Dynamic Mortalities

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Introduction

The Lee-Carter model was designed in 1992 to predict the future mortality probability of the US population.It capture age specific trends from an observed period and extrapolate these trends in the future. Dynamic mortalities that change over time present a serious risk for companies offering life annuities. The source of risk can be :

- Financial risk.
- Liability risk due to inflation and discounting.
- Life table risk

Mortalities since the Second World War

- Life table or Actuarial table.
  - Period life table.
  - Cohort life table or generation life table.
- The force of mortality μ_{xt}.

$$\mu_{xt} = \frac{D_{xt}}{E_{xt}}$$

Where D_{xt} is the number of deaths of people aged x in year t and E_{xt} the exposure of age x in year t.

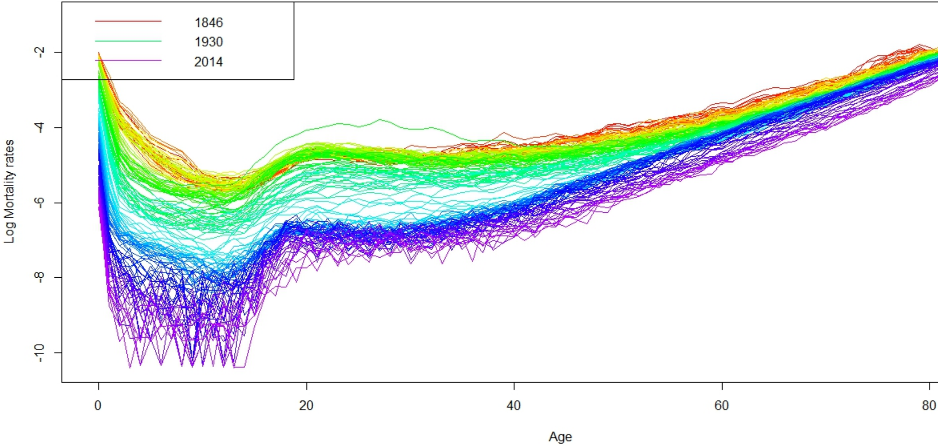
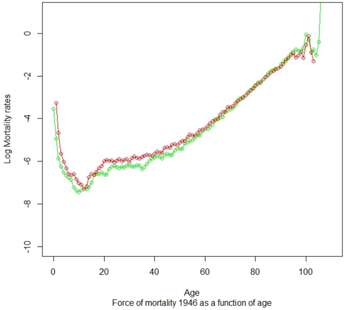


Figure: **Force of Mortality sexe-neutral**



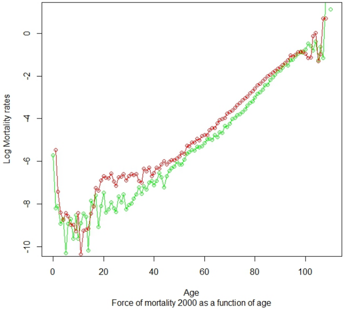


Figure: **Force of Mortality for the years 1946 and 2000**

Mathematical formulation of the model.

- l_0 is the age at the beginning of the contract so $l_0 + k$ will be the age at time k; l_r is the age when retirement start.
- s is the pension benefit until the maximum realistic age l_e.
- One-step survival probabilities ${}_1P_l = P_l$ and mortalities ${}_1q_l = q_l = 1 - P_l$.
- ${}_kP_{l_0}$ is the probability that and individual of age l_0 is alive at age $l + k$
- d $= \frac{1}{1+r}$ is the discount.

- The most popular mathematical description of mortality is the Gomperz-Makeham:

Gomperz-Makeham

$\mu(l) = \theta_0 + \theta_1 e^{\theta_2 l}$ is the intensity.

$$
\begin{aligned}
{}_k p_l &= exp(-\int_{lT}^{(k+l)T} \mu(l) d_l) \\
&= exp(-\int_{lT}^{(k+l)T} \theta_0 + \theta_1 e^{\theta_2 l}) d_l) \\
&= exp(-\theta_0 kT - \tfrac{\theta_1}{\theta_2}(e^{\theta_2 kT} - 1)e^{\theta_2 lT})
\end{aligned}
$$

The current mortalities q_{l0} can then be written as:

$q_{l0} = 1 - P_{l0} = 1 - e^{-\theta_0 - \theta_1 e^{\theta_2 l}}$

Where $\theta_0, \theta_1, \theta_2$ are parameters.

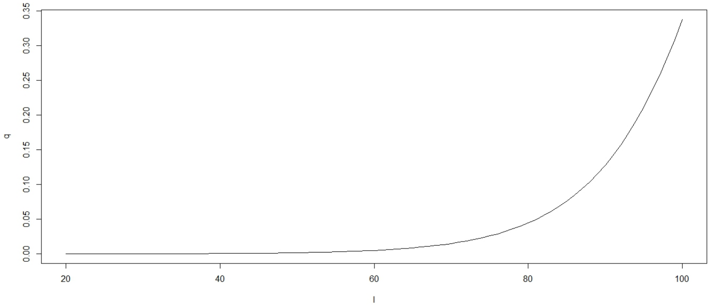


Figure: **Plot of the true mortalities probabilities under the Gomperz-Mekam.**

- The parameters of the Gomperz-Makeham model are difficult to interpret and tell us little about how long we expect people to live.
- Lee- Carter, we supposed for the model that the life expectancy is growing. We take the mortality of an individual of age l in year k.

Simplified version of the Lee-Carter with parameter estimates for Norway.

$$q_{lk} = w_l^k q_{lo}$$

Where

$$\log(w_l) = -b_0 \frac{e^{h_l}}{(1+e^{h_l})^2}$$

and

$$h_l = a_0 + a_1 l + a_2 l^2$$

Dealing with longer lives

- How Mortalities are enter in actuarial calculation.

$$p_l \rightarrow \mathbf{Z} \rightarrow \hat{p}_l \rightarrow {}_k\hat{p}_l \rightarrow \hat{\pi}, \hat{\chi}_k, \widehat{PV_0}$$

- Now people live longer than expected.This can be analyse by introducing a default sequence q_{l0} of mortality. Two approaches can be use her, the cohort version and the time-dynamic version.

Simple model

$$q_l(i) = q_{l0} e^{-\gamma(i)} \text{ , cohort version}$$
$$q_{lk} = q_{l0} e^{-\gamma_k} \text{ , time dynamic-version}$$

- $\gamma(i)$ and γ_k are parameters that make the mortalities deviate from the default sequence q_{l0}.

k-step survival probabilities required for actuarial calculations.

$${}_{k+1}p_l = 1 - q_{l+k}(l).{}_kp_l, \text{ Cohort version.}$$
$${}_{k+1}p_l = (1 - q_{l+k,k}).{}_kp_l, \text{ time-dynamic version.}$$

- For k = 0,1,2,..., and people of age l born at -l, the probability that they survive the periode from k to k+1 is then $1 - q_{l+k}(l)$ and $(1 - q_{l+k,k})$
- By inserting the simple model into the k-step survival probabilities we obtain:

Life table.

$${}_{k+1}p_l = (1 - q_{l0}^{-\gamma(l)}).{}_kp_l, \text{ Cohort version.}$$
$${}_{k+1}p_l = (1 - q_{l0}^{-\gamma_k}).{}_kp_l, \text{ time-dynamic version.}$$

- With the appropriate life table ${}_k p_l$ it is now possible to compute the estimated equivalence premium,liability and present value for k = 1,2,3,...
- The equivalence premium π is the solution of :

$$\pi \sum_{k=0}^{l_r - l_0 - 1} d^k \ {}_k p_{l_0} = s \sum_{k=l_r - l_0}^{l_e - l_0} d^k \ {}_k p_{l_0}$$

Numerically illustrations

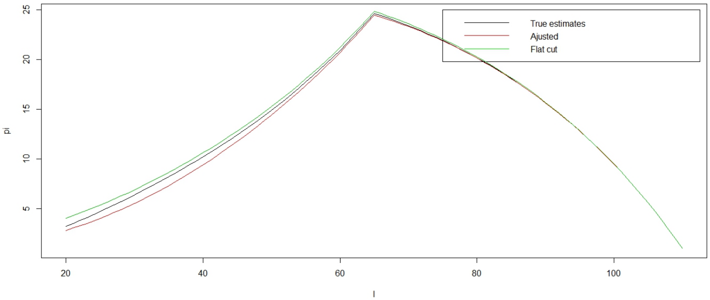
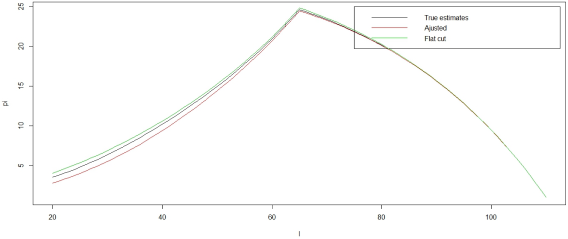


Figure: **One time premia Sex-neutral**



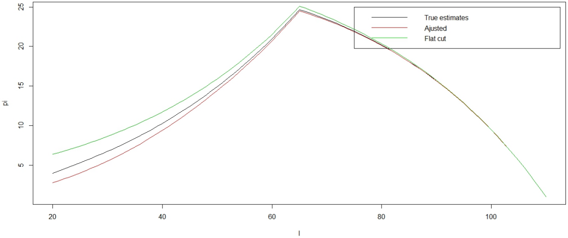
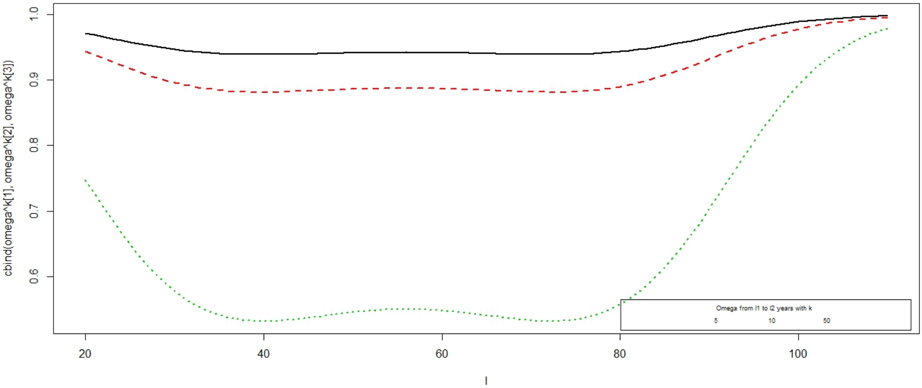


Figure: **One time premia male and female**



cbind(omega^k[1], omega^k[2], omega^k[3])
1.0
0.9
0.8
0.7
0.6
20
40
60
80
100
l
Omega from l1 to l2 years with k
5
10
50

- The lee-carter becaome less smooth over time. The forecast become narrow for people beyon 70.
- The model underpredict futurs gains.

Thank You For your attention !