3.58
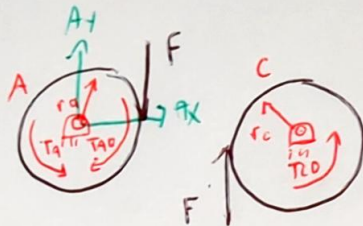
Análisis estatico

$\sum M_A = 0$

O.C.L.

$$\therefore T_A - T_{AB} - F r_A \quad (1)$$

$\sum M_c = 0$

$$\therefore T_{CD} - F r_c = 0 \quad (2)$$

Tenemos
3 incognitas
y 2 ecuaciones
$\therefore$ es
Hiperestatico

De (1) y (2):

$$T_A - T_{AB} - \left[\frac{T_{CD}}{r_c}\right] r_A = 0 \quad (3)$$

Analisis de deformaciones

$$\phi_A = \phi_{A/B} = \frac{T_{AB} \, L_{AB}}{G \, J_{AB}} \quad (4)$$

$$\phi_c = \phi_{c/D} = \frac{T_{CD} \, L_{CD}}{G \, J_{CD}} \quad (5)$$

Analisis de relación cinematica:

$$r_A \, \phi_A = r_c \, \phi_c \quad (6)$$

Sustituyendo (4) y (5) en (6):

$$r_A \frac{T_{AB} \, L_{AB}}{G \, J_{AB}} = r_c \frac{T_{CD} \, L_{CD}}{G \, J_{CD}} \quad (7)$$

Datos:

$G = 77.2 \, [GPa]$

$T_A = 80 \, [nm]$

$\tau_{max} = ¿? = \dfrac{T}{J}$

$J = \dfrac{\pi}{2} r^4$

$J_{AB} = \dfrac{\pi}{2}\left(\dfrac{18\times10^{-3}}{2}\right)^4$

$J_{AB} = 1.0305 \times 10^{-8}$

$$J_{CD} = \frac{\pi}{2}\left(\frac{15\times10^{-3}}{2}\right)^4 = 4.97 \times 10^{-9} \quad (9)$$

$\therefore$ Sustituyendo los valores en (7)

$$\left(\frac{18\times10^{-3}}{2}\right)\frac{T_{AB}(240\times10^{-3})}{(77.2\times10^9)(1.0305\times10^{-8})} = \left(\frac{15\times10^{-3}}{2}\right)\left(\frac{T_{CD}(240\times10^{-3})}{(77.2\times10^9)(4.97\times10^{-9})}\right)$$

$$\therefore \quad 2.7151\times10^{-6} \, T_{AB} = 4.6913\times10^{-6} \, T_{CD} \quad (8)$$

∴ Planteando el sistema de ecuaciones: con (3) y (7)

$$T_A - T_{AB} - \left[\frac{T_{CD}}{r_c}\right] r_A = 0$$

Sustituyendo los valores conocidos

$$80 - T_{AB} - \frac{T_{CD}}{\left(\frac{15 \times 10^{-3}}{2}\right)} \left(\frac{18 \times 10^{-3}}{2}\right) = 0$$

$$\therefore \quad 80 - T_{AB} - 1.2 T_{CD} = 0$$

$$2.7151 \times 10^{-6} T_{AB} - 4.6913 \times 10^{-6} T_{CD} = 0$$

$$\therefore \quad \boxed{T_{AB} = 47.211 \ [Nm]} \qquad \boxed{T_{CD} = 27.324 \ [Nm]}$$

a) $\tau_{CD} = ¿? = \dfrac{T_{CD} \ r_{CD}}{J_{CD}} = \dfrac{(27.324)\left(\frac{15 \times 10^{-3}}{2}\right)}{7 \ 4.97 \times 10^{-9}} = \boxed{41.2334 \ MPa}$

b) $\phi_A = ¿? = \dfrac{T_{AB} \ L_{AB}}{G \ J_{AB}} = \dfrac{(47.211)(240 \times 10^{-3})}{(77.2 \times 10^{9})(1.0305 \times 10^{-8})} = \boxed{0.01424 \ rad}$

$$= \boxed{0.876°}$$

$$2\pi = 360°$$

$$0.01$$

$$300 \ [Nm] = [0.7071] \ F$$

$$\therefore \ F = 424.26$$