6.94

6.25 / 9

Cálculo Integral 2023-1 **Segundo Parcial**

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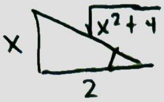
Contesta de forma clara y ordena. Incluye procedimiento, siempre que haya uno para que sea tomado en cuenta tu respuesta.

1 Calcular la Integrales definidas

1. $\int_2^4 \frac{3x}{\sqrt{x^2+4}}dx = \int_2^4 \frac{\overset{6}{\cancel{12}}\tan\theta\sec^2\theta}{2\cancel{\sec\theta}}d\theta = 6\int_2^4 \tan\theta\sec\theta\, d\theta = 6\left[\sec\theta\right]_2^4$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$



$6\tan\theta\, 2\sec^2\theta$

$= 6\left[\frac{2}{\sqrt{x^2+4}}\right]_2^4$

$= 6\left(\frac{\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right)$

$= \boxed{-1.5593}$

2. $\int_1^4 \frac{u-2}{\sqrt{u}}du = \int_1^4 \frac{u}{\sqrt{u}}du - 2\int_1^4 \frac{1}{\sqrt{u}}du = \int_1 \sqrt{u}\,du - 2\int \frac{1}{\sqrt{u}}du$

$= \left[\frac{2u^{3/2}}{3}\right]_1^4 - \left[2\sqrt{u}\right]_1^4$

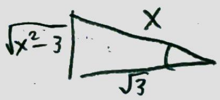
$= \left(\frac{16}{3} - \frac{2}{3}\right) - (4-2)$

$= \boxed{\frac{8}{3}}$

3. $\int \frac{\sqrt{x^2-3}}{x}dx = \int \frac{3\tan^2\theta\,\cancel{\sec\theta}}{\sqrt{3}\cancel{\sec\theta}}d\theta = \sqrt{3}\int \tan^2\theta\, d\theta = \sqrt{3}\int(\sec^2\theta - 1)d\theta$

$x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta\, d\theta$



$\theta = \operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right)$

$= \sqrt{3}\left[\int\sec^2\theta - \int d\theta\right]$

$= \sqrt{3}\left[\tan\theta - \theta\right]$

$= \sqrt{3}\left[\frac{\sqrt{x^2-3}}{\sqrt{3}} - \operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right)\right]$

$= \boxed{\sqrt{x^2-3} - \sqrt{3}\operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right) + C}$

4. $\int \frac{3x^2-5}{x^3+2x}dx$ $\left[\int \frac{3x^2-5}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}\right](x)(x^2+2)$

$3x^2 - 5 = A(x^2+2) + (Bx+C)(x) = A(x^2+2) + Bx^2 + Cx$

$A + B = 3$ $A = -\frac{5}{2}$ $B = 3 - \left(-\frac{5}{2}\right)$

$C = 0$ $B = \frac{11}{2}$

$2A = -5$

$\therefore \int \frac{3x^2-5}{x^3+2x}dx = -\frac{5}{2}\int\frac{1}{x}dx + \frac{11}{2}\int\frac{x}{x^2+2}dx = -\frac{5}{2}\ln|x| + \frac{11}{4}\int\frac{du}{u}$

$u = x^2+2$

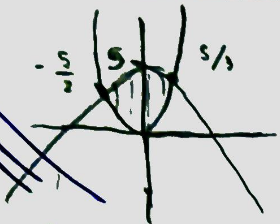
$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= -\frac{5}{2}\ln|x| + \frac{11}{4}\ln|u| + C$

$\therefore \boxed{-\frac{5}{2}\ln|x| + \frac{11}{4}\ln|x^2+2| + C}$

5. Encontrar el área entre las dos curvas $y=-\frac{x^2}{5}+x+5, y=x^2+2x$



$$-\frac{1}{5}x^2+x+5=x^2+2x$$

$$\frac{6}{5}x^2+x-5=0$$

$$x=\frac{-1\pm\sqrt{1^2-4(\frac{6}{5})(-5)}}{2(\frac{6}{5})}$$

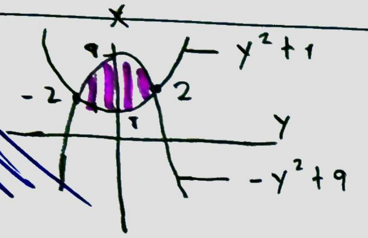
$$x_1=\frac{5}{3} \qquad x_2=-\frac{5}{2}$$

$$\int_{-\frac{5}{2}}^{\frac{5}{3}}\left[\left(-\frac{1}{5}x^2+x+5\right)-(x^2+2x)\right]dx=\int_{-\frac{5}{2}}^{\frac{5}{3}}\left(-\frac{6}{5}x^2-x+5\right)dx$$

$$=\left[-\frac{2}{5}x^3-\frac{x^2}{2}+5x\right]_{-\frac{5}{2}}^{\frac{5}{3}}$$

$$=\left(\frac{275}{54}-\left(-\frac{75}{8}\right)\right)=\boxed{14.4675\ u^2}$$

6. Encontrar el área entre las curvas. Encontrar el área acotada por $x=y^2+1, x=-y^2+9$. Haz un esbozo de la gráfica



$$y^2+1=-y^2+9$$

$$2y^2=8$$

$$y^2=4 \qquad y_1=2$$

$$y=\sqrt{4} \qquad y_2=-2$$

$$\therefore \int_{-2}^{2}(-y^2+9-(y^2+1))\,dy$$

$$=\int_{-2}^{2}(-2y^2+8)\,dy=\left[-\frac{2}{3}y^3+8y\right]_{-2}^{2}=\left[\frac{32}{3}-\left(-\frac{32}{3}\right)\right]=\boxed{\frac{64}{3}\ u^2}$$

7. Resuelve la siguiente ecuación diferencial $x^2y'+\frac{1}{y^2}$, $y(1)=2$

$$x^2y'+\frac{1}{y^2} \qquad y'=\frac{1}{x^2} \qquad -\frac{1}{x}+3$$

...ilizar Sumas de Riemann. Encuentra el área en general para n, utílizando las suma superior y encuentra el ...ite para $f(x) = 2x^3 + 5, [1,2]$

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_i = 1 + \frac{i}{n}$

$f(x_i) = 2\left(1+\frac{i}{n}\right)^3 + 5 = 2\left(1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right) + 5$

$= 7 + \frac{6i}{n} + \frac{6i^2}{n^2} + \frac{2i^3}{n^3}$

$\frac{1}{n}\left[\sum_{i=1}^{n} 7 + \frac{6}{n}\sum_{i=1}^{n} i + \frac{6}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n^3}\sum_{i=1}^{n} i^3\right] = \frac{1}{n}\left[7n + \frac{6}{n}\left(\frac{n(n+1)}{2}\right) + \frac{6}{n^2}\left(\frac{n(2n^2+3n+1)}{6}\right) + \frac{2}{n^3}\left(\frac{n^2(n^2+2n+1)}{4}\right)\right]$

$= \frac{1}{n}\left[7n + 3n + 3 + \frac{2n^2+3n+1}{n} + \frac{n^2+2n+1}{2n}\right]$

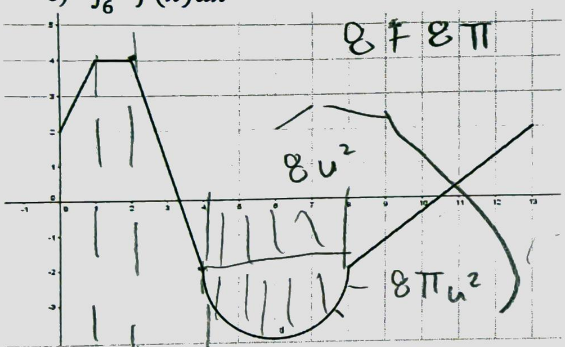
$= \frac{1}{n}\left[10n + 3 + 2n + 3 + \frac{1}{n} + \frac{1}{2}n + 1 + \frac{1}{2n}\right] = \frac{1}{n}\left[\frac{25}{2}n + 7 + \frac{3}{2n}\right] = \frac{25}{2} + \frac{7}{n} + \frac{3}{2n^2}$

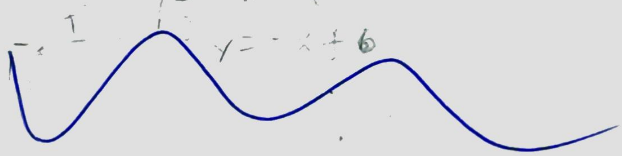
$\lim_{n \to \infty}\left(\frac{25}{2} + \frac{7}{n} + \frac{3}{2n^2}\right) = \boxed{\frac{25}{2}}$

8. Aproxima las integrales que se piden.

a) $\int_1^3 f(x)dx$
b) $\int_3^{11} f(x)dx$
c) $\int_6^{12} f(x)dx$

$(2,4)\ ,\ (4,-2)$





$2x + 2 / 4$

a) $\int_1^3 f(x) = 4 +$