1. Dada la ecuación diferencial:

a. $y' - 3x^3 + 6x^2 - x - 1 = 0$

- Obtenga la solución general.

$$\frac{dy}{dx} = +3x^3 - 6x^2 + x + 1 \Rightarrow \int dy = \int (3x^3 - 6x^2 + x + 1)\,dx$$

$$y = \frac{3x^4}{4} - \frac{6x^3}{3} + \frac{x^2}{2} + x + C \Rightarrow y = \frac{3x^4}{4} - 2x^3 + \frac{x^2}{2} + x + C$$

- Obtenga la familia de curvas que representa a dicha ED.

C: -1 $\quad y = \frac{3x^4}{4} - 2x^3 + \frac{x^2}{2} + x - 1$

x	y
-1	1.25
0	-1
1	-0.75
2	-1
3	13.25

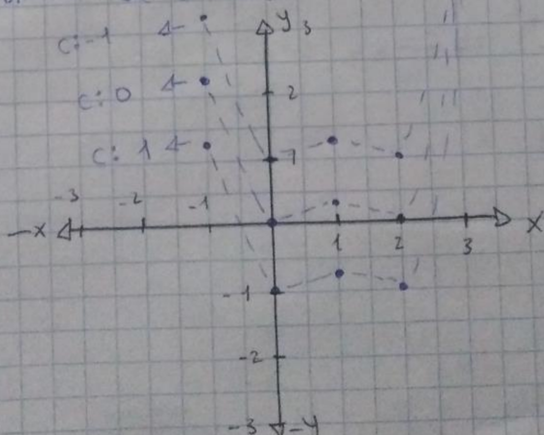
C: 0 $\quad y = \frac{3x^4}{4} - 2x^3 + \frac{x^2}{2} + x$

x	y
-3	116.25
-2	28
-1	2.25
0	0
1	0.25
2	0
3	14.25

C: 1 $\quad y = \frac{3x^4}{4} - 2x^3 + \frac{x^2}{2} + x + 1$

x	y
-3	117.25
-2	29
-1	3.25
0	1
1	1.25
2	1
3	15.25

- grafique la familia de curvas.



Proteger la naturaleza es defender la vida.

② Determine la solución del siguiente problema de valor inicial

$$x^2 \frac{dy}{dx} = y - xy \quad ; \quad y(-1) = 1$$

$$x^2 \frac{dy}{dx} = y(1-x) \Rightarrow \int \frac{dy}{y} = \int \frac{(1-x)}{x^2} dx$$

$$\ln(y) = \frac{x^{-1}}{-1} - \ln(x) + c \Rightarrow e^{\cancel{\ln}(y)} = e^{-\frac{1}{x}} - \cancel{e}^{\cancel{\ln}(x)} + e^{C}$$

$$y = e^{-1/x} - x + C //$$

$$1 = e^{-1/-1} + \cancel{1} + C \Rightarrow C = -e //$$

$$y = e^{-1/x} - x - e //$$

③ Resuelva los siguientes ED.

- $dx + e^{3x} dy = 0 \Rightarrow e^{3x} dy = -dx$

$$\int dy = \int \frac{-dx}{e^{3x}} \Rightarrow dy = -\frac{1}{3} \int \frac{du}{u^2}$$

$$u = e^{3x}$$

$$du = e^{3x} \cdot 3dx$$

$$\frac{du}{3e^{3x}} = dx$$

$$\frac{du}{3u} = dx$$

$$dy = -\frac{1}{3} \left[\frac{u^{-1}}{-1} \right] + c \Rightarrow dy = \frac{1}{3} \left(\frac{1}{e^{3x}} \right) + c$$

$$dy = \frac{1}{3} e^{-3x} + c //$$

- $y\,dy = 4x(y^2+1)^{1/2} dx \; ; \; y(0) = 1$

$$\int \frac{y\,dy}{\sqrt{y^2+1}} = \int 4x\,dx \Rightarrow \int \frac{du}{2\sqrt{u}} = \frac{4x^2}{2} + c$$

$$u = y^2 + 1$$

$$du = 2y\,dy$$

$$\frac{du}{2} = y\,dy$$

$$\frac{1}{2} \left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right) = 2x^2 + c \Rightarrow \left(\sqrt{y^2+1} \right)^2 = (2x^2 + c)^2$$

$$y^2 + 1 = (2x^2 + c)^2 \Rightarrow y = \sqrt{(2x^2+c)^2 - 1} \; //$$

$$1^2 + 1 = (2(0)^2 + c)^2 \Rightarrow C = 2 //$$

$$y = \sqrt{(2x^2+2)^2 - 1} \; //$$

- $\operatorname{sen}(x)(e^{-y}+1)\,dx = (1+\cos(x))\,dy, \quad y(0)=0$

$$\int \frac{\operatorname{sen}(x)\,dx}{1+\cos(x)} = \int \frac{dy}{e^{-y}+1}$$

$u = 1+\cos(x)$
$du = -\operatorname{sen}(x)\,dx$
$-du = \operatorname{sen}(x)\,dx$

$$\int \frac{-du}{u} = \int \frac{1}{v} \cdot \frac{-dv}{v-1}$$

$v = e^{-y}+1$
$dv = e^{-y} \cdot -1 \cdot dy$
$\frac{-dv}{v-1} = dy$

$$-\ln(u) = -\int \frac{dv}{v^2-v} \Rightarrow \frac{1}{v(v-1)} = \frac{A}{v} + \frac{B}{v-1}$$

$$1 = (v-1)A + vB \Rightarrow 1 = -A \Rightarrow A = -1 \,//, \quad B = 1$$

$$-\ln(1+\cos(x)) = -\int \left[\frac{-1}{v} + \frac{1}{v-1}\right] dv \Rightarrow -\ln(1+\cos(x)) = -\left[-\ln(v) + \ln(v-1)\right] + c$$

$$-\ln(1+\cos(x)) = \ln(e^{-y}+1) - \ln(e^{-y}+\cancel{1}-\cancel{1}) + c$$

$$\ln((1+\cos(x))(e^{-y}+1)) = -y + c \,//$$

$$c = 1.38629 \Rightarrow \ln((1+\cos(x))(e^{-y}+1)) = -y + 1.38629 \,//$$

- $(y^2+yx)\,dx - x^2\,dy = 0$

$M(x,y)\,dx \quad N(x,y)\,dy$

$y = xu$
$dy = x\,du + u\,dx$
$u = \frac{y}{x}$

$$(x^2u^2 + ux^2)\,dx - x^2(x\,du + u\,dx) = 0$$

$$(x^2u^2 + \cancel{ux^2})\,dx - x^3\,du - \cancel{ux^2\,dx} = 0$$

$$x^2u^2\,dx - x^3\,du = 0 \Rightarrow u^2x^2\,dx = x^3\,du \Rightarrow \int \frac{dx}{x} = \int \frac{du}{u^2}$$

$$\ln(x) = -\frac{1}{u} + c \Rightarrow \ln(x) = -\frac{1}{\left(\frac{y}{x}\right)} + c$$

$$\ln(x) = -\frac{x}{y} + c \,// \Rightarrow \frac{x}{y} = -\ln(x) + c \Rightarrow \frac{x}{-\ln(x)+c} = y \,//$$

- $xy\,dx - x^2\,dy = y\sqrt{x^2+y^2}\,dy, \quad y(0) = 1 \,//$

$x = yu$
$dx = u\,dy + y\,du$

$$\underbrace{xy\,dx}_{M} = \underbrace{\left(y\sqrt{x^2+y^2} + x^2\right)dy}_{N} \Rightarrow uy^2(u\,dy + y\,du) = \left(y\sqrt{u^2y^2+y^2} + u^2y^2\right)dy$$

$$\cancel{u^2y^2\,dy} + uy^3\,du = \left(y\sqrt{u^2y^2+y^2} + \cancel{u^2y^2}\right)dy \Rightarrow uy^3\,du = y\sqrt{u^2y^2+y^2}\,dy$$

$$u^2y^6\,du = y^2(u^2y^2+y^2)\,dy \Rightarrow u^2y^6\,du = y^4(u^2+1)\,dy$$

$$\int \frac{u^2\,du}{u^2+1} = \int \frac{y^4\,dy}{y^6} \Rightarrow \int 1\,du - \frac{1}{u^2+1}\,du = \int \frac{dy}{y^2}$$

$$\begin{array}{r|l} u^2 & u^2+1 \\ \hline \cancel{-u^2} - 1 & 1 \\ -1 & \end{array}$$

$$u + \operatorname{arccot}(u) = -\frac{1}{y} + c \qquad y(0) = 1$$

$$\frac{x}{y} + \operatorname{arccot}\left(\frac{x}{y}\right) = \frac{-1}{y} + c \Rightarrow \frac{0}{1} + \operatorname{arccot}\left(\frac{0}{1}\right) = -\frac{1}{1} + c$$

$$c = \frac{\pi}{2} + 1 \Rightarrow \frac{x}{y} + \operatorname{arccot}\left(\frac{x}{y}\right) = -\frac{1}{y} + \frac{\pi}{2} + 1 \,//$$

- $\underbrace{(2y^2x-3)}_{M}dx + \underbrace{(2yx^2+y)}_{N}dy = 0$

$$\frac{\partial}{\partial y}\left[2y^2x-3\right] = \frac{\partial}{\partial x}\left[2yx^2+y\right] \Rightarrow 4yx = 4xy \Rightarrow \text{si es exacta}$$

$$\frac{\partial f}{\partial x}(x,y) = M \Rightarrow \int \partial f(x,y) = \int (2y^2x-3)\,\partial x$$

$$f(x,y) = \frac{2y^2x^2}{2} - 3x + K(y) = C$$

$$\frac{\partial}{\partial y}\left[f(x,y)\right] = \frac{\partial}{\partial y}\left[y^2x^2 - 3x + K(y)\right] = 2yx^2 + y$$

$$\cancel{2yx^2} + \frac{\partial}{\partial y}\left[K(y)\right] = \cancel{2yx^2} + y \Rightarrow \int \partial\left[K(y)\right] = \int y\,dy$$

$$K(y) = \frac{y^2}{2} \Rightarrow f(x,y) = y^2x^2 - 3x + \frac{y^2}{2} = C \,//$$

- $\underbrace{(xy^2 + x^2y^2 + 3)}_{M}dx + \underbrace{x^2y}_{N}dy = 0$

$$\frac{\partial}{\partial y}\left[xy^2 + x^2y^2 + 3\right] = \frac{\partial}{\partial x}\left[x^2y\right] \Rightarrow 2yx + 2yx^2 = 2xy \Rightarrow \text{inexacta}$$

$$\frac{\cancel{2yx} + 2yx^2 - \cancel{2yx}}{x^2y} = \frac{2\cancel{x^2y}}{\cancel{x^2y}} = 2 = f(x) \,// \quad FI: e^{\int 2\,dx} = e^{2x}$$

$$\frac{\partial}{\partial y}\left[e^{2x}xy^2 + e^{2x}x^2y^2 + 3e^{2x}\right] = \frac{\partial}{\partial x}\left[e^{2x}x^2y\right]$$

$$2e^{2x}xy + 2e^{2x}x^2y = y(2xe^{2x} + e^{2x}.2x^2) \Rightarrow \text{Exacta} \,//$$

$$\frac{\partial}{\partial x}\left[f(x,y)\right] = e^{2x}xy^2 + e^{2x}x^2y^2 + 3e^{2x}$$

$$f(x,y) = \int (e^{2x}xy^2 + e^{2x}x^2y^2 + 3e^{2x})\,\partial x$$

$$f(x,y) = \frac{1}{2} y^2 e^{2x} x^2 + \frac{3}{2} e^{2x} + k(y) = C$$

$$\frac{\partial}{\partial y}[f(x,y)] = \frac{\partial}{\partial y}\left[\frac{1}{2} y^2 e^{2x} x^2 + \frac{3}{2} e^{2x} + k(y)\right] = e^{2x} x^2 y$$

$$y \cancel{e^{2x} x^2} + \frac{\partial}{\partial y}[k(y)] = \cancel{e^{2x} x^2} y$$

$$k(y) = \int 0 \, \partial y \Rightarrow k(y) = 0 \,//$$

$$f(x,y) = \frac{1}{2} y^2 e^{2x} x^2 + \frac{3}{2} e^{2x} + 0 = C \,//$$

- $3x + 2y + y^2 + (2x + 2xy + 5y^2)y' = 0$

$$(2x + 2xy + 5y^2)\, dy = (-3x - 2y - y^2)\, dx$$

$$\underbrace{(3x + 2y + y^2)}_{m} dx + \underbrace{(2x + 2xy + 5y^2)}_{N} dy = 0$$

$$\frac{\partial}{\partial y}[3x + 2y + y^2] = \frac{\partial}{\partial x}[2x + 2xy + 5y^2] \Rightarrow 2 + 2y = 2 + 2y \,//$$

$$\frac{\partial}{\partial x}[f(x,y)] = 3x + 2y + y^2 \Rightarrow f(x,y) = \int (3x + 2y + 2y^2)\, \partial x$$

$$f(x,y) = \frac{3x^2}{2} + 2yx + 2y^2 x + k(y) = C$$

$$\frac{\partial}{\partial y}[f(x,y)] = \frac{\partial}{\partial y}\left[\frac{3x^2}{2} + 2yx + 2y^2 x + k(y)\right] = 2x + 2xy + 5y^2$$

$$\cancel{2x} + 4yx + \frac{\partial}{\partial y}[k(y)] = \cancel{2x} + 2xy + 5y^2$$

$$k(y) = \int (-2xy + 5y^2)\, \partial y \Rightarrow k(y) = -xy^2 + 5\frac{y^3}{3}$$

$$f(x,y) = \frac{3x^2}{2} + 2yx + 2y^2 x - xy^2 + 5\frac{y^3}{3} = C \,//$$

$$\cdot\ x\frac{dy}{dx}+2y=3\ (\div x)\Rightarrow \frac{dy}{dx}+\frac{2}{x}y=\frac{3}{x}\rightarrow \text{EDO.L.C}$$

$$y=e^{-\int\frac{2}{x}dx}\left[\int e^{\int\frac{2}{x}dx}\cdot\frac{3}{x}\cdot dx+C\right]$$

$$y=\cancel{e}^{\cancel{\ln}x^{-2}}\left[\int \cancel{e}^{\cancel{\ln}x^{2}}\cdot\frac{3}{x}dx+C\right]\Rightarrow y=\frac{1}{x^2}\left[\int x^2\cdot\frac{3}{x}dx+C\right]$$

$$y=\frac{1}{x^2}\left[\frac{3x^2}{2}+C\right]\Rightarrow y=\frac{3}{2}+C\ //$$