UNIT-3

Numerical Integration

Numerical Integration is the numerical evaluation of a definite integral.

$$\int_a^b y\,dx = \int_a^b f(x)\,dx.$$

where a,b are constants and $f(x)$ is a function which covers a curve b/w a and b values.

(1) Trapezodial Rule:

$$T.R \Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots)\right]$$

$$= \frac{h}{2}\left[\begin{pmatrix}\text{Sum of first} \\ \text{\& last terms}\end{pmatrix} + 2\begin{pmatrix}\text{Sum of remaining} \\ \text{all terms}\end{pmatrix}\right]$$

For Trapezoidal rule can be applied to any no:of sub-intervals (n) that is odd or even.

(2) Simpson's 1/3rd Rule:

$$\text{Sim } 1/3^{rd} \Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{h}{3}\ (y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4[y_1+y_3+y_5+\cdots]$$

$$= \frac{h}{3}\left[\begin{pmatrix}\text{Sum of 1st} \\ \text{\& last terms}\end{pmatrix} + 2\begin{pmatrix}\text{Sum of even} \\ \text{terms}\end{pmatrix} + 4\begin{pmatrix}\text{Sum of} \\ \text{odd terms}\end{pmatrix}\right]$$

→ Simpson's 1/3rd rule can be applied only the no:of sub-intervals should be taken as even no:of sub-interval.

(3) Simpson's 3/8th rule:

$$\text{Simp } 3/8^{th} \Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n) + 2\cdot(y_3+y_6+y_9+\cdots) + 3(y_1+y_2+y_4+y_5+\cdots)\right]$$

$$= \frac{3h}{8}\left[\begin{pmatrix}\text{Sum of 1st \&} \\ \text{last terms}\end{pmatrix} + 2\begin{pmatrix}\text{sum of} \\ \text{multiples of } 3\end{pmatrix} + 3\begin{pmatrix}\text{Sum of remaining} \\ \text{all terms}\end{pmatrix}\right]$$

Simpson's $3/8^{th}$ rule can be applied only multiples of 3 number of sub-intervals.

Problems:

1. Evaluate $\int_1^4 e^x\,dx$ using Trapezoidal rule given that

x	1	2	3	4
y	2.72	7.39	20.09	54.6

Sol: By using Trapezoidal rule

x	1	2	3	4
y	2.72	7.39	20.09	54.6
	y_0	y_1	y_2	y_3

$h=1$

$$T.R \Rightarrow \int_1^4 e^x\,dx = \frac{h}{2}\left[(y_0+y_3)+2(y_1+y_2)\right]$$

$$= \frac{1}{2}\left[(2.72+54.6)+2(7.39+20.09)\right]$$

$$\int_1^4 e^x\,dx = 56.14$$

2. Evaluate $\int_0^6 \frac{1}{1+x}\,dx$ by using Simpson's $1/3^{rd}$ rule

x :	0	1	2	3	4	5	6
y :	1	0.5	0.33	0.25	0.2	0.167	0.143

Sol: Given;

x	0	1	2	3	4	5	6
y	1	0.5	0.33	0.25	0.2	0.167	0.143
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By using Simpson's $1/3^{rd}$ rule

$$\int_0^6 \frac{1}{1+x}\,dx = \frac{h}{3}\left[(y_0+y_6)+2(y_2+y_4\cancel{+y_6})+4(0.5+0.25+0.167)\right]$$

$$\int_0^6 \frac{1}{1+x}\,dx = \frac{1}{3}\ (1+0.143) + 2\ (0.33+0.2) + 4\ (0.5+0.25+0.167)$$

$$= \frac{1}{3}\ [1.143 + 1.06 + 3.668]$$

$$= \frac{1}{3}\ [5.871]$$

$$\int_0^6 \frac{1}{1+x}\,dx = 1.957$$

3. Evaluate $\int_4^{5.2} \log x\, dx$ by using Simpson's $3/8^{th}$ rule

Sol: Given; $\int_4^{5.2} \log x\, dx$; $a=4$, $b=5.2$, $n=6$

let; $h = \frac{b-a}{n} = \frac{5.2-4}{6} = 0.2$

$h = 0.2$

x:	4	4.2	4.4	4.6	4.8	5.0	5.2
y:	1.3862	1.4350	1.4816	1.5260	1.5686	1.6094	1.6486
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simp $3/8^{th}$ rule :-

$$\int_4^{5.2} \log x\, dx = \frac{3h}{8}\left[(y_0+y_6) + 2\,(y_3) + 3\,(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3(0.2)}{8}\ (1.3862+1.6486) + 2\ (1.5260) + 3\ (1.4350 + 1.4816 + 1.5686 + 1.6094)$$

$$= \frac{3}{40}\ [3.0348 + 3.052 + 18.2838]$$

$$\int_4^{5.2} \log x\, dx = 1.8277$$

4. Evaluate $\int_0^1 e^{-x^2}\,dx$ by dividing the range of integration into 4 equal parts using (a) Trapezoidal Rule. (b) Simpson's $1/3^{rd}$ rule.

Sol: Given $\int_0^1 e^{-x^2}\,dx$

$n=4$

$$h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$$

x	0	0.25	0.5	0.75	1
y	1	0.9394	0.7788	0.5697	0.3678
	y_0	y_1	y_2	y_3	y_4

(a) $$\int_0^1 e^{-x^2}dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(1+0.3678) + 2(0.9394 + 0.7788 + 0.5697)\right]$$

$$= 0.125\,[5.9436]$$

$$= 0.74295$$

(b) $$\int_0^1 e^{-x^2}dx = \frac{h}{3}\left[(y_0+y_4) + 2(y_2) + 4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(1+0.3678) + 2(0.7788) + 4(0.9394+0.5697)\right]$$

$$= \frac{0.25}{3}\,[18.9618]$$

$$= 1.58015$$

5. Evaluate $\int_0^6 \frac{e^x}{x+1}$ by using Simpson's $\frac{1}{3}^{rd}$ rule with $h=1$

Sol: Given; $h=1$

x	0	1	2	3	4	5	6
y	1	1.35914	2.46301	5.02138	10.91963	24.73552	57.63268
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^6 \frac{e^x}{x+1} = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$= \frac{1}{3}\left[(1+57.63268) + 2(2.46301 + 10.91963) + 4(1.35914 + 5.02138 + 24.73552)\right]$$

$$= \frac{1}{3}\,[209.86212]$$

$$= 69.95404$$

6. Evaluate $\int_0^1 xe^x dx$ taking 4 intervals using.

(i) Trapezoidal rule (ii) Simpson's $1/3^{rd}$ rule (iii) Simpson $3/8^{th}$ rule

Sol: Given; $\int_0^1 xe^x dx$; $a=0, b=1, n=4$

let; $h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$

x :	0	0.25	0.5	0.75	1
y :	0	0.32100	0.82436	1.58775	2.71828
	y_0	y_1	y_2	y_3	y_4

(i) Trapezoidal rule:

$$\int_0^1 xe^x dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(0+2.71828) + 2(0.32100+0.82436+1.58775)\right]$$

$$= \frac{0.25}{2}\left[7.6445\right]$$

$$= \cancel{0.637044}\ 1.02306$$

(ii) Simpson's $1/3^{rd}$ rule:

$$\int_0^1 xe^x dx = \frac{h}{3}\left[(y_0+y_4) + 2(y_2) + 4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(0+2.71828) + 2(0.82436) + 4(0.32100 + 1.58775)\right]$$

$$= \frac{0.25}{3}\left[12.002\right]$$

$$= 1.00016$$

(iii) Simpson's $3/8^{th}$ rule:

$$\int_0^1 xe^x dx = \frac{3h}{8}\left[(y_0+y_4) + 2(y_3) + 3(y_1+y_2)\right]$$

$$= \frac{3(0.25)}{8}\left[(0+2.71828) + 2(1.58775) + 3(0.32100+0.82436)\right]$$

$$= \frac{3(0.25)}{8}\left[9.32986\right] = 0.87467$$

7. Find the value of $\int_0^1 \frac{dx}{1+x^2}$ taking 5 sub intervals by Trapezoidal rule correct to five significant figures. Compare it with exact value.

Sol:

Given; $\int_0^1 \frac{dx}{1+x^2}$

$a=0,\ b=1,\ n=5$

$$h = \frac{b-a}{n} = \frac{1-0}{5} = 0.2$$

x:	0	0.2	0.4	0.6	0.8	1
y:	1	0.96153	0.86206	0.73529	0.60975	0.5
	y_0	y_1	y_2	y_3	y_4	y_5

(i) **Trapezoidal Rule:**

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_5) + 2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{0.2}{2}\left[(1+0.5) + 2\left[0.96153 + 0.86206 + 0.73529 + 0.60975\right]\cancel{+0}\right]$$

$$= \frac{0.2}{2}\left[7.83726\right]$$

$$= 0.78372$$

Exact value:

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \pi/4 - 0$$

$$= 0.78539.$$

8. Evaluate $\int_1^2 \frac{1}{x}\,dx$ by using Trapezoidal rule taking $h=0.25$

Sol:

Given; $\int_1^2 \frac{1}{x}\,dx.$

$a=1,\ b=2,\ \cancel{n=6},\ h=0.25$

$$\cancel{h = \frac{b-a}{n} = \frac{2-1}{6} = 0.16}$$

x :	1	1.25	1.5	1.75	2
y :	1	0.8	0.66666	0.57142	0.5
	y_0	y_1	y_2	y_3	y_4

$$\int_1^2 \frac{1}{x}\,dx = \frac{0.25}{2}\left[1\,(y_0+y_4) + 2\,(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[1\,(1+0.5) + 2\,(0.8+0.66666+0.57142)\right]$$

$$= \frac{0.25}{2}\left[5.57616\right]$$

$$= 0.69702$$

~~9. Evaluate the integr~~

9. Evaluate the following $\int_0^4 e^x\,dx$ 4 intervals using (i) T.R (ii) Simp $1/3^{rd}$ (iii) Simp $3/8^{th}$. Also compare your result with actual integration.

<u>Sol:</u> Given; $\int_0^4 e^x dx$

$a=0,\ b=4,\ n=4$

$$h = \frac{b-a}{4} = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	1	2.71828	7.38905	20.08553	54.59815
	y_0	y_1	y_2	y_3	y_4

(i) T.R $\Rightarrow$ $\frac{1}{2}\left[(y_0+y_4) + 2\,(y_1+y_2+y_3)\right]$

$$\int_0^4 e^x dx = \frac{1}{2}\left[(1+54.59815) + 2\,(2.71828+7.38905+20.08553)\right]$$

$$= \frac{1}{2}\left[115.98387\right]$$

$$= 57.99193$$

(ii) Simp $1/3^{rd}$ =>

$$\int_0^4 e^x dx = \frac{h}{3}\left[(y_0+y_4) + 2(y_2) + 4(y_1+y_3)\right]$$

$$= \frac{1}{3}\left[(1+54.59815) + 2(7.38905) + 4(2.71828 + 20.08553)\right]$$

$$= \frac{1}{3}\left[161.59149\right]$$

$$= 53.86383$$

(iii) Simp $3/8^{th}$ =>

$$\int_0^4 e^x dx = \frac{3h}{8}\left[(y_0+y_4) + 2(y_3) + 3(y_1+y_2)\right]$$

$$= \frac{3}{8}\left[(1+54.59815) + 2(20.08553) + 3(2.71828 + 7.38905)\right]$$

$$= \frac{3}{8}\left[126.0912\right]$$

$$= 47.2842$$

Exact value:

$$\int_0^4 e^x dx = [e^x]_0^4$$

$$= e^4 - e^0$$

$$= 53.59815$$

10. Evaluate $\int_1^2 \frac{e^x}{x} dx$ for $n=4$ by Simpson's $1/3^{rd}$ rule.

Sol: Given; $\int_1^2 \frac{e^x}{x} dx$

$a=1$, $b=2$, $n=4$

$$h = \frac{2-1}{4} = 0.25$$

x :	1	1.25	1.5	1.75	2
y :	2.71828	2.79227	2.98779	3.28834	3.69452
	y_0	y_1	y_2	y_3	y_4

Simp $1/3^{rd}$:-

$$\int_1^2 \frac{e^x}{x} dx = \frac{0.25}{3}\left[(y_0+y_4) + 2(y_2) + 4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(2.71828 + 3.69452) + 2(2.98779) + 4(2.79227 + 3.28834)\right]$$

$$= \frac{0.25}{3}\left[36.71082\right]$$

$$\int_1^2 \frac{e^x}{x}\,dx = 3.05923$$

11. Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by Trapezoidal rule for $n=6$.

Sol:

Given; $\int_0^1 \frac{1}{1+x}\,dx$.

$a=0,\ b=1,\ n=6$

$$h = \frac{b-a}{6} = \frac{1-0}{6} = 0.16666$$

x:	0	0.16666	0.33332	0.49998	0.66664	0.8333
y:	1	0.85714	0.75000	0.66667	0.60000	0.54546
	y_0	y_1	y_2	y_3	y_4	y_5

x:	0.99996
y:	0.50001
	y_6

T-R ⇒ $$\int_0^1 \frac{1}{1+x}\,dx = \frac{h}{2}\left[(y_0+y_6) + 2\left(y_1+y_2+y_3+y_4+y_5\right)\right]$$

$$= \frac{0.16666}{2}\left[(0.85714 + 0.50001) + 2\left(y_1+y_2+y_3+y_4+y_5\right)\right]$$

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{0.16666}{2}\left[(0.85714 + 0.50001) + 2\left(0.75000 + 0.66667 + 0.60000 + 0.54546\right)\right]$$

$$\int_0^1 \frac{1}{1+x}\,dx = 0.69485$$

12. Given that:

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_{4.0}^{5.2} \log x \, dx \quad , \; h = 0.2$$

Simp $3/8$th =)

$$\int_{4.0}^{5.2} \log x \, dx = \frac{3(0.2)}{8} \left[(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5) \right]$$

$$= \frac{3(0.2)}{8} \left[(1.3863 + 1.6487) + 2(1.5261) + 3(1.4351 + 1.4816 + 1.5686 + 1.6094) \right]$$

$$= \frac{3(0.2)}{8} [24.3713]$$

$$\int_{4.0}^{5.2} \log x \, dx = 1.82784$$

13. Evaluate $\int_0^1 \sqrt{1+x^3} \, dx$ using Trapezoidal taking $h = 0.1$

Sol: Given; $\int_0^1 \sqrt{1+x^3} \, dx$, $h = 0.1$

x:	0	0.1	0.2	0.3	0.4	0.5	0.6
y:	1	1.00049	1.00399	1.01341	1.03150	1.06066	1.10272
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

0.7	0.8	0.9	1
1.15887	1.22963	1.31491	1.41421
y_7	y_8	y_9	y_{10}

T.R =)

$$\int_0^1 \sqrt{1+x^3} \, dx = \frac{0.1}{2} \left[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9) \right]$$

$$= \frac{0.1}{2} \left[(1 + 1.41421) + 2[1.00049 + 1.00399 + 1.01341 + 1.03150 + 1.06066 + 1.10272 + 1.15887 + 1.22963 + 1.31491] \right]$$

$= \frac{0.1}{2} \quad [20.04113]$

$= 1.00205 \qquad 1.11232$

$\int_0^1 \sqrt{1+x^3}\,dx = \frac{0.1}{2}\,[22.24657]$

$\int_0^1 \sqrt{1+x^3}\,dx = 1.11232$

14. Evaluate $\int_0^2 e^{-x^2}dx$ with proper no:of sub intervals by using

(i) Simp 1/3rd (ii) Simp 3/8th

Sol:

Given; $\int_0^2 e^{-x^2}dx$

$a=0,\ b=2,\ n=6.$

$h = \frac{2-0}{6} = 0.33333$

x:	0	0.33333	0.66666	0.99999	1.33332	1.66665
y:	1	0.89484	0.64118	0.36778	0.16901	0.06217
		$e^{(0.71653)^2}$	$e^{(0.51342)^2}$	$e^{(0.36788)^2}$	$e^{(0.26360)^2}$	$e^{(0.18887)^2}$
	y_0	y_1	y_2	y_3	y_4	y_5

x: 1.99998

y: 0.01831 $e^{(0.13533)^2}$

y_6

Simp 1/3rd:

$$\int_0^2 e^{-x^2}dx = \frac{0.33333}{3}\left[(y_0+y_6)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$= \frac{0.33333}{3}\ (1+0.01831)+2(0.64118+0.16901)+4[(0.89484+0.36778+0.06217)]$$

$$\int_0^2 e^{-x^2}dx = 0.88201$$

Simp 3/8th:

$$\int_0^2 e^{-x^2}dx = \frac{3(0.33333)}{8}\left[(y_0+y_6)+2(y_3)+3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3\,(0.33333)}{8}\left[(1+0.01831) + 2\,(0.36778) + 3\left[(0.89484 + 0.64118 + 0.16901 + 0.06217)\right]\right]$$

$$= \frac{3\,(0.33333)}{8}\,[7.05547]$$

$$\int_0^2 e^{-x^2}dx = 0.88192$$

5. Evaluate $\int_0^{\pi} \cos x\,dx$ by dividing the range into 6 equal parts by using Trapezoidal & Simp 1/3rd.

Sol:

$n=6;$

$$h = \frac{b-a}{n} = \frac{\pi-0}{6} = \pi/6.$$

x:	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	$6\pi/6=\pi$
y:	1	0.86602	0.5	0	-0.5	-0.86602	-1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

T.R :-

$$\int_0^{\pi}\cos x\,dx = \frac{\pi/6}{2}\left[(y_0+y_6) + 2\,(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{\pi/6}{2}\left[(1+(-1)) + 2\;\; 0.86602 + 0.5 + 0 + (-0.5) - 0.86602\right]$$

$$\int_0^{\pi}\cos x\,dx = 0$$

Simp 1/3rd :- $\Rightarrow \frac{\pi/6}{3}\left[(y_0+y_6) + 2\,(y_2+y_4) + 3\,(y_1+y_3+y_5)\right]$

$$\int_0^{\pi}\cos x\,dx = \frac{\pi/6}{3}\left[1+(-1) + 2\,(0.5-0.5) + 3\,(0.86602 + 0 - 0.86602)\right]$$

$$\int_0^{\pi}\cos x\,dx = 0$$

Curve Fitting:

To find an equation of the curve of the best fit which is most suitable predicting the unknown values. this process of finding of an equ^n of best fit is called curve fitting.

2M:-

1. Write the principle of least squares method.

Sol: The sum of squares of differences is minimum.

i.e. $S = d_1^2 + d_2^2 + \cdots = \min$

2. Write the normal equations for straight line $y = a + bx$

Sol: The general form of straight line is $y = ax + b$

the normal equ^{n}s are $\Rightarrow$ $\Sigma y = an + b\Sigma x$

$\Sigma yx = a\Sigma x + b\Sigma x^2$

3. Certain experimental values of x and y are given below.

x	0	2	5	7
y	-1	5	12	20.

If $y = a_0 + a_1 x$, find the approximate values of a_0 & a_1

Sol: Given; Straight line $y = a_0 + a_1 x$

The normal equ^{n}s are $\Sigma y = a_0 n + a_1 \Sigma x \rightarrow ①$ $\quad \Sigma 1 = n$

$\Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2 \rightarrow ②$

n = no:of x points.

x	y	xy	x^2
0	-1	0	0
2	5	10	4
5	12	60	25
7	20	140	49
$\Sigma x = 14$	$\Sigma y = 36$	$\Sigma xy = 210$	$\Sigma x^2 = 78$

4. Fit

Sol:

of the best
. values. this
ed curve fitting.

is minimum.

below.

a_1

$\Sigma 1 = n$

$\rightarrow$ ②

From ① & ②

$36 = 4a_0 + 14a_1 \Rightarrow 4a_0 + 14a_1 = 36$
$210 = 14a_0 + 78a_1 \Rightarrow 14a_0 + 78a_1 = 210$ } → Solve

$a_0 = -1.137$; $a_1 = 2.8965$

Substitute a_0, a_1 in given line.

$$y = -1.137 + (2.8965)x$$

4. Fit the straight line from the following data:

x	1	2	3	4	5
y	14	27	40	55	68

Sol: ~~Give~~ WKT; $y = ax + b$

the normal equ^n^s are $\sum_1^n y = a\sum_1^n x + bn \rightarrow$ ①

$\sum_1^n xy = a\sum_1^n x^2 + b\sum_1^n x \rightarrow$ ②

n = no:of x points.

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
$\sum_1^n x = 15$	$\sum_1^n y = 204$	$\sum_1^n xy = 748$	$\sum_1^n x^2 = 55$

From ① & ②

$204 = a\,15 + 5b \Rightarrow 15a + 5b = 204$
$748 = a\,55 + b15 \Rightarrow 55a + 15b = 748$ } → solve.

$a = 13.6$; $b = 0$

Substitute. a, b in line.

$$y = (13.6)x + 0$$

5. Fit the straight line for the following data:

x	1	3	5	7	9
y	1.5	2.8	4	4.7	6

Sol:

WKT ; $y = ax + b$

The normal equations are $\Rightarrow$ $\sum y = a \sum x + bn \rightarrow (1)$

$$\sum xy = a \sum x^2 + b \sum x \rightarrow (2)$$

$n =$ no: of x points.

x	y	xy	x^2
1	1.5	1.5	1
3	2.8	8.4	9
5	4	20	25
7	4.7	32.9	49
9	6	54	81
$\sum x = 25$	$\sum y = 19$	$\sum xy = 116.8$	$\sum x^2 = 165$

From (1) & (2)

$$19 = a\,25 + b\,5 \Rightarrow 25a + 5b = 19$$
$$116.8 = a\,165 + b\,25 \Rightarrow 165a + 25b = 116.8$$
$\rightarrow$ solve.

$a = 0.545$; $b = 1.075$

Substitute a, b in S.L equn

$$y = (0.545)x + 1.075$$

Type-II :- Fitting of a second degree polynomial or Parabola:-

1. Write the normal equ^{n}s to best fit the parabola $y = a+bx+cx^2$

Sol: Given; $y = a+bx+cx^2$

the normal equ^{n}s are $\Rightarrow$ $\sum_1 y = an + b\sum_1 x + c\sum_1 x^2$

$\sum_1 xy = a\sum_1 x + b\sum_1 x^2 + c\sum_1 x^3$

(2) $\sum_1 x^2y = a\sum_1 x^2 + b\sum_1 x^3 + c\sum_1 x^4$.

2. Find the least square parabola $y = a+bx+cx^2$ to data.
$f(-1) = -2$, $f(0) = 1$, $f(1) = 2$ & $f(2) = 4$

Sol: Given; equ^n is $y = a+bx+cx^2$

The normal equ^{n}s are $\Rightarrow$ $\sum_1 y = an + b\sum_1 x + c\sum_1 x^2$

$\sum_1 xy = a\sum_1 x + b\sum_1 x^2 + c\sum_1 x^3$

$\sum_1 x^2y = a\sum_1 x^2 + b\sum_1 x^3 + c\sum_1 x^4$

x	-1	0	1	2
y	-2	1	2	4

$n =$ no:of x points.

x	y	x^2	xy	x^3	x^2y	x^4
-1	-2	1	2	-1	-2	1
0	1	0	0	0	0	0
1	2	1	2	1	2	1
2	4	4	8	8	16	16
$\sum_1 x = 2$	$\sum_1 y = 5$	$\sum_1 x^2 = 6$	$\sum_1 xy = 12$	$\sum_1 x^3 = 8$	$\sum_1 x^2y = 16$	$\sum_1 x^4 = 18$

The equ^{n}s $\Rightarrow$

$$\left.\begin{aligned} 5 &= 4a + 2b + 6c \\ 12 &= 2a + 6b + 8c \\ 16 &= 6a + 8b + 18c \end{aligned}\right\} \quad \begin{aligned} a &= 0.55 \\ b &= 2.15 \\ c &= -0.25 \end{aligned}$$

The equ^n is $\Rightarrow$ $y = 0.55 + 2.15x - (0.25)x^2$

3. Find the parabola of the form $y = ax^2 + bx + c$ passing the points $(-1,2)$ $(0,1)$ and $(1,4)$

Sol:- Given; $y = ax^2 + bx + c$.

The normal equ^{n}s are $\Rightarrow$ $\sum_1 y = a\sum_1 x^2 + b\sum_1 x + cn$

$$\sum_1 xy = a\sum_1 x^3 + b\sum_1 x^2 + c\sum_1 x$$

$$\sum_1 x^2y = a\sum_1 x^4 + b\sum_1 x^3 + c\sum_1 x^2$$

x:	-1	0	1
y:	2	1	4

x	y	x^2	xy	x^3	x^2y	x^4
-1	2	1	-2	-1	2	1
0	1	0	0	0	0	0
1	4	1	4	1	4	1
$\sum_1 x = 0$	$\sum_1 y = 7$	$\sum_1 x^2 = 2$	$\sum_1 xy = 2$	$\sum_1 x^3 = 0$	$\sum_1 x^2y = 6$	$\sum_1 x^4 = 2$

The equ^{n}s $\Rightarrow$
$$\left.\begin{aligned} 7 &= 2a + 3c \\ 2 &= 2b \\ 6 &= 2a + 2c \end{aligned}\right\} \rightarrow \text{solving}$$

$a = 2$; $b = 1$; $c = 1$

The required equ^n is $\Rightarrow$ $y = 2x^2 + x$

4. the ... fit a ...

Sol:

x
0
1
2
3
4
5
$\sum_1 x = 15$

1. The following data related drying time of certain varnish. Fit a 2nd degree polynomial by method of least square.

x	0	1	2	3	4	5
y	12	10.5	10	8	7	8

Sol:

$y = ax^2 + bx + c.$

The normal equ^ns ⇒ $\sum_1^6 y = a\sum_1^6 x^2 + b\sum_1^6 x + cn$

$\sum_1^6 xy = a\sum_1^6 x^3 + b\sum_1^6 x^2 + c\sum_1^6 x$

$\sum_1^6 x^2y = a\sum_1^6 x^4 + b\sum_1^6 x^3 + c\sum_1^6 x^2$

x	y	x^2	xy	x^3	x^2y	x^4
0	12	0	0	0	0	0
1	10.5	1	10.5	1	10.5	1
2	10	4	20	8	40	16
3	8	9	24	27	72	81
4	7	16	28	64	112	256
5	8	25	40	125	200	625
$\sum_1^6 x = 15$	$\sum_1^6 y = 55.5$	$\sum_1^6 x^2 = 55$	$\sum_1^6 xy = 122.5$	$\sum_1^6 x^3 = 225$	$\sum_1^6 x^2y = 434.5$	$\sum_1^6 x^4 = 979$

The equ^ns ⇒
$$\left.\begin{aligned} 55.5 &= 55a + 15b + 6c \\ 122.5 &= 225a + 55b + 15c \\ 434.5 &= 979a + 225b + 55c \end{aligned}\right\} \rightarrow \text{solving}$$

$a = 0.1875$; $b = -1.866$; $c = 12.196$

The required equ^n;

$$y = (0.1875)x^2 - (1.866)x + 12.196$$

Type - III: Fitting of a exponential curve

the general form of exponential curve is $y = ae^{bx}$

Taking log on both sides

$$\log y = \log ae^{bx}$$

$$= \log a + \log e^{bx}$$

$$= \log a + bx \log_e e$$

$$\log y = \log a + bx$$

$$Y = A + bx$$

the normal equ^ns $\sum_1 y = nA + b\sum_1 x$, $\sum_1 xy = A\sum_1 x + b\sum_1 x^2$ } by solving we get A, b.

A and b.

$A = \log_e a$

$a = e^A$; b.

1. Find the cure of best fit of type $y = ae^{bx}$ for data:

x	1	5	7	9	12
y	10	15	12	15	21

Sol: the equ^n is $y = ae^{bx}$

Apply log on B.S.

$$\log y = \log ae^{bx}$$

$$= \log a + \log e^{bx}$$

$$= \log a + bx \log_e e.$$

$$\log y = \log a + bx$$

$$Y = A + bx$$

the normal equ^ns $\sum_1 y = nA + b\sum_1 x \rightarrow (1)$

$$\sum_1 xy = A\sum_1 x + b\sum_1 x^2 \rightarrow (2).$$

$y = ae^{bx}$

ng we

A, b.

x	y	$Y = \log y$	xY	x^2
1	10	2.302	2.302	1
5	15	2.708	13.540	25
7	12	2.484	17.388	49
9	15	2.708	24.372	81
12	21	3.044	36.528	144
$\Sigma x = 34$	Σ	$\Sigma Y = 13.246$	$\Sigma xY = 94.130$	$\Sigma x^2 = 300$

the equ^ns $\Rightarrow$

$$\left.\begin{aligned} 13.246 &= 5A + b34 \\ 94.130 &= 34A + 300b. \end{aligned}\right\} \text{ Solving}$$

$$A = 2.248 \quad ; \quad b = 0.058$$

$$a = e^{A}$$

$$a = 9.468 \quad ; \quad b = 0.058$$

$\therefore$ the required equ^n is $y = 9.468\, e^{0.058x}$

2. Fit the curve of form $y = ae^{bx}$ for data.

x	77	100	185	239	285
y	2.4	3.4	7.0	11.1	19.6.

Sol: the equ^n is $y = ae^{bx}$ (Apply log)

$$\log y = \log ae^{bx}$$

$$= \log a + \log e^{bx}$$

$$= \log a + bx \log e^{e}$$

$$= \log a + \cancel{\log}\, bx$$

$$\log y = \log a + bx$$

$$Y = A + bx$$

the normal $eq^{u^n s}$ $\Rightarrow$ $\sum Y = nA + b\sum x$

$\sum xY = A\sum x + b\sum x^2$

x	y	$Y = \log y$	xY	x^2
77	2.4	0.875	67.375	~~529~~ 5929
100	3.4	1.223	122.3	10000
185	7.0	1.945	359.825	34225
239	11.1	2.406	575.034	57121
285	19.6	2.975	847.875	81225
$\sum x = 886$		$\sum Y = 9.424$	$\sum xY = 1972.409$	$\sum x^2 =$ ~~183100~~ 188500

$9.424 = 5A + 886b.$

$1972.409 = 886A + \cancel{183100}\,188500\,b$ $\Rightarrow$ $A = 0.183$; $b = 0.009$

~~$A = -0.168$~~ ~~$b = 0.011$~~

~~$a = e^A$~~

~~$a = 0.845$~~ ; ~~$b = 0.011$~~

~~$y = 0.845\, e^{0.011x}$~~

$a = e^A$

$a = 1.200$

$$\boxed{y = 1.200\, e^{0.009x}}$$

Sol:

3. Fit the exp-cure ae^{bx} for data:

x :	0	1	2	3
y :	1.05	2.10	3.85	8.30

Sol:

$$y = a e^{bx}$$

Apply log on b.s.

$$\log y = \log a e^{bx}$$

$$\log y = \log a + \log e^{bx}$$

$$= \log a + bx \log_e e$$

$$\log y = \log a + bx$$

$$Y = A + bx$$

normal equ^{n}s $\Rightarrow$ $\sum_1 Y = nA + b\sum_1 x$

$$\sum_1 xY = A\sum_1 x + b\sum_1 x^2$$

x	y	$Y = \log y$	xY	x^2
0	1.05	0.048	0	0
1	2.10	0.741	0.741	1
2	3.85	1.348	2.696	4
3	8.30	2.116	6.348	9
$\sum_1 x = 6$		$\sum_1 Y = 4.253$	$\sum_1 xY = 9.785$	$\sum_1 x^2 = 14$

$$4.253 = 4A + 6b$$

$$9.785 = 6A + 14b$$

$A = 0.041$; $b = \cancel{0.04}\ 0.681$

$a = 1.041$; $b = 0.681$

$$\boxed{Y = 1.041 e^{0.681x}}$$

Type-IV: Fitting of a power curve. $y = ab^x$ or $y = ax^b$.

Take $\log_{10}$ on b.s.

$$y \log_{10} = \log_{10}(ab^x)$$

$$y \log_{10} = \log_{10} a + \log_{10} b^x$$

$$y \log_{10} = \log_{10} a + x \log_{10} b$$

$$Y = A + Bx \Rightarrow \quad \Sigma_1^n Y = An + B\Sigma_1^n x$$

$$\Sigma_1^n xY = A\Sigma_1^n x + B\Sigma_1^n x^2$$

$A = \log_{10} a$; $B = \log_{10} b$.

$a = 10^A$; $b = 10^B$.

1. Using least squares fit a curve form $y = ab^x$

x	2	3	4	5	6.
y	8.3	15.4	33.1	65.2	127.4

Sol:

Given: $y = ab^x$.

Apply $\log_{10}$ on B.S.

$$\log_{10} y = \log_{10}(ab^x)$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$Y = A + Bx$$

Normal equ^ns $\Rightarrow$ $\Sigma_1^n Y = An + B\,\Sigma_1^n x$

$$\Sigma_1^n xY = A\Sigma_1^n x + B\,\Sigma_1^n x^2$$

x	y	$Y = \log_{10} y$.	xY	x^2
2	8.3	0.919	1.838	4
3	15.4	1.187	3.561	9
4	33.1	1.519	6.076.	16
5	65.2	1.814	9.070	25
6	127.4	2.105	12.630	36
$\Sigma_1^n x = 20$		$\Sigma_1^n Y = 7.544$	$\Sigma_1^n xY = 33.175$	$\Sigma_1^n x^2 = 90$

The equ^ns ⇒ $7.544 = 5A + 20B$

$33.175 = 20A + 90B$

$A = 0.3092$; $B = 0.2999$

$a = 10^A$; $b = 10^B$

$a = 2.037$; $b = 1.995$

The req. equ^n is $y = 2.037(1.995)^x$

$+B\sum_1^n x$

$x + B\sum_1^n x^2$

~~2. obtain a relation of form $y=ab^x$ for following data.~~

~~x: 2 3 4 5 6.~~

~~y: 8.3 15.4~~

2. Fit the power curve $y=ab^x$ for data:

x :	1	2	3	4
y :	7	11	17	27

Sol:

Given; $y = ab^x$

Apply $\log_{10}$ on b.s.

$\log_{10} y = \log_{10}(ab^x)$

$\log_{10} y = \log_{10} a + \log_{10} b^x$

$Y = A + Bx$

Normal equ^ns ⇒ $\sum_1^n Y = An + B\sum_1^n x$

$\sum_1^n xY = A\sum_1^n x + B\sum_1^n x^2$

x	y	$Y = \log_{10} y$	xY	x^2
1	7	0.845	0.845	1
2	11	1.041	2.082	4
3	17	1.230	3.690	9
4	27	1.431	5.724	16
$\sum_1^n x = 10$		$\sum_1^n Y = 4.547$	$\sum_1^n xY = 12.341$	$\sum_1^n x^2 = 30$

$$4.547 = 4A + 10B$$
$$12.341 = 10A + 30B$$

$A = 0.65$ $\quad$ $B = 0.1947$

$a = 10^A$ $\quad$ $b = 10^B$

$a = 4.4668$ $\quad$ $b = 1.5656$

$$Y = 4.4668\,(1.5656)^x.$$

3. Fit the power curve for data: $y = ax^b$.

x : 61 $\quad$ 26 $\quad$ 7 $\quad$ 2.6

y : 350 $\quad$ 400 $\quad$ 500 $\quad$ 600.

Sol:

Given; $y = ax^b$.

$$\log_{10} y = \log_{10}(ax^b)$$

$$\log_{10} y = \log_{10} a + \log_{10} x^b$$

$$Y = A + bX$$

Nrml eqns. $\Rightarrow \sum Y = An + b\sum X$

$$\sum XY = A\sum X + b\sum X^2$$

x	y	$Y = \log y$	XY
61	350	2.544	155.184 ~~151.097~~

x	y	$X = \log_{10} x$	$Y = \log_{10} Y$	XY	X^2
61	350	1.785	2.544	4.541	~~3721~~ 3.186
26	400	1.414	2.602	3.679	~~18.83~~ 1.999
7	500	0.845	2.698 ~~18.886~~	2.278	0.714
2.6	600	0.414	2.788 ~~7.222~~	1.150	0.171
		$\sum_1^4 X = 4.458$	$\sum_1^4 Y = 10.622$	$\sum_1^4 XY = 11.648$	$\sum_1^4 X^2 = 6.07$

$$10.622 = 4A + 4.458b$$

$$11.648 = 4.458A + 6.07b$$

$A = 2.847$; $b = -0.172$

$a = 10^A$; $b = -0.172$

$a = 703.07$

$y = ab^x \Rightarrow y = 703.07\,(-0.172)^x$

$y = ax^b \Rightarrow y = 703.07\,x^{(-0.172)}$