13/3}(x)\,dx$

$u = \cos(x)$
$du = -\sin x\,dx$
$-du = s\ \ x\,dx$

$= -\int u^{1/3}du + \int u^{7/3}du + \int u^{7/3}du - \int u^{13/3}du$

$= -\int u^{1/3}du + 2\int u^{7/3}du - \int u^{13/3}du$

$= -\frac{3}{4}u^{4/3} + \frac{3}{5}u^{10/3} - \frac{3}{16}u^{16/3} + C$

$$\boxed{= -\frac{3\sqrt[3]{\cos^4(x)}}{4} + \frac{3\sqrt[3]{\cos^{10}(x)}}{5} - \frac{3\sqrt[3]{\cos^{16}(x)}}{16} + C}$$

5: $\int \sec^6(x)\,dx = \int \sec^2(x)\sec^2(x)\sec^2(x)\,dx = \int \sec^2 x(1+\tan^2(x))^2\,dx$

$= \int \sec^2(x)(1 + 2\tan^2(x) + \tan^4(x))\,dx = \int \sec^2(x)\,dx + 2\int \sec^2 x\tan^2 x\,dx + \int \sec^2(x)\tan^4(x)\,dx$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$= \int du + 2\int u^2 du + \int u^4 du$

$= u + \frac{2}{3}u^3 + \frac{1}{5}u^5 + C = \boxed{\tan(x) + \frac{2}{3}\tan^3(x) + \frac{1}{5}\tan^5(x) + C}$