UNIVERSIDAD AUTÓNOMA DE QUERÉTARO
FACULTAD DE INGENIERÍA

Laboratorio de Cálculo Integral

Nombre del Alumno	**Diego Joel Zuñiga Fragoso**	**Grupo**	**514**
Fecha de la Práctica	**24/04/2023**	**No Práctica**	**6**
Nombre de la Práctica	**Fracciones Parciales**		
Unidad	**Métodos de Integración**		

OBJETIVOS
Resolver las integrales y practicar el método de fracciones Parciales.

EQUIPO Y MATERIALES
Computadora y el programa Scientific workplace

DESARROLLO

En cada una de las partes vas a realizar integrales por el método que se te pide. **No puedes realizar la integral directamente**

Integración por Fracciones Parciales.

Realiza cada una de las siguientes integrales utilizando la opción de Scientific Work Place **Compute>Calculus> Partial Fraction** .

1. $\int \frac{7x+3}{x^2+3x-4}$	$\int \frac{7x+3}{x^2+3x-4}dx = \int \frac{7x+3}{(x+4)(x-1)}dx$ $\left[\frac{7x+3}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}\right](x+4)(x-1)$ $7x+3 = A(x-1)+B(x+4)$ $\begin{array}{c} A+B=7 \\ -A+4B=3 \end{array}$, Solution is: $[A=5, B=2]$ $\int \frac{7x+3}{(x+4)(x-1)}dx = \int \frac{5}{x+4}dx + \int \frac{2}{x-1}dx = 2\ln(x-1)+5\ln(x+4)+C$

2. $\int \frac{x+1}{(x-1)(x^2-4x+3)}dx$	$\int \frac{x+1}{(x-1)(x^2-4x+3)}dx = \int \frac{x+1}{(x-1)(x-3)(x-1)}dx$ $\left[\frac{x+1}{(x-1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-3)}\right](x-1)^2(x-3)$ $x+1 = A(x-1)(x-3) + B(x-3) + C(x-1)^2$ $= A(x^2-4x+3) + B(x-3) + C(x^2-2x+1)$ $A + C = 0$ $-4A + B - 2C = 1$ $3A - 3B + C = 1$, Solution is: $[A = -1, B = -1, C = 1]$ $\int \frac{x+1}{(x-1)^2(x-3)}dx = \int \frac{-1}{x-1}dx + \int \frac{-1}{(x-1)^2}dx + \int \frac{1}{(x-3)}dx = \ln(x-3) - \ln(x-1) + \frac{1}{x-1} + C$
3. $\int \frac{2x+1}{(x+1)(x^2-5x-6)}dx$	$\int \frac{2x+1}{(x+1)(x^2-5x-6)}dx = \int \frac{2x+1}{(x+1)(x-6)(x+1)}dx = \int \frac{2x+1}{(x+1)^2(x-6)}dx$ $\left[\frac{2x+1}{(x+1)^2(x-6)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x-6)}\right](x+1)^2(x-6)$ $2x+1 = A(x+1)(x-6) + B(x-6) + C(x+1)^2$ $2x+1 = A(x^2-5x-6) + B(x-6) + C(x^2+2x+1)$ $A + C = 0$ $-5A + B + 2C = 2$ $-6A - 6B + C = 1$, Solution is: $[A = -\frac{13}{49}, B = \frac{1}{7}, C = \frac{13}{49}]$ $\int \frac{2x+1}{(x+1)^2(x-6)} = \int \frac{-\frac{13}{49}}{x+1}dx + \int \frac{\frac{1}{7}}{(x+1)^2}dx = \int \frac{\frac{13}{49}}{x-6}dx = \frac{13}{49}\ln(x-6)$
4. $\int \frac{x-1}{x^2(x^2-4)}dx$	$\int \frac{x-1}{x^2(x^2-4)}dx = \int \frac{x-1}{x^2(x-2)(x+2)}dx =$ $\left[\frac{x-1}{x^2(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2}\right]x^2(x-2)(x+2)$ $x-1 = A(x)(x-2)(x+2) + B(x-2)(x+2) + C(x^2)(x+2) + D(x^2)(x-2)$ $x-1 = A(x^3-4x) + B(x^2-4) + C(x^3+2x^2) + D(x^3-2x^2)$ $A + C + D = 0$ $B + 2C - 2D = 0$ $-4A = 1$ $-4B = -1$, Solution is: $[A = -\frac{1}{4}, B = \frac{1}{4}, C = \frac{1}{16}, D = \frac{3}{16}]$ $\int \frac{x-1}{x^2(x-2)(x+2)}dx = \int \frac{-\frac{1}{4}}{x}dx + \int \frac{\frac{1}{4}}{x^2}dx + \int \frac{\frac{1}{16}}{x-2}dx + \int \frac{\frac{3}{16}}{x+2}dx$ $= -\frac{1}{4}\ln x - \frac{1}{4x} + \frac{1}{16}\ln(x-2) + \frac{3}{16}\ln(x+2)$

5. $\int \frac{11x+2}{2x^2-5x-3}\,dx$

$$\int \frac{11x+2}{2x^2-5x-3}\,dx = \int \frac{11x+2}{(x+3)(2x-1)}\,dx$$

$$\left[\frac{11x+2}{(x+3)(2x-1)} = \frac{A}{x+3} + \frac{B}{2x-1}\right](x+3)(2x-1)$$

$$11x+2 = A(2x-1) + B(x+3)$$

$$\begin{array}{|c|}\hline 2A + B = 11 \\ \hline -A + 3B = 2 \\ \hline \end{array}, \text{ Solution is: } \left[A = \frac{31}{7}, B = \frac{15}{7}\right]$$

$$\int \frac{11x+2}{(x+3)(2x-1)}\,dx = \int \frac{\frac{31}{7}}{x+3}\,dx + \int \frac{\frac{15}{7}}{2x-1}\,dx = \frac{31}{7}\ln(x+3) + \frac{15}{14}\ln\left(x-\frac{1}{2}\right)$$

6. $\int \frac{x^5+4}{x^3-4x^2}\,dx$

$$\int \frac{x^5+4}{x^3-4x^2}\,dx = \int \frac{x^5}{x^2(x-4)}\,dx + \int \frac{4}{x^2(x-4)}\,dx = \int \frac{x^3}{x-4}\,dx + \int \frac{4}{x^2(x-4)}\,dx$$

$$u = x-4 \quad x = u+4 \quad du = dx$$

$$\int \frac{(u+4)^3}{u}\,du = \int \frac{u^3+12u^2+48u+64}{u}\,du = \int u^2\,du + 12\int u\,du + 48\int du + 64\int \frac{1}{u}\,du$$

$$= \frac{u^3}{3} + 6u^2 + 48u + 64\ln|u|$$

$$= \frac{(x-4)^3}{3} + 6(x-4)^2 + 48(x-4) + 64\ln|x-4|$$

$$\left[\frac{4}{x^2(x-4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4}\right]x^2(x-4)$$

$$4 = A(x)(x-4) + B(x-4) + Cx^2$$

$$4 = A(x^2-4x) + B(x-4) + Cx^2$$

$$\begin{array}{|c|}\hline A + C = 0 \\ \hline -4A + B = 0 \\ \hline -4B = 4 \\ \hline \end{array}, \text{ Solution is: } \left[A = -\frac{1}{4}, B = -1, C = \frac{1}{4}\right]$$

$$\int \frac{4}{x^2(x-4)}\,dx = \int \frac{-\frac{1}{4}}{x}\,dx + \int \frac{-1}{x^2}\,dx + \int \frac{\frac{1}{4}}{x-4}\,dx = \frac{1}{4}\ln(x-4) - \frac{1}{4}\ln x + \frac{1}{x}$$

$$\boxed{\int \frac{x^5+4}{x^3-4x^2}\,dx = 48x - \frac{1}{4}\ln x + \frac{1}{4}\ln(x-4) + 64\ln|x-4| + 6(x-4)^2 + \frac{1}{3}(x-4)^3 + \frac{1}{x} - 192}$$

7. $\int \frac{2x+1}{(x+3)^3(x^2+5x+4)}\,dx$

$$\int \frac{2x+1}{(x+3)^3(x^2+5x+4)}\,dx = \int \frac{2x+1}{(x+3)^3(x+4)(x+1)}\,dx$$

$$\left[\frac{2x+1}{(x+3)^3(x+4)(x+1)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3} + \frac{D}{x+4} + \frac{E}{x+1}\right](x+3)^3(x+4)(x+1)$$

$$2x+1 = A(x+3)^2(x+4)(x+1) + B(x+3)(x+4)(x+1) + C(x+4)(x+1) + D(x+3)^3(x+1) + E(x+3)^3(x+4)$$

$$2x+1 = A(x^4+11x^3+43x^2+69x+36) + B(x^3+8x^2+19x+12) + C(x^2+5x+4) + D(x^4+10x^3+36x^2+54x+27) + E(x^4+13x^3+63x^2+135x+108)$$

$$\begin{array}{|c|}\hline A + D + E = 0 \\ \hline 11A + B + 10D + 13E = 0 \\ \hline 43A + 8B + C + 36D + 63E = 0 \\ \hline 69A + 19B + 5C + 54D + 135E = 2 \\ \hline 36A + 12B + 4C + 27D + 108E = 1 \\ \hline \end{array}, \text{ Solution is: } \left[A = \frac{19}{8}, B = -\frac{9}{4}, C = \frac{5}{2}, D = -\frac{7}{3}, E = -\frac{1}{24}\right]$$

$$\int \frac{2x+1}{(x+3)^3(x+4)(x+1)}\,dx = \int \frac{\frac{19}{8}}{x+3}\,dx + \int \frac{-\frac{9}{4}}{(x+3)^2}\,dx + \int \frac{\frac{5}{2}}{(x+3)^3}\,dx + \int \frac{-\frac{7}{3}}{x+4}\,dx + \int \frac{-\frac{1}{24}}{x+1}\,dx = \frac{19}{8}\ln(x+3) - \frac{1}{24}\ln(x+1) - \frac{7}{3}\ln(x+4) + \frac{9}{4(x+3)} - \frac{5}{4(x+3)^2}$$

8. $\int \frac{e^x}{e^{2x}-4}dx$	$\int \frac{e^x}{e^{2x}-4}dx = \int \frac{1}{u^2-4}du = \int \frac{1}{(u+2)(u-2)}du$ $u = e^x \qquad du = e^x dx$ $\left[\frac{1}{(u+2)(u-2)} = \frac{A}{u+2} + \frac{B}{u-2}\right](u+2)(u-2)$ $1 = A(u-2) + B(u+2)$ $\boxed{\begin{array}{c} A+B=0 \\ \hline -2A+2B=1 \end{array}}$, Solution is: $[A = -\frac{1}{4}, B = \frac{1}{4}]$ $\int \frac{1}{(u+2)(u-2)}du = \int \frac{-\frac{1}{4}}{u+2}du + \int \frac{\frac{1}{4}}{u-2}du = \frac{1}{4}\ln(u-2) - \frac{1}{4}\ln(u+2)$ $= \frac{1}{4}\ln(e^x-2) - \frac{1}{4}\ln(e^x+2) + C$
9. $\int \frac{\cos(x)}{\sin^2 x-4}dx$	$\int \frac{\cos x}{\sin^2 x-4}dx = \int \frac{1}{u^2-4}du = \int \frac{1}{(u+2)(u-2)}dx$ $u = \sin x \qquad du = \cos x dx$ $\left[\frac{1}{(u+2)(u-2)} = \frac{A}{u+2} + \frac{B}{u-2}\right](u+2)(u-2)$ $1 = A(u-2) + B(u+2)$ $\boxed{\begin{array}{c} A+B=0 \\ \hline -2A+2B=1 \end{array}}$, Solution is: $[A = -\frac{1}{4}, B = \frac{1}{4}]$ $\int \frac{1}{(u+2)(u-2)}du = \int \frac{-\frac{1}{4}}{u+2}du + \int \frac{\frac{1}{4}}{u-2}du = \frac{1}{4}\ln(u-2) - \frac{1}{4}\ln(u+2)$ $= \frac{1}{4}\ln(\sin x-2) - \frac{1}{4}\ln(\sin x+2) + C$
10. $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$	$\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx = \int \frac{u}{(u^2+3u+2)}du = \int \frac{u}{(u+2)(u+1)}du$ $u = e^x \qquad du = e^x dx$ $\left[\frac{u}{(u+2)(u+1)} = \frac{A}{u+2} + \frac{B}{u+1}\right](u+2)(u+1)$ $u = A(u+1) + B(u+2)$ $\boxed{\begin{array}{c} A+B=1 \\ \hline A+2B=0 \end{array}}$, Solution is: $[A = 2, B = -1]$ $\int \frac{u}{(u+2)(u+1)}dx = \int \frac{2}{u+2}du + \int \frac{-1}{u+1}du = 2\ln(u+2) - \ln(u+1)$ $= 2\ln(e^x+2) - \ln(e^x+1) + C$

11. $\int \frac{2x+1}{x^4+9x^2}dx$

$$\int \frac{2x+1}{x^4+9x^2}dx = \int \frac{2x+1}{x^2(x^2+9)}dx$$

$$\left[\frac{2x+1}{x^2(x^2+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}\right]x^2(x^2+9)$$

$$2x+1 = A(x)(x^2+9) + B(x^2+9) + (Cx+D)(x^2)$$

$$2x+1 = A(x^3+9x) + B(x^2+9) + Cx^3 + Dx^2$$

$A + C = 0$
$B + D = 0$
$9A = 2$
$9B = 1$

, Solution is: $[A = \frac{2}{9}, B = \frac{1}{9}, C = -\frac{2}{9}, D = -\frac{1}{9}]$

$$\int \frac{2x+1}{x^2(x^2+9)}dx = \int \frac{\frac{2}{9}}{x}dx + \int \frac{\frac{1}{9}}{x^2}dx + \int \frac{-\frac{2}{9}x-\frac{1}{9}}{x^2+9}dx = \frac{2}{9}\int x^{-1}dx + \frac{1}{9}\int x^{-2} - \frac{2}{9}\int \frac{x}{x^2+9}dx - \frac{1}{9}\int \frac{1}{x^2+9}dx$$

$$u = x^2+9 \qquad du = 2xdx$$

$$= \frac{2}{9}\ln x - \frac{1}{9x} - \frac{4}{9}\int \frac{1}{u}du - \frac{1}{9}\int \frac{1}{x^2+9}dx \text{ —-> } \frac{2}{9}\ln x - \frac{1}{9x} - \frac{4}{9}\ln|x^2+9| - \frac{1}{9}\int \frac{1}{x^2+9}dx$$

$$x = 3\tan\theta \qquad dx = 3\sec^2\theta d\theta \qquad \theta = \arctan\frac{x}{3}$$

$$\int \frac{1}{x^2+9}dx = \int \frac{3\sec^2\theta}{(3\tan\theta)^2+9}d\theta = \int \frac{3\sec^2\theta}{9\tan^2\theta+9}d\theta = \int \frac{3\sec^2\theta}{9(\tan^2\theta+1)}d\theta = \frac{3}{9}\int \frac{\sec^2\theta}{\sec^2\theta}d\theta = \frac{3}{9}\theta = \frac{3}{9}\arctan\frac{x}{3}$$

$$\frac{2}{9}\ln x - \frac{1}{9x} - \frac{4}{9}\ln|x^2+9| - \frac{1}{9}\left(\frac{3}{9}\arctan\left(\frac{x}{3}\right)\right) = \frac{2}{9}\ln x - \frac{1}{9x} - \frac{4}{9}\ln|x^2+9| - \frac{3}{81}\arctan\left(\frac{x}{3}\right)$$

CONCLUSIONES

Esta práctica me ayudo a desarrollar mas mi facilidad de identificar cual de los 4 casos corresponde el problema que estoy resolviendo, también me parece curioso como voy integrando todos los conocimientos del semestre, como integrar con el método sustitución, sustitución trigonométrica y fracciones parciales en el desarrollo de resolver una sola integral.

EVALUACIÓN DE LA PRÁCTICA

Se evaluará el documento con los datos solicitados, las gráficas y conclusiones enviado a través del Campus Virtual