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Álgebra Lineal 2022-2 **Cuarto Parcial**

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Contesta de forma clara y ordena. Incluye procedimiento, siempre que haya uno para que sea tomado en cuenta tu respuesta. Tener cámara encendida enfocar de manera que se aprecie la mesa de trabajo y su perfil.

$Tu + Tv = (x_1 - y_1, -3x_1) + (x_2 - y_2, -3x_2) = ((x_1+x_2) - (y_1+y_2), -3(x_1+x_2))$

1. **Determinar formalmente si las transformaciones dadas son o no lineales.**

a) $T:\mathbb{R}^2 \to \mathbb{R}^2$

$T(x,y) = (x-y, -3x)$

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P.D. $T(\bar u + \bar v) = T\bar u + T\bar v$

Sean $\bar u, \bar v \in \mathbb{R}^2$, donde $\bar u = (x_1, y_1, z_1)$, $\bar v = (x_2, y_2, z_2)$

$T(\bar u + \bar v) = T((x_1, y_1, z_1) + (x_2, y_2, z_2))$

$= T(x_1+x_2, y_1+y_2, z_1+z_2)$

$= ((x_1+x_2) - (y_1+y_2), -3(x_1+x_2))$ luego

$\therefore \underline{T(\bar u + \bar v) = T\bar u + T\bar v}$

P.D. $T(K\bar u) = KT(u)$

Sea $K \in \mathbb{R}$; $u \in \mathbb{R}^2$, donde $\bar u = (x, y, z)$

$T(K\bar u) = T(K(x, y, z))$

$= T(Kx, Ky, Kz)$

$= (Kx - Ky, -3Kz)$

$= K(x - y, -3z)$, sea $K \in \mathbb{R}$

$T(Ku) = KT(u)$

$\therefore$ $\boxed{\text{Es una transformación lineal}}$

b) $T: M_{mn}(\mathbb{R}) \to M_{mn}(\mathbb{R})$

$T(A) = A^T$

P.D.: $T(A+B) = TA + TB$

Sean $A, B \in M_{mn}(\mathbb{R})$, donde $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$, $K \in \mathbb{R}$

$T(A+B) = T\left(\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}\right)$

$T(A+B) = \begin{bmatrix} a_{11}+b_{11} & a_{21}+b_{21} & \cdots & a_{m1}+b_{m1} \\ a_{12}+b_{12} & a_{22}+b_{22} & \cdots & a_{m2}+b_{m2} \\ \vdots & & & \vdots \\ a_{1n}+b_{1n} & a_{2n}+b_{2n} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$ ✓

P.D.: $T(KA) = KTA$

$T(KA) = T\left(K\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}\right)$

$= \begin{bmatrix} Ka_{11}+Kb_{11} & Ka_{12}+Kb_{12} & \cdots & Ka_{1n}+Kb_{1n} \\ Ka_{21}+Kb_{21} & Ka_{22}+Kb_{22} & \cdots & Ka_{2n}+Kb_{2n} \\ \vdots & & & \vdots \\ Ka_{m1}+Kb_{m1} & Ka_{m2}+Kb_{m2} & \cdots & Ka_{mn}+Kb_{mn} \end{bmatrix}$

Continuación en hoja anexo

2. **Encuentra la ecuación de la transformación lineal $T:\mathbb{R}^3 \to \mathbb{R}^3$ tal que:**

$T(1,0,0) = (2,0,-1)$
$T(0,1,0) = (0,1,2)$
$T(0,1,1) = (5,3,1)$

3. **Sea $T:\mathbb{R}^4 \to \mathbb{R}^2$ una transformación lineal definida como; $T(x,y,z,w) = (x+z, y+w)$, encuentra el núcleo, imagen, rango, nulidad de la transformación. ¿Es Inyectiva?¿Es suprayectiva? ¿Es biyectiva?. Encontrar la matriz asociada. Verificar el teorema de la dimensión.**

Im T

Bases de $\mathbb{R}^4 = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

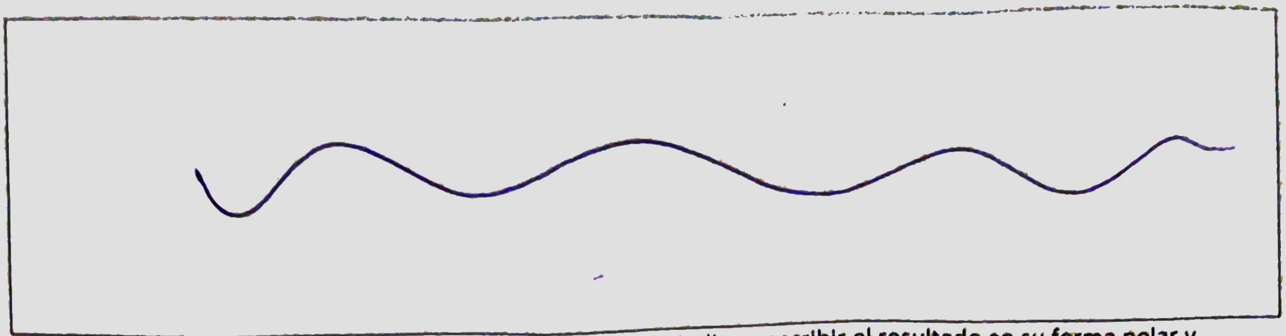
$T(1,0,0,0) = (1,0)$
$T(0,1,0,0) = (0,1)$
$T(0,0,1,0) = (1,0)$
$T(0,0,0,1) = (0,1)$

$\boxed{\begin{array}{l} \text{Im}\,T = \langle (1,0), (0,1) \rangle \\ \text{Dim}(\text{Im}\,T) = 2 \end{array}}$

~~Ultimatex~~

$\boxed{\therefore \text{Es sobreyectiva}}$

Continuación en hoja anexa



4. Elevar el número complejo dado a la potencia que se indica y escribir el resultado en su forma polar y rectangular.

a) $*(1+5i)(2+8i) = (2-40)+(8+10)i$

$\boxed{= -38 + 18i}$

$\boxed{\text{Forma Polar} = 26\sqrt{26}\,(\mathrm{Cis}(843.9°))}$

b) $(1-5i)^3$ 4°

$r = \sqrt{1^2+(-5)^2} = \sqrt{26}$ $\quad \theta = \tan^{-1}\left(-\frac{5}{1}\right) = -78.69 \rightarrow \theta = -78.69 + 360° = 281.3°$

$z = (\sqrt{26})^3 (\cos 3(281.3°) + i\,\mathrm{sen}\, 3(281.3°)) = 26\sqrt{26}\,(\cos(843.9°) + i\,\mathrm{sen}(843.9°))$

Rectangular $\boxed{z = -73.94 + 110.03i}$

c) $(1-5i)^3/(1+2i)^3 = \left(\frac{1-5i}{1+2i}\right)^3$

$\frac{1-5i}{1+2i}\left(\frac{1-2i}{1-2i}\right) = \frac{(1-5i)(1-2i)}{5}$

$= \frac{(1-10)+(-2-5)i}{5} = -\frac{9}{5} - \frac{7}{5}i$

$\boxed{= (-1.8 - 1.4i)^3}$

3° Cuadrante $\quad$ Esta No

$r = \sqrt{(-1.8)^2 + (-1.4)^2} = \sqrt{5.2}$ $\quad \theta = \tan^{-1}\left(\frac{-1.4}{-1.8}\right) + 180° =$

$\theta = 217.87°$

$z = (\sqrt{5.2})^3 (\cos 3(217.87) + i\,\mathrm{sen}\, 3(217.87))$

$z = (\sqrt{5.2})^3 (\cos(653.61) + i\,\mathrm{sen}(653.61))$

$\boxed{z = 4.74 - 10.865i}$ ← Rectangular

$\boxed{z = \frac{26\sqrt{130}}{25}(\mathrm{Cis}(653.61°))}$ ← Polar

5. Encontrar las 3 raíces quintas de $z = 4-3i$ - 4° cuadrante

$\therefore \theta = -36.869 + 360 = 323.13°$

$r = \sqrt{4^2 + (-3)^2} = 5$ $\quad \theta = \tan^{-1}\left(\frac{-3}{4}\right) = -36.869$

$z_0 = 5^{1/5}\left(\cos\left(\frac{323.13° + 360°(0)}{5}\right) + i\,\mathrm{sen}\left(\frac{323.13 + 360(0)}{5}\right)\right) = 5^{1/5}(\cos(64.646°) + i\,\mathrm{sen}(64.646°))$

$\boxed{z_0 = 0.59 + 1.241i}$

$z_1 = 5^{1/5}\left(\cos\left(\frac{323.13 + 360°(1)}{5}\right) + i\,\mathrm{sen}\left(\frac{323.13° + 360°(1)}{5}\right)\right) = 5^{1/5}(\cos(136.626°) + i\,\mathrm{sen}(136.626°))$

$\boxed{z_1 = -1 + 0.947i}$

$z_2 = 5^{1/5}\left(\cos\left(\frac{323.13 + 360(2)}{5}\right) + i\,\mathrm{sen}\left(\frac{323.13 + (360)(2)}{5}\right)\right) = 5^{1/5}(\cos(208.626) + i\,\mathrm{sen}(208.626))$

$\boxed{z_2 = -1.211 - 0.661i}$

6. Sea T una transformación lineal $T(x,y,z) = (-2x+y, x-3y, z)$

a) Ecuación característica, b) valores propios, c) vectores propios y d) matriz semejante D

Base de $\mathbb{R}^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(1,0,0) = (-2,1,0)$

$T(0,1,0) = (1,-3,0)$

$T(0,0,1) = (0,0,1)$

d) $\boxed{D_T = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$ $\det = 0 + 0 + 1(5) = 5$? 0

$k = 1$

$\begin{bmatrix} -2-1 & 1 & 0 \\ 1 & -3-1 & 0 \\ 0 & 0 & 1-1 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (0,0,0)$ X

SEL
$-3x + y = 0$
$x - 4y = 0$

$y = 3x$
$x - 12x \neq 0$ $\therefore$ Solución trivial

$E_1 = (0,0,0)$

$M_T - kI = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} - k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{vmatrix} -2-k & 1 & 0 \\ 1 & -3-k & 0 \\ 0 & 0 & 1-k \end{vmatrix} = 0 + 0 + 1-k\begin{vmatrix} -2-k & 1 \\ 1 & -3-k \end{vmatrix} = (1-k)[(-2-k)(-3-k) - 1]$

$= (1-k)(6 + 2k + 3k + k^2 - 1)$

a) $\boxed{\text{Ecuación Característica} = (1-k)(k^2 + 5k + 5) = 0}$

$1 - k = 0$ $\quad k^2 + 5k + 5 = 0$

$k_3 = 1$

$k = \frac{-5 \pm \sqrt{25 - 4(1)(5)}}{2(1)}$

$k_1 = -\frac{5 - \sqrt{5}}{2}$

$k_2 = -\frac{5 + \sqrt{5}}{2}$

b) $\boxed{\text{Valores Propios: } k_1 = 1 \quad k_2 = -\frac{5-\sqrt{5}}{2} \quad k_3 = -\frac{5+\sqrt{5}}{2}}$

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1)

b)

$$= k\begin{bmatrix} a_{11}+b_{11} & a_{21}+b_{21} & \cdots & a_{m1}+b_{m1} \\ a_{12}+b_{12} & a_{22}+b_{22} & \cdots & a_{m2}+b_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n}+b_{1n} & a_{2n}+b_{2n} & \cdots & a_{mn}+b_{mn} \end{bmatrix} \quad \therefore\ T(kA) = kTA$$

$\boxed{\therefore\ U \text{ es una transformación lineal}}$

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3) $\ker T = T(x, y, z, w) = (0,0)$

$(x+z,\ y+w) = (0,0)$

SEL

$x + z = 0 \qquad y + w = 0$

$x = -z \qquad y = -w$

Solucion SEL $= (-z, -w, z, w)$

$= (-z, 0, z, 0) + (0, -w, 0, w)$

$= z(-1, 0, 1, 0) + w(0, -1, 0, 1)$

$\boxed{\ker T = \langle (-1,0,1,0),\ (0,-1,0,1) \rangle}$ $\boxed{\therefore \text{ No es inyectiva}}$

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6) $k = -\dfrac{5-\sqrt{5}}{2}$

$$\begin{bmatrix} -2-\left(-\frac{5-\sqrt{5}}{2}\right) & 1 & 0 \\ 1 & -3-\left(-\frac{5-\sqrt{5}}{2}\right) & 0 \\ 0 & 0 & 1-\left(-\frac{5-\sqrt{5}}{2}\right) \end{bmatrix} = \begin{bmatrix} \frac{1-\sqrt{5}}{2} & 1 & 0 \\ 1 & -\frac{1+\sqrt{5}}{2} & 0 \\ 0 & 0 & \frac{7-\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

SEL

$\dfrac{1-\sqrt{5}}{2}x + y = 0 \qquad x - \dfrac{1+\sqrt{5}}{2}y = 0 \qquad \dfrac{7-\sqrt{5}}{2}z = 0$

$y = -\dfrac{1-\sqrt{5}}{2}x \qquad x - \dfrac{1+\sqrt{5}}{2}\left(\dfrac{1-\sqrt{5}}{2}x\right) = 0 \qquad z = 0$

$x - x = 0$

$0 = 0$

c)

Solución SEL

$\left(x, -\dfrac{1-\sqrt{3}}{2}x, 0\right)$

$x\left(1, -\dfrac{1-\sqrt{3}}{2}, 0\right)$

$\boxed{E_{-\frac{5-\sqrt{5}}{2}} = \left\langle \left(1, -\dfrac{1-\sqrt{3}}{2}, 0\right) \right\rangle}$

Vector Propio

$\lambda = -\frac{5+\sqrt{3}}{2}$

$$\begin{bmatrix} -2-\left(-\frac{5+\sqrt{3}}{2}\right) & 1 & 0 \\ 1 & -3-\left(-\frac{5+\sqrt{3}}{2}\right) & 0 \\ 0 & 0 & 1-\left(-\frac{5+\sqrt{3}}{2}\right) \end{bmatrix} = \begin{bmatrix} \frac{1+\sqrt{3}}{2} & 1 & 0 \\ 1 & \frac{-1+\sqrt{3}}{2} & 0 \\ 0 & 0 & \frac{7+\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (0,0,0)$$

$\frac{1+\sqrt{3}}{2}x + y = 0 \qquad x + \frac{-1+\sqrt{3}}{2}y = 0 \qquad \frac{7+\sqrt{3}}{2}z = 0$

$y = -\frac{1+\sqrt{3}}{2}x \qquad x + \frac{-1+\sqrt{3}}{2}\left(-\frac{1+\sqrt{3}}{2}x\right) = 0 \qquad z = 0$

$0 = 0$

Solución SEL:

$\left(x, -\frac{1+\sqrt{3}}{2}x, 0\right)$

$x\left(1, -\frac{1+\sqrt{3}}{2}, 0\right)$

$$\boxed{E_{-\frac{5+\sqrt{3}}{2}} = \left\langle \left(1, -\frac{1+\sqrt{3}}{2}, 0\right) \right\rangle}$$

Vector Propio

Vectores Propios: c)