T8. Solidos de Revolucion

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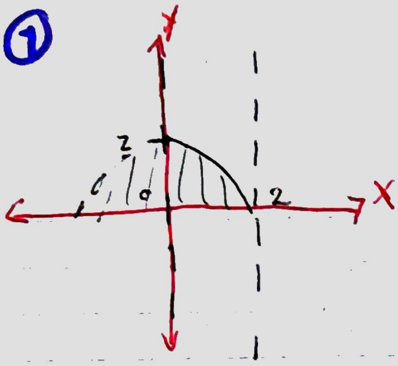


$y = 2 - \frac{x^2}{2}$

$f(2)_y = 2 - \frac{4}{2} = 0$

$-\frac{x^2}{2} = y - 2$

$x^2 = -2y + 4$

$x = \sqrt{-2y+4}$

$$\pi \int_0^2 (\sqrt{2y+4})^2 \, dy = \pi \int_0^2 (2y+4) \, dy$$

$$= \pi \left[-y^2 + 4y\right]_0^2 = \pi(4-0)$$

$$= \boxed{4\pi \, u^3}$$

2 Eje y

$\cancel{y = x - x^2}$ $f(x) = x - x^2$

$y = 0$

$x - x^2 = 0$

$x(1-x) = 0$

$x_1 = 0 \quad x_2 = 1$

$$2\pi \int_0^1 x(x - x^2)\, dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 dx$$

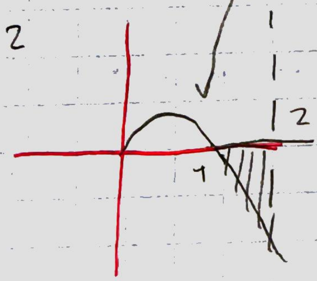
$$= 2\pi\left(\frac{1}{12} - 0\right) = \boxed{\frac{1}{6}\pi \, u^3}$$

3 Eje $x = 2$

$y = x - x^2$

$y = 0$

$\cancel{y} = x^2$



$$-2\pi \int_1^2 x(x - x^2)\, dx$$

$$= -2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_1^2$$

$$= -2\pi\left(-\frac{4}{3} - \frac{1}{12}\right)$$

$$= \boxed{\frac{17}{6}\pi}$$

4 1: Volumen eje $x = 4$

$y = x^2$

$y = 6x - x^2$

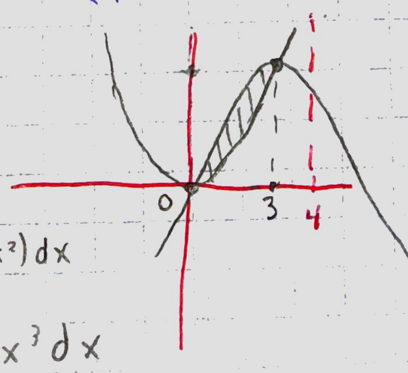
$x^2 = 6x - x^2$

$2x^2 - 6x = 0$

$x(2x - 6) = 0$

$x_1 = 0 \quad 2x = 6$

$x_2 = 3$



$$2\pi \int_0^3 x(6x - x^2)\, dx - 2\pi \int_0^3 x(x^2)\, dx$$

$$= 2\pi \int_0^3 (6x^2 - x^3)\, dx - 2\pi \int_0^3 x^3 \, dx$$

$$2\pi\left[2x^3 - \frac{x^4}{4}\right]_0^3 - 2\pi\left[\frac{x^4}{4}\right]_0^3 = 2\pi\left(\frac{175}{4}\right) - 2\pi\left(\frac{81}{4}\right)$$

$$= 2\pi\left(\frac{132}{4} - \frac{81}{4}\right) = \boxed{27\pi}$$

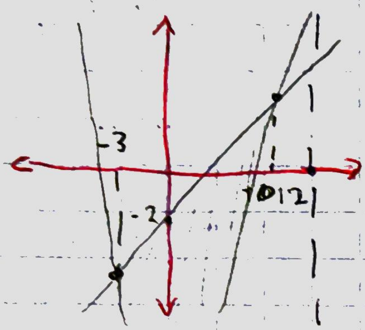
3: Given $x = 12$

$y = 4x^2 - 21x - 122$

$y = 7x - 2$

$$4x^2 - 21x - 122 = 7x - 2$$

$$4x^2 - 28x - 120 = 0$$



$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(4)(-120)}}{2(4)}$$

$x_1 = 10$ $\quad$ $x_2 = -3$

$$2\pi\int_{-3}^{10} (12 - x)\left(7x - 2 - (4x^2 - 21x - 122)\right) dx$$

$$= 2\pi\int_{-3}^{10} (12 - x)(-4x^2 + 28x + 120)\, dx = 2\pi\int_{-3}^{10} \cancel{(-48x^2 + 336x + 1440 + 4x^3 - 28x^2 - \ldots}$$

$$= 2\pi\int_{-3}^{10} (4x^3 - 76x^2 + 216x + 1440)\, dx = 2\pi\left[x^4 - \frac{76}{3}x^3 + 108x^2 + 1440x\right]_{-3}^{10}$$

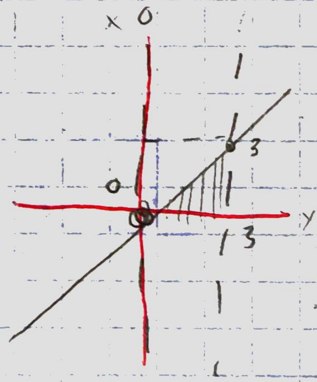
$$= 2\pi\left(\frac{29600}{3} - (-2583)\right) = 2\pi\left(\frac{37349}{3}\right) = \boxed{\frac{74698}{3}\pi}$$

4: eje y

$y = x$

$x = 0$

$x = 3$



$x = y$

$$\pi\int_0^3 (3)^2 - (y)^2\, dy$$

$$= \pi\int_0^3 (9 - y^2)\, dy$$

$$= \pi\left[9y - \frac{y^3}{3}\right]_0^3$$

$$= \pi(24) = 24\pi$$

$$2\pi\int_0^3 x(x)\, dx = 2\pi\left[\frac{x^3}{3}\right]_0^3$$

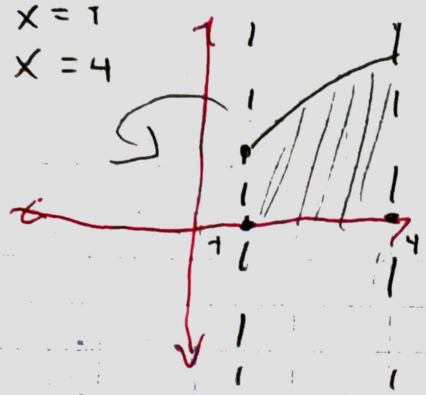
$$= 2\pi(3) = 6\pi$$

⑤ Calcular volumen de solido

$y = \sqrt{x}$ ⟲ eje y

$x = 1$

$x = 4$



$$2\pi\int_1^4 (x+1)(\sqrt{x})\,dx = 2\pi\int_1^4 (x\sqrt{x}+\sqrt{x})\,dx$$

$$= 2\pi\int_1^4 (x^{3/2}+\sqrt{x})\,dx = 2\pi\left[\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}\right]_1^4$$

$$= 2\pi\left(\frac{272}{15}-\frac{16}{15}\right) = \frac{512}{15}\pi$$

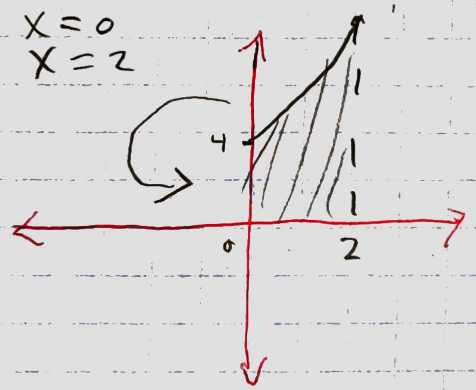
$$= \boxed{107.23\ u^2}$$

6: Calcular volumen de solido

$y = x^2+4$

$x = 0$

$x = 2$



⟲ eje y $\quad 2\pi\int_0^2 x(x^2+4)\,dx =$

$$= 2\pi\int_0^2 (x^3+4x)\,dx = 2\pi\left[\frac{x^4}{4}+2x^2\right]_0^2$$

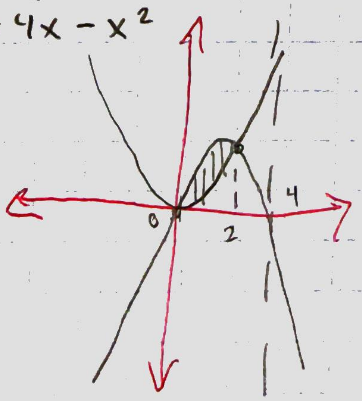
$$= 2\pi(12-0) = 24\pi$$

$$= \boxed{75.3982\ u^2}$$

7: Calcular el volumen del solido

$y = x^2$

$y = 4x - x^2$



⟲ $x = 4$

$$x^2 = 4x - x^2$$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

$2x = 0 \qquad x - 2 = 0$

$\boxed{x_1 = 0} \qquad \boxed{x_2 = 2}$

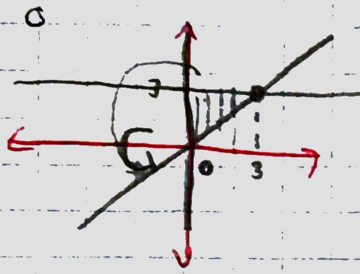
$$2\pi\int_0^2 (4-x)(4x-2x^2)\,dx$$

$= 2\pi \int_0^2 (16x - 8x^2 - 4x^2 + 2x^3)\,dx = 2\pi \int_0^2 (2x^3 - 12x^2 + 16x)\,dx$

$= 2\pi \left[\frac{1}{2}x^4 - 4x^3 + 8x^2\right]_0^2 = 2\pi(8 - 0) = \boxed{16\pi}$

8: Calcular el volumen del solido:

$y = x$
$y = 3$ $\quad$ ↺ eje x $\quad$ $x = 3$
$x = 0$



$\pi \int_0^3 (3^2 - x^2)\,dx$

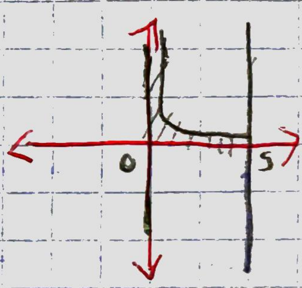
$= \pi \int_0^3 (9 - x^2)\,dx$

$= \pi \left[9x - \frac{x^3}{3}\right]_0^3$

$= \pi(18 - 0) = \boxed{18\pi}$

9: Calcular el volumen de solido

$y = \frac{1}{x}$ $\quad$ ↺ eje x
$x = 0$
$x = 5$

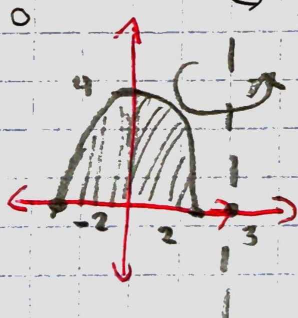


$\pi \int_0^5 \left(\frac{1}{x^2}\right) dx$

Indeterminado //

10: Calcular el volumen del solido

$y = 4 - x^2$ $\quad$ ↺ $x = 3$
$y = 0$



$4 - x^2 = 0 \qquad x_1 = 2$
$\qquad\qquad\qquad x_2 = -2$

$x^2 = 4$

$x = \sqrt{4}$

$2\pi \int_{-2}^{2} (3 - x)(4 - x^2)\,dx$

$= 2\pi \int_{-2}^{2} (12 - 3x^2 - 4x + x^3)\,dx$

$= 2\pi \left[12x - x^3 - 2x^2 + \frac{x^4}{4}\right]_{-2}^{2}$

$= 2\pi(32) = \boxed{64\pi}$