Definition :- A set of numbers are arranged in rows and Columns in the square matrices is Called matrix. In this matrix the set of numbers are Called element.

Ex:-

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Real matrices

1. Symmetric
2. Skew-symmetric
3. orthogonal

Def:- In any matrix Contain all the elements are real number. then that matrix is Called real matrix.

ex:- $\begin{bmatrix} 1 & 2 \\ \sqrt{3}/2 & 5/2 \end{bmatrix}$

Symmetric matrix

A square matrix "A" is said to be symmetric $A^T = A$. we have $A_{ij} = A_{ji}$ $\forall ij$.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 5 \\ 4 & 5 & 3 \end{bmatrix}$$

Skew-Symmetric matrix

A square matrix "A" is said to be Skew-Symmetric $A^T = -A$.

In this we have $A_{ij} = -A_{ji}$ and the diagonal elements of a skew-symmetric are all zero's.

Ex:-

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & +3 \\ -2 & 3 & 0 \end{bmatrix}$$

Orthogonal matrix

A square matrix is said to be orthogonal. if $A \cdot A^T = I$

$$A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

Orthogonal matrix $\Rightarrow A \cdot A^T = I$

$$A \cdot A^T = \frac{1}{4}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1+1+1+1 & -1-1+1+1 & -1+1-1+1 & -1+1+1-1 \\ -1-1+1+1 & 1+1+1+1 & 1-1-1+1 & 1-1+1-1 \\ -1+1-1+1 & 1-1-1+1 & 1+1+1+1 & 1+1-1-1 \\ -1+1-1+1 & 1-1+1-1 & 1+1-1-1 & 1+1+1+1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \frac{\cancel{4}}{\cancel{4}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A \cdot A^T = I}$$

1. Define Symmetric matrix, if $A = \begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ i.e, symmetric then find a, b

Sol:-

In a square matrix "A" is said to be symmetric matrix if $A^T = A$.

G. that "A" is symmetric matrix then

$$A = A^T$$

$$\begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 7 \\ a & 2 & 4 \\ b & 4 & 5 \end{bmatrix}$$

$a = -2 \quad b = 7$

Topic - 1.

Rank of matrix

⇒ Sub-matrix :- A matrix is obtained by Deleting some of it's rows (or) Columns (or) both is called as Sub matrix.

ex:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 8 & 9 & 6 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \quad A_2 = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \end{bmatrix}$$

Minor of matrix :- let 'A' be the matrix. the Determinant of square sub matrix is called the minor of "A".

ex:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 4 & 5 & 6 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix} \quad M_2 = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \quad M_3 = |3|$$

Rank of matrix

let A be any matrix, let the number 'r' is called be the rank of "A", if

1. There is atleast one minor of order "r" is not equal to zero.

2. All the minors of order "r+1" are equal zero.

3. The rank of matrix "A" is denoted by "$\rho(A)$" or "r".

Note :- 1. Every matrix will have a rank.

2. Rank matrix is unique.

3. If $\rho(A) \geq 1$, when A is non zero matrix.

4. rank of the identity matrix $I_{n\times n}$ is n only

5. If A is the matrix of order 'n' and "A" is non-singular [$|A| \neq 0$] then $\rho(A)$ of A is n.

Methods to find rank of matrix

1. echelon form :-

1. The rank of the matrix in echelon form = no. of non-zero rows

2. Every matrix can be transferred to echelon form by applying only row operations.

1. find the rank of $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 5 \\ 2 & 1 & 2 \end{bmatrix}$

G. $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 5 \\ 2 & 1 & 2 \end{bmatrix}$

$R_3 \longrightarrow 3R_3 - 2R_2$

$\sim \begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

$R_3 \to 4R_3 - R_2$

$$\sim \begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 11 \end{bmatrix}$$

$\therefore$ The matrix is in the echelon form.

Rank of $A = 3$

no. of non-zero rows $= 3$

$\rho(A) = 3$

* find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ by reducing echelon form.

G.
$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 5 & -3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non-zero rows = 2

∴ Rank of A $\rho(A) = 2$.

* find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & 3 & -1 \end{bmatrix}$ by reducing in echelon form.

G. $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$

$R_2 \rightarrow 2R_2 - 3R_1$

$R_3 \rightarrow 2R_3 - 1R_1$

$\sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & -13 & -13 \\ 0 & -9 & -9 \end{bmatrix}$

$R_3 \rightarrow 13R_3 - 9R_2$

$\sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & -13 & -13 \\ 0 & 0 & 0 \end{bmatrix}$

∴ no. of rows of non-zero = 2

$\rho(A) = 2$

* Define rank of matrix and find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

G. $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$R_2 - 1R_2 - 2R_1$

$R_3 - 1R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 6 \\ 0 & -5 & 6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ No. of non-zero rows $= 2$

$\rho(A) = 2$

* Reduce the matrix $A = \begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$ in to echelon form and find the matrix.

G. $A = \begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$

$R_3 \leftrightarrow R_1$

$$\begin{bmatrix} -1 & -2 & 10 & 1 \\ 2 & 10 & 2 & 4 \\ 10 & -2 & 8 & 0 \\ 2 & 3 & 4 & 9 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 + 2R_1$

$R_3 \rightarrow 1R_3 + 10R_1$

$R_4 \rightarrow 1R_4 + 2R_1$

$$\sim \begin{bmatrix} -1 & -2 & 10 & 1 \\ 0 & 6 & 22 & 6 \\ 0 & -22 & 103 & 10 \\ 0 & -1 & 24 & 11 \end{bmatrix}$$

$R_3 \rightarrow 6R_3 + 22R_2$

$R_4 \rightarrow 6R_4 + R_2$

$$\sim \begin{bmatrix} -1 & -2 & 10 & 1 \\ 0 & 6 & 22 & 6 \\ 0 & 0 & 1102 & 192 \\ 0 & 0 & 166 & 72 \end{bmatrix}$$

$R_3 \rightarrow R_3/2 \quad R_4 \rightarrow R_4/2$

$$\sim \begin{bmatrix} -1 & -2 & 10 & 1 \\ 0 & 6 & 22 & 6 \\ 0 & 0 & 551 & 96 \\ 0 & 0 & 83 & 38 \end{bmatrix}$$

$R_4 \to 551 R_4 - 83 R_3$

$$\sim \begin{bmatrix} -1 & -2 & 10 & 1 \\ 0 & 6 & 22 & 6 \\ 0 & 0 & 551 & 96 \\ 0 & 0 & 0 & 11868 \end{bmatrix}$$

No. of non-zero rows = 4

$\rho(A) = 4$

* find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$R_4 \to R_4 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{bmatrix}$$

$R_3 \to 2R_3 + 1R_2$

$R_4 \to 2R_4 - 3R_2$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -5 \end{bmatrix}$$

$R_4 \to 1R_4 - 5R_3$

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

No. of non-zero rows $= 4$

$\rho(A) = 4$

* Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ into Echelon form and hence find it's rank?

G. $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \longrightarrow 1R_2 - 2R_1$

$R_3 \longrightarrow 1R_3 - 3R_1$

$R_4 \longrightarrow 1R_4 - 6R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Here, $R_3 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to 4R_4 - 4R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -12 & 8 \end{bmatrix}$$

$R_4 \to 3R_4 - 12R_3$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ The matrix is in Echelon form.

So, the no. of non-zero rows $= 3$

rank $\rho(A) = 3$.

* find the rank $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

G. $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

$R_2 \rightarrow 1R_2 - 2R_1$

$R_3 \rightarrow 1R_3 - 3R_1$

$R_4 \rightarrow 1R_4 + 1R_1$

$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & -1 & 2 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 3R_2$

$R_4 \rightarrow 3R_4 + 3R_2$

$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix is in Echelon form.

No. of non-zero rows = 2

$\therefore$ rank $\rho(A) = 2$

* find the rank of $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{bmatrix}$

G. $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - 2R_1$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 4 & -2 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 - 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & -4 \end{bmatrix}$$

$R_4 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix is in Echelon form.

So, no. of non zero rows $= 3$. $\therefore \rho(A) = 3$

27/2

find l and m Such that $A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 7 & -1 & l & m \end{bmatrix}$ has rank two by using Echelon form

G. that, $A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 7 & -1 & l & m \end{bmatrix}$

$R_2 \to 1R_2 - 2R_1$

$R_3 \to 1R_3 - 7R_1$

$\sim \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 3 & -5 & -5 \\ 0 & 6 & l-14 & m-28 \end{bmatrix}$

$R_3 \to 3R_3 - 6R_2$

$\sim \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 3 & -5 & -5 \\ 0 & 0 & 3l-12 & 3m-54 \end{bmatrix}$

The above matrix is in echelon form.

By Given,
the rank of the matrix is "2"

The last row is zero row

$\therefore 3l-12 = 0 \qquad 3m-54 = 0$

$3l = 12 \qquad 3m = 54$

$l = 4 \qquad m = 18$

* find a and b values such that rank of A = $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is 3 by using Echelon form.

G. that $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$

$R_2 \rightarrow 1R_2 - 2R_1$

$R_3 \rightarrow 1R_3 - 6R_2$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 5 & -7 & 0 \\ 0 & 10 & a-18 & b-6 \end{bmatrix}$$

$R_3 \rightarrow 5R_3 - 10R_2$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 5 & -7 & 0 \\ 0 & 0 & 5a-20 & 5b-30 \end{bmatrix}$$

The above matrix is in echelon form.

By Given,

the rank of matrix is "3"

$\therefore$ The last row is non-zero row

Case (i):-

$5a - 20 \neq 0$ $5b - 30 \neq 0$

$a - 4 \neq 0$ $b - 6 \neq 0$

$a \neq 4$ $b \neq 6$

case-(ii)

$5a-20 = 0 \qquad 5b-30 \neq 0$

$a-4 = 0 \qquad b-6 \neq 0$

$a = 4 \qquad b \neq 6$

case-(iii):-

$5a-20 \neq 0 \qquad 5b-30 = 0$

$a-4 \neq 0 \qquad b-6 = 0$

$a \neq 4 \qquad b = 6$

Normal form (or) Canonical form :-

Every non-matrix can be reduced into the form $\begin{bmatrix} I_{r\times r} & 0 \\ 0 & 0 \end{bmatrix}$ is called normal (or) canonical form.

By using elementary row and column operations.

problems

* Reduce the matrix $A = \begin{bmatrix} 0 & 1 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to its canonical form and find it's rank.

G. that $A = \begin{bmatrix} 0 & 1 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & 1 \\ 3 & 1 & 0 & 2 \end{bmatrix}$

$R_3 \to 1R_3 - 3R_1$

$R_4 \to 1R_4 - 1R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$C_3 \to 1C_3 - 1C_1$

$C_4 \to 1C_4 - 1C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 \to 1R_3 - 1R_2$

$R_4 \to 1R_4 - 1R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 + 3C_2$

$C_4 \to C_4 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{2\times 2} & 0 \\ 0 & 0 \end{bmatrix}$$

Rank of matrix is 2

* find the rank of matrix $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$ by reducing into normal form.

G. that $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$.

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -3 & -1 \\ 3 & -2 & 4 \\ 2 & 3 & 7 \end{bmatrix}$$

$R_2 \to 1R_2 - 3R_1$

$R_3 \to 1R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & -3 & -1 \\ 0 & -7 & 7 \\ 0 & 9 & 9 \end{bmatrix}$$

$C_2 \to 1C_2 + 3C_1$

$C_3 \to 1C_3 + 1C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 7 \\ 0 & 9 & 9 \end{bmatrix}$$

$R_3 \to 7R_3 - 9R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 7C_3 - 7C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2/7$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

rank of matrix is "2"

Q.B

* Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & -5 & 10 \end{bmatrix}$ to it Canonical form and find it's rank.

G. that $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & -5 & 10 \end{bmatrix}$

$R_2 \to 1R_2 - 2R_1$

$R_3 \to 1R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -20 \end{bmatrix}$$

$C_2 \to 1C_2 - 2C_1$

$C_3 \to 1C_3 - 3C_1$

$C_4 \to 1C_4 - 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -20 \end{bmatrix}$$

$R_3 \to 3R_3 - 6R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -36 \end{bmatrix}$$

$C_2 \to C_2/-3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & -36 \end{bmatrix}$$

$C_3 \to 1C_3 + 2C_2$

$C_4 \to 1C_4 + 5C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -36 \end{bmatrix}$$

$C_3 \to C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -36 & 0 \end{bmatrix}$$

$C_3 \to C_3/-36$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} I_{3\times3} & 0 \\ 0 & 0 \end{bmatrix}$$

Rank of matrix is 3.

* Reduce the matrix A into normal form where

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

G. that, $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$.

$C_1 \to C_2$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

$R_3 \to 1R_3 - 1R_1$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$C_3 \to 1C_3 - 2C_1$

$C_4 \to 1C_4 + 2C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$R_3 \to 4R_3 - 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2/4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 1C_3 - 2C_2$

$C_4 \to 1C_4 - 6C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of matrix = 2

* $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

G. that, $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

$C_2 \leftrightarrow C_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 0 & 4 & 1 \\ 3 & 2 & 7 & 5 \\ 5 & 2 & 11 & 6 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 3R_1$

$R_3 \rightarrow 1R_3 - 3R_1$

$R_4 \rightarrow 1R_4 - 5R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & -4 & -2 & -7 \\ 0 & -8 & \cdots & \end{bmatrix}$$

$C_2 \to 1C_2 - 2C_1$

$C_3 \to 1C_3 - 3C_1$

$C_4 \to 1C_4 - 4C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -5 & -11 \\ 0 & -4 & -2 & -7 \\ 0 & -8 & -4 & -4 \end{bmatrix}$$

$R_3 \to 6R_3 - 4R_2$

$R_4 \to 6R_4 - 8R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -5 & -11 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 16 & 4 \end{bmatrix}$$

$C_2 \to C_2 / -6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -5 & -11 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 16 & 4 \end{bmatrix}$$

$C_3 \to 1C_3 + 5C_2$

$C_4 \to 1C_4 + 11C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 16 & 4 \end{bmatrix}$$

$R_4 \rightarrow 8R_4 - 16R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \rightarrow 8C_4 - 2C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \rightarrow C_3/8$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} I_{3\times3} & 0 \\ 0 & 0 \end{bmatrix}$$

Rank of matrix is 3

* $$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

G. that,

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 1R_1$

$R_3 \rightarrow 1R_3 - 2R_1$

$R_4 \rightarrow 1R_4 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 3 & -7 \\ 0 & -7 & -8 & 5 \end{bmatrix}$$

$C_2 \rightarrow 1C_2 - 1C_1$

$C_3 \rightarrow 1C_3 - 1C_1$

$C_4 \rightarrow 1C_4 - 1C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 3 & -7 \\ 0 & -7 & -8 & 5 \end{bmatrix}$$

$R_3 \to 1R_3 - 1R_2$

$R_4 \to 1R_4 + 7R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 6 & -30 \end{bmatrix}$$

$C_3 \to C_3 - 2C_2$

$C_4 \to C_4 + 5C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 6 & -30 \end{bmatrix}$$

$R_4 \to 1R_4 - 6R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -18 \end{bmatrix}$$

$C_4 \rightarrow 1C_4 + 2C_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

$C_4 \rightarrow C_4/_{-8}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is "4".

* $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

G. that, $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

$C_1 \leftrightarrow C_2$

$$\begin{bmatrix} 1 & 8 & 3 & 6 \\ 3 & 0 & 2 & 2 \\ -1 & -8 & -3 & 4 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 3R_1$

$R_3 \rightarrow 1R_3 + 1R_1$

$$\begin{bmatrix} 1 & 8 & 3 & 6 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \rightarrow 1C_2 - 8C_1$

$C_3 \rightarrow 1C_3 - 3C_1$

$C_4 \rightarrow 1C_4 - 6C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \rightarrow C_2/-24$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_3 \rightarrow 1C_3 + 7C_2$

$C_4 \rightarrow 1C_4 + 16C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_3 \rightarrow C_4$

$R_2 \longrightarrow 1R_2 - 3R_1$

$R_3 \longrightarrow 1R_3 + 1R_1.$

$$\begin{bmatrix} 1 & 8 & 3 & 6 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \rightarrow 1C_2 - 8C_1$

$C_3 \rightarrow 1C_3 - 3C_1$

$C_4 \rightarrow 1C_4 - 6C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \rightarrow C_2/-24$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_3 \rightarrow 1C_3 + 7C_2$

$C_4 \rightarrow 1C_4 + 16C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_3 \rightarrow C_4$

$R_3 \to 1R_3 - 5R_1$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix}$$

$C_2 \to C_2 + C_1$

$C_3 \to 1C_3 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & 4 \end{bmatrix}$$

$C_3 \to 1C_3 - 2C_1$

$C_4 \to C_4 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -12 & 4 \end{bmatrix}$$

$C_4 \to 12C_4 + 4C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -12 & 0 \end{bmatrix}$$

$C_3 \to \frac{C_3}{-12}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{2\times 3} & 0 \\ 0 & 0 \end{bmatrix}$$

Rank of $\rho(A) = 3$.

Reduce the matrix $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

Given $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

$C_1 \leftrightarrow C_2$

$$\begin{bmatrix} 1 & 6 & 3 & 8 \\ 2 & 4 & 6 & -1 \\ 3 & 10 & 9 & 7 \\ 4 & 16 & 12 & 15 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 4R_1$

$$\begin{bmatrix} 1 & 6 & 3 & 8 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \end{bmatrix}$$

$C_2 \to C_2 - 6C_1$

$C_3 \to C_3 - 3C_1$

$C_4 \to C_4 - 8C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \end{bmatrix}$$

$C_3 \leftrightarrow C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -8 & -17 & 0 \\ 0 & -8 & -17 & 0 \\ 0 & -8 & -17 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -8 & -17 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 \to 8C_3 - 4C_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2 \to C_2/-8$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{2\times 2} & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

6.

Solving the system of homogeneous and non-homogeneous equation.

Consider the system of non homogeneous and non homogeneous linear equation.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots \qquad\qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots a_{nn}x_n = b_n$$

The above system can be written as $AX = B$.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

The Augmented matrix is

$$AB = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{bmatrix}$$

Condition for Consistance & in-Consistance

The system of equation $AX = B$ is Consistance if and only if the rank of co-efficient matrix is equal to rank of Augmented matrix AB. that is $\rho(AB) = \rho(A)$

(ii) If $\rho(AB) \neq \rho(A)$ then the given system inconsistance.

working rule for finding solutions $Ax = B$

(i) The system of equation can be written as '$Ax = B$'.

(ii) write Augmented matrix.

(iii) Apply row operations, Convert the Aug matrix into Echelon form.

(iv) find the $\rho(A)$ & $\rho(AB)$

① If $\rho(A) \neq \rho(A \cdot B)$ then the system has no solutions.

② If $\rho(A) = \rho(AB) = r = n$ then the system has unique solution.

③ If $\rho(A) = \rho(A \cdot B) = r < n$ and the system has infinitely many solutions.

Q.B
* when the system of non-homogeneous equations will have unique solution, no. solution & infinetly many solution.

1. If $\rho(A) \neq \rho(AB)$ then the system has no solutions.
2. If $\rho(A) = \rho(AB) = r = n$ then the system has unique solution.
3. If $\rho(A) = \rho(A \cdot B) = r < n$ and the system has Infinitely many solutions.

Show that the equa.

$$x+y+z=-3$$
$$3x+y-2z=-2$$
$$2x+4y+7z=-1$$

Sol:

The given equations

$$x+y+z=-3$$
$$3x+y-2z=-2$$
$$2x+4y+7z=-1$$

$$Ax=B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ -1 \end{bmatrix}$$

Augmented values is

$$[A \cdot B] = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & -1 \end{bmatrix}$$

$$R_2 \to 1R_2 - 3R_1$$
$$R_3 \to 1R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 40 \end{bmatrix}$$

$\rho(A) = 2 \quad \rho(B) = 3$

Here, rank of a is not equal to

$\rho(A) \neq \rho(B)$

$\therefore$ The system of equas are inconsistent.

The Given has no solution.

Solve the system of equations

$$\begin{bmatrix} 2x - y + 3z = 8 \\ -x + 2y + z = 4 \\ 3x + y - 4z = 0 \end{bmatrix}$$

The given equas are

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

$AX = B$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

The Augmented values,

$$[A \cdot B] = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 + 1R_1$

$R_3 \rightarrow 2R_3 - 3R_1$

$$\sim \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 5 & -17 & -24 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 5R_2$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -76 & -152 \end{bmatrix}$$

$\rho(A) = 3 \quad \rho(A \cdot B) = 3$

Here, $\rho(A) = \rho(AB) = 3$

$\therefore$ The system of is Consistence.

Here, $\rho(A) = \rho(A \cdot B) = 3 = n$

$n = \text{no. of variables} = 3$

Here, the system has unique solution.

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$A_1 x = B_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -76 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ -152 \end{bmatrix}$$

① $-76z = -152$

$\boxed{z = 2}$

② $3y + 5z = 16$

$3y + 5(2) = 16$

$3y + 10 = 16$

$3y = 16 - 10$

$3y = 6$

$\boxed{y = 2}$

③ $2x - y + 3z = 8$

$2x = 8 + y - 3z$

$2x = 8 + 2 - 6$

$2x = 4$

$\boxed{x = 2}$

prove that (or) solve the non-homogeneous system of Equations

$3x + 3y + z = 1$

$x + 2y = 4$

$10y + z = -2$

$2x - 3y - z = 5$

The given equations are

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

$$AX = B$$

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

The Augmented equa is $(A \cdot B)$

$$= \begin{bmatrix} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix}$$

$$R_2 \rightarrow 3R_2 - 1R_1$$
$$R_4 \rightarrow 3R_4 - 2R_1$$

$$\sim \begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 10 & 3 & -2 \\ 0 & -15 & -7 & 13 \end{bmatrix}$$

$$R_3 \rightarrow 3R_3 - 10R_2$$
$$R_4 \rightarrow 3R_4 + 15R_2$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -51 & 204 \end{bmatrix}$$

$R_4 \rightarrow 29R_4 + 51R_3$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3 \quad \rho(A \cdot B) = 3$

The system is Consistence

$\rho(A) = \rho(A \cdot B) = 3 = n$

no.of Variables $= 3$

The system has unique solution

$A_1 x = B_1$

$$\begin{bmatrix} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 29 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \\ -116 \end{bmatrix}$$

① $\cancel{29}z = -\cancel{116}^{4}$

$z = -4$

③ $3y - 2z = 11$

$3y - 2(-4) = 11$

$3y + 8 = 11$

$$3y = 11 - 8$$

$$\cancel{3}y = \cancel{3}$$

$$y = 1$$

③ $3x + 3y + 2z = 1$

$$3x + 3(1) + 2(-4) = 1$$

$$3x + 3 - 8 = 1$$

$$3x - 5 = 1$$

$$3x = 1 + 5$$

$$\cancel{3}x = \cancel{6}$$

$$\boxed{x = 2}$$

$\therefore$ $x = 2$, $y = 1$, $z = -4$

find whether the following Equa are Consistance If so, solve them.

$$2x - y - z = 2$$

$$x + 2y + z = 2$$

$$4x - 7y - 5z = 2$$

The given equa are

$$2x - y - z = 2$$

$$x + 2y + z = 2$$

$$4x - 7y - 5z = 2$$

$$Ax = B$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

The Augment equa is

$$[A \cdot B] = \begin{bmatrix} 2 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{bmatrix}$$

$R_2 \to 2R_2 - 1R_1$

$R_3 \to 2R_3 - 4R_1$

$$\begin{bmatrix} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & -10 & -6 & -4 \end{bmatrix}$$

$R_3 \to 5R_3 + 10R_2$

$$\begin{bmatrix} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 \quad \rho(A \cdot B) = 2$

Thy system is Consistent

$\rho(A) = \rho(A \cdot B) = 3 <= n$

$n =$ no.of variables $= 3$

$\therefore$ The system has ~~an~~ Infinetly ~~many~~ many Solutions

$A_1 x = B_1$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 5 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

let, $\boxed{z = K}$

$5y + 3z = 2$

$5y = 2 - 3k$

$$\boxed{y = \frac{2-3k}{5}}$$

$2x - y - z = 2$

$2x = 2 + y + z$

$2x = 2 + \left(\frac{2-3k}{5}\right) + k$

$= \frac{10 + 2 - 3k + 5k}{5}$

$x = \frac{12 + 2k}{10}$

$$\boxed{x = \frac{6+k}{5}}$$

3/3/23

Solve the equations, $x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$

Given equas are,

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

The Augmented equation,

$$[A \cdot B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$R_3 \longrightarrow 1R_3 - 1R_1$

$R_2 \longrightarrow 1R_2 - 1R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix}$$

$R_3 \longrightarrow 1R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 \quad \rho(A \cdot B) = 2$

The system is Consistent

$\rho(A) = \rho(A \cdot B) = 2 < n = 3$

no. of variables $= 2$

The system has infinitely many solutions.

$A_1 x = B_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

let, $y + 2z = 8$

$y = 8 - 2k$

(ii) $x + y + z = 6$

$x = 6 - y - k$

$= 6 - (8 - 2k) - k$

$= 6 - 8 + 2k - k$

$x = k - 2$

for what values of λ and μ the equas

$x + y + z = 6$ ① has no solutions.

$x + 2y + 3z = 10$ ② has unique solution

$x + 2y + \lambda z = \mu$ ③ has infinite solution.

Given solutions,

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

$$Ax = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

The Augmented equas is

$$A \cdot B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

The above matrix is in Echelon form.

① has no solutions

$$\rho(A) = 2$$

$$\rho(A \cdot B) = 3$$

$$\rho(A) \neq \rho(A \cdot B)$$

$$\lambda - 3 = 0 \qquad \mu - 10 \neq 0$$

$$\lambda = 3 \qquad \mu \neq 10$$

② has unique solutions

$$\rho(A) = \rho(A \cdot B) = 3 = n = 3$$

$$\rho(A) = 3 \qquad \rho(A \cdot B) = 3$$

$$\lambda - 3 \neq 0 \qquad \mu - 10 = 0$$

$$\lambda \neq 3 \qquad \mu = 10$$

③ has Infinite solutions

$$\rho(A) = \rho(A \cdot B) = 2 < n = 3$$

$$\boxed{\lambda = 3} \qquad \boxed{\mu = 10}$$

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find the Conditions for the system of equas

$$-2x+y+z=a$$
$$x-2y+z=b$$
$$x+y-2z=c$$

① have no solution

② Infinite solution

The given equas are

$$-2x+y+z=a$$
$$x-2y+z=b$$
$$x+y-2z=c$$

$$Ax=B$$

$$\begin{bmatrix}-2 & 1 & 1\\ 1 & -2 & 1\\ 1 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$$

The Augmented equas is

$$A\cdot B=\begin{bmatrix}-2 & 1 & 1 & a\\ 1 & -2 & 1 & b\\ 1 & 1 & -2 & c\end{bmatrix}$$

$R_2 \to 2R_2+1R_1$

$R_3 \to 2R_3+1R_1$

$$\begin{bmatrix}-2 & 1 & 1 & a\\ 0 & -3 & 3 & 2b+a\\ 0 & 3 & -3 & 2c+a\end{bmatrix}$$

$R_3 \to 3R_3+3R_2$

$$\begin{bmatrix} -2 & 1 & 1 & a \\ 0 & -3 & 3 & 2b+a \\ 0 & 0 & 0 & 6a+6b+c \end{bmatrix}$$

The above matrix is in Echelon form.

<u>have no solution</u>

$$\rho(A) \neq \rho(A \cdot B)$$

$$\rho(A) = 2, \; \rho(A \cdot B) = 3$$

$$6a + 6b + c \neq 0$$

<u>have infinite solution</u>:

$$\rho(A) = \rho(A \cdot B) = 2 < n = 3$$

$$6a + 6b + c = 0$$

.. for what value of λ the system

① have solution & Solve the equas

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

The Give equas are

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

$$Ax = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

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The augmented form,

$$(A \cdot B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$R_3 \to 1R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 6 & \lambda^2-\lambda \end{bmatrix}$$

$R_3 \to 1R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

The above matrix is in Echelon form.

By q., the system has solution.

$\rho(A) = \rho(A \cdot B)$

$\rho(A) = 2 \quad \rho(A \cdot B) = 2$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda-2) - 1(\lambda-2) = 0$

$\lambda - 1 = 0 \qquad \lambda - 2 = 0$

$\lambda = 1 \qquad \lambda = 2$

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Case (i) $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \cancel{\lambda-1}\, 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A_1 x = B_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \cancel{\lambda-1}\, 0 \\ 0 \end{bmatrix}$$

Let (i) $z = k$

(ii) $y + 3z = \cancel{\lambda-1}\ 0$

$y = \cancel{\lambda-1} - 3z$

$\boxed{y = -3k}$

$x + y + z = 1$

$x - 3k + k = 1$

$x - 2k = 1$

$\boxed{x = 1 + 2k}$

Case (ii): $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A_1 x = B_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

let, $z = 0$

(i) $y + 3z = 1$ (ii) $x + y + z = 1$

$\boxed{y = 1 - 3k}$

$x = 1 - y - z$

$= 1 - (1 - 3k) - 0$

$\boxed{x = 2k}$

Solve the system $x + y + z = 9$

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$Ax = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

The Augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix} \sim \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{bmatrix}$$

$\rho(A) = 3$, $\rho(A \cdot B) = 3$

no. of non - zero rows $= 3$

$\rho(A) = \rho(A \cdot B) = 3 = n$

It is consistant has unique solution.

$A_1 x = B_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 34 \\ 20 \end{bmatrix}$$

$-4z = 20\,5$

$\boxed{z = 5}$

$3y + 5z = 4$

$3y + 25 = 34$

$3y = 9$

$\boxed{y = 3}$

$x + y + z = 9$

$5 + 3 + z = 9$

$\boxed{x = 1}$

$\therefore x = 1, y = 3, z = 5$

Show that equas $x + 2y - z = 3$
$3x - y + 2z = 1$
$2x - 2y + 3z = 2$ are consistent & find solutions.
$x - y + z = -1$

$Ax = B$

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

The Augmented matrix is, $A \cdot B$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - R_1$

$R_3 \to 7R_3 - 6R_2$

$R_4 \to 7R_4 - 3R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{bmatrix}$$

$R_4 \to 5R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

no. of non zero rows $= 3$

$\rho(A) = 3$. $\rho(A \cdot B) = 3$

$\rho(A) = \rho(A \cdot B) = 3 = n = 3$

The system is consistent has unique Solution.

$5z = 20$

$\boxed{z = 4}$

$-7y + 5z = -8$

$-7y + 20 = -8$

$-7y = -28$

$\boxed{y = 4}$

$x + 2y - z = 3$

$-4 + 8 + x = 3$

$\boxed{x = -1}$

for what value of k, the equas $x + y + z = 1$, $4x + y + 10z = k^2$, $2x + y + 4z = k$ have a solution & solve them.

$Ax = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 1 & 10 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ k^2 \\ k \end{bmatrix}$$

The Augmented matrix is $A \cdot B$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 10 & k^2 \\ 2 & 1 & 4 & k \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -3 & 6 & k^2 - 4 \\ 0 & -1 & 2 & k - 2 \end{bmatrix}$$

$R_3 \to 3R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -3 & 6 & k^2-4 \\ 0 & 0 & 0 & 3k-k^2-2 \end{bmatrix}$$

$\rho(A) = 2$

By given the system has unique solution. i.e, the System is consistent $\rho(A) = \rho(A,B)$

By above matrix we get rank '2' and also $\rho(A,B)$ is '2'.

$3k - k^2 - 2 = 0$

$k^2 - 3k + 2 = 0$

$k^2 - k - 2k + 2 = 0$

$k(k-1) - 2(k-1) = 0$

$(k-1)(k-2) = 0$

$k = 1, 2$

case(i): $k = 1$.

$A_1 X = B_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 1 \\ -3 \end{matrix}$$

Let $z = k$

$-3y + 6z = -3$

$-3y = -3 - 6z$

$y = 1 + 2z$

$\boxed{y = 1 + 2k}$

$x + y + z = 1$

$x = 1 - y - z$

$x = 1 - 1 - 2k - k$

$\boxed{x = -3k}$

Case(ii) $k = 2$

Let $z = k$

$-3y + 6z = 0$

$3y = 6z$

$\boxed{y = 2k}$

$x + y + z = 1$

$x = 1 - 2k - k$

$\boxed{x = 1 - 3k}$

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[Solutions of homogeneous linear Equations

The given system of homogeneous linear equa

$$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n = 0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots a_{nn}x_n = 0 \Big]$$

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* find 'k' the system of equas $x+2y-3z=-2$, $3x-y-2z=1$, $2x+3y-5z=k$ is consistent.

The given equas are

$$x+2y-3z=-2$$

$$3x-y-2z=1$$

$$2x+3y-5z=k$$

$$A\cdot X = B$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ k \end{bmatrix}$$

The Augmented matrix A-B is

$$\begin{bmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & K\ 1 \\ 2 & 3 & -5 & K \end{bmatrix}$$

$R_2 \to 1R_2 - 3R_1$

$R_3 \to 1R_3 - 1R_1$

$$\begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & k+4 \end{bmatrix}$$

$R_3 \to 7R_3 - 1R_2$

$$\begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & 7k+21 \end{bmatrix}$$

the above matrix is in echelon form

By Given the,

$$\rho(A) = \rho(A \cdot B)$$

$$\rho(A) = 2$$

$$\rho(A \cdot B) = 2$$

$7k+21 = 0 \Rightarrow 7k = -21$. $\boxed{k = -3}$

ans

* find the values of a and b, the equations

$x+y+z=3$ 1. have a unique solution.

$x+2y+2z=6$ 2. No solution

$x+ay+3z=b$ 3.

the given equa are

$$x+y+z = 3$$

$$x+2y+2z = 6$$

$$x+ay+3z = b$$

$$Ax = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ b \end{bmatrix}$$

The Augmented matrix is $A \cdot B =$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 6 \\ 1 & a & 3 & b \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & \overset{1}{\cancel{a-1}} & 1 & \cancel{b-6}\,3 \\ 0 & a-1 & 2 & b-\cancel{6}3 \end{bmatrix}$$

$R_3 \to 1R_3 - R_2(a-1)$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & \overset{0}{\cancel{3a}} & 3-a & 6-3a \end{bmatrix}$$

The above equa is in Echelon form.

① have unique solution

$\rho(A) = \rho(A \cdot B) = 3 = n = 3$

$\rho(A) = 3 \quad \rho(A \cdot B) = 3$

$3a \neq 0 \qquad b - 3a = 0$

$a \neq 3 \qquad b = 3a$

(2) No solution

$\rho(A) \neq \rho(A \cdot B)$

$\rho(A) = 2, \quad \rho(A \cdot B) = 3$

$3 - a = 0$ $\qquad$ $b - 3a = 0$

$\boxed{a = 3}$ $\qquad$ $\boxed{b \neq 3a}$

System of homogeneous linear equations

The given system of homogeneous linear equa.

$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = 0$

$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = 0$

$\vdots \qquad \vdots \qquad \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \cdots a_{nn}x_n = 0$

working rule

⇒ The homogeneous equas can be written as $AX = 0$

⇒ write the co-efficient matrix $[A]$.

⇒ By applying row operation convert co-efficient matrix into Echelon form.

⇒ find rank of A

(i) if, $\rho(A) = r = n$ then the system has zero solution. or trivial solution or unique solution.

(ii) if, $\rho(A) = r < n$ then the system has infinite solution or non-trivial or non-unique solution.

Note:-

1. If $\det |A| = 0$ then the system has infinite, non-zero, non-trivial solutions.
2. If $\det |A| \neq 0$ then the system has zero, unique, trivial solution.

Solve the system of Equas $x+2y+3z=0$
$3x+4y+4z=0$
$7x+10y+12z=0$

The given Equas are

$$x+2y+3z=0$$
$$3x+4y+4z=0$$
$$7x+10y+12z=0$$

$$Ax=0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix is $[A]$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 3R_1$
$R_3 \rightarrow 1R_3 - 7R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix}$$

$R_3 \rightarrow 2R_3 - 4R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 2 \end{bmatrix}$$

$\rho(A) = 3 = n = 3$

$\therefore$ The system has unique solution

$x = 0 \quad y = 0 \quad z = 0.$

Q.8

Determine whether the following equas will have a non-trivial solution. If so solve them $4x + 2y + z + 3w = 0$

$6x + 3y + 4z + 7w = 0$

$2x + y + w = 0$

The 4 equas are

$4x + 2y + z + 3w = 0$

$6x + 3y + 4z + 7w = 0$

$2x + y + w = 0$

$AX = 0$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix $[A]$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \to 4R_2 - 6R_1$

$R_3 \to 4R_3 - 2R_1$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$R_3 \to 10R_3 + 2R_2$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

$\rho(A) = 2 < n = 4$

let,
$A_1 x = 0$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$10z + 10w = 0$

let $w = k_1$

$10z = -10k_1$

$\boxed{-k_1 = +z}$

$4x + 2y + z + 3w = 0$

let, $y = k_2$

$4x + 2k_2 - k_1 + 3w = 0$

$4x = -2k_2 + k_1 - 3w$

$4x = -2k_2 + k_1 - 3k_1$

$= -2k_2 - 2k_1$

$x = -\underline{(k_2+k_1)}$

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Solve the System of Equations $x+3y-2z=0$
$2x-y+4z=0$
$x-11y+14z=0$

The G. Equas, $x+3y-2z=0$
$2x-y+4z=0$
$x-11y+14z=0$

$$AX=0$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix [A] is

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$R_2 \to 1R_2 - 2R_1$

$R_3 \to 1R_3 - 1R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 0 \\ 0 & -14 & 16 \end{bmatrix}$$

$R_3 \to 7R_3 - 14R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < n = 3$

The system has ~~unique~~ Infinite solution

$A_1 x = 0$

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$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-7y + 8z = 0$$

let, $z = k$

$$-7y + 8k = 0$$

$$-7y = -8k$$

$$\boxed{y = 8k/7}$$

$$x + 3y - 2z = 0$$

$$x = 2z - 3y$$

$$= 2k - 3(8k/7)$$

$$= 2k - \frac{24}{7}k$$

$$= \frac{14k - 24k}{7}$$

$$\boxed{x = \frac{-10k}{7}}$$

Solve the system of Equas

$$x + y + z = 0$$
$$2x - y - 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

The given Equa are

$$x + y + z = 0$$
$$2x - y - 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$$Ax = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix is,

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

$R_2 \to 1R_2 - 2R_1$

$R_3 \to 1R_3 - 3R_1$

$R_4 \to 1R_4 - 1R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & -8 & 1 \\ 0 & 16 & 3 \end{bmatrix}$$

$R_3 \to 3R_3 - 8R_2$

$R_4 \to \cancel{8}\,3R_4 - 16R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 43 \\ 0 & 0 & 89 \end{bmatrix}$$

$R_4 \to 43R_4 - 89R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 43 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

The system has unique solutions

find the value of λ for which the following Equa[tions]

$x + 2y + 3z = \lambda x$ have a non-zero solution

$3x + y + 2z = \lambda y$

$2x + 3y + z = \lambda z$.

The given Equas are

$x - \lambda x + 2y + 3z = 0$

$3x + y - \lambda y + 2z = 0$

$2x + 3y + z - \lambda z = 0$

$\Rightarrow (1-\lambda)x + 2y + 3z = 0$

$3x + (1-\lambda)y + 2z = 0$

$2x + 3y + (1-\lambda)z = 0$.

$AX = 0$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix is $[A]$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix}$$

By given the system has non-zero solution

$|A| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$R_1 = R_1 + R_2 + R_3$

$$\begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$6-\lambda \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix}$$

$$1\left[(1-\lambda)^2 - 6\right] - 1\left[3(1-\lambda) - 4\right] + 1\left[9 - 2(1-\lambda)\right] = 0$$

$$1 + \lambda^2 - 2\lambda - 6 - 1\left[3 - 3\lambda - 4\right] + 9 - 2 + 2\lambda = 0$$

$$1 + \lambda^2 - 2\lambda - 6 - 3 + 3\lambda + 4 + 9 - 2 + 2\lambda = 0$$

$$\lambda^2 + 3\lambda + 3 = 0$$

$a = 1, \; b = 3, \; c = 3$

$$\lambda = \frac{-3 \pm \sqrt{9-12}}{2}$$

$$= \frac{-3 \pm \sqrt{-3}}{2}$$

$$\boxed{\lambda = \frac{-3 \pm i\sqrt{3}}{2}}$$

Q.B
2M

find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.

G. Matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$

$R_2 \rightarrow 1R_2 - 2R_1$

$R_3 \rightarrow 1R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & k-4 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow 1R_3 + 1 \cdot R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & k-4 & 1 \\ 0 & -(k-4) & 0 \end{bmatrix}$$

By Give the rank of matrix is 2

∴ The last row is zero row

∴ $-(k-4) = 0$

$k-4 = 0$

$\boxed{k=4}$

Q. Solve the system of equas $x+y+z+\omega=0$

$x+y+z-\omega=4$

$x+y-z+\omega=-4$

$x-y+z+\omega=2$

The given equas are

$$x+y+z+\omega=0$$
$$x+y+z-\omega=4$$
$$x+y-z+\omega=-4$$
$$x-y+z+\omega=2$$

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -4 \\ 2 \end{bmatrix}$$

The Augmented-matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{bmatrix}$$

$R_4 \to R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & 0 & 0 & -2 & 4 \end{bmatrix}$$

$\rho(A) = 4 \quad \rho(A \cdot B) = 4$

$\therefore \rho(A) = \rho(A \cdot B)$

The system is Consistent.

$\rho(A) = \rho(A \cdot B) = 4 = n = 4$

$\therefore$ The system has unique Solution.

$A_1 X = B$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -4 \\ 4 \end{bmatrix}$$

(i) $\cancel{-2}w = \cancel{4}\,2$

$w = -2$

(ii) $\cancel{-2}z = \cancel{-4}\,2$

$z = 2$

(iii) $\cancel{-2}y = \cancel{2}$

$y = -1$

(iv) $x + y + z + w = 0$

$x - 1 + \cancel{2} - \cancel{2} = 0$

$x = 1$

$\therefore x = 1,\ y = -1,\ z = 2,\ w = -2.$

Topic-3

Eigen values and Eigen vectors (characteristic roots and characteristic vector

Let A be a square matrix and I be the unit matrix of order same and λ be a constant then $[A-\lambda I]$ is called as characteristic of A. the determinant of $|A-\lambda I| = 0$ is called the characteristic equa of A.

Eigen value (latent roots)

The values of the equa $|A-\lambda I| = 0$ are called as Eigen values (or) characteristic roots of A.

Spectrum

The set of eigen values of A is known as spectrum of A.

Eigen vector

the non-zero solution $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ of the equation $[A-\lambda I]x = 0$ is called the eigen vector of A.

problems

find Eigen values and eigen vectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$G.A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \quad \text{let, } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A-\lambda I] = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

The characteristic equa $|A-\lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10-5\lambda-2\lambda+\lambda^2-4 = 0$$

$$\lambda^2-7\lambda+6 = 0$$

$$\lambda^2-6\lambda-\lambda+6 = 0$$

$$\lambda(\lambda-6)-1(\lambda-6) = 0$$

$$(\lambda-1)(\lambda-6) = 0$$

$$\boxed{\lambda = 1, 6}$$

$\therefore$ The Eigen values $\lambda = 1, 6$

<u>case (i) :-</u> $\lambda = 1$

$$[A-\lambda I]x = 0$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

put $\lambda = 1$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 4x_1+4x_2 &= 0 \\ x_1+x_2 &= 0 \end{aligned}\right\} \begin{aligned} x_1+x_2 &= 0 \\ x_1+x_2 &= 0 \end{aligned}$$

$x_1+x_2 \rightarrow$

$$x_1 = -x_2$$

Let, $x_2 = k$; $x_1 = -k$

$$\left[x_1 = \begin{bmatrix} -k \\ k \end{bmatrix}\right]$$

Case (iii): $\lambda = 6$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} 5-1 & 4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -x_1 + 4x_2 &= 0 \\ x_1 - 4x_2 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_1 - 4x_2 &= 0 \\ x_1 - 4x_2 &= 0 \end{aligned}$$

$$x_1 - 4x_2 = 0$$

$$x_1 = 4x_2$$

Let, $x_2 = k$; $x_1 = 4k$

put $k=2$

$$X_2 = \begin{bmatrix} 4k \\ k \end{bmatrix} \Rightarrow X_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Q.B
Obtain the eigen values and eigen vectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

G.t. $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

$$[A - \lambda I] = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}$$

The characteristics equa $|A - \lambda I| = 0$

$$(-5-\lambda)(-2-\lambda) - 4 = 0$$

$$10 + 5\lambda + 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2+7\lambda+6=0$$

$$\lambda^2+6\lambda+\lambda+6=0$$

$$\lambda(\lambda+6)+1(\lambda+6)=0$$

$$(\lambda+6)(\lambda+1)=0$$

$$\boxed{\lambda=-1,-6}$$

$\therefore$ Eigen values $\lambda = -1, -6$

Case(i): $\lambda=-1$

$$[A-\lambda I]X=0$$

$$\begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

put $\lambda=-1$

$$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -4x_1+2x_2 &= 0 \\ 2x_1 - x_2 &= 0 \end{aligned}\right\} \quad \begin{aligned} 2x_1 - x_2 &= 0 \\ 2x_1 - x_2 &= 0 \end{aligned}$$

$$2x_1 = x_2$$

let, $x_1 = k$, $x_2 = 2k$

let, $k=1$

$$X_1 = \begin{bmatrix} k \\ 2k \end{bmatrix} \Rightarrow X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Case (ii): $\lambda=-6$

$$[A-\lambda I]X=0$$

$$\begin{bmatrix} -8-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

put $\lambda = -6$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 + 2x_2 &= 0 \\ 2x_1 + 4x_2 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_1 + 2x_2 &= 0 \\ x_1 + 2x_2 &= 0 \end{aligned}$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

let, $x_2 = k$

$x_1 = -2k$,

$$x_2 = \begin{bmatrix} -2k \\ k \end{bmatrix} \Rightarrow x_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

obtain the Eigen values and Eigen vectors $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$

G. $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$

let, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$[A - \lambda I] = \begin{bmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix}$$

$$(2-\lambda)(5-\lambda) - 4 = 0$$

$$10 - 2\lambda - 5\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$$

$$(\lambda - 1)(\lambda - 6) = 0$$

$$\lambda = +1, 6$$

Eigen values $\lambda = 1, 6$

Case (i):

$\lambda = 1$

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

put $\lambda = 1$

$$\begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 + x_2 &= 0 \\ 4x_1 + 4x_2 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_1 + x_2 &= 0 \\ x_1 + x_2 &= 0 \end{aligned}$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

Let, $x_2 = k$

$$x_1 = -k$$

$$x_1 = \begin{bmatrix} -k \\ k \end{bmatrix} \Rightarrow x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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case(iii): $\lambda = 6$

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -4x_1 + x_2 &= 0 \\ 4x_1 - x_2 &= 0 \end{aligned}\right\} \quad \begin{aligned} 4x_1 - x_2 &= 0 \\ 4x_1 - x_2 &= 0 \end{aligned}$$

$$4x_1 - x_2 = 0$$

$$4x_1 = x_2$$

let $x_2 = k$, $x_1 = k/4$

$$x_2 = \begin{bmatrix} k/4 \\ k \end{bmatrix} \Rightarrow x_2 = \begin{bmatrix} 1/4 \\ 1 \end{bmatrix}.$$

find the Eigen values and Eigen vectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 3 & -4 & 3 \end{bmatrix}$$

G. that, $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 3 & -4 & 3 \end{bmatrix}$ $\therefore I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$[A - \lambda I] = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 3 & -4 & 3-\lambda \end{bmatrix}$$

$$8-\lambda\left[(7-\lambda)(3-\lambda)-16\right] + 6\left[-6(3-\lambda)+8\right]$$

$$+2\left[\overset{24}{-4(8)(-1\times4)} - 3(7-\lambda)\right] = 0$$

$$8-\lambda\left[21-7\lambda-3\lambda+\lambda^2-16\right]$$
$$+6\left[-18+6\lambda+8\right]+2\left[24-14+2\lambda\right]=0$$

$$8-\lambda\left[\lambda^2-10\lambda+5\right]+6\left[6\lambda-10\right]+2\left[2\lambda+10\right]=0$$

$$(8-\lambda)\lambda^2-10\lambda(8-\lambda)+5(8-\lambda)+36\lambda-60$$
$$+4\lambda+20=0$$

$$8\lambda^2-\lambda^3-80\lambda+10\lambda^2+40-5\lambda+36\lambda-60$$
$$+4\lambda+20=0$$

$$-\lambda^3+18\lambda^2-45\lambda=0$$

$$+\lambda=0 \qquad \lambda^2+18\lambda-45=0$$

$$\lambda^2-15\lambda-3\lambda-45=0$$

$$\lambda(\lambda-15)-3(\lambda-15)=0$$

$$(\lambda-15)(\lambda-3)=0$$

$$\lambda=15,\ \lambda=3$$

Eigen values $\lambda=0, 3, 15$

Case (i) $\lambda=0$

$$[A-\lambda I]x=0$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1-6x_2+2x_3=0 \rightarrow (1)$$
$$-6x_1+7x_2-4x_3=0 \rightarrow (2)$$
$$2x_1-4x_2+3x_3=0 \rightarrow (3)$$

1 Q 2

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{array}$$

$$\frac{x_1}{24-14} = \frac{x_2}{-12+32} = \frac{x_3}{56-36}$$

$$\frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

case (ii) :- $\lambda = 3$

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 6x_2 + 2x_3 = 0 \rightarrow (1)$

$-6x_1 + 4x_2 - 4x_3 = 0 \rightarrow (2)$

$2x_1 - 4x_2 = 0$

Solve (1) & (2)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 2 & 5 & -6 \\ 4 & -4 & -6 & 4 \end{array}$$

$$\frac{x_1}{24-8} = \frac{x_2}{-12+20} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Case(iii) $\lambda = 15$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 - 6x_2 + 2x_3 = 0 \quad — (1)$

$-6x_1 - 8x_2 - 4x_3 = 0 \quad — (2)$

$2x_1 - 4x_2 - 12x_3 = 0 \quad — (3)$

Solve (1) & (2)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 2 & -7 & -6 \\ -8 & -4 & -6 & -8 \end{array}$$

$$\frac{x_1}{24+16} = \frac{x_2}{-12-28} = \frac{x_3}{+56-36}$$

$$\frac{x_1}{40} = \frac{x_2}{-40} = \frac{x_3}{20}$$

$$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$x_3 = \begin{bmatrix} +2 \\ -2 \\ 1 \end{bmatrix}$$

Eigen values $\lambda = 0, 3, 15$

Eigen vectors $x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$; $x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$; $x_3 = \begin{bmatrix} +2 \\ -2 \\ 1 \end{bmatrix}$

Determine the Eigen values and Eigen vectors

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

G. that, $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Let, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$$

$(-2-\lambda)$ $\cancel{+(2+\lambda)}$ $[(1-\lambda)(-\lambda) - 12]$ $-2[2\lambda - 6] - 3$

$[-4 + (1-\lambda)]$

$(-2-\lambda)\left[-\lambda+\lambda^2-12\right]-2\left[2\lambda-6\right]-3\left[-4+(1-\lambda)\right]$

$(-2-\lambda)(-\lambda)+(-2-\lambda)\lambda^2-12(-2-\lambda)-$
$4\lambda+12+12-3+3\lambda=0$

$+2\lambda+\lambda^2-2\lambda^2-\lambda^3+24+12\lambda$
$+4\lambda+24-3+3\lambda=0$

$-\lambda^3-\lambda^2+21\lambda+45=0$

$\lambda^3+\lambda^2-21\lambda-45=0$

$$\lambda=-3\quad\left|\begin{array}{cccc} 1 & 1 & -21 & -45 \\ 0 & -3 & 6 & 45 \\ \hline 1 & -2 & -15 & \boxed{0} \end{array}\right.$$

$\lambda^2-2\lambda-15=0$

$\lambda^2-5\lambda+3\lambda-15=0$

$\lambda(\lambda-5)+3(\lambda-5)=0$

$(\lambda-5)(\lambda+3)=0$

Eigen v $\Rightarrow$ $\lambda=-3,-3,5$

Case(i): $\lambda=-3$

$[A-\lambda I]x=0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 + 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 + 3x_3 = 0$$

The system has same equations i.e,

$$x_1 + 2x_2 - 3x_3 = 0$$
$$x_1 + 2x_2 - 3x_3 = 0$$
$$x_1 + 2x_2 - 3x_3 = 0$$

let, $x_2 = k_1$, $x_3 = k_2$

$$x_1 + 2x_2 - 3x_3 = 0$$

$$x_1 = -2x_2 + 3x_3$$

$$= -2k_1 + 3k_2$$

put $k_1 = 0$, $k_2 = 1$

$$x_1 = -20 + 3(1)$$

$$= 3$$

$$x_1 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Case (ii):- $\lambda = 5$

$$[A - \lambda B] x = 0$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x_1 + 2x_2 - 3x_3 = 0 \rightarrow (1)$$
$$2x_1 - 4x_2 - 6x_3 = 0 \rightarrow (2)$$
$$x_1 - 2x_2 - 5x_3 = 0 \rightarrow (3)$$

Solve (1) & (2)

x_1	x_2	x_3	
2	-3	-7	2
-4	-6	2	-4

$$\frac{x_1}{-12-12} = \frac{x_2}{-6-42} = \frac{x_3}{28-4}$$

$$\frac{x_1}{-24} = \frac{x_2}{-48} = \frac{x_3}{24}$$

$$\frac{x_1}{-1} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$x_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

find Eigen values and Eigen vectors $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

G.that, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Let,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix}$$

$$1-\lambda\left[(1-\lambda)(1-\lambda)-1\right]-1\left[1(1-\lambda)-1\right]+1\left[1-(1-\lambda)\right]=0$$

$$1-\lambda\left[\cancel{1}-\lambda-\lambda+\lambda^2-\cancel{1}\right]-1\left[1-\lambda-1\right]+1\left[1-1+\lambda\right]=0$$

$$1-\lambda\left[\lambda^2-2\lambda\right]-1\left[-\lambda\right]+1\left[\lambda\right]=0$$

$$(1-\lambda)\lambda^2-2\lambda(1-\lambda)+\lambda+\lambda=0$$

$$\lambda^2-\lambda^3-\cancel{2\lambda}+2\lambda^2+\cancel{2\lambda}=0$$

$$-\lambda^3+3\lambda^2\cancel{+2\lambda}=0$$

$$-\lambda^3+3\lambda^2=0$$

$$-\lambda^2\left[\lambda-3\right]=0$$

$$\lambda=0,\ \lambda=0,\ \lambda=3$$

case(i): $\lambda=0$

$$x_1+y+z=0$$

let,
$$x+y+z=0$$
$$x+y+z=0$$

let, $y=k_1$, $z=k_2$

$$x_1=-y-z$$
$$=-k_1-k_2$$

let, $k_1 = 0$; $k_2 = 1$

$$x_1 = -1$$

$$x_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

case (iii):- $\lambda = 3$

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + x_3 = 0 \rightarrow (1)$$

$$x_1 + -2x_2 + x_3 = 0 \rightarrow (2)$$

$$x_1 + x_2 - 2x_3 = 0 \rightarrow (3)$$

Solve (1) & (2)

x_1	x_2	x_3	
1	1	-2	1
-2	1	1	-2

$$\frac{x_1}{1+2} = \frac{x_2}{1+2} = \frac{x_3}{4-1}$$

$$\frac{x_1}{3} = \frac{x_2}{3} = \frac{x_3}{3}$$

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$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Eigen values = 0, 0, 3

Eigen vectors $x_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$; $x_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$; $x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

find Eigen values and Eigen vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

G. that

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

let,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A - \lambda I] = \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$$

$$6 - \lambda \left[(3-\lambda)(3-\lambda) - 1\right] + 2\left[-2(3-\lambda) + 2\right]$$
$$+ 2\left[2 - 2(3-\lambda)\right]$$

$$6-\lambda\left[9-3\lambda-3\lambda+\lambda^2-1\right]+2\left[-6+2\lambda+2\right]$$
$$+2\left[2-6+2\lambda\right]$$

$$6-\lambda\left[9-6\lambda+\lambda^2-1\right]+2\left[2\lambda-4\right]$$
$$+2\left[2\lambda-4\right]$$

$$6-\lambda\left[\lambda^2-6\lambda+8\right]+2\left[2\lambda-4\right]+2\left[2\lambda-4\right]$$

$$(6-\lambda)\lambda^2-(6-\lambda)6\lambda+8(6-\lambda)+4\lambda-8+4\lambda-8$$

$$6\lambda^2-\lambda^3-36\lambda+6\lambda^2+48-8\lambda+4\lambda+4\lambda-16$$

$$-\lambda^3+12\lambda^2-36\lambda+32=0$$

$$\lambda^3-12\lambda^2+36\lambda-32=0$$

$$\begin{array}{r|rrrr} & 1 & -12 & 36 & -32 \\ \lambda=2 & 0 & 2 & -20 & 32 \\ \hline & 1 & -10 & 16 & \boxed{0} \end{array}$$

$$\lambda^2-10\lambda+16=0$$

$$\lambda^2-8\lambda-2\lambda+16=0$$

$$\lambda(\lambda-8)-2(\lambda-8)=0$$

$$(\lambda-8)(\lambda-2)=0$$

$$\lambda=8 \quad \lambda=2$$

$$\lambda=2,2,8$$

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case (i) :- $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_1 - 2x_2 + 2x_3 = 0$$
$$-2x_1 + x_2 - x_3 = 0$$
$$2x_1 - x_2 + x_3 = 0$$

$$2x_1 - x_2 + x_3 = 0$$
$$2x_1 - x_2 + x_3 = 0$$
$$2x_1 - x_2 + x_3 = 0$$

$2x_1 - x_2 + x_3 = 0$

let, $\boxed{x_2 = k_1}$ $\boxed{x_3 = k_2}$

$$2x_1 = x_2 - x_3$$
$$2x_1 = k_1 - k_2$$
$$x_1 = \frac{k_1 - k_2}{3}$$

case (iii) $\lambda = 8$

$[A - \lambda I]x = 0$

$$\begin{pmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 \mp 2x_2 + 2x_3 = 0 \Rightarrow x_1, x_2, x_3 \neq 0$$

$$-2x_1 - 5x_2 - x_3 = 0$$

$$2x_1 - x_2 - 5x_3 = 0$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -2 & 2 & -2 & -2 \\ -5 & -1 & -2 & -5 \end{array}$$

$$\frac{x_1}{+2+10} = \frac{x_2}{-4-2} = \frac{x_3}{10 \mp 4}$$

$$\frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$x_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

find Eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

G. that,

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

$$[A - \lambda I] = \begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 2-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

$$= 1-\lambda\left[(2-\lambda)(-2-\lambda)-0\right]-2\left[0-0\right]$$

$$-1\left[0-0\right]$$

$$= 1-\lambda\left[(2-\lambda)(-2-\lambda)\right]$$

$$= 1-\lambda\left[-4-\cancel{2\lambda}+\cancel{2\lambda}+\lambda^2\right]$$

$$= 1-\lambda\left[\lambda^2-4\right]$$

$$= (1-\lambda)\lambda^2-(1-\lambda)4$$

$$= \lambda^2-\lambda^3-4+4\lambda$$

$$= -\lambda^3+\lambda^2+4\lambda-4=0$$

$$\lambda^3-\lambda^2-4\lambda+4=0$$

$$\lambda=1,\quad \lambda=2,\quad \lambda=-2$$

Case (i) :- $\lambda=1$

$$[A-\lambda I]X=0$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_2-x_3=0$$

$$x_2-2x_3=0$$

$$-3x_3=0$$

$$x_1=\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Case (ii):- $\lambda = 2$

$$\begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x_3 = 0$$

$$\boxed{x_3 = 0}$$

$$-x_1 + 2x_2 - x_3 = 0$$

$$\cancel{-}x_1 = \cancel{-}2x_2$$

Let, $x_2 = k$

$$x_1 = 2k$$

$$X_2 = \begin{bmatrix} 2k \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Case (iii):- $\lambda = -2$

$$\begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_2 + 2x_3 = 0$$

$$\cancel{4}^{2}x_2 = -\cancel{2}x_3$$

$$x_3 = -2x_2$$

Let, $x_2 = k$

$$x_3 = -2k$$

$$3x_1 = -2x_2 + x_3$$

$$3x_1 = -2k - 2k$$

$$x_1 = \frac{-4k}{3}$$

$$x_3 = \begin{bmatrix} -4k/3 \\ k \\ -2k \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} \\ 1 \\ -2 \end{bmatrix}$$

a/03
properties

We have $[A - \lambda I]^T = [A^T - \lambda I^T]$

$$= [A^T - \lambda I]$$

$$|[A-\lambda I]^T| = |A^T - \lambda I|$$

$$= |A - \lambda I| \qquad (\because |A^T| = |A|)$$

λ is the eigen value A and A^T.

Prove that if λ is an eigen value of A and then prove that $1/\lambda$ is an eigen value of A^{-1}!

Let, A be the non-singular matrix. λ is the Eigen value & x is the Eigen vector. $(\lambda \neq 0)$

we know that,

$$[A - \lambda I]x = 0$$

$$Ax - \lambda x I = 0$$

$$\boxed{Ax = \lambda x}$$

Multiply A^{-1} on both sides

$$A^{-1}Ax = \lambda x A^{-1}$$

$$Ix = \lambda x A^{-1}$$

$$x \, {}^{1}/_{\lambda} = x A^{-1}$$

$$\boxed{A^{-1}x = \lambda^{-1}x}$$

${}^{1}/_{\lambda}$ is the Eigen value of A^{-1}.

If 'λ' is the Eigen value of A, then λ^m is an Eigen value of A^m.

we know that,

$$[A - \lambda I]x = 0$$

$$Ax - \lambda x I = 0$$

$$\boxed{Ax = \lambda x} \quad \text{—} (1)$$

Multiply 'A' on both sides

$$A \cdot A x = \lambda x A$$

$$A^2 x = \lambda A x$$

from equa (1)

$$A^2 x = \lambda(\lambda x)$$

$$A^2 x = \lambda^2 x$$

λ^2 is the Eigen value of A^2.

$\parallel^{ly}$ $\boxed{A^m x = \lambda^m x}$

λ^m is the eigen value of A^m.

If 'λ' is an Eigen value of A then prove the Eigen value of $B = a_0A^2 + a_1A + a_2I$ is $a_0\lambda^2 + a_1\lambda + a_2$.

Let, 'λ' is an eigen value of A.

$$[A - \lambda I]x = 0$$

$$Ax - \lambda Ix = 0$$

$$\boxed{Ax = \lambda x} \quad \text{—— (1)}$$

multiply 'A' on both sides

$$AAx = \lambda x A$$

$$A^2x = \lambda x A \quad [\text{from Eq (1)}]$$

$$A^2x = \lambda(\lambda x)$$

$$\boxed{A^2x = \lambda^2 x} \quad \text{—(2)}$$

Now,

$$[a_0A^2 + a_1A + a_2I]x = a_0A^2x + a_1Ax + a_2Ix$$

from Eq (1) & 2

$$= a_0\lambda^2x + a_1\lambda x + a_2x$$

$$[a_0A^2 + a_1A + a_2I]x = (a_0\lambda^2 + a_1\lambda + a_2)x$$

$[a_0\lambda^2 + a_1\lambda + a_2]$ is the eigen value of $a_0A^2 + a_1A + a_2I$.

The values of a triangular matrix (upper and lower triangular) are just the diagonal element of the matrix.

Let, $A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ 0 & a_{22} \cdots & a_{2n} \\ 0 & 0 & a_{nn} \end{bmatrix}$ be a triangular matrix

of order n.

the characteristic equa $|A - \lambda I| = 0$

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$$\begin{vmatrix} a_{11}-\lambda & a_{12} \cdots & a_{1n} \\ 0 & a_{22}-\lambda \cdots & a_{2n} \\ 0 & 0 & a_{nn}-\lambda \end{vmatrix}$$

$$a_{11}\left((a_{22}-\lambda)(a_{nn}-\lambda)\ldots(a_{nn}-\lambda)\right) = 0$$

$$\therefore \lambda = a_{11}, a_{22}, a_{nn}$$

Any

The sum of the eigen values of square matrix is equal to trace of A.

And the product of Eigen values is equal $|A|$.

problems

find the Eigen values of matrix A and its A^{-1}.

where, $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

The given matrix upper triangular matrix.

(i) The Eigen values are $\lambda = 1, 2, 3$.

(ii) 'λ' is the Eigen value of A then $1/\lambda$ is the Eigen value of A^{-1}.

$\therefore 1/\lambda = 1/1, 1/2, 1/3$

find the Eigen values of $A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 12 \end{bmatrix}$

The Eigen values of triangular matrix are diagonal only.

$\therefore$ Eigen values are $\lambda = 7, 9, 12$.

find the trace and adjoint of A of $A = \begin{bmatrix} 1 & 9 & 9 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

The given matrix is upper triangular matrix are principle of diagonal elements only.

$\therefore$ Eigen values are $\lambda = 1, 2, 4$

Trace of $A = 1 + 2 + 4$ [$\because$ Sum of Eigen values]

$= 7$

Adj of $A = 1 \times 2 \times 4$ [$\because$ product of Eigen values]

$= 8$

find the Eigen values of $B = 2A^2 - \frac{1}{2}A + 3I$ where

$A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

G: Matrix $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

$[A - \lambda I] = \begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = (8-\lambda)(2-\lambda) - (-4)(2)$$

$$= 16 - 8\lambda - 2\lambda + \lambda^2 + 8$$

$$= \lambda^2 - 10\lambda + 24$$

$$= \lambda^2 - 6\lambda - 4\lambda + 24$$

$$= \lambda^2(\lambda - 6) - 4(\lambda - 6)$$

$$= (\lambda - 6)(\lambda - 4)$$

$$\lambda = 6, 4$$

∴ The eigen values $\lambda = 4, 6$ of A

The Eigen value of $B = 2A^2 - \frac{1}{2}A + 3I$

$$= 2\lambda^2 - \frac{1}{2}\lambda + 3I$$

(i) $\lambda = 4 \Rightarrow 2(4)^2 - \frac{1}{2}4 + 3$

$$= 32 - 2 + 3$$

$$= 33$$

(ii) $\lambda = 6 \Rightarrow 2(6)^2 - \frac{1}{2}(6) + 3$

$$= 72 - 3 + 3 = 72$$

find the, Eigen values of A^2 where $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

G. that $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.

The Eigen values of diagonal matrix are diagonal elements

$\lambda = 2, 3, 6$

The values of A^2 matrix is λ^2

$\therefore \lambda^2 = 4, 9, 36$

prove that the sum of the Eigen values of a matrix is trace of matrix. and product of the Eigen values is equal to it's determant.

proof:- Let, A be the square matrix of order $n \times n$.

Let, $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$ are n Eigen values of 'A'.

Sum of the Eigen values is $\lambda_1 + \lambda_2 \ldots + \lambda_n$ = Trace.

we know that,

Trace(A) is sum of diagonal elements.

Let, $A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$

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The characteristic equa is $|A-\lambda I| = 0$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn}-\lambda \end{vmatrix} = 0$$

$(a_{11}-\lambda)\left[(a_{22}-\lambda)(a_{33}-\lambda)\ldots(a_{nn}-\lambda)\right] - a_{12}$ (a polynomial of degree $n-2$) $+ \ldots\ldots = 0$

$$(-1)^n \left[(\lambda-(a_{11}))(\lambda-a_{22})(\lambda-a_{33})\ldots(\lambda-a_{nn})\right] - a_{12}(\ ) = 0$$

$(-1)^n \left[\lambda^n - (a_{11}+a_{22}+a_{33}\ldots a_{nn})\lambda^{n-1} + \right.$ a polynomial of degree $\lambda^{n-2} = 0$

$(-1)^n \lambda^n + (-1)^{n+1}(-\text{Trace}(A))\lambda^{n-1} +$ a polynomial of degree $\lambda^{n-2} = 0$

∴ Sum of the Eigen values $\left[\begin{array}{l}\text{Sum of roots} \Rightarrow \\ \alpha+\beta = b/a\end{array}\right]$

$$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 \ldots \lambda_n = \frac{-(-1)^{n+1}\text{Trace}(A)}{(-1)^n}$$

$$= \frac{-\cancel{(-1)^n}(-1)\text{Trace}(A)}{\cancel{(-1)^n}} = \text{Trace of } A$$

Sum of E.V $= \text{Trace}(A)$

(iii) $|A-\lambda I| = (-1)^n\lambda^n + \dots + a_0$

put $\lambda = 0$

$|A| = a_0$

[product of roots $= (-1)^n c/a$]

$\lambda_1 . \lambda_2 . \lambda_3 \dots \lambda_n = \frac{\cancel{(-1)^n} a_0}{\cancel{(-1)^n}}$

$= a_0 \ |A|$

product of Eigen values $= |A|$

9. Equations,

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 4z = 0$

$Ax = 0$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The co-efficient matrix is

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 4 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 2R_1$

$R_3 \rightarrow 1R_3 - 1R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 6 \end{bmatrix}$$

$R_3 \rightarrow 7R_3 - 14R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & -70 \end{bmatrix}$$

$\therefore \rho(A) = 3 = n = 3$

$x=0, \ y=0, \ z=0$

The system has zero solution..

Solve the equas, $x + ay + z = 3$; $x + 2y + 2z = b$; $x + 5y + 3z$

G. Equations,

$$x + ay + z = 3$$
$$x + 2y + 2z = b$$
$$x + 5y + 3z = 9$$

$AX = B$

~~The Arithmetic~~ Eq

Augmented matrix is

$$[A \cdot B] = \begin{bmatrix} 1 & a & 1 & 3 \\ 1 & 2 & 2 & b \\ 1 & 5 & 3 & 9 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 5 & 3 & 9 \\ 1 & 2 & 2 & b \\ 1 & a & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & -3 & -1 & b-9 \\ 0 & a-5 & -2 & -6 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2(a-5)$

$$\begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & -3 & -1 & b-9 \\ 0 & a-5 & -1-a & ab-9a-5b-27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & -3 & -1 & b-9 \\ 0 & 0 & -1-a & ab-9a-5b-27 \end{bmatrix}$$

$\rho(A) = 3 = n = 3$

$\rho(A) = \rho(AB)$

$\therefore$ the system is consistent, and has unique solution.

$-1-a \neq 0$ $\qquad ab-9a-5b-27=0$

$a \neq -1$ $\qquad a = 1$

$b-9-5b-27=0$

$-4b = 36$

$\boxed{b=-9}$

11/03

Topic - 4

25th

Cayley Hamilton theorem (without proof)

Statement :- Every square matrix satisfies it's characteristic equation.

Applications

1. To calculate powers of A.
2. To calculate A^{-1}!

State Cayley Hamilton theorem, and verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ find A^{-1}, A^4.

Every square matrix satisfies it's characteristic equation.

$$[A - \lambda I] = 0$$

$$= \begin{bmatrix} 2-\lambda & 1 & 2 \\ 5 & 3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{bmatrix}$$

Characteristic equa,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 5 & 3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix}$$

$$2-\lambda\left[(3-\lambda)(-2-\lambda)-0\right]-1\left[5(-2-\lambda)+3\right]$$
$$+2\left[0+3-\lambda\right]=0$$

$$2-\lambda\left[-6-3\lambda+2\lambda+\lambda^2\right]-1\left[-10-5\lambda+3\right]$$
$$+2\left[3-\lambda\right]=0$$

$$2-\lambda\left[\lambda^2-\lambda-6\right]+5\lambda+7+6-2\lambda=0$$

$$(2-\lambda)\lambda^2-(2-\lambda)\lambda-(2-\lambda)6+3\lambda+13=0$$

$$2\lambda^2-\lambda^3-2\lambda+\lambda^2-12+6\lambda+3\lambda+13=0$$

$$-\lambda^3+3\lambda^2+7\lambda+1=0$$

$$\boxed{\lambda^3-3\lambda^2-7\lambda-1=0}\longrightarrow(1)$$

To show that A satisfies Equa (1) i.e.,

$$A^3-3A^2-7A-I=0$$

① $A=\begin{bmatrix}2&1&2\\5&3&3\\-1&0&-2\end{bmatrix}$ ② $A^2=\begin{bmatrix}7&5&3\\22&14&13\\0&-1&2\end{bmatrix}$

③ $A^3=\begin{bmatrix}36&22&23\\101&64&60\\-7&-3&-7\end{bmatrix}$

$$A^3-3A^2-7A-I=0$$

$$\begin{bmatrix}36&22&23\\101&64&60\\-7&-3&-7\end{bmatrix}-3\begin{bmatrix}7&5&3\\22&14&13\\0&-1&2\end{bmatrix}-7\begin{bmatrix}2&1&2\\5&3&3\\-1&0&-2\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&1&0\end{bmatrix}$$
$$=0$$

$$\begin{bmatrix} 36 & 22 & 23 \\ 101 & 64 & 60 \\ -7 & -3 & -7 \end{bmatrix} - \begin{bmatrix} 21 & 15 & 9 \\ 66 & 42 & 39 \\ 0 & -3 & 6 \end{bmatrix} - \begin{bmatrix} 14 & 7 & 14 \\ 35 & 21 & 21 \\ -7 & 0 & -14 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cayley Hamilton theorem is verified.

To find A^4

$$A^3 - 3A^2 - 7A - I = 0$$

multiply A on both side

$$A\left[A^3 - 3A^2 - 7A - IA\right] = 0$$

$$A^4 - 3A^3 - 7A^2 - A = 0$$

$$A^4 = 3A^3 + 7A^2 + A$$

$$= \begin{bmatrix} 108 & 66 & 69 \\ 303 & 192 & 180 \\ -21 & -9 & -21 \end{bmatrix} + \begin{bmatrix} 49 & 35 & 21 \\ 154 & 98 & 91 \\ 0 & -7 & 14 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 159 & 102 & 92 \\ 462 & 293 & 274 \\ -22 & -16 & -9 \end{bmatrix}$$

To find A^{-1}

$$A^3 - 3A^2 - 7A - I = 0$$

$$A^3 = 3A^2 + 7A + I$$

multiply A^{-1} on both side

$$A^2 = 3A + \cancel{7A}^{\,7AA^{-1}} + A^{-1}I$$

$$A^2 - 3A - 7I = A^{-1}$$

$$\therefore A^{-1} = A^2 - 3A - 7I$$

$$= \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 3 & 6 \\ 15 & 7 & 9 \\ -3 & 0 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 2 & -3 \\ 7 & -2 & 4 \\ 3 & -1 & 1 \end{bmatrix}$$

State Cayley-Hamilton theorem, and verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}, A^4.

Every square matrix is ~~which~~ satisfies its characteristic equations.

The characteristic equa,

$$[A-\lambda I] = 0$$

$$\begin{bmatrix} 1-\lambda & -2 & 2 \\ 1 & 2-\lambda & 3 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

characteristic equa is,

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 & 2 \\ 1 & 2-\lambda & 3 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$1-\lambda\,[(2-\lambda)(2-\lambda)+3] + 2[2-\lambda] + 2[-1] = 0$$

$$1-\lambda\,[4-2\lambda-2\lambda+\lambda^2+3] + 4-2\lambda-2 = 0.$$

$$1-\lambda\,[\lambda^2-4\lambda+7] - 2\lambda+2 = 0$$

$$(1-\lambda)\lambda^2 - (1-\lambda)4\lambda + (1-\lambda)7 - 2\lambda + 2 = 0$$

$$\lambda^2 - \lambda^3 - 4\lambda + 4\lambda^2 + 7 - 7\lambda - 2\lambda + 2 = 0$$

$$-\lambda^3 + 5\lambda^2 - 13\lambda + 9 = 0 \longrightarrow (1)$$

To show that A satisfies equa (1) i.e.

$$A^3 - 5A^2 - 13A + 9 = 0$$

① $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ ② $A^2 = \begin{bmatrix} -1 & 8 & 0 \\ 3 & -1 & 14 \\ -1 & -4 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} -9 & -14 & -26 \\ 2 & -22 & 31 \\ -5 & -4 & 12 \end{bmatrix}$$

$$A^3 - 5A^2 + 13A - 9B = 0$$

$$\begin{bmatrix} -9 & -14 & -26 \\ 2 & -22 & 31 \\ -5 & -4 & -12 \end{bmatrix} - \begin{bmatrix} -5 & 40 & 0 \\ 15 & -5 & 70 \\ -5 & -20 & 5 \end{bmatrix} + \begin{bmatrix} 13 & -26 & 26 \\ 13 & 26 & 39 \\ 0 & -13 & 26 \end{bmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To find A^4

$$A^3 - 5A^2 + 13A - 9 = 0$$

multiply 'A' on both sides

$$A^4 = 5A^3 + 13A^2 + 9A$$

$$= 5\begin{bmatrix} -9 & -14 & -26 \\ 2 & -22 & 31 \\ -5 & -4 & -12 \end{bmatrix} - 13\begin{bmatrix} -1 & 8 & 0 \\ 3 & -1 & 14 \\ -1 & -4 & 1 \end{bmatrix} + 9\begin{bmatrix} 9 & -18 & 18 \\ 9 & 18 & 27 \\ 0 & -9 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 16 & -172 \\ -20 & -79 & 0 \\ -12 & 8 & -85 \end{bmatrix}$$

To find A^{-1}

$$A^3 - 5A^2 + 13A - 9I = 0$$

$$A^3 = 5A^2 - 13A + 9I$$

multiply A^{-1} on both sides

$$A^2 = 5A - 13 - 9A^{-1}$$

$$9A^{-1} = A^2 - 5A + 13I$$

$$9A^{-1} = \begin{bmatrix} -1 & 8 & 0 \\ 3 & -1 & 14 \\ -1 & -4 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

$$A^{-1} = 1/9 \begin{bmatrix} 7 & 2 & -10 \\ -2 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$

Verify cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. and hence find its A^{-1} & A^4 ?

The characteristic equa, $[A - \lambda I] = 0$

$$[A - \lambda I] = \begin{bmatrix} 1-\lambda & 0 & 3 \\ 2 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 3 \\ 2 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)(1-\lambda)-1\right] - 0 + 3\left[(2(-1)-0) - 1(1-\lambda)\right] = 0$$

$$(1-\lambda)\left[1-\lambda-\lambda+\lambda^2-1\right] + 3\left[(-2) - 1+\lambda\right] = 0$$

$$(1-\lambda)(\lambda^2-2\lambda) + 3\left[(-3+\lambda)\right] = 0$$

$$\lambda^2-2\lambda-\lambda^3+2\lambda^2-9+3\lambda = 0$$

$$-\lambda^3+3\lambda^2+\lambda-9 = 0$$

$$\lambda^3-3\lambda^2-\lambda+9 = 0$$

$$A^3-3A^2-A+9I = 0$$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix}$$

$$A^3-3A^2-A+9I =$$

$$\begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -9 & 18 \\ 9 & 6 & 12 \\ 0 & -6 & 15 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Cayley hamilton theorem is verified.

To find A^4 :-

$$A^3-3A^2-A+9I = 0$$

$A^3 = 3A + A - 9\mathbb{I}$

Multiply 'A' on both sides

$A^4 = 3A^3 + A^2 - 9\mathbb{I}A$

$= 3A^3 + A^2 - 9A$

$$= \begin{bmatrix} 12 & -27 & -63 \\ 33 & -6 & 33 \\ 3 & -21 & 21 \end{bmatrix} + \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 27 \\ 18 & 9 & -9 \\ 9 & -9 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -30 & 42 \\ 18 & -13 & 46 \\ -6 & -14 & 17 \end{bmatrix}$$

To find A^{-1} :-

$A^3 - 3A^2 - A + 9\mathbb{I} = 0$

multiply 'A^{-1}' $\Rightarrow$

$A^{-1}A^3 - 3A^2 \cdot A^{-1} - A \cdot A^{-1} + 9\mathbb{I}A^{-1} = 0$

$A^2 - 3A - \mathbb{I} + 9A^{-1} = 0$

$9A^{-1} = -A^2 + 3A + \mathbb{I}$

$$= -\begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 9 \\ 6 & 3 & -3 \\ 3 & -3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 1/9 \begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$$

To find A^{-2} :-

$A^3 - 3A^2 - A + 9\mathbb{I} = 0$

multiply A^{-2}

$A^{-2} \cdot A^3 - 3A^{-2}A^2 - A^{-2}A + 9A^{-2}\mathbb{I} = 0$

$$A - 3I - A^{-1} + 9A^{-2} = 0$$

$$-9A^{-2} = A - 3I - A^{-1}$$

$$-9A^{-2} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 3 \\ 2 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 0.3333 & 0.3333 \\ 0.3333 & 0.2222 & -0.777 \\ 0.3333 & -0.111 & -0.111 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -0.33 & 2.66 \\ 1.66 & -2.22 & -0.22 \\ 0.66 & -0.88 & -1.88 \end{bmatrix}$$

$$= -\frac{1}{9}\begin{bmatrix} -2 & -0.33 & 2.66 \\ 1.66 & -2.22 & -0.22 \\ 0.66 & -0.88 & -1.88 \end{bmatrix}$$

Diagnolization

A matrix 'A' is diagnolizable if there exist a matrix "p" such that $p^{-1}Ap = D$ where d is a diagonal matrix and 'p' is the modal matrix.

Modal matrix

Any matrix it contains Eigen vectors as it columns, that matrix is called modal matrix 'p'. It transforms 'A' into diagonal matrix.

spectral matrix

~~The diagonal matrix~~

After diagnolization the resultant diagonal matrix 'd' is called spectral matrix.

working rule for diagnolization

step 1:- find eigen values of 'A'.

2 :- find Eigen vectors corresponding Eigen values 'λ'.

3 :- write modal matrix $p = [x_1 \cdot x_2 \cdots x_n]$

4 :- Calculate $p^{-1} = \frac{\text{adj } p}{\det p}$

5 :- find the diagonal matrix $d = p^{-1} \cdot A \cdot p$

Application

Calculate A^n from $P \cdot D^n \cdot P^{-1}$

problems

Diagnalize the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

Given that, $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

$$[A-\lambda I] = \begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix}$$

$$8-\lambda\left[(-3-\lambda)(1-\lambda)-8\right]+8\left[4(1-\lambda)+6\right]$$
$$-2\left[-16-3(-3-\lambda)\right]$$

$$8-\lambda\left[-3+3\lambda-\lambda+\lambda^2-8\right]+8\left[4-4\lambda+6\right]$$
$$-2\left[-16+9+3\lambda\right]$$

$$8-\lambda\left[\lambda^2+2\lambda-11\right]+8\left[-4\lambda+10\right]$$
$$-2\left[3\lambda-7\right]$$

$$8-\lambda(\lambda^2)+8-\lambda(2\lambda)+(8-\lambda)(-11)+-32\lambda+80$$
$$-6\lambda+14$$

$$-\lambda^3+6\lambda^2-11\lambda+6=0$$

$$\lambda^3-6\lambda^2+11\lambda-6=0$$

$$\lambda=1 \quad \begin{array}{|cccc} 1 & -6 & 11 & -6 \\ 0 & 1 & -5 & 6 \\ \hline 1 & -5 & 6 & \boxed{0} \end{array}$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda^2-3\lambda-2\lambda+6=0$$

$$\lambda(\lambda-3)-2(\lambda-3)=0$$

$$\lambda-3=0 \quad \lambda-2=0$$

$$\lambda=3 \quad \lambda=2$$

$$\boxed{\lambda=1,2,3}$$

Eigen vectors

let $\lambda=1$

$$[A-\lambda I]=\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7x_1-8x_2-2x_3=0 \rightarrow (1)$$

$$4x_1-4x_2-2x_3=0 \rightarrow (2)$$

$$3x_1-4x_2=0 \rightarrow (3)$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -8 & -2 & 7 & -8 \\ -4 & -2 & 4 & -4 \end{array}$$

$$\frac{x_1}{16-8} = \frac{x_2}{-8+14} = \frac{x_3}{-28+32}$$

$$\frac{x_1}{8} = \frac{x_2}{6} = \frac{x_3}{4}$$

$$x_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

Case (ii) :- $\lambda = 2$

$$[A - \lambda I]x = 0$$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$6x_1 - 8x_2 + 2x_3 = 0 \longrightarrow (1)$$
$$4x_1 - 5x_2 - 2x_3 = 0 \longrightarrow (2)$$
$$3x_1 - 4x_2 - x_3 = 0 \longrightarrow (3)$$

Solving (1) & (2)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -8 & -2 & 6 & -8 \\ -5 & -2 & 4 & -5 \end{array}$$

$$\frac{x_1}{16-10} = \frac{x_2}{-8+12} = \frac{x_3}{-30+32}$$

$$\frac{x_1}{6} = \frac{x_2}{4} = \frac{x_3}{2}$$

$$x_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

<u>Case (iii)</u> $\lambda = 3$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 - 8x_2 - 2x_3 = 0 \longrightarrow (1)$$
$$4x_1 - 6x_2 - 2x_3 = 0 \longrightarrow (2)$$
$$3x_1 - 4x_2 - 2x_3 = 0 \longrightarrow (3)$$

Solving (1) & (2)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -8 & -2 & 5 & -8 \\ -6 & -2 & 4 & -6 \end{array}$$

$$\frac{x_1}{16-12} = \frac{x_2}{-8+10} = \frac{x_3}{-30+32}$$

$$\frac{x_1}{4} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$x_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Modal matrix $(P) = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

$$P^{-1} = \frac{\text{Adj } P}{\det P} \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\det P = \begin{vmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -1$$

$$P = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 2 \\ -1 & 2 & -1 \end{bmatrix} \quad \text{adj } P = [\text{Co-factors } P]^{T}$$

$$P^{-1} = \frac{\text{Co-factors}}{|P|} = -1 \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$d = P^{-1} A P.$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

find modal matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ also find A^4.

(or)

Diagnalize $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ and also find A^4.

G.that,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

let, I be the Identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A - \lambda I] = \begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix}$$

$$1-\lambda\left[(2-\lambda)(3-\lambda)-2\right] - 1\left[2-2(2-\lambda)\right] = 0$$

$$1-\lambda\left[6-2\lambda-3\lambda+\lambda^2-2\right] - 1\left[2-4+2\lambda\right] = 0$$

$$1-\lambda\left[\lambda^2-5\lambda+4\right] - 2+4-2\lambda = 0$$

$$(1-\lambda)\lambda^2 - 5\lambda(1-\lambda) + 4(1-\lambda) - 2+4-2\lambda = 0$$

$$\cancel{\lambda^2} - \lambda^3 - 5\lambda + 5\lambda^2 + 4 - 4\lambda + 2 - \cancel{2\lambda} = 0$$

$$-\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\begin{array}{r|rrrr} \lambda = 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$

$$\lambda(\lambda - 3) - 2(\lambda - 3) = 0$$

$$(\lambda - 2)(\lambda - 3) = 0$$

$$\lambda = 2,\ \lambda = 3$$

$$\therefore \lambda = 1, 2, 3$$

Case(i):

Let, $\lambda = 1$ then

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_3 = 0 \longrightarrow (1)$$

$$x_1 + x_2 + x_3 = 0 \longrightarrow (2)$$

$$2x_1 + 2x_2 + 2x_3 = 0 \longrightarrow \cancel{(3)}$$

$$x_1 + x_2 + x_3 = 0 \longrightarrow (3)$$

$$\cancel{x_1} \quad \cancel{x_2} \quad \cancel{x_3}$$

$$x_1 \qquad x_2 \qquad x_3$$

$$\begin{array}{cccc} 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}$$

$$\frac{x_1}{0+1} = \frac{x_2}{-1+0} = \frac{x_3}{0+0}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{0}$$

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

case (iii): $\lambda = 2$ then

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 - x_3 = 0 \rightarrow (1)$$
$$x_1 + x_3 = 0 \rightarrow (2)$$
$$2x_1 + 2x_2 + \cancel{2}x_3 = 0 \rightarrow (3)$$

Solving (2) & (3)

$$\begin{array}{cccccc} & x_1 & & x_2 & & x_3 \\ 0 & & 1 & & 1 & 0 \\ 2 & & 1 & & 2 & 2 \end{array}$$

$$\frac{x_1}{0-2} = \frac{x_2}{2-1} = \frac{x_3}{2-0}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$x_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

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Case(iii): $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 - x_3 = 0 \rightarrow (1)$

$x_1 - x_2 + x_3 = 0 \rightarrow (2)$

$2x_1 + 2x_2 = 0 \rightarrow (3)$

Solving (1) & (2)

$$\begin{array}{cccccc} x_1 & & x_2 & & x_3 & \\ 0 & & -1 & & -2 & 0 \\ -1 & \times & 1 & \times & 1 & \times \; -1 \end{array}$$

$$\frac{x_1}{0-1} = \frac{x_2}{-1+2} = \frac{x_3}{2-0}$$

$$\frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$x_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

modal matrix (P) $= \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

$\det P = -2$

$$\text{Co-factors } P = \begin{bmatrix} 0 & -2 & 2 \\ 2 & -2 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{adj } P = [\text{Co-factors } P]^T$$

$$= \begin{bmatrix} 0 & 2 & -1 \\ +2 & +2 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

$$P^{-1} = +\tfrac{1}{2} \begin{bmatrix} 0 & -2 & 1 \\ -2 & -2 & 0 \\ +2 & +2 & +1 \end{bmatrix}$$

$$D = P^{-1} A \; P$$

$$= \tfrac{1}{2} \begin{bmatrix} 0 & -2 & 1 \\ -2 & -2 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$= \tfrac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

To find $A^4 = P.D^4.P^{-1}$

$$= \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix} \begin{bmatrix} 0 & -2 & 1 \\ -2 & -2 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -98 & -100 & -80 \\ 130 & 132 & 80 \\ 260 & 260 & 162 \end{bmatrix}$$