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PROBABILITY DISTRIBUTIONS

There are two types of Theoritical distributions or probability distributions.

1. Discrete probability distribution
2. Continuous probability distribution.

1. Discrete probability distribution:
   1. Binomial distribution
   2. Poison distribution

   3. ~~CB~~

2. Continuous probability distribution:
   1. Normal distribution

In this chapter, we will study only the three distributions Binomial, Poison and normal distributions.

Before we discuss these distributions, we first give a brief introduction of uniform and Bernoulli's distribution.

Discrete uniform distribution:-

A random variable x has a discrete uniform distribution if and only if its probability distribution is given by

$$P(x) = \frac{1}{k} \qquad x = x_1, x_2, \ldots, x_k$$

The random variable x is called discrete uniform random variable.

Ex:

x	0	1
$P(x)$	$1/2$	$1/2$

Bernoulli's distribution:

A random variable x which takes two values 0 and 1 which probability q and p respectively.

x	0	1
$P(x)$	q	p

i.e, $P(x=0)=q$

$P(x=1)=p$

where $q=1-p$

$p+q=1$

This is called Bernoulli's distribution.

The probability function of Bernoulli's distribution can be written as $P(x)=p^x q^{1-x}$

→ Mean of the Bernoulli's distribution

$$\mu = E(x) = \Sigma p_i x_i$$

$$= 0(q)+1(p)$$

$$\boxed{\mu = p}$$

→ Variance of Bernoulli's distribution

$$V(x) = \Sigma p_i x_i^2 - \mu^2$$

$$= 0(q)+1(p)-p^2$$

$$= p-p^2$$

$$= p(1-p)$$

$$\boxed{V(x) = pq}$$

Note:

1. Bernoulli's theorem:

If the probability of the occurance of the event (success) in a single trial is p, then the probability that it will occur exactly r times out of n independent trials is

$$\boxed{P(r) = {}^nC_r\, p^r q^{n-r}}$$

2. Binomial theorem:

$$(q+p)^n = q^n C_0 p^0 q^{n-0} + {}^nC_1 p^1 q^{n-1} + {}^nC_2 p^2 q^{n-2} + \cdots + {}^nC_n p^n q^{n-n}$$

$$(q+p)^n = q^n + {}^nC_1 p^1 q^{n-1} + {}^nC_2 p^2 q^{n-2} + \cdots + p^n.$$

Binomial distribution:

Definition: (Imp)

A random variable x has a binomial distribution if it assumes only non-negative values and it's probability density function is given by

$$P(X=r) = P(r) = \begin{cases} {}^nC_r p^r q^{n-r} & ; r=0,1,2,\ldots,n \\ & \quad q=1-p \\ 0 & ; \text{otherwise.} \end{cases}$$

Examples of binomial distribution:

1. The no. of defective bolts in a box containing n bolts.
2. The no. of postgraduates in a group of n men.

Conditions of Binomial distribution:

1. Trials are repeated under identical conditions for a fixed no. of times say n times.
2. There are only two possible outcomes.
   Ex: success or Failure of each trial.
3. The probability of success in each trial remains constant and does not change from trial to trial.
4. The trials are independent i.e, probability of a event in any events is not affected

by the results of any other trial.

Constants of Binomial distribution:

1. Mean of the Binomial distribution:

The binomial distribution is given by

$P(r) = {}^nC_r p^r q^{n-r}$ where $r = 0, 1, 2, \dots, n$.

$$\text{Mean } E(X) = \mu = \sum P_i x_i$$

$$= \sum_{r=0}^{n} P(r) \cdot r$$

$$= 0 P(0) + 1 P(1) + 2 \cdot P(2) + \dots + n \cdot P(n)$$

$$= {}^nC_1 p q^{n-1} + 2 \cdot {}^nC_2 p^2 q^{n-2} + \dots + n \cdot {}^nC_n p^n q^{n-n}$$

$$= npq^{n-1} + 2\frac{(n)(n-1)}{2!} p^2 q^{n-2} + \dots + np^n$$

$$= np\left[q^{n-1} + p(n-1) q^{n-2} + \dots + p^{n-1}\right]$$

$$= np(p+q)^{n-1} \quad (\because \text{Binomial theorem})$$

$$= np(1)^{n-1} \quad (\because \text{Bernoulli's theorem})$$

$$= np(1)$$

$$= np$$

$$\therefore \boxed{\text{Mean} = np}$$

2. Variance of Binomial distribution:

The binomial distribution is given by

$P(r) = {}^nC_r p^r q^{n-r}$ where $r = 0, 1, 2, \dots, n$.

$$\text{variance } V(X) = E(X^2) - [E(X)]^2$$

$$= \sum P(x_i) x_i^2 - \mu^2$$

$$= \sum P(r) r^2 - \mu^2$$

$$= \Sigma(P(r))(r^2 + r - r) - \mu^2$$

$$= \Sigma r(r-1) P(r) + \Sigma r P(r) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1) {}^nC_r p^r q^{n-r} + \mu - \mu^2$$

$$= 0 + 0 + {}^nC_2 (2) p^2 q^{n-2} + 3(2) {}^nC_3 p^3 q^{n-3} + \dots + n(n-1) p^n + \mu - \mu^2$$

$$= \cancel{2} \frac{n(n-1)}{\cancel{2}!} p^2 q^{n-2} + \cancel{6} \cdot \frac{n(n-1)(n-2)}{\cancel{3}!} p^3 q^{n-3} + \dots + n(n-1) p^n + \mu - \mu^2$$

$$= n(n-1) p^2 \left[\cancel{p^2} q^{n-2} + (n-2) q^{n-3} p + \dots + p^{n-2} \right] + \mu - \mu^2$$

$$= n(n-1) p^2 (p+q)^{n-2} + \mu - \mu^2$$

$$= n(n-1) p^2 (1)^{n-2} + np - n^2p^2$$

$$= (n^2 - n) p^2 + np - n^2 p^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np(1-p)$$

$$= np(q)$$

$$\boxed{\text{variance} = npq}$$

3. Mode of the Binomial distribution:

Mode of the distribution is the value of x at which P(X) has maximum value.

$$\text{Mode} = \begin{cases} \text{integral part of } (n+1)p; & \text{if } (n+1)p \text{ is not an integer} \\ (n+1)p \text{ and } (n+1)(p-1) : & \text{if } (n+1) \text{ is an integer} \end{cases}$$

4. Recurrance relation of the binomial distribution:

We know that,

$$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r} \rightarrow ①$$

$$P(r+1) = {}^{n}C_{r+1}\, p^{r+1} q^{n-(r+1)} \rightarrow ②$$

$$\frac{②}{①} \Rightarrow \frac{P(r+1)}{P(r)} = \frac{{}^{n}C_{r+1}\, p^{r+1} q^{n-(r+1)}}{{}^{n}C_{r}\, p^{r} q^{n-r}}$$

$$= \frac{\frac{n!}{(n-r-1)!(r+1)!} p^{r}.p\, q^{n-r}.q^{-1}}{\frac{n!}{(n-r)!\,r!} p^{r}.q^{n-r}}$$

$$\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \frac{p}{q}$$

$$\boxed{P(r+1) = \left(\frac{n-r}{r+1}\right)\frac{p}{q} P(r)}$$

Binomial frequency distribution:

We know that,

The Binomial frequency distribution is.

$N\,{}^{n}C_{r}\, p^{r} q^{n-r}$ (or) $N(p+q)^{n}$

Problems:

A fair coin is tossed 6 times. Find the probability of getting four heads.

n = no. of trials = 6

p = probability of getting head in a single throw

$\boxed{p = 1/2}$

$p + q = 1 \Rightarrow q = 1 - p$

$q = 1 - 1/2$

$\boxed{q = 1/2}$

i.e, probability of not getting head in a single throw$(q) = 1/2$.

$r = 4$

We know that,

$$P(r) = {}^nC_r \, p^r q^{n-r}$$

probability of getting 4 heads

$$P(4) = {}^nC_4 \, p^4 q^{n-4}$$

$$P(4) = {}^6C_4 \, p^4 q^{6-4}$$

$$= {}^6C_2 \, p^4 q^2$$

$$= \frac{6 \times 5}{2 \times 1} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$$

$$= \frac{15}{2^6} = \frac{15}{64}.$$

10 coins are thrown simultaneously. Find the probability of getting atleast

i) 7 heads ii) 6 heads.

n = no. of trials = 10

p = probability of getting head in single throw

$p = 1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

i) $r = 7$

probability of getting atleast 7 heads

$= P(r \geq 7)$

$= P(r=7) + P(r=8) + P(r=9) + P(r=10)$

we know that,

Binomial distribution is given by

$P(r) = {}^nC_r \, p^r q^{n-r}$

$P(r \geq 7) = {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$

$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0\right]$

$= \frac{1}{2^{10}} (120 + 45 + 10 + 1)$

$= \frac{176}{1024}$

$= \frac{11}{64}$

$= 0.1718$

ii) $r = 6$.

probability of getting atleast 6 heads.

$= P(r \geq 6)$

$= P(r=6) + P(r=7) + P(r=8) + P(r=9) + P(r=10)$

$= {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4 + {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 {}^{10}C_8$
$+ {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_4 + {}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0\right]$

$= \frac{1}{2^{10}} (210 + 120 + 45 + 10 + 1)$

$= \frac{386}{1024} = \frac{193}{512} = 0.3769$

Two dice are thrown 5 times. Find the probability of getting 7 as sum.
i) Atleast once ii) 2 times iii) $P(1 < X < 5)$

n = No. of trials = 5.

probability of getting 7 as sum in a single trial $= \frac{6}{36} = 1/6 = p$

$q = 1 - p = 1 - 1/6 = 5/6$

i) probability of getting 7 atleast once

$= P(r \geq 1)$

$= 1 - P(r < 1)$

$= 1 - P(r = 0)$

We know that,

$$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$= 1 - {}^{5}C_{0}\, p^{0} q^{5-0}$

$= 1 - 1\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{5}$

$= 1 - \left(\frac{5}{6}\right)^{5}$

$= 1 - \frac{3125}{7776}$

$= \frac{4651}{7776}$

$= 0.598$

ii) probability of getting 7 2 times $= P(r = 2)$.

$= {}^{5}C_{2}\, p^{2} q^{5-2}$

$= 10\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{3}$

$= 10 \frac{1}{6^3} \frac{5^3}{6^3}$

$= 10 \frac{5^3}{6^6}$

$= \frac{10 \times 125}{46656}$

$= 0.026$

iii) $P(1<X<5) = P(X=2)+P(X=3)+P(X=4)$

We know that,

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$$P(1<X<5) = {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{5-2} + {}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3} + {}^5C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{5-4}$$

$$= 10\,\frac{1}{36}\,\frac{5^3}{6^3} + {}^5C_2\,\frac{5^2}{6^5} + {}^5C_1\,\frac{5}{6^5}$$

$$= \frac{10(5^3)}{7776} + \frac{10(25)}{7776} + 5\times\frac{5}{7776}$$

$$= \frac{1525}{7776}$$

$$= 0.1961.$$

Determine the binomial distribution for which mean is 4 and variance is 3.

Given that,

Mean $= 4$

$np = 4 \rightarrow (1)$

Variance $= 3$

$npq = 3 \rightarrow (2)$

From (1) & (2)

$4q = 3$

$q = 3/4$

$p = 1-q = 1-3/4 = 1/4$

$np = 4$

$n\left(\frac{1}{4}\right) = 4$

$\boxed{n = 16}$

Binomial distribution $= (p+q)^n$

$= \left(\frac{1}{4}+\frac{3}{4}\right)^{16}$

$= 1^{16} = 1.$

The mean and variance of Binomial distribution are 4 and 4/3 respectively. Find $P(x \geq 1)$.

Given that,

$$\text{Mean} = 4$$
$$np = 4 \rightarrow (1)$$
$$\text{variance} = 4/3$$
$$npq = \frac{4}{3} \rightarrow (2)$$
$$4q = \frac{4}{3}$$
$$q = 1/3$$
$$p = 1-q = 1-1/3 = 2/3.$$
$$np = 4 \Rightarrow n\left(\frac{2}{3}\right) = 4$$
$$\boxed{n=6}$$

$$P(X \geq 1) = 1 - P(X<1)$$
$$= 1 - P(X=0)$$
$$= 1 - 6C_0\, p^0 q^{6-0}$$
$$= 1 - (1)(1)\left(\frac{1}{3}\right)^6$$
$$= 1 - \frac{1}{3^6}$$
$$= \frac{728}{729}$$
$$= 0.9986$$

The mean and variance of binomial distribution with parameters n and p are 16 and 8. Find $P(x \geq 1)$ and $P(x > 2)$.

Given that,

$$\text{Mean} = 16$$
$$np = 16. \rightarrow (1)$$
$$\text{variance} = 8$$
$$npq = 8 \rightarrow (2)$$
$$16q = 8$$
$$q = 1/2$$
$$p = 1-q = 1-1/2 = 1/2$$
$$np = 16 \Rightarrow n\left(\frac{1}{2}\right) = 16$$
$$n = 32.$$

$$P(X\geq 1) = 1-P(X<1)$$
$$= 1-P(X=0)$$
$$= 1-{}^{32}C_0\, p^0 q^{32-0}$$
$$= 1-(1)(1)\left(\tfrac{1}{2}\right)^{32}$$
$$= 1-\frac{1}{2^{32}}$$
$$= 0.999$$

$$P(X>2) = 1-P(X\leq 2)$$
$$= 1-P(X=0)-P(X=1)-P(X=2)$$
$$= 1-{}^{32}C_0\, p^0 q^{32-0} - {}^{32}C_1\, p^1 q^{32-1} - {}^{32}C_2\, p^2 q^{32-2}$$
$$= 1-(1)(1)\frac{1}{2^{32}} - 32\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^{31} - 496\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^{30}$$
$$= 1-\frac{1}{2^{32}}(1+32+496)$$
$$= 0.999.$$

The probability of a defective bolt is $1/8$. Find i) mean ii) variance for the distribution of defective bolts of 640.

Given that,

Probability of defective bolt $(P) = 1/8$

No. of ~~b~~ ~~de~~ bolts $(n) = 640$.

$q = 1-P = 1-1/8 = 7/8$.

Mean $= nP = 640\left(\frac{1}{8}\right) = 80$

variance $= npq = 80\left(\frac{7}{8}\right) = 70$.

20% of items produced from a factory are defective. Find the probability that in a sample of 5 choosen at random.

i) None is defective ii) 1 is defective iii) $P(1<X<4)$

Given that,

$$P = \frac{20}{100} = 1/5$$
$$q = 1-P = 1-1/5 = 4/5$$
$$n = 5.$$

i) / $p(\gamma=0)$ =

Binomial distribation is given by

$$P(X=\gamma) = {}^{n}C_{\gamma}\, p^{\gamma} q^{n-\gamma}$$

i) $P(X=0) = {}^{n}C_{0}\, p^{0} q^{n-0}$

$= {}^{5}C_{0} \left(\frac{1}{5}\right)^{0} \left(\frac{4}{5}\right)^{5}$

$= 1 \cdot \frac{4^5}{5^5}$

$= 0.32768.$

ii) $P(X=1) = {}^{n}C_{1}\, p^{1} q^{n-1}$

$= {}^{5}C_{1}\, p^{1} q^{5-1}$

$= 5 \left(\frac{1}{5}\right)^{1} \left(\frac{4}{5}\right)^{4}$

$= 5 \cdot \frac{4^4}{5^5}$

$= 0.4096.$

ii) $P(1<X<4) = P(X=2) + P(X=3)$

$= {}^{n}C_{2}\, p^{2} q^{n-2} + {}^{n}C_{3}\, p^{3} q^{n-3}$

$= {}^{5}C_{2}\, p^{2} q^{5-2} + {}^{5}C_{3}\, p^{3} q^{5-3}$

$= 10\, p^{2} q^{3} + {}^{5}C_{2}\, p^{3} q^{2}$

$= 10 \left(\frac{1}{5}\right)^{2} \left(\frac{4}{5}\right)^{3} + 10 \left(\frac{1}{5}\right)^{3} \left(\frac{4}{5}\right)^{2}$

$= \frac{4^3(5)}{5^5} + \frac{10(4^2)}{5^5}$

$= \frac{480}{3125}$

$= \frac{96}{625}$

$= 0.1536.$

The probability of hitting the target is $\frac{1}{3}$ a man

1) If he hits 5 times, what is the probability of his hitting the target atleast twice.

ii) How many times must he fire so that the probability of his hitting the target atleast once is more than 90%.

Given that,

$$P = \frac{1}{3}$$

$$q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$$

i) Here, n = no. of times = 5.

Probability of hitting target atleast twice

$$= P(x \geq 2)$$

$$= 1 - P(x < 2)$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - {}^5C_0 P^0 q^{5-0} - {}^5C_1 P^1 q^{5-1}$$

$$= 1 - 1\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 - 5\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{1}{3^5}\left(32 + 5(16)\right)$$

$$= 1 - \frac{1}{3^5}(32+80)$$

$$= \frac{1\cancel{-118}}{3^5} = 1 - \frac{112}{3^5}$$

$$= 0.539$$

ii) $$P(x \geq 1) > \frac{90}{100}.$$

$$1 - P(x<1) > \frac{9}{10}$$

$$1 - P(x=0) > \frac{9}{10}$$

$$1 - {}^nC_0 P^0 q^{n-0} > \frac{9}{10}$$

$$1 - (1)(1)\left(\frac{2}{3}\right)^n > \frac{9}{10}.$$

$$1 - \left(\frac{2}{3}\right)^n > \frac{9}{10}$$

$1-\frac{9}{10} > \left(\frac{2}{3}\right)^n$

$\frac{1}{10} > \left(\frac{2}{3}\right)^n$

$0.1 > \left(\frac{2}{3}\right)^n.$

$n=1 \Rightarrow 0.666 < 0.1$

$n=2 \Rightarrow 0.444 < 0.1$

$n=3 \Rightarrow 0.296 < 0.1$

$n=6 \Rightarrow 0.087 < 0.1$

$\therefore \boxed{n=6}$

In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 success are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

Given that,

No. of trials $(n) = 5$.

$P(X=1) = 0.4096 = \frac{4096}{10000}$

$P(X=2) = 0.2048 = \frac{2048}{10000}$

$P(X=1) = \frac{4096}{10000}$

${}^5C_1 p^1 q^4 = 0.4096$

$pq^4 = \frac{0.4096}{5} \rightarrow (1)$

$P(X=2) = 0.2048$

${}^5C_2 p^2 q^3 = 0.2048$

$(10) p^2 q^3 = 0.2048$

$p^2 q^3 = \frac{0.2048}{10} \rightarrow (2)$

$\frac{(1)}{(2)} \Rightarrow \frac{pq^4}{p^2q^3} = \frac{\frac{0.4096}{5}}{\frac{0.2048}{10}}$

$\frac{q}{p} = \frac{\frac{256}{3125}}{\frac{64}{3125}}$

$\frac{1-p}{p} = 4$

$1-p = 4p$

$1 = 5p$

$\boxed{p = 1/5 = 0.2}$

In a family of 5 children, Find the probability that there are i) 2 boys ii) Atleast one boy iii) All are boys iv) No boys.

Given that,

Total no. of children $(n) = 5$

p = probability of child being a boy $= 1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

i) Probability that there are 2 boys $= P(X=2)$

$$= 5C_2 p^2 q^3$$

$$= 10\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$$

$$= \frac{10}{32} = \frac{5}{16} = 0.3125.$$

ii) probability that there is atleast one boy $= P(X \geq 1)$

$$= 1 - P(X<1)$$

$$= 1 - P(X=0)$$

$$= 1 - 5C_0 p^0 q^5$$

$$= 1 - (1)(1)\left(\frac{1}{2}\right)^5$$

$$= 1 - \frac{1}{32}$$

$$= \frac{31}{32}$$

iii) Probability that all are boys $= P(X=5)$

$$= 5C_5 q^0 p^5$$

$$= 5C_0 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{32}.$$

iv) probability that there are no boys

$$= P(X=0)$$

$$= 5C_0 p^0 q^5$$

$$= 1\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5$$

$$= 1/32.$$

In 256 sets of 12 tosses of coin. In how many cases one can expect 8 heads and 4 tails.

n = No. of tosses = 12

p = probability of getting a head = $1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

probability of getting 8 heads and 4 tails

$= P(x=8)$

$= 12_{C_8}\, p^8\, q^{12-8}$

$= 12_{C_4} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^4$

$= \frac{495}{2^{12}}$

$= \frac{495}{4096}$

$= 0.1208$

The Expected no. of such cases in 256 sets

$= 256\, P(x=8)$

$= 256\,(0.1208)$

$= 30.9248$.

Four coins are tossed 160 times. The no. of times x heads occurs is given below.

x :	0	1	2	3	4
No. of times :	8	34	69	43	6

Fit a binomial distribution to this data on the hypothesis that coins are unbiased.

$n = 4$

p = probability of getting head = $1/2$

$q = 1 - p = 1 - 1/2 = 1/2$.

Binomial distribution is $P(r) = n_{C_r}\, p^r q^{n-r}$.

x	f	P(x)	f(x) = N P(x)
0	8	$P(0)=\frac{1}{16}$	$f(0)=160\times\frac{1}{16}=10$
1	34	$P(1)=1/4$	$f(1)=160\times\frac{1}{4}=40$
2	69	$P(2)=3/8$	$f(2)=160\times\frac{3}{8}=60$
3	43	$P(3)=1/4$	$f(3)=160\times 1/4=40$
4	6	$P(4)=1/16$	$f(4)=160\times 1/16=10.$

$$P(x=0)={}^4C_0\,p^0q^{4-0}=1(1)\left(\tfrac{1}{2}\right)^4=1/16$$
$$P(x=1)={}^4C_1\,p^1q^{4-1}=4\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^3=\frac{4}{16}=1/4$$
$$P(x=2)={}^4C_2\,p^2q^{4-2}=6\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2=\frac{6}{16}=3/8$$
$$P(x=3)={}^4C_3\,p^3q^{4-3}={}^4C_1\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^1=\frac{4}{24}=1/4$$
$$P(x=4)={}^4C_4\,p^4q^{4-4}={}^4C_0\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^0=1\left(\tfrac{1}{16}\right)=1/16.$$

Fit a binomial distribution to the following data.

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

x	f	$f_i x_i$
0	2	2
1	14	28
2	20	40
3	34	~~98~~ 102
4	22	88
5	8	40
	$\Sigma f=100$	$\Sigma f_i x_i=$ ~~296~~ 300

$\boxed{N=100}$

$$\text{Mean}=\frac{\Sigma f_i x_i}{\Sigma f_i}$$

$np = \frac{\cancel{298}\;300}{100}$

Here, $n = 5$

$5p = \frac{\cancel{298}\;300}{100}$

$5p = 3$

$p = 3/5$

$q = 1 - p = 1 - 3/5 = 2/5$.

Binomial distribution is $P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$.

x	f	$P(x)$	$f(x) = N\,P(x)$
0	2	$P(0) = \frac{32}{3125}$	$f(0) = 100 \times \frac{32}{3125} = \frac{128}{125}$
1	14	$P(1) = 240/3125$	$f(1) = \frac{96}{125}$
2	20	$P(2) = 720/3125$	$f(2) = 288/125$
3	34	$P(3) = 1080/3125$	$f(3) = \frac{864}{25}$
4	22	$P(4) = 810/3125$	$f(4) = \frac{324}{125}$
5	8	$P(5) = 243/3125$	$f(5) = \frac{972}{125}$

$P(x=0) = {}^{5}C_{0}\, p^{0} q^{5} = (1)(1)\left(\frac{2}{5}\right)^{5} = \frac{2^{5}}{5^{5}} = \frac{32}{3125}$

$P(x=1) = {}^{5}C_{1}\, p^{1} q^{4} = 5\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^{4} = \frac{15(2^{4})}{5^{5}} = \frac{240}{3125}$

$P(x=2) = {}^{5}C_{2}\, p^{2} q^{3} = 10\left(\frac{3}{5}\right)^{2}\left(\frac{2}{5}\right)^{3} = \frac{90(2^{3})}{5^{5}} = \frac{720}{3125}$

$P(x=3) = {}^{5}C_{3}\, p^{3} q^{2} = {}^{5}C_{2}\left(\frac{3}{5}\right)^{3}\left(\frac{2}{5}\right)^{2} = \frac{540(2)}{5^{5}} = \frac{1080}{3125}$

$P(x=4) = {}^{5}C_{4}\, p^{4} q^{1} = {}^{5}C_{1}\left(\frac{3}{5}\right)^{4}\left(\frac{2}{5}\right)^{1} = \frac{810}{5^{5}} = \frac{810}{3125}$

$P(x=5) = {}^{5}C_{5}\, p^{5} q^{0} = {}^{5}C_{0}\left(\frac{3}{5}\right)^{5}\left(\frac{2}{5}\right)^{0} = \frac{243}{5^{5}} = \frac{243}{3125}$

If the probability that a man aged 60 will leave to be 70 is 0.65. What is the probability that out of 10 men now 60 atleast 7 will live to be 70.

$n=10$, $P=0.65=\frac{65}{100}=\frac{13}{20}$

$q=1-P=1-0.65=0.35=7/20$

Now, $P(X\geq 7)=P(X=7)+P(X=8)+P(X=9)+P(X=10)$

$=10_{C_7}P^7q^{10-7}+10_{C_8}P^8q^2+10_{C_9}P^9q^1+10_{C_{10}}P^{10}q^0$

$=10_{C_3}\left(\frac{13}{20}\right)^7\left(\frac{7}{20}\right)^3+10_{C_2}\left(\frac{13}{20}\right)^8\left(\frac{7}{20}\right)^2+10_{C_1}\left(\frac{13}{20}\right)^9\left(\frac{7}{20}\right)^1$
$+10_{C_0}\left(\frac{13}{20}\right)^{10}$

$=0.252+0.175+0.072+0.013$

$=\frac{64}{125}$

$=0.512$

6 dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 (or) 6.

Given that, $n=6$

$P=\frac{2}{6}=1/3$

$q=1-P=1-1/3=2/3$.

Now, $P(X\geq 3)=1-P(X<3)$

$=1-P(X=0)-P(X=1)-P(X=2)$

$=1-{}^6C_0P^0q^6-{}^6C_1P^1q^5-{}^6C_2P^2q^4$

$=1-(1)\left(\frac{2}{3}\right)^6-6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5-15\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$

$=1-\frac{1}{36}[64-192-240]$

$=1+\frac{368}{36}$

$=1+\frac{368}{729}$

$=1.504$

Determine the probability of getting a sum of 9 exactly twice in 3 throws with a pair of fair dice.

Here, $n=3$

$$p = P\{(3,6),(4,5),(5,4),(6,3)\} = \frac{4}{36} = 1/9$$

$$q = 1-p = 1-1/9 = 8/9$$

$$P(x=r) = P(x=2) = 3_{C_2}\, p^2 q^{3-2}$$

$$= 3_{C_1}\left(\frac{1}{9}\right)^2\left(\frac{8}{9}\right)^1$$

$$= 3\left(\frac{8}{9^3}\right)$$

$$= \frac{24}{729}$$

$$= 0.0329.$$

A dice is tossed thrice. A success is getting 1 (or) 6 on a toss. Find mean and variance of the no. of success.

$n=3.$ $p = 1/6 + 1/6 = 2/6 = 1/3$

$$q = 1-p = 1-1/3 = 2/3$$

Mean $= np = 3\left(\frac{1}{3}\right) = 1$

Variance $= npq = 1\left(\frac{2}{3}\right) = \frac{2}{3}.$

Poisson distribution:

A random variable x is said to be a poisson distribution if it assume only non-negative values and its probability density function is given by.

$$P(X=x) = P(x,\lambda) = \begin{cases} \frac{e^{-\lambda}\lambda^{x}}{x!} & ; x=0,1,2,\dots\infty \\ 0 & ; \text{otherwise} \end{cases}$$

Here $\lambda>0$

λ is called the parameter of the distribution.

In this distribution occurance of p is very small and no. of trials is very lage where np is finite.

Note:

1. The poisson distribution function is

$$F(x) = P(X\leq x) = \sum_{r=0}^{x} P(r)$$

$$= \sum_{r=0}^{x} \frac{e^{-\lambda}\lambda^{r}}{r!}$$

2. $\displaystyle\sum_{x=0}^{\infty} \frac{\lambda^{x}}{x!} = e^{\lambda}$

Examples:

1. The no of defective electric bulbs manufactured by reputed Company.
2. The no. of telephone calls per minute.
3. The no of cars passing a certain point in one minute.

Conditions on Poisson distribution:

1. The variable (no. of occurances) is a discrete Variable.
2. The occurance are rare.

3. The No. of trials n is large.

4. The probability of success p is very small (very close to zero).

5. $\lambda = np$ is finite.

Constants of Poisson distribution:

1. Mean of the poisson distribution:

We know that,

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\text{Mean} = \mu = E(x) = \sum P(x)\, x.$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!}\, x$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-1)!\, \cancel{x}} \times \cancel{x}$$

$$= \sum_{x=01}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-1)!}$$

put $x-1 = y$.

$x = y+1$

$$= \sum_{y=0}^{\infty} \frac{e^{-\lambda} (\lambda)^{y+1}}{y!}$$

$$= e^{-\lambda} \lambda \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}$$

$$= e^{-\lambda} \lambda e^{\lambda}$$

$$= e^{0} \lambda$$

$$\boxed{\mu = \lambda}$$

2. Variance of the poisson distribution:

We know that, $P(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$

$$\text{Variance } V(x) = E(x^2) - [E(x)]^2$$

$$= \sum P(x)\, x^2 - \mu^2$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} x^2 - \lambda^2$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x x^2}{x(x-1)!} - \lambda^2$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x x}{(x-1)!} - \lambda^2$$

$$= \sum_{x=1}^{\infty} (x+1-1)\frac{e^{-\lambda}\lambda^x}{(x-1)!} - \lambda^2$$

$$= \sum_{x=1}^{\infty} (x-1)\frac{e^{-\lambda}\lambda^x}{(x-1)(x-2)!} + \sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^x}{(x-1)!} - \lambda^2$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!} + \sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^x}{(x-1)!} - \lambda^2$$

put $x-2=y$ $\quad x-1=z$

$x=y+2$ $\quad x=z+1$

$$= \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^{2+y}}{y!} + \sum_{z=0}^{\infty}\frac{e^{-\lambda}\lambda^{z+1}}{z!} - \lambda^2$$

$$= e^{-\lambda}\cdot\lambda^2 e^{\lambda} + e^{-\lambda}\cdot\lambda\cdot e^{\lambda} - \lambda^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

$$\boxed{V(x) = \lambda}$$

Mode of the poisson distribution:

Mode is the value of x for which the probability $P(x)$ is maximum.

1. If λ is an integer then $\lambda-1$ is also an integer so we have two maximum values and two modes are λ and $\lambda-1$.
2. If λ is not an integer then the mode is integral part of λ.

Reccurrance relation for the poisson distribution:

We know that,

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$= \frac{e^{-\lambda}\lambda^{x}\lambda}{(x+1)x!}$$

$$= \frac{e^{-\lambda}\lambda^{x}}{x!} \times \frac{\lambda}{x+1}$$

$$\boxed{P(x+1) = \frac{\lambda}{x+1} P(x)}$$

Properties of poisson Distribution:

1. Range of the variable is from 0 to ∞.
2. Mean and variance are equal.
3. Distribution gets more and more symmetrical about the mean as λ increases and tends to Normal distribution.

2% of the items of the factory are defective. The items are packed in the boxes. What is the probability that there will be

(i) 2 defective items.

(ii) Atleast 3 defective items in a box of 100 items.

Given that,

$$n = 100$$

Probability of getting defective item

$$P = \frac{2}{100} = 0.02$$

$$\lambda = np = 100\left(\frac{2}{100}\right) = 2.$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

i) $P(X=2) = \frac{e^{-2}(2)^2}{2!} = \frac{e^{-2}(4)}{2} = \frac{2}{e^2} = 0.2706.$

ii) $P(X \geq 3) = 1 - P(X<3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!} + \frac{e^{-2}(2)^2}{2!}\right]$

$= 1 - e^{-2}\left(1 + 2 + \frac{4}{2}\right)$

$= 1 - 0.1353(5)$

$= 1 - 0.6767$

$= 0.3233.$

Average number of accidents on any day on a National highway is 1.8. Determine the probability that the no. of accidents are (i) atleast 1 (ii) Atmost 1.

Given,

Mean $= \lambda = 1.8$

i) $P(\text{atleast } 1) = P(X \geq 1)$

$= 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-\lambda} \lambda^x}{x!}$

$= 1 - \frac{e^{-1.8}(1.8)^0}{0!}$

$= 0.8347.$

ii) $P(\text{atmost } 1) = P(X \leq 1)$

$= P(X=0) + P(X=1)$

$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}$

$= e^{-1.8} + e^{-1.8}(1.8)$

$= 0.1653 + 0.2975$

$= 0.4628.$

Using recursion formula, find the probabilities when $x = 0, 1, 2, 3, 4, 5$. If the mean of the poisson distribution is 3.

Given, $\lambda = 3$.

Recursion formula is $P(x+1) = \frac{\lambda}{x+1} P(x) \rightarrow ①$

~~Probo~~ Poisson distribution is $P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \rightarrow ②$

$P(0) = \frac{e^{-3} 3^0}{0!} = e^{-3} = 0.04978 = 0.0498.$

put $x = 0$ in ①.

$P(1) = \frac{3}{1} \cdot e^{-3} = 3(0.0498) = 0.1494$

put $x = 1$ in ①

$P(2) = \frac{3}{2} P(1) = \frac{3}{2}(0.1494) = 0.2241$

put $x = 2$ in ①.

$P(3) = \frac{3}{3} P(2) = 0.2241$

put $x = 3$ in ①

$P(4) = \frac{3}{4} P(3) = \frac{3}{4}(0.2241) = 0.16807$
$= 0.1681.$

put $x = 4$ in ①

$P(5) = \frac{3}{5} P(4) = \frac{3}{5}(0.1681) = 0.10086.$

If the variance of a poisson variate is 3 then find the probability that i) $x=0$ ii) $0<x\leq 3$ iii) $1<x<4$.

Given that,

Mean = variance = $\lambda = 3$.

poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

i) $P(x=0) = \frac{e^{-3}3^{0}}{0!} = e^{-3} = 0.04978 = 0.0498$

ii) $P(0<x\leq 3) = P(x=1)+P(x=2)+P(x=3)$

$= \frac{e^{-3}(3)^{1}}{1!} + \frac{e^{-3}(3)^{2}}{2!} + \frac{e^{-3}(3)^{3}}{3!}$

$= 3e^{-3} + \frac{9}{2}e^{-3} + \frac{\cancel{27}^{9}}{\cancel{6}_{2}}e^{-3}$

$= 3e^{-3} + 9e^{-3}$

$= 12e^{-3}$

$= 12(0.0498)$

$= 0.5976$.

iii) $P(1<x<4) = P(x=2) + P(x=3)$

$= \frac{e^{-3}(3)^{2}}{2!} + \frac{e^{-3}(3)^{3}}{3!}$

$= \frac{9}{2}e^{-3} + \frac{27e^{-3}}{6}$

$= \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3}$

$= 9e^{-3}$

$= 9(0.0498)$

$= 0.4482$.

If x is a poisson variate such that $3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$. find i) Mean

ii) $P(x \leq 2)$

we know that,

Poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given that,

$$3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$$

$$\frac{3e^{-\lambda}\lambda^4}{4!} = \frac{1}{2}\frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^0}{0!}$$

$$\frac{3e^{-\lambda}}{\cancel{24}\,8}\lambda^4 = \frac{e^{-\lambda}}{4}\lambda^2 + e^{-\lambda}$$

$$\frac{\lambda^4}{8} = \frac{\lambda^2}{4} + 1$$

$$\frac{\lambda^4}{\cancel{8}\,2} = \frac{\lambda^2+4}{\cancel{4}}$$

$$\lambda^4 = 2\lambda^2 + 8$$

$$\lambda^4 - 2\lambda^2 - 8 = 0$$

$$(\lambda^2-4)(\lambda^2+2) = 0$$

$$\lambda^2 = 4 \quad (or) \quad \lambda^2 = -2.$$

$\lambda^2 = 4$ since, λ is always +ve and real number.

$\lambda = 2$ $\therefore \boxed{\lambda=2}$

$\therefore \boxed{\lambda = 2}$

i) In poisson distribution,

Mean $= \lambda = 2$.

$\therefore \boxed{\text{Mean} = 2}$

ii). $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= e^{-2}[1+2+2] = 5e^{-2} = 0.67667$$

$$= 0.6767.$$

It has been found that 2% of the tools produced by a certain machine are defective. what is the probability that in a shipment of 400 such tools

i) 3% (or) more ii) 2% (or) less will prove defective.

Given that,

$$P = \frac{2}{100} = 1/50$$

$$n = 400$$

$$\text{Mean} = \lambda = np = 400 \times \frac{1}{50} = 8.$$

Poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

i) $P(x \geq 3) = 1 - P(x<3)$

$$= 1 - P(x=0) - P(x=1) - P(x=2)$$

$$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} - \frac{e^{-\lambda}\lambda^1}{1!} - \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= 1 - e^{-8}\left[1 + 8 + \frac{64}{2}\right]$$

wrong

$$= 1 - e^{-8}[1 + 8 + 32]$$

$$= 1 - 41e^{-8}$$

$$= 1 - 0.0137$$

$$= 0.9863.$$

ii) $P(x \leq 2) = P(x=0)$

i) P(3% or more)

$$= P\left(\frac{3}{100}(400) \text{ or more}\right)$$

$$= P(12 \text{ or more}) = P(x \geq 12)$$

$$= 1 - P(x<12)$$

$$= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$$

$$- P(x=4) - P(x=5) - P(x=6) -$$

$$\ldots - P(x=11).$$

$$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} - \frac{e^{-\lambda}\lambda^1}{1!} - \frac{e^{-\lambda}\lambda^2}{2!} - \cdots - \frac{e^{-\lambda}\lambda^{11}}{11!}$$

$$= 1 - e^{-\lambda}\left[\lambda^0 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots + \frac{\lambda^{11}}{11!}\right]$$

$$= 1 - e^{-8}\left[8^0 + 8 + \frac{64}{2} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} + \frac{8^6}{6!} + \frac{8^7}{7!} + \frac{8^8}{8!} + \frac{8^9}{9!} + \frac{8^{10}}{10!} + \frac{8^{11}}{11!}\right]$$

$$= 1 - e^{-8}\left[1 + 8 + 32 + 85.33 + 170.66 + 273.06 + 364.08 + 416.10 + 416.10 + 369.86 + 295.89 + 215.19\right]$$

$$= 1 - 2647.27e^{-8}$$

$$= 1 - 0.888$$

$$= 1 - 0.888$$

$$= 0.112.$$

ii) $P(2\% \text{ or less})$ $\qquad 2\% = \frac{2}{100} \times 400 = 8.$

$$= P(x \le 8)$$

$$= P(x=0) + P(x=1) + \cdots + P(x=8)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \cdots + \frac{e^{-\lambda}\lambda^8}{8!}$$

$$= e^{-\lambda}\left[1 + \lambda + \cdots + \frac{\lambda^8}{8!}\right]$$

$$= e^{-8}\left[1 + 8 + \frac{64}{2} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} + \frac{8^6}{6!} + \frac{8^7}{7!} + \frac{8^8}{8!}\right]$$

$$= e^{-8}\left[1 + 8 + 32 + 85.33 + 170.66 + 273.06 + 364.08 + 416.10 + 416.10\right]$$

$$= e^{-8}(1766.33)$$

$$= 0.5925.$$

In a company electric lamps 5% are defective. If a random sample of 8 lamps are inspected. What is the probability that one or more lamps are defective.

Given that,

$$P = \frac{5}{100} = 1/20$$

$$n = 8$$

$$q = 1 - P = 1 - 1/20 = 19/20$$

probability distribution is $P(x) = {}^nC_r p^r q^{n-r}$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^8C_0 p^0 q^8$$

$$= 1 - (1)\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^8$$

$$= 1 - \frac{19^8}{20^8}$$

$$= 0.3365$$

If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals, more than 2 will get a bad reaction.

Given that,

$$p = 0.001 = 1/1000$$

$$n = 2000$$

$$\text{Mean} = \lambda = np = \frac{2000}{1000} = 2$$

Poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - P(X = 0) - P(X = 1) - P(X = 2)$$

$$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} - \frac{e^{-\lambda}\lambda^1}{1!} - \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= 1 - e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2}\right]$$

$$= 1 - e^{-2}[1 + 2 + 2]$$

$$= 1 - 5e^{-2}$$

$$= 1 - 0.67667$$

$$= 0.32333$$

A hospital switch board recieves an average of 4 emergency calls in a 10 minute interval. What is the probability that (i) there are atmost 2 emergency calls in a 10 minute interval.

(ii) there are exactly 3 emergency calls in a 10 minute interval.

Given that,

$$\text{Mean} = \lambda = 4.$$

i) $P(X \le 2)$ Poisson distribution $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$\text{i)}\ P(x \le 2) = P(x=0)+P(x=1)+P(x=2)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= e^{-4}[1+4+8]$$

$$= 13e^{-4}$$

$$= 0.2381$$

$$\text{ii)}\ P(x=3) = \frac{e^{-\lambda}\lambda^3}{3!}$$

$$= \frac{e^{-4}(4)^3}{6}$$

$$= 0.19536.$$

The average no. of phone calls per minute coming into a switch board between 2 p.m and 4 p.m is 2.5. Determine the probability that during one particular minute there will be

(i) 4 or few ii) More than 6 calls.

Given that,

$$\text{Mean} = \lambda = 2.5.$$

$$\text{i)}\ P(x \le 4) = P(x=0)+P(x=2)+P(x=1)+P(x=3)+P(x=4)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!} + \frac{e^{-\lambda}\lambda^4}{4!}$$

$$= e^{-2.5}\left[1+2.5+\frac{(2.5)^2}{2}+\frac{(2.5)^3}{6}+\frac{(2.5)^4}{24}\right]$$

$= e^{-2.5}(1+2.5+3.125+2.6041+1.6276)$

$= 0.89117$

ii) $P(x>6) = 1-P(x \leq 6)$

$= 1-P(x=0)-P(x=1)-P(x=2)-P(x=3)-P(x=4)$
$-P(x=5)-P(x=6)$

$= 1-\frac{e^{-\lambda}\lambda^0}{0!}-\frac{e^{-\lambda}\lambda^1}{1!}-\frac{e^{-\lambda}\lambda^2}{2!}-\frac{e^{-\lambda}\lambda^3}{3!}-\frac{e^{-\lambda}\lambda^4}{4!}$
$-\frac{e^{-\lambda}\lambda^5}{5!}-\frac{e^{-\lambda}\lambda^6}{6!}$

$= 1-e^{-2.5}(1+2.5+3.125+2.6041+1.6276+0.8138+0.339)$

$= 0.0142$

Suppose 2% of the people on the average are left handed. Find (i) The probability of finding 3 or more left handed. (ii) The probability that none or 1 are left handed.

Given that,

$\text{Mean} = \lambda = \frac{2}{100} = 1/50 = 0.02$

i) $P(x \geq 3) = 1-P(x<3)$

$= 1-P(x=0)-P(x=1)-P(x=2)$

$= 1-\frac{e^{-\lambda}\lambda^0}{0!}-\frac{e^{-\lambda}\lambda^1}{1!}-\frac{e^{-\lambda}\lambda^2}{2!}$

$= 1-e^{-0.02}[1+0.02+0.0002]$

$= 1-0.9999$

$= 0.0001.$

ii) $P(x=0)+P(x=1)$

$= \frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}$

$= e^{-0.02}[1+0.02]$

$= e^{-0.02}(1.02)$

$= 0.9998.$

If 2% of the light bulbs are defective. Find (i) atleast one is defective. (ii) $P(1<X<8)$ in a sample of 100.

Given that,

$$P = \frac{2}{100} = 0.02 \quad , n=100$$

$$\text{Mean} = \lambda = np = \frac{2}{100}(100) = 2. \quad \therefore \boxed{\lambda = 2}$$

Poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

i) $P(X \geq 1) = 1 - P(X<1)$

$$= 1 - P(X=0)$$

$$= 1 - \frac{e^{-\lambda}\lambda^{0}}{0!}$$

$$= 1 - e^{-2}$$

$$= 0.86466.$$

ii) $P(1<X<8) = P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7)$

~~In~~ na,

$$= \frac{e^{-\lambda}\lambda^{2}}{2!} + \frac{e^{-\lambda}\lambda^{3}}{3!} + \frac{e^{-\lambda}\lambda^{4}}{4!} + \frac{e^{-\lambda}\lambda^{5}}{5!} + \frac{e^{-\lambda}\lambda^{6}}{6!} + \frac{e^{-\lambda}\lambda^{7}}{7!}$$

$$= e^{-2}\left[\frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \frac{64}{720} + \frac{128}{5040}\right]$$

$$= e^{-2}[2 + 1.33 + 0.66 + 0.26 + 0.08 + 0.025]$$

$$= e^{-2}(4.355)$$

$$= 0.5893.$$

In a poisson variate, $2P(X=0) = P(X=2)$. Find the probability that (i) $P(2<X\leq 5)$. (ii) $P(X\leq 3)$

Given that,

$$2P(X=0) = P(X=2)$$

Poisson distribution is $P(X) = \frac{e^{-\lambda}\lambda^{X}}{X!}$

$$\frac{2e^{-\lambda}\lambda^{0}}{0!} = \frac{e^{-\lambda}\lambda^{2}}{2!}$$

$$2 = \frac{\lambda^2}{2}$$

$$\lambda^2 = 4$$

$$\boxed{\lambda = 2}$$

i) $P(2<X\le 5) = P(X=3)+P(X=4)+P(X=5)$

$$= \frac{e^{-\lambda}\lambda^3}{3!}+\frac{e^{-\lambda}\lambda^4}{4!}+\frac{e^{-\lambda}\lambda^5}{5!}$$

$$= e^{-2}\left(\frac{8}{6}+\frac{16}{24}+\frac{32}{120}\right)$$

$$= e^{-2}(1.33+0.66+0.26)$$

$$= 0.3045$$

ii) $P(X\le 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$$= \frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}+\frac{e^{-\lambda}\lambda^2}{2!}+\frac{e^{-\lambda}\lambda^3}{3!}$$

$$= e^{-2}(1+2+2+1.33)$$

$$= 0.8566$$

Fit poisson distribution for the following data and calculate expected frequencies.

x : 0 1 2 3 4

$f(x)$: 109 65 22 3 1.

x	f	$f_i x_i$
0	109	0
1	65	65
2	22	44
3	3	9
4	1	4.

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{122}{200} = 0.61.$$

$\boxed{n=4}$ $\boxed{\lambda=0.61}$

$np = 0.61$

$4p = 0.61$

$\boxed{p=0.1525}$

$q = 1-p = 0.8475.$

P(x) = Poisson distribution is $P(X) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$P(0) = \frac{e^{-0.61}(0.61)^0}{0!} = 0.5433.$$

$$P(1) = \frac{e^{-0.61}(0.61)^1}{1!} = 0.3314$$

$$P(2) = \frac{e^{-0.61}(0.61)^2}{2!} = 0.1010$$

$$P(3) = \frac{e^{-0.61}(0.61)^3}{3!} = 0.0205$$

$$P(4) = \frac{e^{-0.61}(0.61)^4}{4!} = 0.0003$$

x	$f(x)$	$P(x)$	$F(x) = N \times P(x)$
0	109	0.5433	$F(x) = 200 \times 0.5433 = 108.66$
1	65	0.3314	$F(x) = 200 \times 0.3314 = 66.28$
2	22	0.1010	$F(x) = 200 \times 0.1010 = 20.2$
3	3	0.0205	$F(x) = 200 \times 0.0205 = 4.1$
4	1	0.0003	$F(x) = 200 \times 0.0003 = 0.06$

Normal distribution:

A random variable x is said to have a normal distribution if its density function or probability distribution is given by

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ is the mean

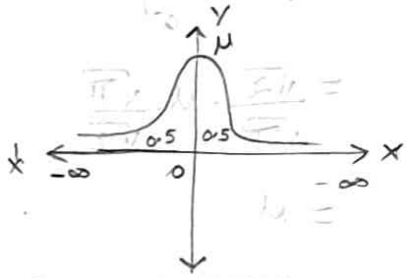
σ is standard deviation of x

This function is called probability density function of the normal distribution, depends on two values μ and σ.

Here μ and σ are parameters of normal distribution.

The curve representing the normal distribution is called the normal curve. And the total area bounded by the curve and x-axis is 1.

i.e, $\int_{-\infty}^{\infty} f(x)\,dx = 1.$



Constants of normal distribution:

1. Mean of the normal distribution:

We know that,

Normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z$

$x = \sigma z + \mu$

$dx = \sigma dz$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{\frac{-z^2}{2}} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{-z^2/2} dz$$

$$= \frac{1}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{\infty} \sigma z e^{-z^2/2} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu e^{-z^2/2} dz$$

$$= 0 + \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} \mu e^{-z^2/2} dz$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \mu \int_{0}^{\infty} e^{-z^2/2} dz$$

Let $\frac{z^2}{2} = t$

$z = \sqrt{2t}$

$dz = \frac{\sqrt{2}}{2\sqrt{t}} dt$

$dz = \frac{1}{\sqrt{2}\sqrt{t}} dt$

$\frac{1}{\sqrt{2}} \int_0^{\infty} t^{-1/2} e^{-t} dt$

$= \frac{1}{\sqrt{2}} \int_0^{\infty} t^{1/2 - 1} e^{-t} dt$

$= \frac{1}{\sqrt{2}} \Gamma(1/2) = \frac{\sqrt{\pi}}{\sqrt{2}}$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \mu \cdot \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$= \mu$$

$\therefore \boxed{\text{Mean} = \mu}$

2. Variance of normal distribution:

We know that,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\text{variance } V(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$= E(x-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z$

$x-\mu = z\sigma$

$dx = \sigma dz$

$$= \int_{-\infty}^{\infty} (z\sigma)^2 \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \cancel{\sigma} dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2} dz$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}} dz$$

Let $\frac{z^2}{2} = t$

$z^2 = 2t \Rightarrow z = \sqrt{2}\cdot\sqrt{t}$

$\cancel{2} z dz = \cancel{2} dt$

$dz = \frac{1}{\sqrt{2}\sqrt{t}} dt$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} 2t e^{-t} \cdot \frac{1}{\sqrt{2}\sqrt{t}} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t^{1/2} e^{-t} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t^{3/2-1} e^{-t} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \Gamma(3/2) \qquad \left(\because \Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx\right).$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2}\Gamma(1/2)$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \cdot \sqrt{\pi}$$

$$= \sigma^2$$

$\therefore \boxed{\text{Variance} = \sigma^2}$

3. Mode of the normal distribution:

Mode is the value of x for which $f(x)$ is maximum.

i.e, Mode is the solution of $f'(x)=0$, $f''(x)<0$.

Proof:

We know that,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \rightarrow (1)$$

Differentiate with respect to x.

$$f'(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot -\frac{1}{2}\cdot 2\left(\frac{x-\mu}{\sigma}\right)\times\frac{1}{\sigma}$$

$$f'(x) = \frac{-1}{\sigma} f(x)\left(\frac{x-\mu}{\sigma}\right) \quad (\because \text{From (1)})$$

$$\boxed{f'(x) = -\frac{f(x)(x-\mu)}{\sigma^2}} \rightarrow (2)$$

Now, $f'(x)=0$

$$-\frac{f(x)(x-\mu)}{\sigma^2} = 0$$

$$\boxed{x=\mu}$$

$$f'(x) = -\frac{f(x)(x-\mu)}{\sigma^2}$$

Differentiate with respect to x.

$$f''(x) = -\frac{f'(x)(x-\mu)}{\sigma^2} - \frac{f(x)}{\sigma^2}$$

$$f''(x) = -\left[\frac{-f(x)(x-\mu)^2}{\sigma^4}\right] - \frac{f(x)}{\sigma^2}$$

$$f''(x) = f(x)\left[\frac{(x-\mu)^2}{\sigma^4} - \frac{1}{\sigma^2}\right]$$

At $x=\mu$,

$$f''(x) = f(x)\left[0 - \frac{1}{\sigma^2}\right]$$

$$= \left[\frac{-f(x)}{\sigma^2}\right]_{\text{at } x=\mu}$$

$$= -\frac{1}{\sigma^2}\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}\right]_{x=\mu}$$

$$= \frac{-1}{\sigma^3\sqrt{2\pi}} e^{-1/2(0)}$$

$$= \frac{-1}{\sigma^3\sqrt{2\pi}} < 0$$

$\therefore f''(x) < 0$ at $x = \mu$.

$\therefore \boxed{\text{Mode} = \mu}$

4. Median of normal distribution:

Median of normal distribution is

$$\int_{-\infty}^{M} f(x)\,dx = 1/2 .$$

$$\int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{M} f(x)\,dx = 1/2$$

$$\underbrace{\int_{-\infty}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx}_{①} + \underbrace{\int_{\mu}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx}_{②} = 1/2 \quad \to \text{I}$$

Consider ①

$$\Rightarrow \int_{-\infty}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z$

$x = \sigma z + \mu$

$dx = \sigma\,dz$.

If $x = -\infty \Rightarrow z = -\infty$

If $x = \mu \Rightarrow z = 0$.

$$= \int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2} \cdot \sigma\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2}\,dz$$

$$= \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad \left(\because \text{By symmetry } \int_{-\infty}^{0} = \int_0^\infty\right).$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \int_0^\infty e^{-z^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$= 1/2 .$$

From (I).

$$\frac{1}{2} + \int_\mu^M \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1/2$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_\mu^M e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx = 0 .$$

$$\int_\mu^M e^{-1/2\left(\frac{x-\mu}{2}\right)^2} = 0$$

$\left[\because \text{since if } \int_a^b f(x) = 0 \text{ then } a = b \text{ where } f(x) > 0.\right]$

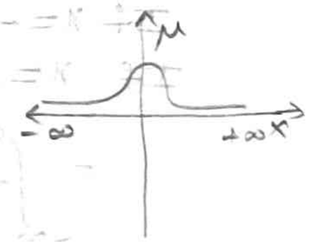
$$\therefore \boxed{\mu = M}$$

$$\therefore \boxed{\text{Median} = \mu}$$

** Show that Mean = median = mode in normal distribution.

Characteristics of normal distribution:

1. The graph of the normal distribution $y = f(x)$ in the xy-plane is known as the normal curve.



2. Area under the normal curve represents the total population.

3. Mean, median and mode of the distribution coincide at $x = \mu$ as the distribution is

symmetrical

4. The probability that the normal variate x with mean μ and standard deviation (σ) lies between x_1 and x_2 is given by

$$P(x_1 \le x \le x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

5. The curve is well shaped and symmetrical with respect to mean. i.e, About the line $x=\mu$ and the two tailes tails on the right and left sides of the mean extends to the infinity. The top of the bell is directly above the mean(μ).

Importance and Applications of the normal distribution:

How to find probability density of normal curve:

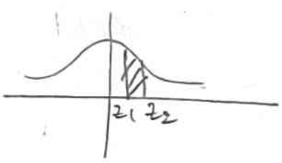
The probability that the normal variate X with mean μ and standard deviation σ lies between two specific values x_1 and x_2 with $x_1 \leq x_2$ can be obtained using the area under the standard normal curve as follows.

1. Perform the change of scale $z = \frac{x-\mu}{\sigma}$ and find z_1 and z_2 corresponding to the values of x_1 and x_2 respectively.

2. a. To find $P(x_1 \leq X \leq x_2) = P[z_1 \leq z \leq z_2]$.

case i: If both z_1 and z_2 are positive or both negative then

$$P(x_1 \leq X \leq x_2) = |A(z_2) - A(z_1)|$$



= (Area under the normal curve from 0 to z_2) - (Area under the normal curve from 0 to z_1)

case ii: If $z_1 < 0$ and $z_2 > 0$ then

$$P(x_1 \leq X \leq x_2) = A(z_2) + A(z_1)$$

b. To find $P(z > z_1)$

case i: If $z_1 > 0$ then $P(z > z_1) = 0.5 - A(z_1)$

case ii: If $z_1 < 0$ then $P(z > z_1) = 0.5 + A(z_1)$

c. To find $P(z < z_1)$

case i: If $z_1 > 0$ then $P(z < z_1) = 0.5 + A(z_1)$

case ii: If $z_1 \leq 0$ then $P(z < z_1) = 0.5 - A(z_1)$.

For a normally distributed variate with mean 1 and standard deviation 3. Find the probabilities that (i) $3.43 \le x \le 6.19$ (ii) $-1.43 \le x \le 6.19$.

Given, $\mu = 1$, $\sigma = 3$

i) $P(3.43 \le x \le 6.19)$

when $x_1 = 3.43$ then

$$z = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{3.43-1}{3} = 0.81$$

when $x_2 = 6.19$ then

$$z = \frac{x-\mu}{\sigma}$$

$$z_2 = \frac{6.19-1}{3} = 1.73.$$

$P(3.43 \le x \le 6.19)$

$= P(0.81 \le z \le 1.73)$

Here, z_1, z_2 are positive.

$\therefore P(0.81 \le z \le 1.73) = |A(1.73) - A(0.81)|$

$= |0.4582 - 0.2910|$ ($\because$ From ND table).

$= 0.1672.$

ii) $P(-1.43 \le x \le 6.19)$

when $x_1 = -1.43$

$$z = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{-1.43-1}{3} = -0.81$$

when $x_2 = 6.19$

$$z = \frac{x-\mu}{\sigma}$$

$$z_2 = \frac{6.19-1}{3} = 1.73$$

$P(-1.43 \le x \le 6.19) = P(-0.81 \le z \le 1.73)$.

Here, $z_1 < 0, z_2 > 0$.

$\therefore P(-0.81 \le z \le 1.73) = A(z_2) + A(z_1)$

$= A(1.73) + A(-0.81)$

$= A(1.73) + A(0.81) \quad (\because A(-z) = A(z))$

$= 0.4582 + 0.2910$

$= 0.7492$

If X is a normal variate with mean 30 and standard deviation 5 find (i) $P(26 \le x \le 40)$

(ii) $P(x \ge 45)$.

Given that,

$\mu = 30, \sigma = 5$

(i) $P(26 \le x \le 40)$

when $x_1 = 26$

$z = \frac{x - \mu}{\sigma}$

$z_1 = \frac{26 - 30}{5} = -0.8$

when $x_2 = 40$

$z = \frac{x - \mu}{\sigma}$

$z_2 = \frac{40 - 30}{5} = 2$.

$P(26 \le x \le 40) = P(-0.8 \le x \le 2)$

Here, $z_1 = -0.8 < 0$, $z_2 = 2 > 0$.

$P(-0.8 \le x \le 2) = A(z_2) + A(z_1)$

$= A(2) + A(-0.8)$

$= A(2) + A(0.8)$

$= 0.4772 + 0.2881$

$= 0.7653$.

ii) $P(X \geq 45)$

when $x = 45$

$$z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = 3$$

$z = 3 > 0$.

$$\therefore P(X \geq 45) = P(z \geq 3)$$
$$= 0.5 - A(3)$$
$$= 0.5 - 0.4987$$
$$= 0.0013$$

$$\therefore \boxed{P(X \geq 45) = 0.0013}$$

The mean and standard deviation of the marks obtained by 1000 students in an examination are 34.5 and 16.5 respectively. Assuming the normality of the distribution, find the approximate no. of students expected to obtain marks between 30 and 60.

Given, $\mu = 34.5, \sigma = 16.5$

$P(30 \leq x \leq 60)$

when $x_1 = 30$

$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{30-34.5}{16.5} = -0.273 = -0.27$$

when $x_2 = 60$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{60-34.5}{16.5} = 1.545 = 1.54$$

$$P(30 \leq x \leq 60) = P(-0.27 \leq x \leq 1.54)$$

Here, $z_1 = -0.27 < 0$, $z_2 = 1.54 > 0$.

$$P(-0.27 \leq x \leq 1.54) = A(z_2) + A(z_1)$$
$$= 0.4382 + A(-0.27)$$
$$= 0.4382 + A(0.27)$$
$$= 0.4382 + 0.1084$$
$$= 0.5466$$

$\therefore$ The No. of students who got marks b/w ~~$P(-0.27 <$~~ 30 and 60 $= 0.5466 \times 1000$

$$= 546.6$$

$= 547.$

Here, 547 students out of 100 got marks between 30 and 60.

Suppose the weights of 800 male students are normally distributed with mean $\mu = 140$ lbs and standard deviation $\sigma = 10$ lbs. Find the no. of students whose weights are (i) between 138 and 148 pounds (ii) More than 152 lbs.

Given that,

Mean $\mu = 140$

standard deviation $\sigma = 10.$

i) $P(138 \leq x \leq 148)$

when $x_1 = 138$

$$Z = \frac{x-\mu}{\sigma}$$

$$Z_1 = \frac{138-140}{10} = -0.2.$$

when $x_2 = 148$

$$Z = \frac{x-\mu}{\sigma}$$

$$Z_2 = \frac{148-140}{10} = 0.8.$$

$P(138 \leq x \leq 148) = P(-0.2 \leq Z \leq 0.8)$

$Z_1 = -0.2 < 0$, $Z_2 = 0.8 > 0$

$$P(-0.2 \leq Z \leq 0.8) = A(-0.2) + A(0.8)$$
$$= A(0.2) + A(0.8)$$
$$= 0.0793 + 0.2881$$
$$= 0.3674.$$

ii) $P(X \geq 152)$

$$Z = \frac{x-\mu}{\sigma}$$

$$Z = \frac{152-140}{10} = 1.2 > 0.$$

$P(X \geq 15) = P(Z \geq 1.2)$
$= 0.5 - A(1.2)$
$= 0.5 - 0.3849$
$= 0.1151$.

Given that the mean height of students in a class is 158 centimeters and standard deviation 20cms. Find howmany students heights lie between 150 cm and 170 cm, If there are 100 students in a class.

Given that, Mean $= \mu = 158$
standard deviation $(\sigma) = 20$.

$P(150 \leq X \leq 170)$
when $X_1 = 150$
$Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{150-158}{20} = -0.4$

when $X_2 = 170$
$Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{170-158}{20} = 0.6$

$P(150 \leq X \leq 170) = P(-0.4 \leq Z \leq 0.6)$
$= A(-0.4) + A(0.6)$
$= A(0.4) + A(0.6) = 0.1554 + 0.2258$
$= 0.3812$.

No. of students $= 0.3812 \times 100 = 38.12 \simeq 38$.

If X is normally distributed with mean 2 and variance 0.1 then find $P(|X-2| \geq 0.01)$!
S.D

Given that,
$\mu = 2$
$\sigma^2 = 0.1$
$\sigma = \sqrt{0.1} = 0.3162$

Now consider, $|X-2| = 0.01$
$X - 2 = \pm 0.01$

case(i): $X - 2 = 0.01$
$X = 2.01$

case(ii): $X - 2 = -0.01$
$X = 1.99$.

$P(1.99 \le x \le 2.01)$

when $x_1 = 1.99$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1.99 - 2}{0.3162} = -0.0316 \simeq 0.03$$

when $x_2 = 2.01$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{2.01 - 2}{0.3162} = 0.0316 = 0.03.$$

$$P(1.99 \le x \le 2.01) = P(-0.03 \le x \le 0.03)$$

$$z_1 = -0.03 < 0, \quad z_2 = 0.03$$

$$= A(0.03) + A(-0.03)$$

$$= 0.120 + A(0.03)$$

$$= 0.12 + 0.12$$

$$= 0.24$$

Now, $P(|x-2| \ge 0.01) = 1 - 0.24$
$= 0.76.$

$$\therefore \boxed{P(|x-2| \ge 0.01) = 0.76}$$

1000 students have written an examination, the mean of test is 35 and standard deviation is 5. Assuming the distribution is normal. Find

(i) How many students marks lie b/w 25 and 40.

(ii) How many students marks lie more than 40.

(iii) below 20

Given that, Mean $= \mu = 35$

Standard deviation$(\sigma) = 5.$

1) $P(25 \le x \le 40)$

$$\boxed{\cancel{s}\, z = \frac{x - \mu}{\sigma}}$$

when $x_1 = 25$,

$$\cancel{s_1} = , \ z_1 = \frac{x_1 - \mu}{\sigma} = \frac{25 - 35}{5} = -2.$$

when $x_2 = 40$,

$$z_2 \cancel{s_2} = \frac{x_2 - \mu}{\sigma} = \frac{40 - 35}{5} = 1$$

$$P(25 \le X \le 40) = P(-2 \le Z \le 1)$$
$$= A(-2) + A(1)$$
$$= A(2) + A(1)$$
$$= 0.4772 + 0.3413$$
$$= 0.8185$$

No. of students marks lie b/w 25 and 40 $= 1000 \times 0.8185$
$= \cancel{81.85}\ 818.5$
$\cong 818$

ii) $P(X \ge 40)$

$\boxed{Z = \dfrac{X-\mu}{\sigma}}$

when $x_1 = 40$ then $z_1 = \dfrac{40-35}{5} = 1 > 0$.

$$P(X \ge 40) = P(Z \ge 1)$$
$$= 0.5 - A(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

No. of students marks lie more than 40 ~~a~~
$= 1000 \times 0.1587$
$= \cancel{15.87}\ 158.7$
$\cong 158$

iii) $P(X \le 20)$

when $x_1 = 20$ then $z_1 = \dfrac{20-35}{5} = -3 < 0$.

$$P(X \le 20) = P(Z \le -3)$$
$$= 0.5 - A(-3)$$
$$= 0.5 - A(3)$$
$$= 0.5 - 0.4987$$
$$= 0.0013$$

No. of students marks below 20 $= 1000 \times 0.0013$
$= 1.3$
$\cong 1$

If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kg, How many students have masses (i) greater than 72 kg (ii) less than or equal to 64 kgs. (iii) Between 65 and 71.

Given,

$\mu = 68$, $\sigma = 3$

i) $P(X > 72)$

we know that $\boxed{z = \frac{X-\mu}{\sigma}}$

$z_1 = \frac{72-68}{3} = \cancel{0.8}\ 1.33$

$\cancel{z_1 > 0.8}$ $z_1 > 0$

$\cancel{P(X>72) = P(z>0.8)}$

$\cancel{= 0.5 - P(0.8)}$

$\cancel{= 0.5 - 0.2881}$

$P(X>72) = P(z > 1.33)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

$\therefore \boxed{P(X>72) = 0.0918}$

ii) $P(\leq 64)$

we know that, $\boxed{z = \frac{X-\mu}{\sigma}}$

$\cancel{z_2}$ $z_1 = \frac{64-68}{3} = -1.33$

$z = -1.33 < 0.$

$P(X \leq 64) = P(z \leq -1.33)$

$= 0.5 - A(-1.33)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

$\therefore \boxed{P(X \leq 64) = 0.0918}$

No. of students

$= 300 \times 0.0918$

$= 27.54$

$= 27$

iii) $P(65 \le x \le 71)$

we know that, $\boxed{z = \frac{x-\mu}{\sigma}}$

when $x_1 = 65$ then $z_1 = \frac{65-68}{3} = -1$

we know that, $x_2 = 71$

$$z_2 = \frac{71-68}{3} = 1$$

$P(65 \le x \le 71) = P(-1 \le z \le 1)$

$z_1 = -1 < 0,\ z_2 = 1 > 0$

$= A(-1) + A(1)$

$= A(1) + A(1)$

$= 0.3413 + 0.3413$

$= 0.6826.$

$\therefore \boxed{P(65 \le x \le 71) = 0.6826}$

No. of students $= 300 \times 0.6826$

$= 204.78$

$\simeq 204.$

Model-2:

In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance and standard Deviation.

Let μ be the mean and σ be the standard deviation of normal curve.

Given, 7% of items are under 35

89% are under 63

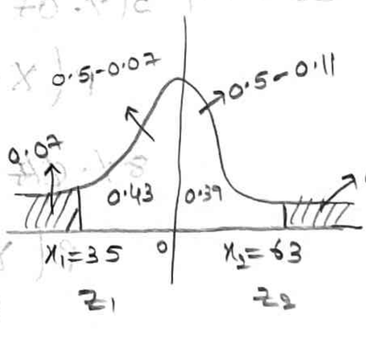
i.e, $P(X < 35) = 7\% = \frac{7}{100} = 0.07$

$P(X < 63) = 89\% = \frac{89}{100} = 0.89$

$P(X > 63) = 1 - P(X \le 63)$

$= 1 - 0.89$

$P(X > 63) = 0.11.$



When $x_1 = 35$

$\Rightarrow z = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = -z_1$ (say)

when $x_2 = 63 \Rightarrow z = \frac{63-\mu}{\sigma} = z_2$ (say).

$P(0<Z<Z_1)=0.43 \Rightarrow Z_1=1.48$ (From table)

$P(0<Z<Z_2)=0.39 \Rightarrow Z_2=1.23$ (From table)

$\frac{35-\mu}{\sigma}=-1.48 \rightarrow ①$. $\frac{63-\mu}{\sigma}=1.23 \rightarrow ②$

$35-\mu=-1.48\sigma$

$63-\mu=1.23\sigma$

$-28=-2.71\sigma$

$\sigma=\frac{28}{2.71}$

$\boxed{\sigma=10.33}$

~~35 - 10.3~~

$35-\mu=-15.2884$

$\mu=35+15.2884$

$\boxed{\mu=50.2884}$

$\text{variance}=\sigma^2=(10.33)^2=106.7089.$

$\therefore \boxed{\mu=50.29, \sigma=10.33, \sigma^2=106.71}$

In normal distribution, 31% of the items are under 45 and 8% are over 64. Find Mean and variance, standard deviation of the distribution.

Let μ be the Mean

σ be the standard deviation.

Given, 31% of items are under 45.

$P(X<45)=31\%=\frac{31}{100}=0.31$

8% ~~of~~ are over 64.

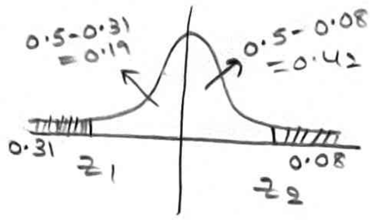
$P(X>64)=8\%=\frac{8}{100}=0.08.$

when $x_1 = 45$ then $z = \frac{45-\mu}{\sigma} = -z_1$ (say)

when $x_1 = 64$ then $z = \frac{64-\mu}{\sigma} = z_2$ (say).

$P(0 < z < z_1) = 0.19$

$\Rightarrow z_1 = 0.5$



$P(0 < z < z_2) = 0.42$

$\Rightarrow z_2 = 1.41$

$\frac{45-\mu}{\sigma} = -0.5 \rightarrow (1)$ $\qquad$ $\frac{64-\mu}{\sigma} = 1.41 \rightarrow (2)$.

$45-\mu = -0.5\sigma$ $\qquad$ $64-\mu = 1.41\sigma$

$$
\begin{array}{rcl}
45-\mu &=& -0.5\sigma \\
64-\mu &=& 1.41\sigma \\
- \quad + & & - \\
\hline
-19 &=& -1.91\sigma
\end{array}
$$

$\sigma = \frac{19}{1.91}$

$\sigma = 9.947$

$\boxed{\sigma = 9.95}$

$45 - \mu = -4.975$

$\boxed{\mu = 49.975}$

$\sigma^2 = (9.95)^2 = 99.0025$

$\therefore \boxed{\mu = 49.975, \ \sigma = 9.95, \ \sigma^2 = 99.0025}$