University of Oslo: Bachelor Project RBNS(Reported But Not Settled) Predictions Using Data From an Insurance Company

Joël Fomete Tankmo October 20, 2020

1 Introduction

Claims are never settled immediately, and for some types of injuries or damage, delays are rather long. For non life insurance it is an obligation to set aside technical reserve to settle claims that have occurred in the past calendar years.

A lack of enough reserve can have a direct impact on an insurance company downfall. Downfall that can have a negative effect on policy holders, shareholders and the insurance company employees. Between the time that the incident is reported by the customer to the insurance company until the final settlement of the case, there is a delay. An estimate of the payment to be made in the future may therefore affect management, investor and shareholders decision.

In order to give a realistic financial picture of all the claims that are in the process of being liquidated but not finished we need to set up a RBNS-reserve (reported, but not settled) base on the distribution of the amount that will eventually be pay.

The paper is set out as follows. In Section 2 we define the notation and describe the data which we will assume is available. In Section 3, the model which we will actually apply is given. We have the conclusion in section 4, the reference in section 5 and in section 6 we have the appendix R-Programs.

2 Data Analysis

The data set contains information from 2354 customers. They are from RBNS-type and come from one of the property insurance companies in Oslo. Each customer is identified by an incident number allocated by the insurance company. The data is divided into eight categories: incident number, year of incident, delay, number of incidents (0 or 1), estimated compensation when the incident occurred, regress from the company, what was eventually paid and the RBNS reserve put aside by the company. The years the incidents were reported vary between 2003 and 2015 and the maximum number of year it took to cover all the claims is 7 years.

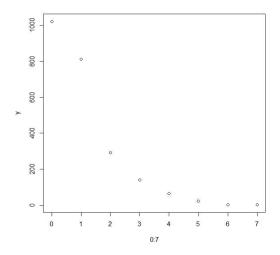


Figure 1: Empirical Distribution of the Delay

The scatter-plot of the delay in figure 1 gives us information about the number of claims needed to be settled according to delay. We can see that the graph decreases considerably from the first period to the last period. During the first period we have that 1022 claims were pending versus 1 claim during the last period. That is why the expected delay needed to settled a claim is under 1 year (0.9341546). The number of claims pending during each period is as follow: 1022; 811; 292; 141; 63; 22; 2; 1.

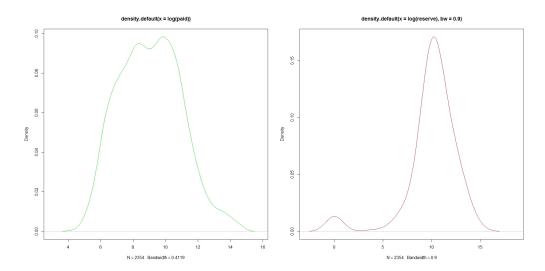


Figure 2: Distribution of the Logarithm of Paid (left) and RBNS-reserve (right.)

In figure 2, it is not easy to identify the type of distribution. One can see that the stability resulting from a large volume of data produces a rather narrow range. With such stability, in this instance, the log normal distribution approaches the shape of a normal distribution.

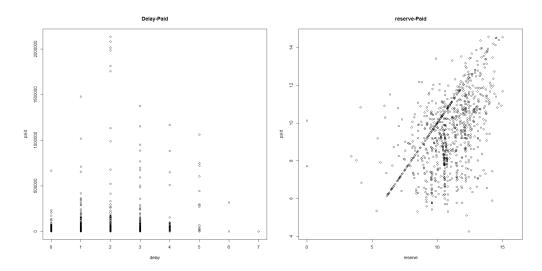


Figure 3: Delay (left)and RBNS-reserve (right) against final payment.

To check if there is a connection between the size of the remunerations and the delay we did in Figure 3 a scatter plot of the delay against what was paid and a scatter plot of the RBNS reserve agains the delay. We can see that across the entire scatter plot(on the left), there is a huge variation among the data. The correlation between the delay and the final payement gave us 0.23 very far from 1. We can therefore say that there is no linear correlation between the delay and the size of the remunerations.

The same procedure was made to check if there is a connection between what was paid and the reserve. In the graph on the right we can see that the dots are tightly clustered around a line. The correlation of 0.45 was obtain. We can therefore say that there is a relationship between what was eventually paid and the RBNS-reserve put aside by the company.

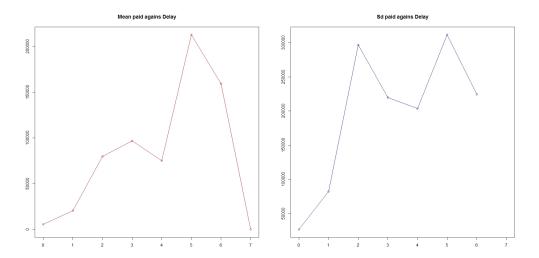


Figure 4: Mean (left) and standard deviation (right) of final Paid against Delay.

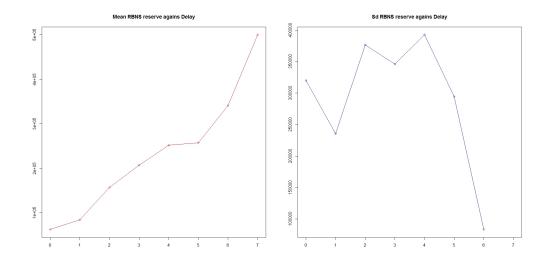


Figure 5: Mean (left) and standard deviation (right) of RBNS-reserve against Delay.

Table 1: Explanatory variables and descriptive statistics.

	Mean	SD
Accident year (1=2003; 2=2004;; 12=2015)	-	-
Delay (number of years)	0.93	1.1
Number of incidents: 1 if an incident occurs; $0=$ otherwise	0.60	0.49
Estimated compensation when the incident occurred	57771.96	305401.9
What was eventually paid by the insurance company	29449.85	140109
The RBNS reserve put aside by the company	97951.09	309500.2

Table2: Mean and Standard deviation of Paid and RBNS-reserve according the delay(measure in 1000 NOK).

Delay	year 0	year 1	year2	year3	year4	year5	year6	year7
Mean paid	5.75	20.69	79.76	96.71	75.30	212.73	159.31	0.00
SD paid	26.78	82.51	296.60	219.82	203.62	311.48	224.32	NA
Mean RBNS	62.79	84.59	157.14	206.96	252.34	257.40	341.02	500.00
SD RBNS	320.06	235.51	376.63	346.06	392.44	294.47	83.40	NA

From table 2 we can see that the smallest expected amount that was paid by the insurance company was 5.75 thousand NOK and that was at the beginning. The biggest expected amount paid was for a claim with 6 years of duration and that was 212 thousand NOK. We also observe that the standard or error increase with time. The mean of the RNS reserve put aside by the company increase with the delay. In the beginning, the amount that was put aside by the company is 62.79 thousand NOK, After a delay of 7 years the amount is 500 thousand NOK. We have estimated α from the data by using two methods.

Method 1:
$$\hat{\xi}_d = z_d$$
 $\hat{\alpha}_d = \frac{\xi_d^2}{Sd^2}$ z_d is the initial estimates available on the file.

With the help of R, we got the following values of alpha for each period of delay:

α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7
0.05	0.09	0.1	0.3	0.6	0.7	0.4	3

We can see in the table above that α increases with the period of delay, except for the sixth period where we have 2 claims needed to be settled. The low variation in the beginning is due to the fact that they are many claims needed to be settled.

Method 2:

$$E[\sum (n_d - 1)Sd^2] = \sum (n_d - 1)\frac{\xi_d^2}{\alpha}$$

$$\sum (n_d - 1)Sd^2 = (\sum (n_d - 1)\frac{\xi_d^2}{Sd^2}$$

$$\hat{\alpha} = \frac{\sum (n_d - 1)\xi_d^2}{\sum (n_d - 1)Sd^2}$$

$$\alpha = 0.606$$

Table3: Mean and standard variation of the amount paid and the RBNS reserve.

year	Mean paid	SD paid	Mean RBNS reserve	SD reserve	RBNS
2003	52295	123846	193770	194586	
2004	19239	64766	59359	61811	
2006	67521	186350	123393	145128	
2007	8222	25453	56211	200644	
2008	15657	62166	101043	298131	
2009	74519	236188	195122	684771	
2010	34054	156690	112983	339780	
2011	45759	161621	145151	275433	
2012	30103	158217	102646	281801	
2013	22891	128904	71429	235591	
2014	3670	14649	26590	70975	
2015	260	1125	24763	38675	

From table 2 and 3 we can observe that when the number of claims is high, then there is a better estimation of the mean . This can influence the SD by making it smaller. For example when the delay is 7 we only have one data for it and therefore cannot make a good estimation of either the mean nor the SD.

3 RBNS-evaluation with uncertainty

The aim here is to develop and implement a Monte Carlo method which returns the distribution of the amount that will be paid.

Let J be the number of incidents needed to be settled in the future, N = 1,...,n

The delay d varies between the present value k = 0 and the final value k = 7.

 z_1, \dots, z_n are the estimate of the claims that have been delay, we note it z_d n.

We have converted the delay vector that were generated into value and we got for each period of delay the following number of claims:

 $1022\ 811\ 292\ 141\ 63\ 22\ 2\ 1$

We now assume that $\alpha_k = \alpha$, that means we assume the variability is the same for all period of delay.

 $z_1,...,z_n \sim z_i = \xi \times Y_\alpha$

where E[Y] = 1 and $Y \sim gamma(\alpha)$

 $E[z_i] = \xi$ and $var(z_i) = \frac{\xi^2}{\alpha}$

With m = 10 000 simulations we got $\overline{Y} = 1.000946$ very close to 1.

 $z_d^* = \xi_d \times Y$

With the help of R we obtained the following values of z_d^* , that means the expected amount that the company will eventually pay in the future period of delay.

z_0^*	z_1^*	z_2^*	z_3^*	z_4^*	z_5^*	z_6^*	z_7^*
68205	87224	120559	155902	228223	242359	181017	498104

Figure 6 is the density function of the RBNS with 10000 simulations. The result is a probability distribution showing reserve outcomes at varying probabilities or confidence levels. The result has the shape of a normal (or Gaussian) distribution. The distribution produces a range of possible outcomes.

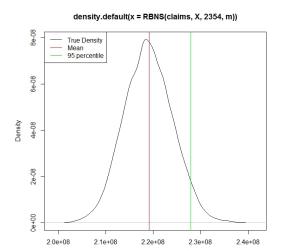


Figure 6: Density function of the RBNS

The statistical mean of this distribution represent the technical reserve that the company will need in order to cover all the claims. A 95% reserve was obtained, that is the level at which there is 95% chance that the amount paid by the insurance company will not exceed the estimate.

The Insurance company may wish to focus on a mean of approximately 219 millions NOK with a standard deviation of 5198190 or error of 2.4% when deciding for the reserve. If instead of the mean the company choose the 95 percentile, that is 228 millions NOK, there will have an error of 2.3% which is lower and therefor a lower risk of insolvency.

4 Conclusion

The time period between an accident until the compensation is usually very long. As a consequence, the insurer is supposed to calculate the reserve for reported but not settled claims. With the aim of estimating a better reserve, insurers are encouraged to follow the Solvency II project. The Solvency II Directive (2009/138/EC) is an EU Directive that codifies and harmonises the EU insurance regulation. Primarily this concerns the amount of capital that EU insurance companies must hold to reduce the risk of insolvency. In a challenging market, which the insurance company faces, there is a necessity to develop more and better models to estimate the RBNS-reserve . This paper has developed a stochastic framework for claims reserving. The Method for the RBNS predictions were discussed in this paper, using data from an insurance company. The approach allows for explicit consideration of the random nature of the claims process.

Acknowledgments. The author thanks his supervisor, prof. Erik Bølviken for valuable comments, remarks and overall help with the research.

5 References

- Erik Bølviken (2014) : Computation and Modelling in Insurance and Finance
- Peter Dalgaard: $Introductory\ Statistics\ with\ R$
- $-\ wikipedia.org/wiki/Solvency IID irective$

6 Appendix - R-program

```
Data from the insurance company.
x=read.table('data_mat_stk_project.txt',header=T)$
names(x)
"SkadeNr"
                                 "avvikling"
                "SkadeAr"
                                                 "antallskader"
"SkadeEstimat" "Regress"
                                 "UtbetaltBelop" "RBNS"
Figure 1
delay=x$avvikling
n=length(delay)
mean(x$avvikling)
var(x$avvikling)
sd(x$avvikling)
max(delay)
y=1:8
for (i in y){y[i]=nrow(subset(x,avvikling==(i-1)))}
plot(0:7,y)
#print y to see how many had a delay 0, 1, 2, 3, ...
Figure 2
paid=x$UtbetaltBelop*(-1)
Mean_paid=mean(paid)
sd(paid)
max(paid)
min(paid)
par(mfrow=c(1,2))
plot(density(log(paid)),col="green")
reserve=x$RBNS
reserve_square =reserve^2
reserve=sqrt(reserve_square)
plot(density(log(reserve),bw=0.9),col="red")
Figure 3
# Plot of the delay against what is paid
cor(paid,delay)
#The correlation coefficient of the delay and what was eventually paid is 0.2300282.
#Since it is very small compare to 1, we can conclude that the variables are not positivel
par(mfrow=c(1,2))
plot(delay,paid,xlab="delay",ylab="paid",main="Delay-Paid")
# plot reserve against paid
plot(log(reserve),log(paid),xlab="reserve",ylab="paid",main="reserve-Paid")
cor(reserve,paid)
Figure 4
# we plot the mean and sd of paid against delay
paid=x$UtbetaltBelop*(-1)
mean(paid)
paid[x$avvikling==7]
e=rep(0,8)
for (i in 1:8)\{e[i]=mean(paid[x\$avvikling==i-1])\} # we plot the mean of paid again the ele
```

```
par(mfrow=c(1,2))
plot(0:7,e,col=2,main="Mean paid agains Delay",type="o",xlab=" ",ylab=" ")
#sum(e)
#sum(paid)
#print e to see what was paid every year
sd(paid)
j=rep(0,8)
for (i in 1:8){j[i]=sd(paid[x$avvikling==i-1])}
plot(0:7,j,col=4,main="Sd paid agains Delay",xlab=" ",ylab=" ",type="o")
Figure 5
# we plot the mean and sd of the reserve again de delay
reserve=x$RBNS
reserve_square =reserve^2
reserve=sqrt(reserve_square)
mean(reserve)
reserve[x$avvikling==7]
e = rep(0,8)
for (i in 1:8){e[i]=mean(reserve[x$avvikling==i-1])}
par(mfrow=c(1,2))
plot(0:7,e,col=2,main="Mean RBNS reserve agains Delay",type="o",xlab=" ",ylab=" ")
#range(x$RBNS)
sd(reserve)
j=rep(0,8)
for (i in 1:8){j[i]=sd(reserve[x$avvikling==i-1])}
plot(0:7,j,col=4,main="Sd RBNS reserve against Delay",type="o",xlab=" ",ylab=" ")
#plot(0:7,e,col="red")
We extracted the delay, the year, paid and reserve from the data to build the table.
df=data.frame(year,delay)
aggregate(.~year,data=df,mean)
df=data.frame(year,Reserve)
aggregate(.~year,data=df,mean)
df=data.frame(year,paid)
aggregate(.~year,data=df,sd)
We generated value from the delay vector
y=1:8
for(i in y){y[i]=nrow(subset(x,delay==(i-1)))}
plot(0:7,y)
```

```
# 1022 811 292 141
                        63
                             22
                                   2
                                        1 claims needed to be settled for each period of
dim(x) = 2354
#p_0=1022/2354; p_1=811/2354; p_2=292/2354; p_3=141/2354; p_4=63/2354; p_5=22/2354; p_
#mysample=sample(c("0year","1year","2year","3year","4year","5year","6year","7year"),10,pro
p=c(1022,811,292,141,63,22,2,1)/2354
mysample=sample(0:7,2354,prob=p,replace=T)
ant=rep(0,8)
for (i in 1:8){ant[i]=sum(mysample==i-1)}
plot(0:7,ant)
#1028 798 299 137
                       77
                            13
                                  2
                                       0
                                           number of incidents (J)that will be settled dur
We estimated alfa by two method
methode 1
z=sqrt((x$SkadeEstimat)^2)
var(z) = 87948050776
psi=mean(z)
xi_d= rep(0,8)
for(i in 1:8)\{xi_d[i]=mean(z[delay==(i-1)])\}
#### xi_d the the expected compensation for each period of delay
### value of xi_d
xi_d0=68464.98;xi_d1=87556.20;xi_d2 =121017.62;xi_d3= 156495.39;xi_d4= 229091.47;xi_d5= 24
alfa=xi_d^2/var(z)
alfa0=0.05329798; alfa1=0.0871661; alfa2=0.1665218; alfa3=0.278469; alfa4=0.5967489
methode 2 # we assume that all the alfa are equall
expectation=mean(sum(2352-1)*var(z))
expectation=A
A=2.067659e+14
mean(z)=psi
psi=93046.16
B=(sum(2352-1)*psi^2)
B= 2.035399e+13
alfa=B/A
alfa = 0.6059003
We do the following to obtain the reserve.
m=100000
y=rgamma(m,alfa)/alfa
mean(y) #
alfa=1.000946
xi_d=c(68464.98,87556.20,121017.62,156495.39,229091.47,243281.62,181705.50,500000.00)
for(i in 1:8){ z_d[i]=mean(y)*xi_d[i]}
### z_d the expected compensation for each periode of delay tomorrow . z_d= xi_d*mean(y)
### z_d: 68393.03 87464.18 120890.44 156330.92 228850.71 243025.95 181514.54 499474.53
m=10000
N[0]=2354 # number of claims to be settled
#p=c(1028 , 798 , 299 , 137 , 77 , 13 , 2
                                              ,0 )/2354
p=c(1022,811,292,141,63,22,2,1)/2354
```

```
X=0:7
N_d=number of delay of length d
claims=c(1022,811,292,141,63,22,2,1)
RBNS=function(claims, X, NO=2354, m){
  rbns=rep(0,m)
  p=claims/2354
  for(i in 1:m){
    D=sample(X,N0,replace=T,prob=p)
    N=rep(0,8)
    for(d in 1:8){
      N[d]=sum(D==(d-1))
      z_d=sum(rgamma(N[d],alfa)/alfa)*xi_d[d]
      rbns[i]=rbns[i]+z_d
   }
return(rbns)
head(RBNS(claims, X, 2354, m)) # 218175104 209596859 2155
reserve = sort(RBNS(claims,X,2354,m))[m*(1-e)]
plot(density(RBNS(claims,X,2354,m)))
abline(v=219090529,col="red")
abline(v=227909617,col="green")
legend("topleft",c("True Density","Mean","95 percentile"),lty=1,col=1:3)
 \mathtt{z\_d} \ \# \ 68205.39 \ 87224.23 \ 120558.78 \ 155902.03 \ 228222.86 \ 242359.21 \ 181016.55 \ 498104.23 \ \ldots 
mean(RBNS(claims,X,2354,m))# 219082571 # by repeating the above described procedure a large
var(RBNS(claims,X,2354,m)) # 4.247832e+13
sd(RBNS(claims, X, 2354, m)) # 6498224
```