Tarea 1:

- Comprueba que $F(x)$ es una antiderivada de $f(x)$

1: $F(x) = \frac{1}{3}x^3 + 2x^2 - x + 2$; $f(x) = x^2 + 4x - 1$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d\left(\frac{1}{3}x^3 + 2x^2 - x + 2\right)}{dx} = x^2 + 4x - 1 = f(x)$$

$\therefore F(x)$ es una antiderivada de $f(x)$

- 2: $F(x) = xe^x + \pi$; $f(x) = e^x(1 + x)$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(xe^x + \pi)}{dx} = e^x + xe^x = e^x(1 + x) = f(x)$$

$\therefore F(x)$ es una antiderivada de $f(x)$

3: $F(x) = \sqrt{2x^2 - 1}$; $f(x) = \frac{2x}{\sqrt{2x^2 - 1}}$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d\left(\sqrt{2x^2 - 1}\right)}{dx} = \frac{4x}{\cancel{2}\sqrt{2x^2 - 1}} = \frac{2x}{\sqrt{2x^2 - 1}} = f(x)$$

$\therefore F(x)$ es una antiderivada de $f(x)$

4: $F(x) = x\ln x - x$; $f(x) = \ln x$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(x\ln x - x)}{dx} = \ln x + \cancelto{1}{\frac{x}{x}} - 1 = \ln x = f(x)$$

$\therefore F(x)$ es la antiderivada de $f(x)$

5: $F(x) = -\cos x$; $f(x) = \sin x$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(-\cos x)}{dx} = -(-\sin x) = \sin x = f(x)$$

$\therefore F(x)$ es la antiderivada de $f(x)$

6: $F(x) = \ln x + \sin^2 x + 3x$; $f(x) = \frac{1}{x}(3x + x\cos x \sin x + 1)$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(\ln x)}{dx} + \frac{d(\sin^2 x)}{dx} + \frac{d(3x)}{dx}$$

$$= \frac{1}{x} + 2\cos x \, \text{sen}\, x + 3 = \frac{2x\cos x \, \text{sen}\, x + 1}{x} + 3 = \frac{3x + 2x\cos x \sin x + 1}{x}$$

$$= \frac{1}{x}(3x + x\cos x \sin x + 1) \neq f(x)$$

$\therefore$ F(x) no es una antiderivada de f(x)

7: $F(x) = e^x + 3\sin 3x + \sqrt{x}$; $f(x) = e^x + 9\cos 3x + \frac{1}{2\sqrt{x}}$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(e^x)}{dx} + 3\frac{d(\sin 3x)}{dx} + \frac{d(\sqrt{x})}{dx}$$

$$= e^x + 3(3\cos(3x)) + \frac{1}{2\sqrt{x}} = e^x + 9\cos(3x) + \frac{1}{2\sqrt{x}} = f(x)$$

$\therefore$ F(x) es la antiderivada de f(x)

8: $F(x) = \tan^3 x^2$; $f(x) = 6x\tan^2 x^2 (\sec^2 x^2)$

$$\frac{d(F(x))}{dx} = f(x) \rightarrow \frac{d(\tan^3 x^2)}{dx} = 3\tan^2 x^2 (2x\sec^2 x^2)$$

$$= 6x\tan^2 x^2 (\sec^2 x^2) = f(x)$$

$\therefore$ F(x) es la antiderivada de f(x)

Hallar la integral indefinida en los siguientes ejercicios:

1: $\int 3x^2 dx = 3\int x^2 dx = 3\left(\frac{x^3}{3}\right) = \boxed{x^3 + C}$

2: $\int (3x^3 + 4x^2 + x + 3)\, dx = 3\int x^3 dx + 4\int x^2 dx + \int x\,dx + 3\int dx$

$= 3\left(\frac{x^4}{4}\right) + 4\left(\frac{x^3}{3}\right) + \frac{x^2}{2} + 3x + C = \boxed{\frac{3}{4}x^4 + \frac{4}{3}x^3 + \frac{1}{2}x^2 + 3x + C}$

3: $\int \left(\frac{2}{x^2}\right) dx = 2\int \frac{1}{x^2} dx = 2\int x^{-2} dx = 2\left(\frac{x^{-1}}{-1}\right) = 2\left(-\frac{1}{x}\right) + C$

$\boxed{= -\frac{2}{x} + C}$

4: $\int \frac{1}{3x^5} dx = \frac{1}{3}\int \frac{1}{x^5} dx = \frac{1}{3}\int x^{-5} dx = \frac{1}{3}\left(\frac{x^{-4}}{-4} + C\right)$

$= \frac{1}{3}\left(-\frac{1}{4x^4}\right) = \boxed{-\frac{1}{12x^4} + C}$

5: $\int \frac{3}{\sqrt{t}} dt = 3\int \frac{1}{t^{1/2}} dt = 3\int t^{-1/2} dt = 3(2\sqrt{t}) = \boxed{6\sqrt{t} + C}$

6: $\int (3 - 2x)\, dx = 3\int dx - 2\int x\,dx = 3x - 2\left(\frac{x^2}{2}\right) + C$

$= \boxed{3x - x^2 + C}$

7: $\int \pi \sqrt{t}\, dt = \pi \int \sqrt{t}\, dt = \pi\left(\frac{t^{3/2}}{\frac{3}{2}}\right) = \pi\left(\frac{2t^{3/2}}{3}\right) = \boxed{\frac{2\pi}{3}\sqrt{t^3} + C}$

8: $\int (x^2 + x + x^{-3})\, dx = \int x^2 dx + \int x\,dx + \int x^{-3} dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^{-2}}{-2} + C = \boxed{\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2x^2}}$

9: $\int (0.3t^2 + 0.003t + 2)\, dt = 0.3\int t^2 dt + 0.003\int t\,dt + 2\int dt$

$= \frac{3}{10}\left(\frac{t^3}{3}\right) + \frac{3}{1000}\left(\frac{t^2}{2}\right) + 2t + C = \boxed{\frac{1}{10}t^3 + \frac{3}{2000}t^2 + 2t + C}$

10: $\int (1 + u + u^2)\,du = \int du + \int u\,du + \int u^2\,du$

$= u + \frac{u^2}{2} + \frac{u^3}{3} + C = \boxed{\frac{1}{3}u^3 + \frac{1}{2}u^2 + u + C}$

11: $\int (4x^3 - \frac{2}{x^2} - 1)\,dx = 4\int x^3\,dx - 2\int x^{-2}\,dx - \int dx$

$= 4(\frac{x^4}{4}) - 2(-x^{-1}) - x + C = \boxed{x^4 + \frac{2}{x} - x + C}$

12: $\int (6x^3 + \frac{3}{x^2} - x)\,dx = 6\int x^3\,dx + 3\int x^{-2}\,dx - \int x\,dx$

$= 6(\frac{x^4}{4}) + 3(-x^{-1}) - \frac{x^2}{2} + C = \boxed{\frac{3}{2}x^4 - \frac{3}{x} - \frac{1}{2}x^2 + C}$

13: $\int (\sqrt{x} + \frac{3}{\sqrt{x}})\,dx = \int x^{1/2}\,dx + 3\int x^{-1/2}\,dx$

$= \frac{x^{3/2}}{\frac{3}{2}} + 3(\frac{x^{1/2}}{\frac{1}{2}}) = \boxed{\frac{2}{3}\sqrt{x^3} + 6\sqrt{x} + C}$

14: $\int \left(\frac{u^3 + 2u^2 - u}{3u}\right) du = \int \frac{u^3}{3u}\,du + \int \frac{2u^2}{3u}\,du - \int \frac{u}{3u}\,du$

$= \frac{1}{3}\int u^2\,du + \frac{2}{3}\int u\,du - \frac{1}{3}\int du = \frac{1}{3}(\frac{u^3}{3}) + \frac{2}{3}(\frac{u^2}{2}) - \frac{1}{3}u$

$\boxed{= \frac{1}{9}u^3 + \frac{1}{3}u^2 - \frac{1}{3}u + C}$

15: $\int \left(\frac{x^4 - 1}{x^2}\right) dx = \int \frac{x^4}{x^2}\,dx - \int x^{-2}\,dx = \int x^2\,dx - \int x^{-2}\,dx$

$= \boxed{\frac{x^3}{3} + \frac{1}{x} + C}$

16: $\int \sqrt{t}\,(t^2+t-1)\,dt = \int t^{5/2}dt + \int t^{3/2}dt - \int t^{1/2}dt$

$= \frac{t^{7/2}}{\frac{7}{2}} + \frac{t^{5/2}}{\frac{5}{2}} - \frac{t^{3/2}}{\frac{3}{2}} + C = \boxed{\frac{2}{7}\sqrt{t^7} + \frac{2}{5}\sqrt{t^5} - \frac{2}{3}\sqrt{t^3} + C}$

17: $\int \frac{ds}{(s+1)^{-2}} = \int (s+1)^2 ds = \int (s^2+2s+1)ds$

$= \int s^2 ds + 2\int s\,ds + \int ds = \frac{s^3}{3} + 2\left(\frac{s^2}{2}\right) + s + C$

$= \boxed{\frac{1}{3}s^3 + s^2 + s + C}$

18: $\int \left(\frac{x^3+x^2-x+1}{x^2}\right)dx = \int \frac{x^3}{x^2}dx + \int \frac{x^2}{x^2}dx - \int \frac{x}{x^2}dx + \int \frac{1}{x^2}dx$

$= \int x\,dx + \int dx - \int \frac{1}{x}dx + \int x^{-2}dx = \boxed{\frac{x^2}{2} + x - \ln|x| - \frac{1}{x} + C}$

19: $\int e^t dt = \boxed{e^t + C}$

20: $\int \left(\frac{1}{x^2} - \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt{x}}\right)dx = \int x^{-2}dx - \int x^{-3/2}dx + \int x^{-1/2}dx$

$= -\frac{1}{x} + \frac{x^{-1/2}}{\frac{1}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C = \boxed{-\frac{1}{x} + \frac{1}{2\sqrt{x}} + 2\sqrt{x} + C}$

21: $\int \left(\frac{t^3+\sqrt[3]{t}}{t^2}\right)dt = \int \frac{t^3}{t^2}dt + \int \frac{\sqrt[3]{t}}{t^2}dt = \int t\,dt + \int t^{-5/3}dt$

$= \frac{t^2}{2} + \left(\frac{t^{-2/3}}{-\frac{2}{3}}\right) = \frac{t^2}{2} - \frac{3}{2\sqrt[3]{t^2}} + C = \boxed{\frac{1}{2}t^2 - \frac{3}{2\sqrt{t^3}} + C}$

22: $\int \left(\frac{\sin x}{\cos^2 x}\right)dx = \int \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right)dx = \int (\tan x \cdot \sec x)dx$

$= \boxed{\sec x + C}$

$23: \int (t^2 - \sin t)\, dt = \int t^2 dt - \int \sin t\, dt$

$= \frac{t^3}{3} - (-\cos x) + C = \boxed{\frac{1}{3}t^3 + \cos x + C}$

Ejercicios:

$V(0) = -4.9(0)^2 + 60(0) + C = 6 \;\therefore V(x) = -4.9x^2 + 60x + 6$

$C = 6$

1: $V(0) = 6$ $\quad V(x) = -9.81\int x\,dx + 60\int dx = -4.9x^2 + 60x + C$

$V'(0) = 60$ $\quad V'(0) = -9.81(0) + C = 60 \;\therefore V'(x) = -9.81x + 60$

$C = 60$

$V''(x) = -9.81$ $\quad \therefore V'(x) = -9.81\int dx = -9.81x + C$

$V'(x) = 0$ $\quad V(6.1162) = -4.9(6.1162)^2 + 60(6.1162) + 6$

$= 215.89$ metros

$-9.81x + 60 = 0$

$-9.81x = -60$

$x = 6.1162$

2: $s(t) = -4.9t^2 + V_0 t + S_0$

$s(0) = S_0$

$s'(0) = V_0$

$\boxed{s''(t) = -9.8}$

$s'(t) = \int -9.8\, dt$

$s'(t) = -9.8t + C$ $\quad \boxed{s'(t) = -9.8t + V_0}$

$s'(0) = -9.8(0) + C = V_0$

$C = V_0$

$s(t) = \int (-9.8t + V_0)\, dt$

$s(t) = -9.8\frac{t^2}{2} + V_0 t + C = -4.9t^2 + V_0 t + C$

$s(0) = -4.9(0)^2 + V_0(0) + C = S_0$ $\quad \therefore \boxed{s(t) = -4.9t + V_0 t + S_0}$

$C = S_0$

Si esta relacionada

3:

$s(0) = 2\ m$

$s'(0) = 10\ m/s$

$s''(x) = -9.81\ m/s^2$

$s'(x) = -9.81 \int dx = -9.81x + c$

$s'(0) = -9.81(0) + c = 10 \quad \therefore\ s'(x) = -9.81x + 10$

$c = 10$

$s(x) = -9.81 \int x\,dx + 10 \int dx = -4.9x^2 + 10x + c$

$s(0) = -4.9(0)^2 + 10(0) + C = 2 \quad \therefore\ s(x) = -4.9x^2 + 10x + 2$

$C = 2$

$s'(x) = 0$

$-9.81x + 10 = 0$

$-9.81x = -10$

$x = 1$

$\boxed{s(1) = -4.9(1)^2 + 10(1) + 2 = 7.1\ m}$

4:

$s(20) = 0$

$s''(x) = -1.6$

$s'(0) = 0$

$s'(x) = -1.6 \int dx = -1.6x + c$

$s'(0) = -1.6(0) + c = 0 \quad \therefore\ s'(x) = -1.6x$

$c = 0$

$s(x) = -1.6 \int x\,dx = -0.8x^2 + C$

$s(20) = -0.8(20)^2 + c = 0 \quad \therefore\ s(x) = -0.8x^2 + 320$

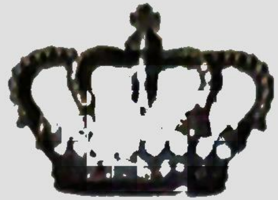
$-320 + c = 0$

$c = 320$

$s(0) = -0.8x^2 + 320 = \boxed{320\ m}$

$s'(20) = -1.6(20) = \boxed{-32\ m/s}$

cayo desde 320 m y
impacto a -32 m/s



5: $s(0) = 0$

$s'(0) = 25 \, km/h = 6.9\bar{4} \, m/s$

$s'(13) = 80 \, km/h = 22.\bar{2} \, m/s$

$$m = \frac{22.2 - 6.94}{13 - 0} = 1.1752$$

$y - y_1 = m(x - x_1)$

$y - 6.944 = 1.1752(x - 0)$

$f'(x) = 1.1752x + 6.944$

$$\boxed{f''(x) = 1.1752 \, m/s^2}$$

$$f(x) = 1.1752 \int x \, dx + 6.944 \int dx = 0.5876x^2 + 6.944x + C$$

$f(0) = 0 \longrightarrow f(0) = 0.5876(0)^2 + 6.944(0) + C = 0$

$C = 0$

$\therefore f(x) = 0.5876x^2 + 6.944x$

$$f(13) = 0.5876(13)^2 + 6.944(13) = \boxed{189.5764 \, m}$$