

Descriptive Statistics & Methods for Data Science

Data Science:- Data science is the science which is used in computer science, statistics, machine learning, visualization and human computer interactions to collect, clean, integrate, analyze, interact with data to create data products.

* Data science is a multi disciplinary field that uses scientific methods, processes, algorithms and systems to extract knowledge from structured and unstructured data.

Eg:- Data science and Machine learning can be used in cyber security programs to identify threats, attack, scam, malware and also to prevent fraud.

* Data science provides meaningful information based on large amounts of complex data (or) big data.

Data science is about solving business problems. Data science deals with enriching the data and making it better for their company. To analyze the data and improving its quality.

Statistics:- Statistic is a branch that deals with the study of collection, analysis, interpretation, organization and presentation of data. Mathematically, statistics is defined as the set of equations which are used to analyze the science of data.

Eg:- statistics used in agriculture, biology, business, hospitals, colleges etc.

There are two types of statistics:

1. Descriptive

2. Inferential

Descriptive Statistics:- Descriptive Statistics Summarizes (or) Describes characteristics of a Data science. Descriptive Statistics consists of two basic categories of measures

1. Measures of central tendency

2. Measures of variability. (or) Spread (or) Methods of Dispersion.

Measures of central tendency include the mean, median and mode. while measures of variability include the standard deviation, variance, minimum, maximum, skewness and kurtosis.

Inferential Statistics:- Inferential statistics that gives sample data to make decision (or) prediction. the most common methodologies are hypothesis test, Confidence intervals and regression Analysis

Population:- Population refers to the total set of observations that can be made. population includes all the elements from a set of data.

Eg:- Total students in a college.

Sample:- A sample consists of one (or) more observations brought from the population.

Statistical data:- A Sequence of observations made on a set of objects included in a sample drawn from population is known as statistical data.

i) ungrouped data:- The data which have been arranged in a systematic order is called ungrouped data (or) raw data

Eg:- 0, 1, 2, 3, ...

ii) grouped data:- Grouped data presented in the form of frequency distribution.

Eg:-

Classes	frequency
0-10	15
10-20	20
20-30	5

Collection of data:- the first step in an investigation is the collection of data, the data may be collected for the whole population (or) for the sample only. It is mostly collected on a sample basis.

Types of data:- There are two types for the collection of data

i) primary data

ii) Secondary data

1) Primary data:- Primary data is the first hand information which is directly collected from one source. It can be obtained from

- * Direct personal Observation

- * Direct / Indirect oral Interviews

- * Administrative Questionnaires

ii) Secondary data:- Secondary data is the second hand information which is already collected by an organisation for some purpose and are available for the present study. It can be obtained from

- * Official [Applications, Agriculture, Industries]

- * Semi-Official [Bank, railways etc.]

- * Technical, ~~Gender~~, News papers, Journals.

Type of variables:- i) Independent variable ii) dependent variable

i) Independent variable:- It is a variable that is the cause (or) reason of any situation which can be manipulated. This is also known as experimental (or) predictor variable.

ii) Dependent variable:- It is a variable something that depends on other factors. It is also known as outcome variable.

Eg:- Time spent on study causes a change in test mark.

Independent

Dependent

Categorical variable:- Categorical variables represent the types of data. This is also known as discrete (or) qualitative variable.

Continuous variable:- This variable is not restricted to particular values. It is also known as quantitative variable.

Measures of Central tendency:- Central tendency is that value which is most representative of that data set. It is a statistical measure and calculates the location (or) position of a central point to explain the central tendency of the whole quantity of data. Measures of central tendency is also known as measure of central value (or) Measure of location (or) Average of first order.

Measures of central tendency are often called as averages.

Eg:- Kohli is representative of Cricket team of India.

The three most common measures of central tendency are the mean, median and mode.

Mean (Arithmetic mean):- Arithmetic mean of set of observations is their sum divided by the No. of observations.

$$\bar{x} = \frac{\text{Sum of observations}}{\text{Total no. of observations}} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x_i}{n}$$

Direct Method:- In case of frequency distribution

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum f_i x_i}{\sum f} = \frac{\sum f_i x_i}{N}$$

Shortcut Method:- If the values of x_i are large

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \text{ where } d = x_i - A, A = \text{Assumed mean}$$

Step-Deviation Method:-

$$\bar{x} = A + \frac{\sum f_i u_i}{N} \times h$$

where, h = length of interval

$$u = \frac{x_i - A}{h}$$

* If \bar{x}_1, \bar{x}_2 be the means of two samples of size n_1, n_2 then the mean \bar{x} of the combined sample of size $n_1 + n_2$ is given by $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$.

Calculate mean of the frequency distribution relating to the weight of 120 articles

Weight	0-10	10-20	20-30	30-40	40-50	50-60
NO. of Articles	14	17	22	26	23	18

A) Direct method:- $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

class	Midvalue (x)	f req	$f_i x_i$	d_i $x_i - A$	$f_i d_i$	h	$f_i h_i$
0-10	5	14	70	-20	-280	-2	-28
10-20	15	17	255	-10	-170	-1	-17
20-30	(25)	22	550	0	0	0	0

30-40	35	26	910	10	260	1	26
40-50	45	23	1035	20	460	2	46
50-60	55	18	990	30	540	3	54
		<u> </u>	<u> </u>		<u> </u>		<u> </u>
		$N = \sum f_i = 120$	$\sum f_i x_i = 3810$		$\sum f_i d_i = 810$		81

$$= \frac{3810}{120} = 31.75$$

Short cut method: $\bar{x} = A + \frac{\sum f_i d_i}{N}$ where $d_i = x_i - A$

$$= 25 + \frac{810}{120} = 31.75$$

Step deviation method: $\bar{x} = A + \frac{\sum f_i u_i}{N} \times h$ where $u_i = \frac{x_i - A}{h}$

$$= 25 + \frac{81}{120} \times 10 = 31.75$$

Median:- In a group of n observations arranged in ascending, (or) descending order of magnitude then the ^{middle} value is called median. It is denoted by Me .

Note:- When we calculate median,

if n is even, then median is $\frac{(\frac{n}{2})^{th} + (\frac{n}{2} + 1)^{th}}{2}$
if n is odd then $\frac{n+1}{2}$

2. Find the median of discrete data

x	1	2	3	4	5	6
f	7	12	17	19	21	24

x	f	cf
1	7	7
2	12	19
3	17	36
④	19	55
5	21	76
6	24	100
	<u>$\Sigma f = 100$</u>	

Median = size of $\left(\frac{n+1}{2}\right)^{th}$

$$= \left(\frac{100+1}{2}\right)^{th} = (50.5)^{th} \text{ item} \\ = 4.$$

Continuous data:-
$$l + \frac{\frac{N}{2} - c}{f} \times h = \text{Median}$$

where, N = Total frequency

l = lower limit of Median class

f = frequency of median class.

c = cumulative frequency of the class

preceding to the median class.

h = class size.

1. Find the median wage of the following distribution

wages	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of workers	3	5	20	10	5

C.I.	f	C.f
2000-3000	3	3
3000-4000	5	8
<u>4000-5000</u>	<u>20</u>	28
5000-6000	10	38
6000-7000	5	43
	<u>$\Sigma f = 43$</u>	

$$\frac{N}{2} = \frac{43}{2} = 21.5$$

$$4000 + \frac{21.5 - 8}{20} \times 1000$$

$$4000 + \frac{13.5}{20} \times 1000$$

$$4000 + 675$$

$$= 4675$$

mode:- The value of the variable for which the frequency is maximum is called mode (or) modal value. It is denoted by z (or) M .

1. Find the mode of series:

19, 17, 16, 19, 7, 9, 8, 9, 7, 9, 19, 9.

A) 7, 7, 9, 9, 9, 9, 19, 19, 19, 16, 17.

mode = 9.

find the mode of discrete data

x	8	7	10	14	22	80
f	1	3	9	17	14	5

mode = 17.

continuous data:- $z = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

l = lower limit of modal data.

f = frequency of modal data.

f_1 = frequency of the class preceding to the modal data.

f_2 = frequency of the class succeeding to the modal data.

h = size of class

1. Find mode of the following data

class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	4	13	21	44	33	22	7

Class	frequency	cf
0-10	4	4
10-20	13	17
20-30	21	38
30-40	44	82
40-50	33	115
50-60	22	137
60-70	7	144
	<u>$\Sigma f = 144$</u>	

$$\text{mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 30 + \frac{44 - 21}{2(44) - 21 - 33} \times 10$$

$$= 30 + \frac{23}{34} \times 10$$

$$= 30 + 6.76$$

$$\boxed{\text{mode} = 36.76}$$

Find the mode of the following distributions

class	frequency
0-10	5
10-20	8
20-30	7
30-40	12
40-50	(28)
50-60	20
60-70	10
70-80	10

$$\text{mode} = 40 + \frac{28 - 12}{2(28) - 12 - 20} \times 10$$

$$40 + \frac{16}{24} \times 10$$

$$40 + 6.6$$

$$= 46.6$$

find mean, median, mode

An incomplete frequency distribution is given as below.

Grade	frequency	midvalue	fixi	cf
40-49	3	44.5	133.5	3
50-59	5	54.5	272.5	8
60-69	6	64.5	387	14
70-79	(9)	74.5	670.5	(23)
80-89	8	84.5	676	31
90-99	7	94.5	661.5	38
	<u>$\Sigma f_i = 38$</u>		<u>$\Sigma f_i x_i = 2801$</u>	

$$\frac{N}{2} = \frac{19}{2} = 9.5$$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2801}{38} = 73.71$$

$$\text{median} = 70 + \frac{19 - 14}{9} \times 10 = 70 + 5 = 75$$

$$\begin{aligned} \text{mode} &= 70 + \frac{9-6}{2(9)-6-8} \times 9 \\ &= 70 + \frac{3}{4} \times 9 = 70 + \frac{27}{4} = 70 + 6.75 \\ &= 76.75 \end{aligned}$$

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean.}$$

Geometric mean:- Geometric mean of a set of n observations is n th root of their product.

$$\text{ungrouped data } G.M = \text{Antilog} \left(\frac{1}{N} \sum \log x_i \right)$$

$$\text{Grouped data } = G.M = \text{Antilog} \left(\frac{\sum f \log m}{N} \right)$$

where M is mid value.

1. Daily income of 10 families of a particular place is given below. Find G.M

x	$\log x$
85	1.9294
70	1.8450
15	1.1760
75	1.8750
500	2.6989
8	0.9030
45	1.6532
250	2.3979
40	1.6020
60	1.7781

$$N = 10$$

$$G.M = \text{Antilog} \left(\frac{1}{10} \times 17.8585 \right)$$

$$= \text{Antilog} (1.78585)$$

$$= \underline{61.073} \left(10^{1.78585} \right)$$

$$\underline{\sum \log x = 17.8585}$$

Q. Calculate G.M for the following data

Marks	freq	$\frac{M}{\log m_i}$	$\log m_i$	$G.M = \text{Antilog} \left(\frac{\sum f \log m_i}{N} \right)$	$f_i \log m_i$
4-8	6	6	0.7781		4.6686
8-12	10	10	1		10
12-16	18	14	1.1461		20.6298
16-20	30	18	1.2552		37.656
20-24	15	22	1.3424		20.136
24-28	12	26	1.4149		16.9796
28-32	10	30	1.4771		14.771
	<u>101</u>				<u>124.84124</u>

$$G.M = \text{Antilog} \left(\frac{124.84124}{101} \right)$$

$$= \text{Antilog} (1.236051)$$

$$= 17.22070$$

Harmonic mean:- Harmonic mean is the reciprocal of the Average of the reciprocal values.

For raw data, $H.M = \frac{N}{\sum \frac{1}{x}}$

For ungrouped data, $H.M = \frac{N}{\sum \frac{f_i}{x_i}}$

For grouped data, $H.M = \frac{N}{\sum f_i / m_i}$

1. Find the H.M of the following

125, 130, 75, 10, 45, 5, 0.5, 0.4, 500, 150

A) $H.M = \frac{1}{\frac{1}{x}} = \frac{1}{0.008, 0.0076, 0.013, 0.1, 0.022, 0.2, 2, 2.5, 0.002, 0.0066}$

$$H.M = \frac{10}{4.8592} = 2.05795$$

2. Calculate H.M for the following

Marks	students	f_i/x_i
10	20	2
20	30	1.5

25	50	2
40	15	0.375
50	5	0.1
<u>N = 120</u>		<u>5.975</u>

$$H.M = \frac{120}{5.975} = 20.0836.$$

3. <u>class</u>	<u>frequency</u>	<u>m</u>	<u>f_i/m</u>
10-20	4	15	0.266
20-30	6	25	0.24
30-40	10	35	0.2857
40-50	7	45	0.1555
50-60	3	55	0.05454
	<u>30</u>		<u>1.00174</u>
H.M = $\frac{30}{1.00174}$			
		= 29.947	

Measures of variability / dispersion:- The measure of the scatterness of the mass of figures in a series about an average is called Measure of variation / dispersion.

Dispersion can be classified into two categories:

1. The Measures which express the spread of observations in terms of distance b/w the values of selected observations. Eg:- range and inter quartile range.
2. The measure which express the spread of observations in terms of the average of deviations of observations from some central value. Eg:- Mean deviations and standard deviation.

Range:- The range is the difference between largest and smallest value in the series. It

$$\therefore \text{Range} = \text{largest value} - \text{smallest value}$$

$$R = L - S$$

Coefficient of Range:- $\frac{L-S}{L+S}$

Quartile deviation:- Quartile are those value which divide the frequency into four equal parts when the values are arranged in the Ascending order of magnitude. The lower quartile (Q₁) is at mid way between the lower

extreme and the median.

The upper quartile (Q_3) is midway b/w median and the upper extreme.

$Q_3 - Q_1$ is called interquartile Range.

$$Q_1 = l + \frac{\frac{N}{4} - c}{f} \times h, \quad Q_3 = l + \frac{\frac{3N}{4} - c}{f} \times h$$

Co-efficient of Quartile deviation / semi inter quartile

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Semi inter quartile range $Q = \frac{Q_3 - Q_1}{2}$

Mean deviation:- Mean deviation is defined as the arithmetic Average of the absolute deviations of a series computed from any one of the Measures of central tendency.

$$M.D(\bar{x}) = \frac{\sum |x - \bar{x}|}{n} \quad \text{where } \bar{x} = \frac{\sum x}{n}$$

ungrouped, $M.D(\bar{x}) = \frac{\sum f |x - \bar{x}|}{n}$ where $\bar{x} = \frac{\sum f_i x_i}{n}$

grouped, $M.D(\bar{x}) = \frac{\sum f |m_i - \bar{x}|}{n}$, $\bar{x} = \frac{\sum f_i m_i}{n}$

step deviation:-

$$M.D = A + \frac{\sum f_i u_i}{n} \times h \quad \text{where, } u_i = \frac{x_i - A}{h}$$

Co-efficient of mean deviation:-

$$\frac{M.D(\bar{x})}{\bar{x}}$$

Standard deviation:- (S.D) This measure of dispersion was represented by Karl Pearson in 1893. SD is the positive square root of the A.M of the squares of the deviations of the given values.

$$\sigma = \sqrt{\frac{1}{n} \sum f_i |x - \bar{x}|^2}$$

$$\sigma = \sqrt{\frac{\sum f x^2}{n} - \left(\frac{\sum f x}{n} \right)^2}$$

co-efficient of standard deviation:- $\frac{\sigma}{\bar{x}}$

shortcut Method:- $\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$

where, $d = x_i - A$

Step deviation:-

$$\sigma = h \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \quad d = \frac{x_i - A}{h}$$

Variance:- $\sigma^2 = (S.D)^2 \Rightarrow \sigma^2 = \frac{1}{N} \sum f_i |x_i - \bar{x}|^2$

1. Find the range and co-efficient of range for the following.

x f

10 2

12 4

13 6

9 4

15 6

20 8

Largest x value = 20

Smallest x value = 9

$$R = L - S = 20 - 9 = 11$$

$$\text{co-efficient of range} = \frac{L - S}{L + S} = \frac{20 - 9}{20 + 9} = \frac{11}{29}$$

$$= 0.3793$$

2. class

frequency

Largest x value = 17.5

8.5-11.5

4

Smallest x value = 2.5

11.5-14.5

8

$$R = L - S = 17.5 - 2.5 = 15$$

14.5-17.5

3

$$\text{C.R.} = \frac{17.5 - 2.5}{17.5 + 2.5} = \frac{15}{20} = 0.75$$

2. calculate median lower, upper quartiles, from the following distribution obtained by 49 students in a class. find class frequency cf Semi-interquartile range and mode

5-10

5

5

10-15

6

11

15-20

15

26

20-25

10

36

25-30

5

41

30-35

4

45

35-40

2

47

40-45

2

49

Given $N = 49 \Rightarrow \frac{N}{2} = 24.5$

$$Me = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$= 15 + \frac{24.5 - 11}{15} \times 5$$

$$= 15 + 4.5 = 19.5$$

$$\text{Lower quartile} = Q_1 = l + \frac{\frac{N}{4} - c}{f} \times h \quad \frac{N}{4} = \frac{49}{4} = 12.25$$

$$= 15 + \frac{12.25 - 11}{15} \times 5$$

$$= 15 + 0.41 = 15.41$$

$$\text{Upper Quartile } Q_3 = l + \frac{\frac{3N}{4} - C}{f} \times h$$

$$= 25 + \frac{36.75 - 36}{5} \times 5$$

$$= 25 + 0.75 = 25.75$$

$$\text{Semi interquartile} = \frac{1}{2} (Q_3 - Q_1) = \frac{1}{2} (25.75 - 15.04)$$

$$= 5.35$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h \quad \text{Highest frequency} = 15$$

$$15 + \frac{15 - 6}{2(15) - 6 - 10} \times 5 = 15 + \frac{9}{14} \times 5$$

$$= 15 + 3.214$$

$$= 18.214$$

Find the mean deviation and coefficient of mean deviation from the mean of the following data.

x 38 70 48 40 42 55 63 46 54 44

$$\text{Mean deviation } (\bar{x}) = \frac{\sum |x - \bar{x}|}{n}$$

$$\bar{x} = \frac{\sum x}{n} = \frac{500}{10} = 50$$

$$= \frac{12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6}{10}$$

$$= 8.4$$

$$\text{Coefficient of M.D} = \frac{M.D(\bar{x})}{\bar{x}} = \frac{8.4}{50} = 0.168$$

2. Find the M.D from median for the data 34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

A) 30, 34, 38, 40, 42, 44, 50, 51, 60, 66

$$\frac{42 + 44}{2} = 43 = \text{me}$$

$$M.D(\text{me}) = \frac{\sum |x_i - \text{me}|}{n} = \frac{13 + 9 + 5 + 3 + 1 + 1 + 7 + 8}{10} = \frac{47}{10} = 4.7$$

calculate co-efficient of variation of the following data.

Item	freq	d_i	d_i^2	fid_i	fid_i^2
10	4	-6	36	-24	144
12	5	-4	16	-20	80
14	10	-2	4	-20	40
A 16	14	0	0	0	0
18	9	2	4	18	36
20	4	4	16	16	64
22	2	6	36	12	72
	<u>48</u>			<u>-18</u>	<u>436</u>

$$\bar{x} = A + \frac{\sum fid_i}{N}$$

$$= 16 + \frac{-18}{48} = 16 - 0.375 = 15.625$$

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \sqrt{\frac{436}{48} - \left(\frac{-18}{48}\right)^2}$$

$$= \sqrt{9.0833 - \frac{324}{2304}}$$

$$= \sqrt{9.0833 - 0.140625}$$

$$= \sqrt{8.942675} = 2.9904$$

$$\text{co-efficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.9904}{15.625} = 19.138$$

Skewness:- skewness is a measure of symmetric in a statistical distribution in which the curve appears bend (or) skewed either to the left or right
 $\text{mean} \neq \text{median} \neq \text{mode}$.

Types of skewness:-

Positive skewness:- If the distribution curve is stretched to wards right we say that there is +ve skewness in the data.

Negative skewness:- If the distribution curve is stretched to ~~wards~~ wards left we say that there is -ve skewness in the data.

measures of skewness:-

(i) Karl Pearson's co-efficient of skewness

(ii) Bowley's

(iii) Kelly's

Karl Pearson's co-efficient of skewness:- is widely used method

* It is denoted by Skp

$$Skp = \frac{\bar{x} - z}{\sigma}$$

$$Skp = \frac{3(\bar{x} - Me)}{\sigma}$$

Problem:-

1. calculate the co-efficient of skewness from the following data.

size	30	40	50	60	70	80	90	100
frequency	7	10	14	35	102	136	43	8

A)	size	f _{real}	$d_i = \frac{x_i - A}{h}$	$f d$	$f d^2$
	30	7	-4	-28	112
	40	10	-3	-30	90
	50	14	-2	-28	56
	60	35	-1	-35	35
(70) A	102		0	0	0
	80	136	1	136	136
	90	43	2	86	172
	100	8	3	24	2
		<u>335</u>		<u>125</u>	<u>673</u>

$$\text{mean } \bar{x} = A + \frac{\sum f d_i}{N} \times h$$

$$= 70 + \frac{125}{335} \times 10 = 70 + 3.731$$

$$\bar{x} = 73.731$$

Mode \bar{z} = Highest frequency = 136

$$\text{SD } \sigma = \sqrt{\frac{\sum f d^2}{N} - \left(\frac{\sum f d}{N}\right)^2} \times h$$

$$\sigma = \sqrt{\frac{673}{335} - \left(\frac{125}{335}\right)^2} \times 10$$

$$\sigma = \sqrt{2.008 - \frac{15625}{112225}} \times 10$$

$$\sigma = \sqrt{2.008 - 0.1392} \times 10$$

$$\sigma = \sqrt{1.868} \times 10$$

$$\sigma = 1.3666 \times 10 \therefore \sigma = 13.6$$

Karl Pearson's co-efficient of skewness

$$\text{Skp} = \frac{\bar{x} - \bar{z}}{\sigma} = \frac{73.731 - 80}{13.6} = -0.487 \sim -0.4609$$

kurtosis:-

moments:- moments are a set of statistical parameters to measure a distribution

moments about mean:-

$$d = x - A$$

$$\mu_1' = \frac{\sum fd}{N}$$

$$\mu_2' = \frac{\sum fd^2}{N}$$

$$\mu_3' = \frac{\sum fd^3}{N}$$

$$\mu_4' = \frac{\sum fd^4}{N}$$

$$d = \frac{x - A}{h}$$

$$= \frac{\sum fd}{N} \times h$$

$$= \frac{\sum fd^2}{N} \times h^2$$

$$= \frac{\sum fd^3}{N} \times h^3$$

$$= \frac{\sum fd^4}{N} \times h^4$$

first moment:-

$$\mu_1 = \mu_1' - \mu_1' = 0$$

second moment: $\mu_2 = \mu_2' - \mu_1'^2$

third moment: $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

fourth moment: $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

Moment ratios:- Ratios in between moments are called moment ratio. we can measure skewness and kurtosis of the distribution

Skewness based on moments:-

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

If $\beta_1 = 0$, then the distribution is symmetric.

If $\beta_1 > 0$, then the distribution is positively skewed.

If $\beta_1 < 0$, then the distribution is negatively skewed.

kurtosis:- kurtosis explains about the shape of a frequency distribution.


$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

If $\beta_2 = 3$ then the distribution is said to be normal and the curve is mesokurtic.



If $\beta_2 > 3$ then the distribution is said to be more peaked and the curve is leptokurtic.



If $\beta_2 < 3$ then the distribution is platykurtic 

1. calculate the first four moments of the following distributions about the mean and hence find β_1 and β_2 .

x	f	$d = x - A$	fd	fd^2	fd^3	fd^4	
0	1	-4	-4	16	-64	256	$\mu_1' = \frac{\sum fd}{N} \times h$
1	8	-3	-24	72	-216	648	$\mu_2' = \frac{\sum fd^2}{N} \times h^2$
2	28	-2	-56	112	-224	448	$\mu_3' = \frac{\sum fd^3}{N} \times h^3$
3	56	-1	-56	56	-56	56	$\mu_4' = \frac{\sum fd^4}{N} \times h^4$
4	70	0	0	0	0	0	
5	56	1	56	56	56	56	
6	28	2	56	112	224	448	
7	8	3	24	72	216	648	
8	1	4	4	16	64	256	
	<u>256</u>	<u>0</u>	<u>0</u>	<u>512</u>	<u>0</u>	<u>2816</u>	

If we get big values, then we add $d = \frac{x-A}{h}$
else $d = x - A$

Moment about $x = 4$

$$\mu_1' = \frac{\sum fd}{N} = 0$$

$$\mu_2' = \frac{\sum fd^2}{N} = \frac{512}{256} = 2$$

$$\mu_3' = \frac{\sum fd^3}{N} = 0$$

$$\mu_4' = \frac{\sum fd^4}{N} = \frac{2816}{256} = 11$$

Moments about the mean:-

$$\mu_1 = \mu_1' - \mu_1'^2 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 0$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 11$$

Moment fact, $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$, $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{11}{4} = 2.75$

the first four central moments of a distribution are 0, 2.5, 0.7, 18.75. Examine the kurtosis of the distribution.

A) $\mu_1 = 0$, $\mu_2 = 2.5$, $\mu_3 = 0.7$, $\mu_4 = 18.75$

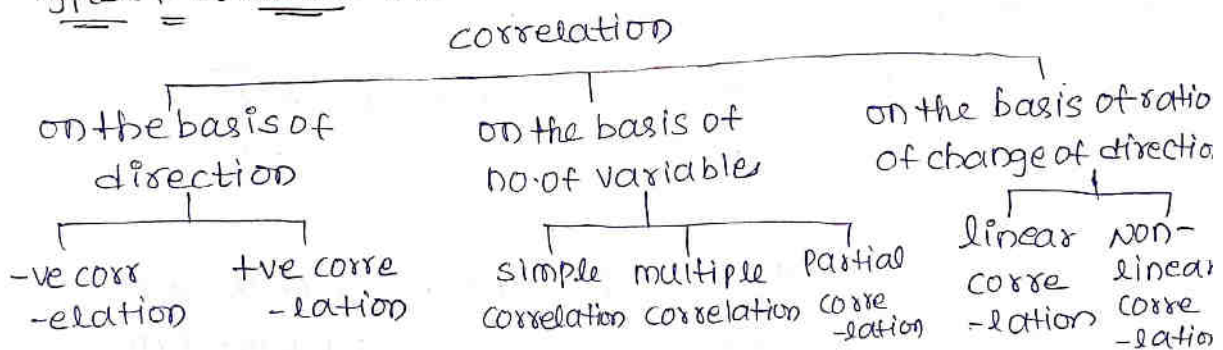
$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{0.49}{15.625} = 0.03136$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.75}{(2.5)^2} = 3$$

Correlation:- correlation is a statistical tool used to measure the relationship between two sets of variables and express each in a precise manner.

An Analysis of the covariance of two (or) more variables is usually called correlation.

Types of correlation:-



Negative correlation (inverse correlation):- Two variables are said to be correlated when both the variables vary in opposite direction.

Eg:- price and demand.

Positive correlation (Direct correlation):- Two variables are said to be positively correlated when both the variables vary in the same direction.

Eg:- Demand and supply

simple correlation:- It is a measure used to determine the relationship between two variables.

Eg:- price & demand, demand and supply.

Multiple correlation:- It is a measure used to determine the relationship among several variables.

Eg:- rainfall, temperature, yield of crops.

Partial correlation:- The study of variables, excluding some other variable is called partial correlation.

Eg:- Relation between study of two variables price and demand eliminating supply.

linear correlation:- If the ratio of change between two variables is uniform then there can be linear co-relation between them. Such variables are plotted on a graph paper. We get a straight line.

Non-linear relation:- (curvilinear):- The amount of change in one variable doesn't bear a constant ratio to the amount of change in the other variable. Then correlation is said to be curvilinear. If such variables plotted on a graph, the points would fall on a curve.

Karl Pearson co-efficient of correlation:-

When deviation is taken from A.M. the formula for coefficient of correlation is

$$r = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}} \quad \text{Where } x = x - \bar{x} \\ y = y - \bar{y}$$

$$= \frac{\sum xy}{n \sigma_x \sigma_y} \quad \text{Where, } \sigma_x = \text{s.d of } x\text{-series} \\ \sigma_y = \text{s.d of } y\text{-series}$$

$$= \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}} \quad n = \text{no. of observations}$$

A Psychological test of intelligence and of engineering ability were applied to 10 students. Here is a record of data showing intelligence ratio (IR) and engineering ratio (ER). calculate the co-efficient of correlation.

Student	Intelligence ratio	ER	$x = X - \bar{X}$	x^2	$y = Y - \bar{Y}$	y^2	xy
A	105	101	6	36	3	9	18
B	104	103	5	25	5	25	25
C	102	100	3	9	2	4	6
D	101	98	2	4	0	0	0
E	100	95	1	1	-3	9	-3
F	99	96	0	0	-2	4	0
G	98	94	-1	1	6	36	-6
H	96	104	9	81	-6	36	-54
I	93	92	-3	9	-1	1	3
J	92	97	-4	16	1	1	-4
	<u>990</u>	<u>980</u>	<u>-7</u>	<u>170</u>	<u>140</u>	<u>92</u>	

$$\bar{X} = \frac{\sum x}{n} = \frac{990}{10} = 99 \quad \bar{Y} = \frac{\sum y}{n} = \frac{980}{10} = 98$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}} = \frac{92}{\sqrt{170} \sqrt{140}} = 0.596$$

When deviations are taken from Assumed mean

$$r = \frac{n \sum d_x d_y - \sum d_x \sum d_y}{\sqrt{n \sum d_x^2 - (\sum d_x)^2} \sqrt{n \sum d_y^2 - (\sum d_y)^2}} \quad \begin{matrix} d_x = X - A_x \\ d_y = Y - A_y \end{matrix}$$

calculate co-efficient of correlation in the following

Height of father (x)	Height of son (y)	$d_x = x - 67$	$d_y = y - 68$	$d_x d_y$	d_x^2	d_y^2
65	67	-2	-1	2	4	1
66	68	-1	0	0	1	0
67	64	0	-4	0	0	16
A (67)	A (68)	0	0	0	0	0
68	72	1	4	4	1	16
69	70	2	2	4	4	4
70	69	3	1	3	9	1
71	70	4	2	8	16	4
		<u>6</u>	<u>4</u>	<u>12</u>	<u>36</u>	<u>42</u>

$$r = \frac{8 \times 26 - 26}{\sqrt{8 \times 62 - \frac{26^2}{160}} \times \sqrt{8 \times 42 - \frac{1764}{160}}}$$

$$= 0.471$$

total sales turn over and net profit of 7 medium sized companies. calculate the kare Pearson correlation

Sales turn over	Net profit	$dx = x - 400$	$dy = y - 80$	$dx dy$	dx^2	dy^2
100	30	-300	-50	15000	90000	2500
200	50	-200	-30	6000	40000	900
300	60	-100	-20	2000	10000	400
400	80	0	0	0	0	0
500	100	100	20	2000	10000	400
600	110	200	30	6000	40000	900
700	130	300	50	15000	90000	2500
		0	0	46000	280000	7600

$$r = \frac{7 \times 46000 - 46000 \times 0}{\sqrt{7 \times 280000 - 0} \sqrt{7 \times 7600 - 0}} = \frac{322000}{14100 \times 230.65}$$

$$= \frac{322000}{322911.75} = 0.9971$$

variance-covariance method:- When co-variance and variance are given then

$$r = \frac{\text{Cov}(x, y)}{\sqrt{\text{Var}(x)} \sqrt{\text{Var}(y)}}$$

If covariance between x and y variables is 12.5 and variance of x and y are 16.4 and 13.8 respectively find the co-efficient of correlation between them.

$$A) \text{Cov}(x, y) = 12.5 \quad \text{Var}(x) = 16.4 \quad \text{Var}(y) = 13.8$$

$$= \frac{12.5}{\sqrt{16.4 \times 13.8}} = 0.830$$

Spearman's rank correlation.

Charles Edward Spearman found the method of finding the coefficient of correlation by ranks. This method is useful in dealing with qualitative characteristics such as character intelligence and beauty. The value of Rank correlation coefficient always lies between -1 and 1. Formula for rank correlation is

$$r = \frac{1 - 6 \sum d^2}{n(n^2 - 1)} \quad \text{where, } d = \text{difference between two ranks (x-y)}$$

r = rank co-efficient of correlation.

n = no. of pairs of observations.

1. When ranks are not given,

i) random sample of 5 college students selected and their grades in Mathematics and Statistics are found to be. calculate Spearman's rank correlation coefficient

Here $n = 5$

given data

<u>Mathematics (x)</u>	<u>Rank(x)</u>	<u>Statistics (y)</u>	<u>Rank(y)</u>
given 85	2	73	1
data 60	4	75	3
73	3	65	4
40	5	50	5
90	1	80	2

$$d = x - y$$

$$1 \quad 1 \quad -1 \quad 0 \quad -1 \quad d^2 \quad 1 \quad 1 \quad 0 \quad 1 \quad = 4$$

spearman's rank correlation is
$$\rho = \frac{1 - 6 \sum d^2}{n(n^2 - 1)}$$

$$= \frac{1 - 6(4)}{5(5^2 - 1)}$$

$$= 1 - 0.2 = 0.8$$

i) calculate the coefficient of correlation by rank method

X	83	88	95	70	60	90	81	50
Y	120	134	130	115	110	140	140	100

<u>X</u>	<u>Rank(x)</u>	<u>Y</u>	<u>Rank(y)</u>	<u>d = x - y</u>	<u>d²</u>
83	4	120	3	-1	1
88	3	134	4	-1	0
95	1	130	2	0	0
70	6	115	6	0	0
60	7	110	7	-1	1
90	2	140	3	3	9
81	5	140	2	0	0
50	8	100	8	0	0
					<u>12</u>

$$\rho = \frac{1 - 6 \times 12}{8(8^2 - 1)} = \frac{1 - 72}{504} = 1 - 0.14 = 0.86$$

3) when ranks are repeated (or) equal :-

$$P = 1 - \frac{6[\sum d^2 + CF]}{n(n^2-1)}$$

where, $CF = \frac{\sum m^3 - m}{12}$

m = no. of times an item is repeated

A sample of 12 fathers and their elder sons gave the following data about their elder son's. calculate of rank correlation.

<u>Father</u>	<u>Rank x</u>	<u>son</u>	<u>Rank y</u>	<u>$d = x - y$</u> <u>d^2</u>
65	9	68	5.5	3.5 12.25
63	11	66	7.5	1.5 2.25

67	6.5	68	5.5	67 1	1
64	10		11.5	-1.5	2.25
68	4.5	65	3	1.5	2.25
62	12	69	9.5	2.5	6.25
70	2	68	5.5	-3.5	12.25
66	8	65	11.5	-3.5	12.25
68	4.5	71	1	3.5	12.25
67	6.5	67	8	-1.5	2.25
69	3	68	5.5	-2.5	6.25
71	1	70	2	-1	1
					<hr/>
					72.5

~~dx dy~~

In x-series, 67 & 68 repeated twice

$$m = 2, 2. \quad C.F = \frac{\sum m^3 - m}{12} = \frac{2^3 - 2}{12} + \frac{2^3 - 2}{12}$$

$$= \frac{8 - 2 + 8 - 2}{12} = 1.$$

In y series, 68 is repeated 4 times and 66 repeated 2 times and 65 is repeated 2 times

$$m = 4, 2, 2, \quad C.F = \frac{4^3 - 4}{12} + \frac{2^3 - 2}{12} + \frac{2^3 - 2}{12}$$

$$= \frac{60}{12} + 1 = 6.$$

$$C.F = 6 + 1 = 7.$$

$$p = \frac{1 - 6[\sum d^2 + C.F]}{n(n-1)} = \frac{1 - 6[72.5 + 6]}{12(12-1)}$$

$$= 1 - \frac{471}{1716} = 1 - 0.2744 = 0.7256$$

$$= 0.726.$$

Find the rank correlation

Regression:- The statistical method which helps us to estimate the unknown value of one variable from the known value of the related variable is called regression.

Lines of regression:- The line described in the average relationship between two variables is known as line of regression.

Regression line of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$
 X on y is $x - \bar{x} = b_{xy}(y - \bar{y})$

Here, $\bar{x} = \frac{\sum x}{n}$, $\bar{y} = \frac{\sum y}{n}$

$$b_{yx} = \frac{\sum xy}{\sum x^2} \quad (\text{or}) \quad \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$(\text{or}) \quad \frac{\text{cov}(xy)}{\sigma_x}$$

$$(\text{or}) \quad r \frac{\sigma_x}{\sigma_y}$$

Assumed mean $b_{yx} = \frac{n\sum dx dy - \sum dx \sum dy}{n\sum dx^2 - (\sum dx)^2}$

$$b_{xy} = \frac{n\sum dx dy - \sum dx \sum dy}{n\sum dy^2 - (\sum dy)^2}$$

$$b_{xy} = \frac{\sum xy}{\sum y^2} \quad (\text{or}) \quad \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$(\text{or}) \quad \frac{\text{cov}(xy)}{\sigma_y}$$

$$(\text{or}) \quad r \frac{\sigma_x}{\sigma_y}$$

Formula

(The geometric mean b/w the regression coefficients)
 $r_{xy} = \pm \sqrt{b_{xy} \cdot b_{yx}}$ for the following data for following regression equations

X	Y	XY	X ²	Y ²
1	15	15	1	225
2	25	50	4	625
3	35	105	9	1225
4	45	180	16	2025
5	55	275	25	3025
			<u>55</u>	<u>7125</u>

$$b_{yx} = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2} = \frac{5(625) - 15(175)}{5(55) - (15)^2} = 10$$

$$b_{yx} = 0.1$$

Reg of X on Y is, $X - \bar{X} = b_{yx}(Y - \bar{Y})$

$$X - 3 = 0.1(Y - 35)$$

$$X - 3 = 0.1Y - 3.5$$

$$X = 0.1Y - 0.5$$

Reg of Y on X

$$Y - \bar{Y} = b_{xy}(X - \bar{X})$$

$$Y - 35 = 10(X - 3) \quad Y = 10X - 30 + 35$$

$$Y - 35 = 10X - 30 \quad Y = 10X + 5$$

using the following bivariate data i) find the two regression lines. ii) estimate x when $y = 7$.

iii) estimate y when $x = 4$. iv) calculate r_{xy}

A)	X	Y	XY	X ²	Y ²
	1	6	6	1	36
	5	1	5	25	1
	3	0	0	9	0
	2	0	0	4	0
	1	1	1	1	1
	2	2	4	4	4
	7	1	7	49	1
	3	5	15	9	25
	<u>$\sum X = 24$</u>	<u>$\sum Y = 16$</u>	<u>$\sum XY = 38$</u>	<u>$\sum X^2 = 102$</u>	<u>$\sum Y^2 = 68$</u>

$$\bar{X} = \frac{\sum X}{n} = \frac{24}{8} = 3, \quad \bar{Y} = \frac{\sum Y}{n} = \frac{16}{8} = 2$$

$$b_{xy} = \frac{n \sum XY - \sum X \sum Y}{n \sum Y^2 - (\sum Y)^2} = \frac{8 \times 38 - 24 \times 16}{8 \times 68 - (16)^2} = \frac{-80}{288} = -0.278$$

$$b_{yx} = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2} = \frac{8 \times 38 - 24 \times 16}{8 \times 102 - (24)^2}$$

$$b_{yx} = -0.33.$$

regression line of x on y is $x - \bar{x} = b_{xy}(y - \bar{y})$

$$x - 3 = -0.278(y - 2)$$

$$x = -0.278y + 3.556$$

Regression line of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$$y - 2 = -0.33(x - 3)$$

$$y = -0.33x + 2.99$$

ii) x when y = 7.

$$x = -0.278(7) + 3.556 = 1.61$$

iii) y when x = 4.

$$y = -0.33 \times 4 + 2.99 = 1.67$$

$$iv) r_{xy} = \pm \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{(-0.278)(-0.33)}$$

$$r_{xy} = -0.3029 \quad [\because \text{both the regression co-efficients } b_{xy} \text{ and } b_{yx} \text{ are -ve}]$$

from the following data write down two regression equations estimate the marks in x when $y = 70$.

	mean	s.d	
X	48.4	8.4	$r = 0.62$
Y	35.6	10.5	

A) Average, $\bar{X} = 48.4$ $\bar{Y} = 35.6$

S.D, $\sigma_x = 8.4$ $\sigma_y = 10.5$

$r = 0.62$

Regression line of x on y .

$$X - \bar{X} = 0.62 \times \frac{8.4}{10.5} (Y - \bar{Y})$$

$$X - 48.4 = 0.496 (Y - 35.6)$$

$$X - 48.4 = 0.496Y - 17.6576$$

$$X = 0.496Y - 17.6576 + 48.4$$

$$X = 0.496Y + 30.7424$$

when $y = 70$

$$X = 34.72 + 30.7424 = 65.4624$$

Regression line of Y on X : $Y - \bar{Y} = r \frac{\sigma_Y}{\sigma_X} (X - \bar{X})$

$$Y - 35.6 = 0.62 \times \frac{10.5}{8.4} (X - 48.4)$$

$$Y - 35.6 = 0.775 (X - 48.4)$$

$$Y = 0.775X - 37.5 + 35.6$$

$$\boxed{Y = 0.775X - 1.91}$$

Angle between two regression lines:-

Let θ be the angle between the regression lines

Regression line of Y on X is

$$Y - \bar{Y} = b_{YX} (X - \bar{X}) = r \frac{\sigma_Y}{\sigma_X} (X - \bar{X})$$

Regression line of X on Y is

$$X - \bar{X} = b_{XY} (Y - \bar{Y}) = r \frac{\sigma_X}{\sigma_Y} (Y - \bar{Y})$$

$$\text{Then, } \tan \theta = \left(\frac{1-r^2}{r} \right) \cdot \frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}, \theta \text{ is acute}$$

$$= \left(\frac{r^2-1}{r} \right) \frac{\sigma_X \cdot \sigma_Y}{\sigma_X^2 + \sigma_Y^2}, \theta \text{ is obtuse.}$$

Note:- If $r=0$, $\tan \theta = \infty \Rightarrow \theta = \frac{\pi}{2}$

Hence, there is no relation between the two variables. i.e. they are independent.

If $r = \pm 1$, then $\tan \theta = 0 \Rightarrow \theta = 0$ or π

then the two regression lines are parallel (or) coincident.

1. If θ is the angle between two regression lines and S.D of Y is twice the S.D of X and $r = 0.25$. Then find $\tan \theta$.

A) Given, $r = 0.25$

$$\sigma_Y = 2\sigma_X$$

θ be the angle between two regression line,

$$\tan \theta = \left(\frac{1-r^2}{r} \right) \frac{\sigma_x \sigma_y}{\sigma_{x'} + \sigma_{y'}}$$

$$= \left[\frac{1-(0.25)^2}{0.25} \right] \times \frac{2\sigma_{x'}}{\sigma_{x'} + (2\sigma_x)^2}$$

$$= 3.75 \times \frac{2\sigma_{x'}}{\sigma_{x'} + 4\sigma_{x'}}$$

$$= 3.75 \times \frac{2\sigma_{x'}}{5\sigma_{x'}}$$

$$= 3.75 \times 0.4 = 1.5$$

4) Test whether the equations $2x+3y=4$ and $x-y=5$ represent valid regression lines.

A) Let the regression line of x on y is $2x+3y=4$

$$x = \frac{4}{2} - \frac{3y}{2} = 2 - \frac{3}{2}y$$

Let the regression line of y on x is $x-y=5$

$$y = x - 5 \rightarrow (2)$$

Comparing the lines with

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$\text{i.e.; } \frac{r \sigma_x}{\sigma_y} = -\frac{3}{2} \rightarrow (3)$$

$$r \frac{\sigma_y}{\sigma_x} = 1 \rightarrow (4)$$

$$(3) \times (4) \Rightarrow \frac{r \sigma_x}{\sigma_y} \times r \frac{\sigma_y}{\sigma_x} = -\frac{3}{2}$$

$$r^2 = -\frac{3}{2}$$

Since, $-1 \leq r \leq 1$

\therefore These two regression lines are not valid equations.

5) If $x = 2y + 3$, $y = kx + 6$ are the regression lines of x on y and y on x respectively. s.t. ① $0 \leq k \leq \frac{1}{2}$

② IF $k = \frac{1}{8}$ find $r \in (\bar{x}, \bar{y})$.

A) let the regression line of x on y is

$$X = 2Y + 3$$

let the regression line of y on x is

$$Y = kX + 6.$$

$$b_{xy} = 2 \quad b_{yx} = k$$

$$\therefore r = \pm \sqrt{b_{xy} \cdot b_{yx}} = \pm \sqrt{2k}$$

S.O.B.S

$$r^2 = 2k$$

$$-1 \leq r \leq 1$$

$$(-1)^2 = 2k \Rightarrow k = \frac{1}{2}$$

$$0 = k \Rightarrow k = 0$$

$$(1)^2 = 2k \Rightarrow k = \frac{1}{2}$$

$$\therefore 0 \leq k \leq \frac{1}{2}.$$

② $r^2 = 2k$
 $r^2 = 2 \times \frac{1}{8} \quad \text{when } k = \frac{1}{8}$

$$r^2 = \frac{1}{4}$$

$$r = \pm \frac{1}{2} = \pm \frac{1}{2}$$

let (\bar{x}, \bar{y}) is passing through the equations then

$$\bar{x} = 2\bar{y} + 3 \quad \text{--- ①}$$

$$\bar{y} = k\bar{x} + 6 \Rightarrow \frac{\bar{x}}{8} + 6. \quad \text{②}$$

solving ① & ②

$$\begin{array}{r} \text{②} \times 8 \\ \bar{x} - 2\bar{y} - 3 = 0 \\ \bar{x} - 8\bar{y} + 48 = 0 \\ \hline 6\bar{y} - 51 = 0 \\ \bar{y} = \frac{51}{6} = 8.5 \end{array}$$

$$\begin{aligned} \bar{x} &= 2 \times 8.5 + 3 \\ &= 17 + 3 \\ \bar{x} &= 20 \end{aligned}$$

Method of least squares:- Method of least square is a device for finding the equation of a specified type of curve, which best fits for a given set of observations. This method ~~defines~~ depends on principle of least square.

"The sum of squares of difference between the observed and corresponding estimated values should be the minimum."

Fitting of a straight line:-

Let $x = A + Bx$ be the line of x on y find a, b value with the following two normal equations to be solved. The normal equations are

$$\sum x = na + b \sum y$$

$$\sum xy = a \sum y + b \sum y^2$$

Let $y = A + Bx$, be then the normal equations are

$$\sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

1. consider the following data to obtain ^{two} regression equations find a and b

x	6	2	10	4	8
y	9	11	5	8	7

X	Y	X ²	Y ²	XY	Here, n=5
6	9	36	81	54	
2	11	4	121	22	
10	5	100	25	50	
4	8	16	64	32	
8	7	64	49	56	
<u>Σx=30</u>	<u>Σy=40</u>	<u>220</u>	<u>340</u>	<u>214</u>	

~~Normal~~ equations of a straight line $y = a + bx$ -

$$\text{Normal eqns are } \Sigma y = na + b \Sigma x \quad \text{--- (1)}$$

$$\Sigma xy = a \Sigma x + b \Sigma x^2 \quad \text{--- (2)}$$

$$214 = a(30) + b(220) \quad \text{--- (3)}$$

$$40 = 5a + 30b \quad \text{--- (4)}$$

Solving (3) & (4)

$$a = 11.9, \quad b = -0.65.$$

$$\text{Sub } y = a + bx$$

$$y = 11.9 - 0.65x$$

$$0.65x + y = 11.9$$

equation of a straight line, $x = a + by$ --- (1)

$$\text{normal eqns are } \Sigma x = na + b \Sigma y$$

$$\Sigma xy = a \Sigma y + b \Sigma y^2$$

$$214 = 40a + b(340) \quad \text{--- (2)}$$

$$30 = 5a + b40$$

$$a = 16.4, \quad b = -1.3$$

$$16.4 = a - b + 3x \quad x = 16.4 - 1.3y$$

1. find the equation of regression line for y on x for the following data. Also estimate y if x=75

A)

x	y	x ²	xy
65	68	4225	4420
63	66	3969	4158
67	68	4489	4556
64	65	4096	4160
68	69	4624	4692

62	66	3844	66 4096
70	68	4900	4760
66	65	4356	4990
68	71	4624	4828
67	67	4489	4489
<u>660</u>	<u>673</u>	<u>43616</u>	<u>44445</u>

Eq. of a st line in $y = a + bx$

The normal eqns are $\sum y = na + b \sum x$

$$\sum xy = a \sum x + b \sum x^2$$

$$673 = 10a + 660b$$

$$44445 = a 660 + b 43616$$

$$a = 35.48 \quad b = 0.4821$$

$$y = 35.48 + 0.4821x$$

$$\text{If } x = 75 \quad y = 35.48 + 0.4821 \times 75$$

$$\boxed{y = 71.6375}$$

3) From a sample of 200 pairs of observation the following quantities were calculated $\sum x = 11.34$, $\sum y = 20.78$, $\sum x^2 = 12.16$, $\sum y^2 = 84.96$, $\sum xy = 22.13$.

From the above data show how to compute the co-efficients of the equation $y = a + bx$.

A) $y = a + bx$

$$\sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$20.78 = 200a + 11.34b$$

$$22.13 = 11.34a + b 12.16$$

$$a = 7.513 \times 10^{-4} \quad b = 1.82$$

$$= 0.00075$$

4. Determine the equation of a st. line which best fits the data

$$a=0.82 \quad b=1.00$$

<u>X</u>	<u>Y</u>	<u>X²</u>	<u>Y²</u>	<u>XY</u>
10	10	100	100	100
12	22	144	484	264
13	24	169	576	312
16	27	256	729	432
17	29	289	841	493
20	33	400	1089	660
25	37	625	1369	925
<u>113</u>	<u>182</u>	<u>1983</u>	<u>5188</u>	<u>3186</u>

Eq of a st line in $y = a + bx$

The normal equations are $\sum y = na + b \sum x$

$$\sum xy = a \sum x + b \sum x^2$$

$$3186 = a(113) + b(1983)$$

$$182 = 7a + b(113)$$

$$a = 0.799 \quad b = 1.56$$

Eq of a straight line in $x = a + by$

Normal equation are $\sum x = a \sum 1 + b \sum y$

$$\sum xy = a \sum y + b \sum y^2$$

$$3186 = a(182) + b(5188)$$

$$113 = 7a + b(182)$$

$$a = 2.00 \quad b = 0.54$$