Team #1

Homework #4: Asymptotes

Subject: Differential Calculus

Teacher: Oscar Martínez Hernández

Career: Mechatronic 3°D

- Garrido Castro Joel
- Guerrero Rodríguez Sebastián Hilario
- Marrero Montiel Sergio
- Silva Rosas Pablo Adrián
- Vargas Ramiro Daniel

Differential Calculus

Derivative Definition

Homework: Evaluate the derivative of the next functions applying the definition of derivative

$\boldsymbol{a)}\ \boldsymbol{f(x) = 37}$

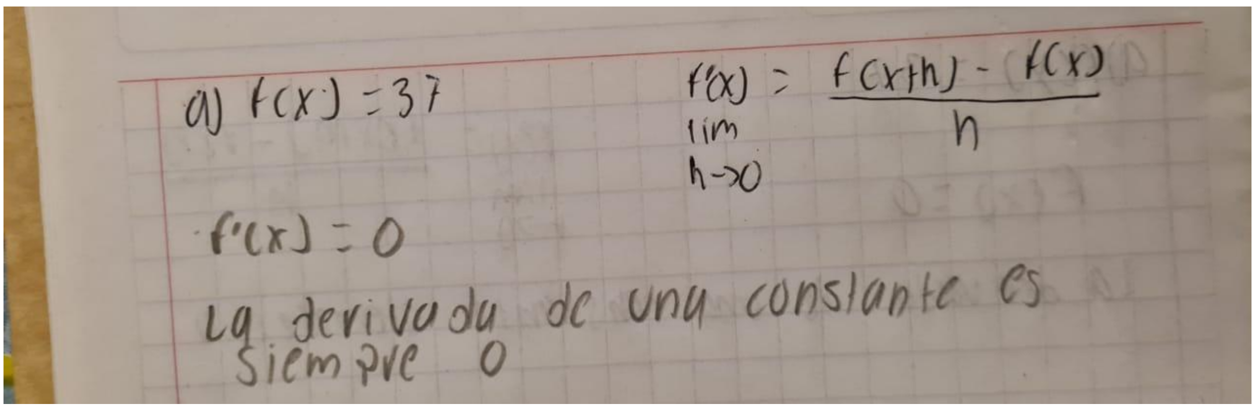
$f'(x) = \frac{f(x+h)-f(x)}{h}$

lim

h->0

$f'(x) = 0$

The derivative of a constant is always 0



$\boldsymbol{b)}\ \boldsymbol{f(x) = 3x^2 - 5x + 4}$

$f'(x) = \frac{f(x+h)-f(x)}{h}$

lim

h->0

$f(x+h) = 3(x+h)^2 - 5(x+h) + 4$

First, we obtain f(x+h) by substituting x by (x+h)

$$f'(x) = \frac{3(x+h)^2 - 5(x+h) + 4 - (3x^2 - 5x + 4)}{h}$$

$$f'(x) = \frac{\boldsymbol{3(x^2 + 2xh + h^2)} - 5x - 5h + 4 - 3x^2 + 5x - 4}{h}$$

$$f'(x) = \frac{3x^2 + 6xh + 3h^2 - 5x - 5h + 4 - 3x^2 + 5x - 4}{h}$$

Then we substitute in the equation the values and we resolve them using algebra.

Convert the first operation with the formula of the perfect square trinomial and multiply to have only adds and subtractions.

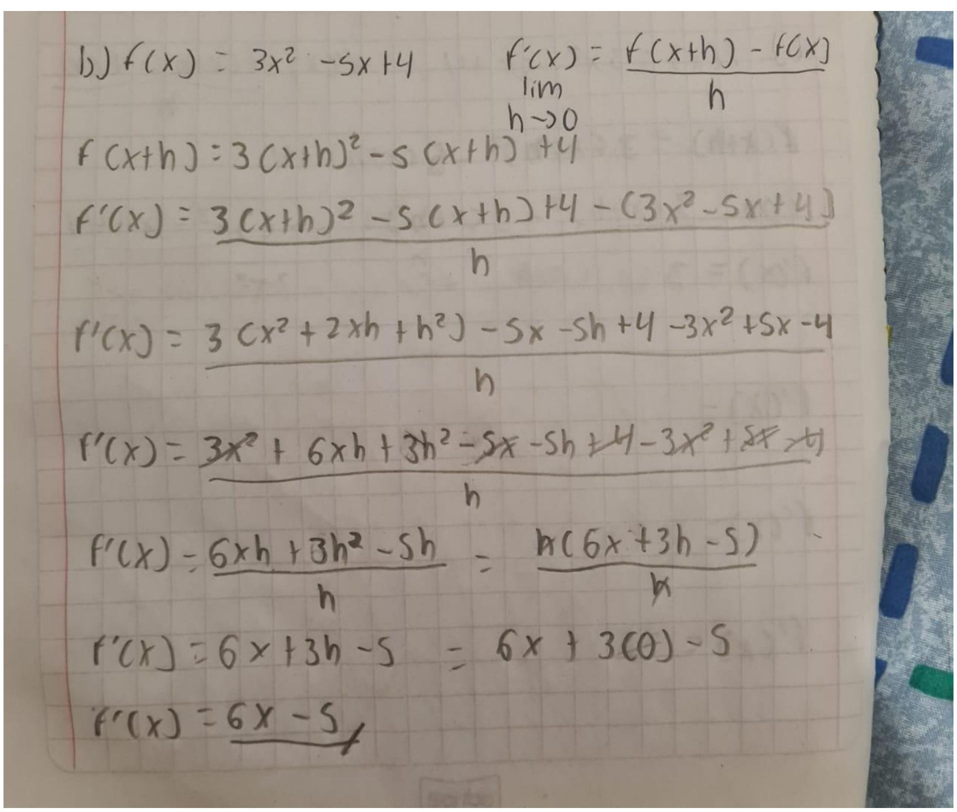
$$f'(x) = \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x + 3h - 5)}{h}$$

$$f'(x) = 6x + 3h - 5 = 6x + 3(0) - 5$$

$$f'(x) = 6x - 5$$

Then, you cancel the positive ones with the negatives and you obtain a trinomial which you factor it and eliminate x,

Finally you substitute the h for the limit that is 0 and resolve it and the result is 6x-5.



$c)\ \boldsymbol{f(x) = \frac{1}{x}}$ $\qquad\qquad \lim\limits_{h=0} \frac{f(x+h)-f(x)}{h}$

First, we obtain f(x+h) by substituting x by (x+h)

$$\boldsymbol{f(x+h)} = \frac{1}{x+h}$$

Then we substitute in the equation the values and we resolve them.

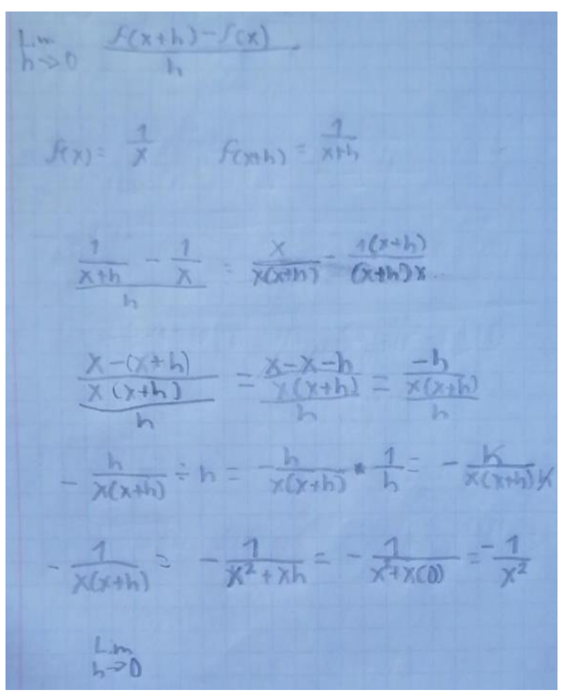
$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{1(x)}{(x+h)(x)} - \frac{1(x+h)}{(x)(x+h)}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\frac{x-x-h}{x(x+h)}}{h}$$

$$\frac{-h}{x(x+h)} \div h = \frac{-h}{x(x+h)} \times \frac{1}{h} = -\frac{h}{x(x+h)h} = -\frac{1}{x^2+xh}$$

Finally you substitute the h for the limit that is 0 and resolve it

$$\lim_{h=0} -\frac{1}{x^2+xh} = -\frac{1}{x^2+x(0)} = -\frac{1}{x^2}$$

$$\boldsymbol{f'(x)} = -\frac{1}{x^2}$$



$\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = \frac{1}{x}$ $\quad f(x+h) = \frac{1}{x+h}$

$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x}{x(x+h)} - \frac{1(x+h)}{(x+h)x}$

$\frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\frac{x-x-h}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h}$

$-\frac{h}{x(x+h)} \div h = -\frac{h}{x(x+h)} * \frac{1}{h} = -\frac{h}{x(x+h)h}$

$-\frac{1}{x(x+h)} = -\frac{1}{x^2+xh} = -\frac{1}{x^2+x(0)} = -\frac{1}{x^2}$

$\lim\limits_{h\to 0}$

d)= $\sqrt{x}$

First, we obtain f(x+h) by substituting x by (x+h)

f(x+h)=$\sqrt{x+h}$

Then we substitute in the equation the values and we resolve them.

The equation is rationalized by multiplying by the same expression, but with the sign changed and divided by itself

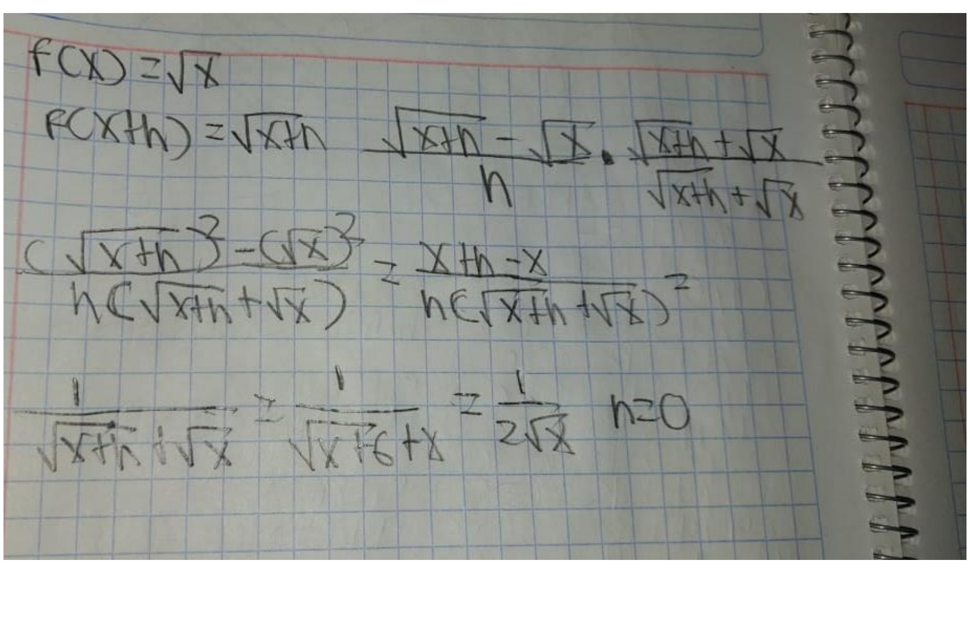
$$\frac{\sqrt{x+h}-\sqrt{x}}{h} * \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

The square root cancels with the power squared

$$\frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\ \sqrt{x})}=\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

Finally you substitute the h for the limit that is 0 and resolve it.

$$\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$



$f(x)=\sqrt{x}$

$f(x+h)=\sqrt{x+h}$ $\quad \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$$\frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}=\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=$$

$$\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x+0}+\sqrt{x}}=\frac{1}{2\sqrt{x}} \quad h=0$$

$$\boldsymbol{e})\ \boldsymbol{f}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{x}-\mathbf{2}} \qquad \lim_{h=0} \frac{f(x+h)-f(x)}{h}$$

First, we obtain f(x+h) by substituting x by (x+h).

$$\boldsymbol{f}(\boldsymbol{x}+\boldsymbol{h}) = \frac{1}{x+h-2}$$

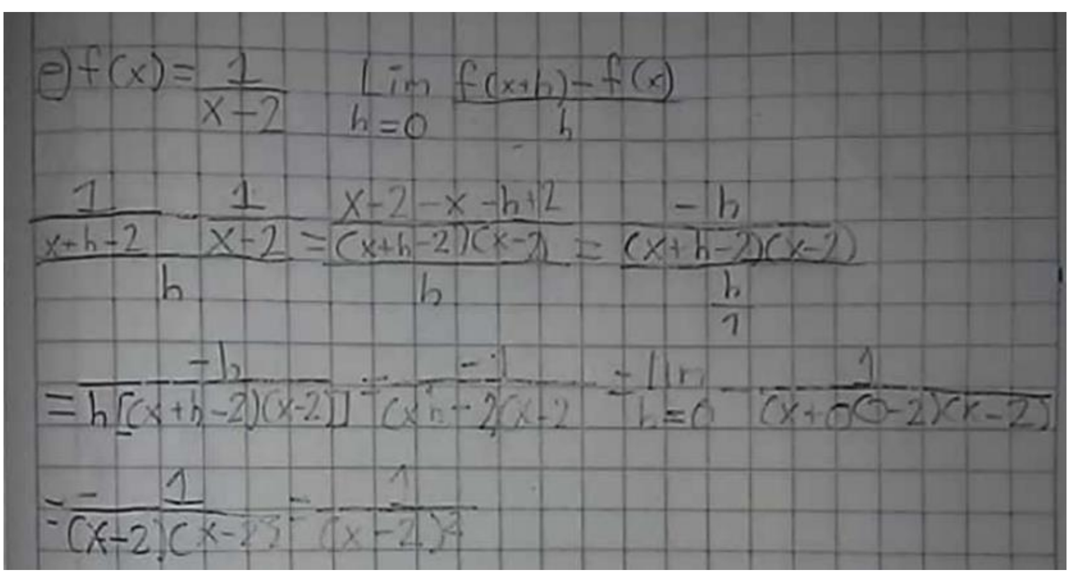
Then we substitute in the equation the values and we solve them.

$$\frac{\frac{1}{x+h-2}-\frac{1}{x-2}}{h} = \frac{\frac{x-2-x-h+2}{(x+h-2)(x-2)}}{h} = \frac{\frac{-h}{(x+h-2)(x-2)}}{\frac{h}{1}}$$

$$\frac{-h}{h[(x+h-2)(x-2)]} = \frac{-1}{(x+h-2)(x-2)}$$

Finally you substitute the h for the limit that is 0 erasing them and solve the problem.

$$\lim_{h=0} -\frac{1}{(x+0-2)(x-2)} = -\frac{1}{(x-2)(x-2)} = -\frac{1}{(x-2)^2}$$



$$\boldsymbol{f)\ f(x) = \frac{7}{\sqrt{x}}} \qquad \lim_{h=0} \frac{f(x+h)-f(x)}{h}$$

First, we obtain f(x+h) by substituting x by (x+h)

$$\boldsymbol{f(x+h)} = \frac{7}{\sqrt{x}}$$

Then we substitute in the equation the values and we resolve them.

The equation is rationalized by multiplying by the same expression, but with the sign changed and divided by itself

$$\frac{\frac{7}{\sqrt{x+h}} - \frac{7}{\sqrt{x}}}{h} = \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{(\sqrt{x} \times \sqrt{x+h})}}{h} = \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{(\sqrt{x} \times \sqrt{x+h})} \times \frac{7\sqrt{x} + 7\sqrt{x+h}}{7\sqrt{x} + 7\sqrt{x+h}}}{h}$$

$$\frac{\frac{49x - 49x - 49h}{(\sqrt{x} \times \sqrt{x+h}) \times (7\sqrt{x} + 7\sqrt{x+h})}}{h} = \frac{\frac{-49h}{(\sqrt{x} \times \sqrt{x+h}) \times 7(\sqrt{x} + \sqrt{x+h})}}{h}$$

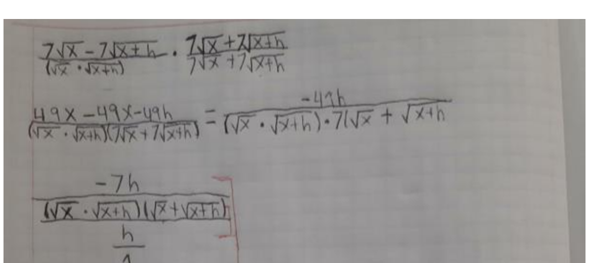
we do the multiplication of the sandwich(extreme*extreme/midle*midle)

$$\frac{\frac{-7h}{(\sqrt{x} \times \sqrt{x+h}) \times (\sqrt{x} + \sqrt{x+h})}}{\frac{h}{1}} = \frac{-7h}{h[(\sqrt{x} \times \sqrt{x+h}) \times (\sqrt{x} + \sqrt{x+h})}$$

Finally we substitute the h for the limit that is 0 and resolve it

$$\lim_{h=0} \frac{-7}{(\sqrt{x} \times \sqrt{x+0}) \times (\sqrt{x} + \sqrt{x+0})}$$

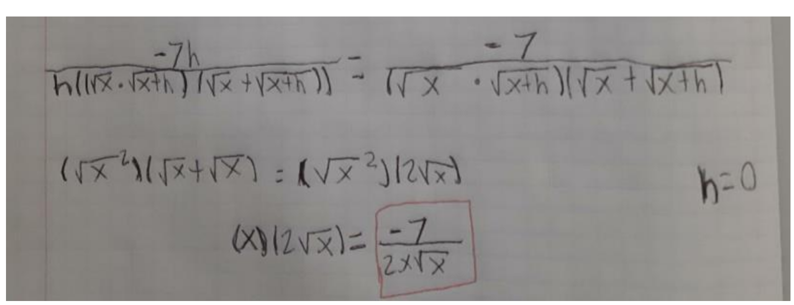
$$\frac{-7}{(\sqrt{x}^2)(2\sqrt{x})} = -\frac{7}{(x)(2\sqrt{x})} = -\frac{7}{2x\sqrt{x}}$$



$$\frac{7\sqrt{x} - 7\sqrt{x+h}}{(\sqrt{x} \cdot \sqrt{x+h})} \cdot \frac{7\sqrt{x} + 7\sqrt{x+h}}{7\sqrt{x} + 7\sqrt{x+h}}$$

$$\frac{49x - 49x - 49h}{(\sqrt{x} \cdot \sqrt{x+h})(7\sqrt{x} + 7\sqrt{x+h})} = \frac{-49h}{(\sqrt{x} \cdot \sqrt{x+h}) \cdot 7(\sqrt{x} + \sqrt{x+h}}$$

$$\frac{\frac{-7h}{(\sqrt{x} \cdot \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}}{\frac{h}{1}}$$



$$\frac{-7h}{h((\sqrt{x} \cdot \sqrt{x+h})(\sqrt{x} + \sqrt{x+h}))} = \frac{-7}{(\sqrt{x} \cdot \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$$

$$(\sqrt{x}^2)(\sqrt{x} + \sqrt{x}) = (\sqrt{x}^2)(2\sqrt{x})$$

$h = 0$

$$(x)(2\sqrt{x}) = \boxed{\frac{-7}{2x\sqrt{x}}}$$