T11. Sucesiones y Series

a) $a_n = 2^n$ $\quad a_1 = 2 \quad a_2 = 4 \quad a_3 = 8 \quad a_4 = 16$
$a_5 = 32$

b) $a_n = \frac{3^n}{n!}$ $\quad a_1 = 3 \quad a_2 = \frac{9}{2} \quad a_3 = \frac{27}{6} \quad a_4 = \frac{81}{24}$
$a_5 = \frac{243}{120}$

c) $a_n = \frac{2n}{n+3}$ $\quad a_1 = \frac{2}{4} \quad a_2 = \frac{4}{5} \quad a_3 = 1 \quad a_4 = \frac{8}{7}$
$a_5 = \frac{10}{8}$

d) $a_n = 10 + \frac{2}{n} + \frac{6}{n^2}$ $\quad a_1 = 18 \quad a_2 = \frac{25}{2} \quad a_3 = \frac{34}{3}$
$a_4 = \frac{87}{8} \quad a_5 = \frac{266}{25}$

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a) $\lim\limits_{n\to\infty} \frac{5n^2}{n^2+2} = \lim\limits_{n\to\infty} \frac{\frac{5n^2}{n^2}}{\frac{n^2}{n^2}+\frac{2}{n^2}} = \lim\limits_{n\to\infty} \frac{5}{1+0} = \boxed{5}$

b) $\lim\limits_{n\to\infty} \frac{2n}{\sqrt{n^2+1}} = \lim\limits_{n\to\infty} \frac{2}{\sqrt{1+\frac{1}{n^2}}} = \boxed{2}$

c) $\lim\limits_{n\to\infty} 5 - \frac{1}{n^2} = \boxed{5}$

d) $\lim\limits_{n\to\infty} \cos\left(\frac{2}{n}\right) = \cos\left(\lim\limits_{n\to\infty} \frac{2}{n}\right) = \cos(0) = \boxed{1}$

$a = \frac{8n}{n+1}$ $b = 4(0.5)^{n-1}$

$c = \frac{(-1)^n}{n}$ $d = (-1)^n$

$e = \frac{4^n}{n!}$ $f = \frac{4}{n+1}$

4:

a) $\lim_{n\to\infty} \frac{5n^2+2n+6}{7n^2-6} = \lim_{n\to\infty} \frac{5+\frac{2}{n}+\frac{6}{n^2}}{7-\frac{5}{n^2}} = \boxed{\frac{5}{7}}$

Converge a $\frac{5}{7}$

b) $\lim_{n\to\infty} n\sin\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) \lim_{n\to\infty} n = \boxed{\infty}$

~~Converge~~

Diverge a infinitivo

c) $\lim_{n\to\infty} (5)^{\frac{1}{n}} = \lim_{n\to\infty} (5)^0 = \boxed{1}$ tiende a 0

Converge a 1

d) $\lim_{n\to\infty} \frac{4n+7}{2n+1} = \lim_{n\to\infty} \frac{4+0}{2+0} = \boxed{2}$

Converge a 2

e) $\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = \lim_{n\to\infty} \frac{n^2-1}{n^2+1} = \lim_{n\to\infty} \frac{1-0}{1+0} = \boxed{1}$

f) $\lim_{an\to\infty} \frac{3\sqrt{n}}{\sqrt{n}+2} = \lim_{n\to\infty} \frac{3}{1+0} = \boxed{3}$

9) $\lim_{n\to\infty} \frac{3+5n^2}{1+n} = \lim_{n\to\infty} \frac{\frac{3}{n^2}+5}{\frac{1}{n^2}+\frac{1}{n}} = \frac{5}{0}$

Diverge a infinito

5:

a) $\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$ | $S_1 = \frac{1}{2}$ $S_2 = 0.55$ $S_3 = 0.56\overline{1}$ $S_4 = 0.5647$

$S_5 \doteq 0.5663$ $S_6 = 0.56708$ $S_7 = 0.5674$

$S_8 = 0.5677$

Converge a

b) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ | $S_1 = 1.4427$ $S_2 = 2.3529$ $S_3 = 3.0743$

$S_4 = 3.6956$ $S_5 = 4.2537$

$S_6 = 4.7676$ $S_7 = 5.2489$ $S_8 = 5.7036$

Converge

c) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ | $S_1 = 0.\overline{3}$ $S_2 = 0.4583$ $S_3 = 0.525$

$S_4 = 0.5666$ $S_5 = 0.5952$ $S_6 = 0.6160$

$S_7 = 0.6319$ $S_8 = 0.6\overline{4}$

6:

a) $\sum_{n=0}^{\infty} 3\left(\frac{3}{n}\right)^n = 3\sum_{n=0}^{\infty} \left(\frac{3}{n}\right)^n$

Criterio de ~~divergencia~~ Raiz

$3\lim_{n\to\infty} \sqrt[n]{\left(\frac{3}{n}\right)^n} = 3\lim_{n\to\infty} \frac{3}{n} = 0$

$\therefore$ Converge por que $L < 1$

b) $\sum_{n=0}^{\infty} (-1)^n \underbrace{\cot^n\left(\frac{\pi}{3}\right)}_{r}$

Criterio de la razón

$|r| = 0.57$

$|r| < 1$

Convergente

c) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ | L'Hopital $\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1} = 1$

$1 \neq 0 \therefore$ Diverge

d) $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$ | $\lim_{n\to\infty} \frac{2^n}{2^n \cdot 2} + \frac{1}{2^n \cdot 2} = \lim_{n\to\infty} \left(\frac{1}{2} + 0\right) = \frac{1}{2}$

$\frac{1}{2} \neq 0 \therefore$ Diverge

e) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ | $\lim_{n\to\infty} \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1} = 1$

$1 \neq 0 \therefore$ Diverge

f) $\sum_{n=0}^{\infty} \frac{15}{4} \underbrace{\left(-\frac{1}{4}\right)^n}_{r}$ $\quad |r| = \frac{1}{4}$

$|r| < 1 \therefore$ Converge

7:

a) $4.\overline{61}$ $\quad 4 \sum_{n=0}^{\infty} 0.61(10^{-2})^n$

$$\boxed{4+\frac{0.61}{1-(10^{-2})}} = \frac{457}{99}$$

b) $0.\overline{15}$ $\quad \sum_{n=0}^{\infty} 0.15(10^{-2})^n$

$$\boxed{\frac{0.15}{1-(10^{-2})} = \frac{5}{33}}$$

c) $2.\overline{01}$ $\quad 2 + \sum_{n=0}^{\infty} 0.01(10^{-2})^n$

$$\boxed{2+\frac{0.01}{1-(10^{-2})} = \frac{199}{99}}$$