Tarea 7 Sumas de Riemann:

1: $\int_2^6 (3x+2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

$\Delta x = \frac{6-2}{n} = \frac{4}{n}$

$x_i = 2 + \frac{4i}{n}$

$\sum_{i=1}^{n} \left(8 + \frac{12i}{n}\right)\frac{4}{n} =$

$f(x_i) = 3\left(2+\frac{4i}{n}\right) + 2 = 8 + \frac{12i}{n}$

$= \frac{4}{n}\sum_{i=1}^{n} 8 + \frac{12}{n}\sum_{i=1}^{n} i = \frac{4}{n}\left(8n - \frac{\cancel{12}^{6}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right)\right)$

$= \frac{4}{n}\left(8n - (6n+6)\right) = \frac{4}{\cancel{n}}(2n-6) = 8 - \frac{24}{n}$

$\lim_{n\to\infty} 8 - \frac{24}{n} = 8 - \lim_{n\to\infty} \frac{24}{n} = \boxed{8}$

2: $\int_{-1}^{4} |x|\,dx = \int_{-1}^{0} (-x)\,dx + \int_0^4 x\,dx$

$\Delta x = \frac{0-(-1)}{n} = \frac{1}{n}$

$x_i = -1 + \frac{i}{n}$

$f(x_i) = -\left(-1+\frac{i}{n}\right) = 1 - \frac{i}{n}$

$\sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n}\left(1-\frac{i}{n}\right)\frac{1}{n} = \frac{1}{n}\left(\sum_{i=1}^{n} 1 - \frac{1}{n}\sum_{i=1}^{n} i\right)$

$= \frac{1}{n}\left(n - \frac{1}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{2}\right)\right) = \frac{1}{\cancel{n}}\left(n - \frac{n+1}{2}\right) = \frac{1}{\cancel{n}}\left(\frac{\cancel{n}}{2} - \frac{1}{2}\right)$

$= \frac{1}{2} - \frac{1}{2n}$ | $\lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{2n}\right) = \frac{1}{2} - \lim_{n\to\infty}\frac{1}{2n} = \frac{1}{2}$

$\Delta x = \frac{4}{n}$ $\quad \sum_{i=1}^{n} \left(\frac{4i}{n}\right)\frac{4}{n} = \frac{16}{n^2}\sum_{i=1}^{n} i = \frac{\cancel{16}^{8}}{\cancel{n^2}n}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right)$

$x_i = \frac{4i}{n}$ $\quad = \frac{8n+8}{n} = 8 + \frac{8}{n}$

$f(x_i) = \frac{4i}{n}$

$\lim_{n \to \infty} 8 + \frac{8}{n} = 8$ $\quad | \quad \therefore \int_{-1}^{4} |x| = 8 + \frac{1}{2} = \boxed{\frac{17}{2}}$

3: $\int_2^4 (x^3 + 2x)\,dx$

$\Delta x = \frac{4-2}{n} = \frac{2}{n}$

$x_i = 2 + \frac{2i}{n}$

$f(x_i) = \left(2 + \frac{2i}{n}\right)^3 + 2\left(2 + \frac{2i}{n}\right) = 8 + \frac{24i}{n} + \frac{24i^2}{n^2} + \frac{8i^3}{n^3} + 4 + \frac{4i}{n}$

$= 12 + \frac{28i}{n} + \frac{8i^3}{n^3} + \frac{24i^2}{n^2}$

$\frac{2}{n}\left(\sum_{i=1}^{n} 12 + \frac{28}{n}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^3 + \frac{24}{n^2}\sum_{i=1}^{n} i^2\right)$

$= \frac{2}{n}\left[12n + \frac{\cancel{28}^{14}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + \frac{\cancel{8}^{2}}{\cancel{n^3}n}\left(\frac{\cancel{n^2}(n+1)^2}{\cancel{4}}\right) + \frac{\cancel{24}^{4}}{\cancel{n^2}n}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right)\right]$

$= \frac{2}{n}\left[12n + 14n + 14 + \frac{2(n^2+2n+1)}{n} + \frac{4(2n^2+3n+1)}{n}\right]$

$= \frac{2}{n}\left[26n + \cancel{14} + 2n + \cancel{2} + \frac{1}{n} + \cancel{8n} + \cancel{12} + \frac{1}{n}\right]$

$= \frac{2}{\cancel{n}}\left[36\cancel{n} + 28 + \frac{2}{n}\right] = 72 + \frac{56}{n} + \frac{4}{n^2}$

$\lim_{n \to \infty} \left(72 + \cancel{\frac{56}{n}}^{0} + \cancel{\frac{4}{n^2}}^{0}\right) = \boxed{72}$

4: $\int_{-1}^{1} (x-1)^2\,dx = \int_{-1}^{1} (x^2 - 2x + 1)\,dx$

$\Delta x = \frac{1+1}{n} = \frac{2}{n}$ $\quad x_i = -1 + \frac{2i}{n}$

$f(x_i) = \left(-1+\frac{2i}{n}\right)^2 - 2\left(-1+\frac{2i}{n}\right)+1 = 1-\frac{4i}{n}+\frac{4i^2}{n^2}+2-\frac{4i}{n}+1$

$= \frac{4i^2}{n^2} - \frac{8i}{n} + 4$

4.

$\frac{2}{n}\left[\frac{4}{n^2}\sum_{i=1}^{n} i^2 - \frac{8}{n}\sum_{i=1}^{n} i + \sum_{i=1}^{n} 4\right]$

$= \frac{2}{n}\left[\frac{4}{n^2}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right) - \frac{8}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + 4n\right]$

$= \frac{2}{n}\left(\frac{4(2n^2+3n+1)}{6n} - \cancel{4n} - 4 \cancel{+4n}\right) = \frac{2}{\cancel{n}}\left(\frac{4}{3}\cancel{n} - 2 + \frac{2}{3n}\right)$

$= \frac{8}{3} - \frac{4}{n} + \frac{4}{3n^2}$ | $\lim_{n\to\infty}\left(\frac{8}{3} - \cancel{\frac{4}{n}} + \cancel{\frac{4}{3n^2}}\right) = \boxed{\frac{8}{3}}$

5: $\int_{-1}^{3} x^5\,dx = \left[\frac{x^6}{6}\right]_{-1}^{3} = \left(121.5 - \frac{1}{6}\right) = \boxed{121.\overline{33}}$

6: $\int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \boxed{\frac{1}{2}}$

7: $\int_0^1 (2 + x\sqrt{x})\,dx = \int_0^1 \left(2 + x^{3/2}\right)dx$

$= \left[2x + \frac{2}{5}x^{5/2}\right]_0^1 = \boxed{\frac{12}{5}}$

8: $\int_1^9 \frac{1}{2x}\,dx = \left[\frac{1}{2}\ln|x|\right]_1^9 = \boxed{1.0986} - 0$

9: $\int_2^8 (4x+3)\,dx = \left[2x^2 + 3x\right]_2^8 = 152 - 14$

$= \boxed{138}$

10: $\int_0^4 (1+3y-y^2)\,dy = \left[y + \frac{3}{2}y^2 - \frac{y^3}{3}\right]_0^4$

$= 20/3 - 0 = \boxed{\frac{20}{3} = 6.\overline{6}}$

11: $\int_0^4 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^4 = \boxed{\frac{16}{3}}$

12: $\int_1^4 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^4 = 4-2 = \boxed{2}$

13: $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin t\,dt = \left[-\cos t\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = -\frac{1}{2} - \left(-\frac{\sqrt{2}}{2}\right)$

$= \boxed{0.2071}$