unit

Solutions of ALGEbraic and transcendental Equation

Equation :-

Any polynomial is equal to Zero is called an equation.

$$x^3 5x+1=0$$

1. Algebraic Equa:-

Any Equa contains only variables that equa is called an algebraic Equation.

$$x^3-5x+1=0$$

2. Transcendental Equa :-

Any Equa contains logarithmic, trigonometric, Exponential etc.. functions that equa is called as transcendental equa.

$$x \log x = 1.2$$

I. Bisection method :-

This method is an simple iteration method to solve an equation, this method is also known as successive bisection method (or) half-interval method.

working rule :-

1. Let, $f(x)=0$ be the given function.
2. find an interval $[a,b]$.
3. the bisection method to use to find $x_0, x_1, \ldots x_n$ like $x_0 = \frac{a+b}{2}$

4: put x_0 in $f(x)$ i.e, $f(x_1) = 0$, Again Calculate x_1 repeating this process we obtain Successive sub intervals which are Smaller.

problems

find a real root of the equa $x^3 5x+1=0$ using the Bisection method in 5 stages.

G. Equation, $f(x) = x^3 5x + 1$

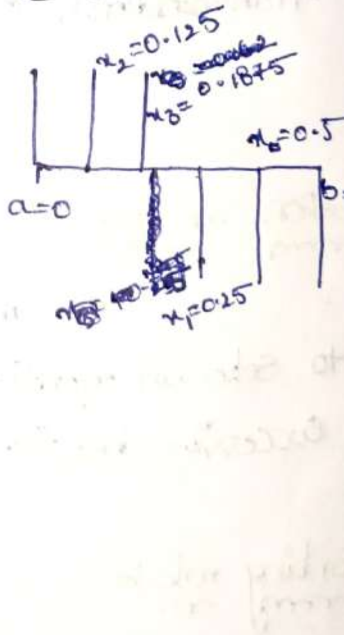
To find an interval

① $x = 0 \Rightarrow f(0) = (0)^3 - 5(0) + 1$
$= 1 \quad (+ve)$

② $x = 1 \Rightarrow f(1) = (1)^3 - 5(1) + 1$
$= 1 - 5 + 1$
$= -3 \quad (-ve)$

$\therefore$ The interval $[a, b] = [0, 1]$

I_1 :- $\boxed{x_0 = \frac{a+b}{2}}$



$= \frac{0+1}{2} = 0.5$

$f(0.5) = (0.5)^3 - 5(0.5) + 1$
$= -1.375 \quad (-ve)$

I_2 :- $x_1 \in [0, 0.5]$
$= \frac{0+0.5}{2} = 0.25$

$f(0.25) = (0.25)^3 - 5(0.25) + 1$
$= -0.23 \quad (-ve)$

I_3 :- $x_2 \in [0, 0.25]$

$$= \frac{0+0.25}{2} = \underline{0.125} \ (+ve)$$

$$f(0.125) = (0.125)^3 - 5(0.125) + 1$$

$$= 0.376 \ (+ve)$$

I_4 :- $x_3 \in [0.125, 0.25]$

$$= \frac{0.25+0.125}{2} = 0.1875 \ (+ve)$$

$$f(0.1875) = (0.1875)^3 - 5(0.1875) + 1$$

$$= 0.071$$

I_5 :- $x_4 \in [0.1875, 0.25]$

$$= \frac{(0.1875+0.25)}{2} = 0.2185$$

$$f(0.2185) = (0.2185)^3 - 5(0.2185) + 1$$

$$= -0.0832$$

3
5)
Find a real root of equa $x^3 - x - 11 = 0$ by bisection method

q. Equa $f(x) = x^3 - x - 11 = 0$

To find an interval,

$x = 0 \Rightarrow f(0) = (0)^3 - (0) - 11$

$= -11 \ (-ve)$

$x = 1 \Rightarrow f(1) = (1)^3 - (1) - 11$

$= 1 - 1 - 11 = -11 \ (-ve)$

$x=2 \Rightarrow f(2) = (2)^3 - 2 - 11$

$= 8 - 2 - 11$

$= -5$ (-ve)

$x=3 \Rightarrow f(3) = (3)^3 - 3 - 11$

$= 27 - 3 - 11$

$= 13$ (+ve)

$\therefore$ Interval $[a,b] = [2,3]$

$\therefore$ B_1 :- $\boxed{x_1 = \frac{a+b}{2}}$

$= \frac{2+3}{2} = 2.5$

Q.B

Determine a real root of the equa $x \sin x - 1 = 0$ using the Bisection method.

G. Equa, $f(x) = x \sin x - 1$

① $x=0 \Rightarrow f(0) = 0 \sin(0) - 1$

$= -1$ (-ve)

②, $x=1 \rightarrow f(1) = 1 \sin(1) - 1$

$= -0.158$ (-ve)

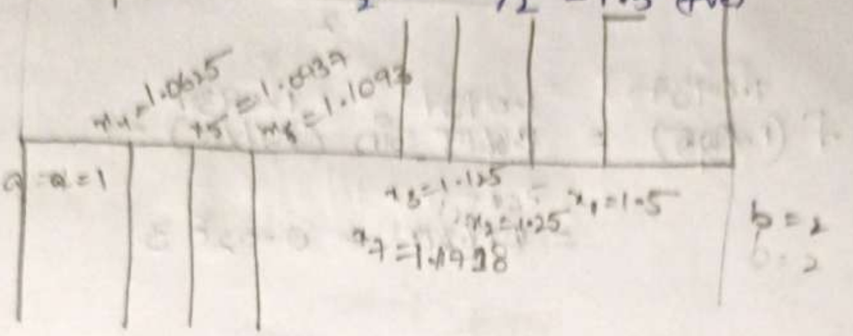
③ $x=2 \rightarrow f(2) = 2 \sin(2) - 1$

$= 0.818$ (+ve)

$\therefore$ Interval $[a,b] = [1,2]$

$\therefore$ I$_1$:- $\boxed{x_1 = \frac{a+b}{2}}$

$= \frac{1+2}{2} = 3/2 = 1.5$ (+ve)



$f(x_1) = 1.5 \sin(1.5) - 1$

$= 0.496$ (+ve)

I$_2$:- $x_2 \in (1, 1.5)$

$= \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$ (+ve)

$f(1.25) = 1.25 \sin(1.25) - 1$

$= 0.186$

I$_3$:- $x_3 \in (1, 1.25)$

$= \frac{1+1.25}{2} = 1.125$ (+ve)

$f(1.125) = 1.125 \sin(1.125) - 1$

$= 0.0150$

I$_4$:- $x_4 \in (1, 1.125)$

$= \frac{1+1.125}{2} = 1.0625$ (-ve)

$f(1.0625) = 1.0625 \sin(1.0625) - 1$

$= \cancel{0.0115} = -0.0718$

I_5 :- $x_5 \in (1.0625, 1.125)$

$$\frac{1.0625 + 1.125}{2} = \overset{1.0937}{\cancel{1.1625}} \; \overset{(-ve)}{\cancel{(+ve)}}$$

$$f(\overset{1.0937}{\cancel{1.0625}}) = \overset{1.0937}{\cancel{1.1625}} \sin(\overset{1.0937}{\cancel{1.1625}}) - 1$$

$$= \cancel{0.1620} - 0.0283$$

I_6 :- $x_6 \in (1.0937, 1.125)$

$$= \frac{1.0937 + 1.125}{2}$$

$$= 1.1093$$

$$f(1.1093) = 1.1093 \sin(1.1093) - 1$$

$$= -.0066$$

I_7 :- $x_7 \in (1.1093, 1.125)$

$$= \frac{(1.1093 + 1.125)}{2} = 1.1171$$

$$f(1.1171) = 1.1171 \sin(1.1171) - 1$$

$$= 0.0042 \; (+ve)$$

I_8 :- $x_8 \in (1.1093, 1.1171)$

$$= \frac{1.1093 + 1.1171}{2} = 1.1132$$

The solution of the equa = 1.1132

find the real root of the equa $x^3 - x - 1 = 0$ by using bisection method Correct to two decimal places.

q. Equa,

$$f(x) = x^3 - x - 1$$

To find interval,

① $x = 0 \Rightarrow f(0) = (0)^3 - 0 - 1$

$= -1$ (-ve)

② $x = 1 \Rightarrow f(1) = (1)^3 - 1 - 1$

$= 1 - 1 - 1$

$= -1$ (-ve)

③ $x = 2 \Rightarrow f(2) = (2)^3 - 2 - 1$

$= 8 - 3$

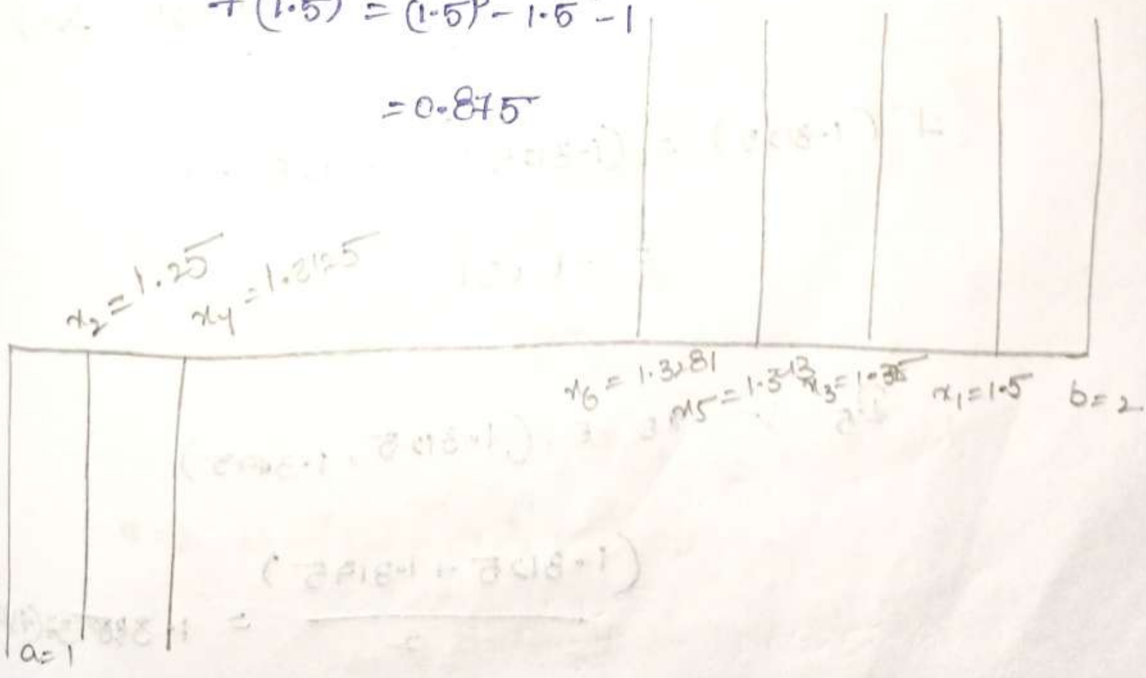
$= 5$ (+ve)

Interval $[a, b] = [1, 2]$

I_1 :- $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 3/2 = 1.5$

$f(1.5) = (1.5)^3 - 1.5 - 1$

$= 0.875$



I_2 :- $x_2 \in (1, 1.5)$

$$\frac{(1+1.5)}{2} = 1.25 \; (-ve)$$

$$f(1.25) = (1.25)^3 - 1.25 - 1$$

$$= -0.296$$

I_3 :- $x_3 \in (1.25, 1.5)$

$$\frac{(1.25+1.5)}{2} = \underline{1.375} \; (+ve)$$

$$f(1.375) = (1.375)^3 - 1.375 - 1$$

$$= 0.2246$$

I_4 :- $x_4 \in (1.25, 1.375)$

$$= \frac{(1.25+1.375)}{2} = \underline{1.3125} \; (-ve)$$

$$f(1.3125) = (1.3125)^3 - 1.3125 - 1$$

$$= -0.051$$

I_5 :- $x_5 \in (1.3125, 1.3175)$

$$\frac{(1.3125+1.3175)}{2} = 1.3437 \; (+ve)$$

$$f(1.343) = (1.343)^3 - 1.343 - 1$$

0.003

I_6 :- $x_6 \in (1.3125, 1.3437)$

$$= \frac{(1.3125 + 1.3437)}{2} = 1.32815 \text{ (+ve)}$$

$$f(1.32815) = (1.32815)^3 - 1.32815 - 1$$

$$= 0.01457$$

I_7 => $x_7 \in (1.3125 + 1.3281)$

$$\frac{(1.3125 + 1.3281)}{2} = 1.3203125$$

∴ The solution of the Equa

$$= 1.32031$$

find the real root of the equa $x^3 - 6x - 4 = 0$ by using bisection method.

G. Equa, $x^3 - 6x - 4 = 0$

To find Interval,

$$x = 0 \Rightarrow f(0) = (0)^3 - 6(0) - 4$$

$$= -4 \text{ (-ve)}$$

$$x = 1 \Rightarrow f(1) = (1)^3 - 6(1) - 4$$

$$= 1 - 6 - 4$$

$$= -9 \text{ (-ve)}$$

$$x = 2 \Rightarrow f(2) = (2)^3 - 6(2) - 4$$

$$= 8 - 12 - 4$$

$$= -8 \text{ (-ve)}$$

$x_3 \Rightarrow f(3) = (3)^3 - 6(3) - 4$

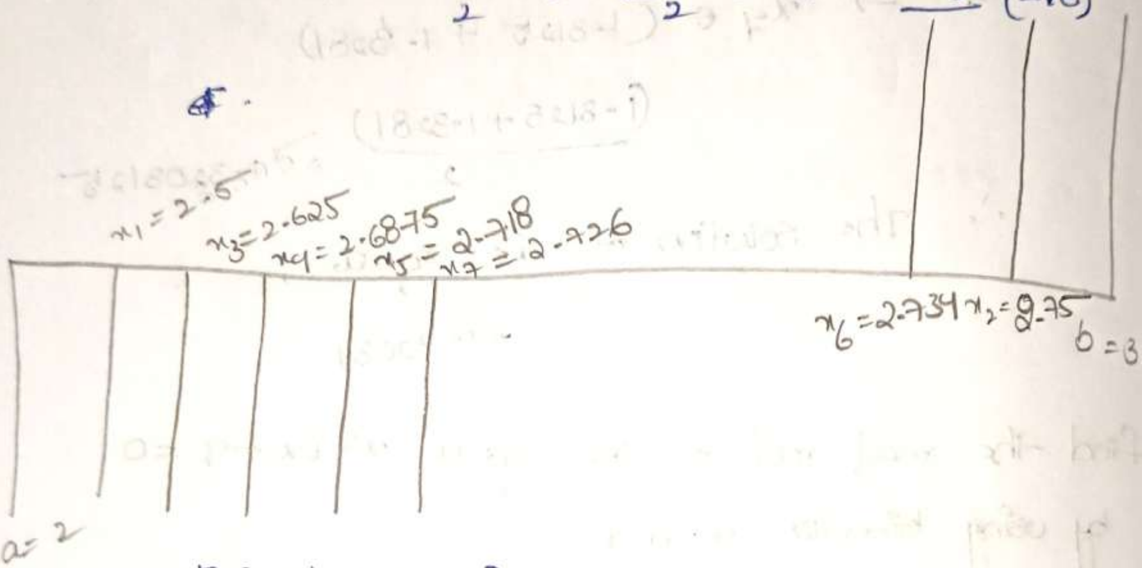
$= 27 - 18 - 4$

$= 27 - 22$

$= 5 \; (+ve)$

$\therefore$ Interval $[a, b] = [2, 3]$

I_1 :- $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 5/2 = \underline{2.5} \; (-ve)$



$f(2.5) = (2.5)^3 - 6(2.5) - 4$

$= -3.375$

$I_2 \Rightarrow x_2 = (2.5, 3)$

$= \frac{(2.5 + 3)}{2} = 2.75 \; (+ve)$

$f(2.75) = (2.75)^3 - 6(2.75) - 4$

$= 0.296$

$I_3 \Rightarrow x_3 = 2.5, 2.75$

$$= \frac{2.5 + 2.75}{2}$$

$$= 2.625 \ (-ve)$$

$$f(2.625) = (2.625)^3 - 6(2.625) - 4$$

$$= -1.662$$

$I_4 \Rightarrow x_4 = (2.625; 2.75)$

$$= \frac{2.625 + 2.75}{2}$$

$$= 2.6875 \ (-ve)$$

$$f(2.6875) = (2.6875)^3 - 6(2.6875) - 4$$

$$= +0.714$$

$I_5 \Rightarrow x_5 = (2.6875, 2.75)$

$$= \frac{(2.6875 + 2.75)}{2}$$

$$= 2.718 \ (-ve)$$

$$f(2.718) = (2.718)^3 - 6(2.718) - 4$$

$$= -0.212$$

$I_6 \Rightarrow x_6 = (2.718, 2.75)$

$$= \frac{(2.718 + 2.75)}{2}$$

$$= 2.734 \ (+ve)$$

$f(2.734) = (2.734)^3 - 6(2.734) - 4$

$= 0.031$

$I_7 \Rightarrow x_7 \in [2.718, 2.734]$

$= \left(\frac{2.718 + 2.734}{2}\right)$

$= 2.726$ (-ve)

$f(2.726) = (2.726)^3 - 6(2.726) - 4$

$= -0.098$

$I_8 \Rightarrow x_8 \in [2.726, 2.734]$

$= \frac{2.726 + 2.734}{2} = 2.73$

The solution of the equation $= 2.73$

23/03,

Find the real root of the equa $3x = \cos x + 1$ by using bisection method.

G. that, $f(x) = 3x - \cos x - 1$

To find Interval,

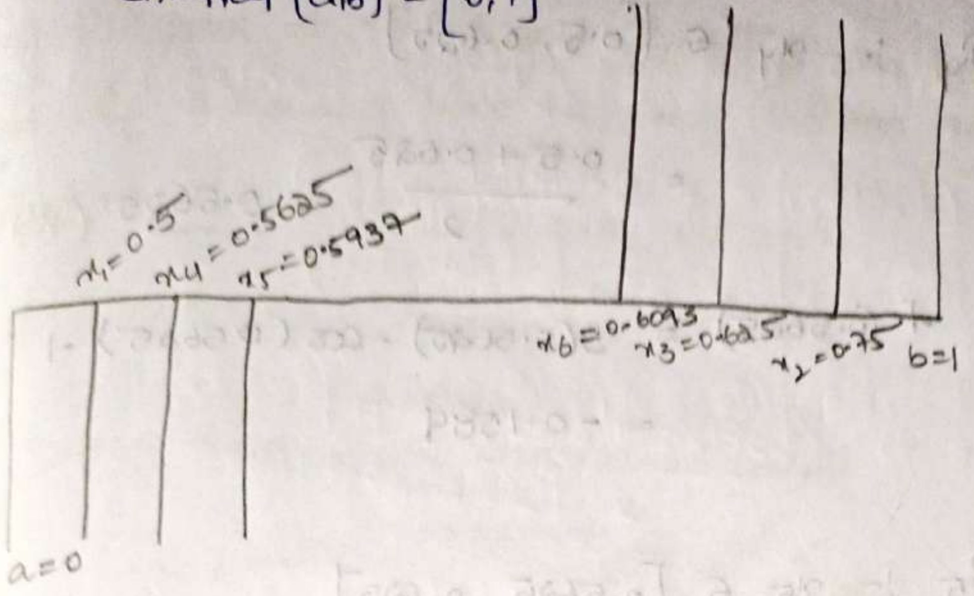
$x = 0 \Rightarrow f(0) = 3(0) - \cos 0 - 1$

$= -2$

$x = 1 \Rightarrow f(1) = 3(1) - \cos(1) - 1$

$= 1.459$

Interval $[a,b] = [0,1]$



I_1 :- $x_1 \in [0,1]$

$$= \frac{(0+1)}{2} = \underline{0.5} \ (-ve)$$

$$f(0.5) = 3(0.5) - \cos(0.5) - 1$$

$$= -0.3775$$

I_2 :- $x_2 \in [0.5, 1]$

$$= \frac{0.5+1}{2} = \underline{0.75} \ (+ve)$$

$$f(0.75) = 3(0.75) - \cos(0.75) - 1$$

$$= 0.5183$$

I_3 :- $x_3 \in [0.5, 0.75]$

$$= \frac{0.5+0.75}{2} = \underline{0.625} \ (+ve)$$

$$f(0.625) = 3(0.625) - \cos(0.625) - 1$$

$$= \cancel{0.19444} \ 0.0640$$

I_4 :- $x_4 \in [0.5, 0.625]$

$$= \frac{0.5 + 0.625}{2} = \underline{0.5625} \ (+ve)$$

$$f(0.5625) = 3(0.5625) - \cos(0.5625) - 1$$

$$= -0.1584$$

I_5 :- $x_5 \in [0.5625, 0.625]$

$$= \frac{0.5625 + 0.625}{2} = \underline{0.5937} \ (-ve)$$

$$f(0.5937) = 3(0.5937) - \cos(0.5937) - 1$$

$$= -0.0477$$

I_6 :- $x_6 \in [0.5937, 0.625]$

$$= \frac{0.5937 + 0.625}{2}$$

$$= \underline{0.6093} \ +ve$$

$$f(0.6093) = 3(0.6093) - \cos(0.6093) - 1$$

$$= 0.0078$$

I_7 :- $x_7 \in [0.5937, 0.6093]$

$$= \frac{0.5937 + 0.6093}{2} = 0.6015$$

The solution of equa $= 0.6015$.

Finding the root of the equa $f(x) = e^{-x}[3.2\sin x - 0.5\cos x]$ that lies $x=3$ and $x=4$ by using bisection method.

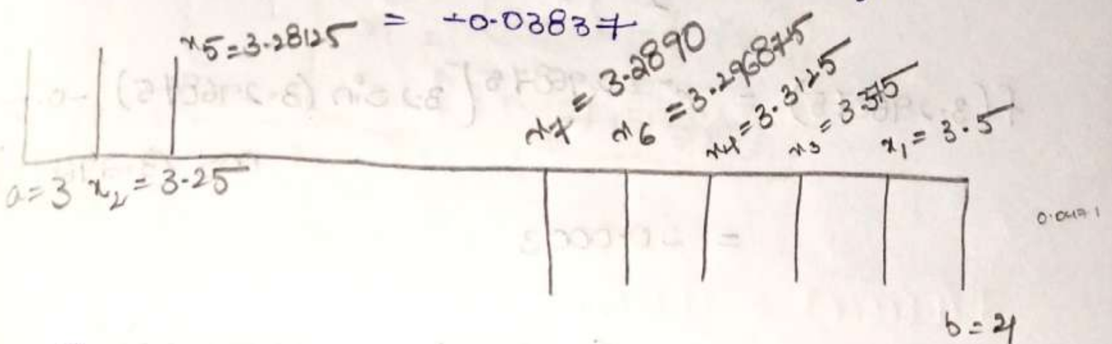
G. that, $f(x) = e^{-x}[3.2\sin x - 0.5\cos x]$ [3-4]

$x = 3 \Rightarrow f(3) = e^{-3}[3.2\sin(3) - 0.5\cos(3)]$

$= 0.04712$

$x = 4 \Rightarrow f(4) = e^{-4}[3.2\sin(4) - 0.5\cos(4)]$

$= -0.03837$



$I_1 \Rightarrow x_1 = \frac{a+b}{2} = \frac{3+4}{2} = 3.5$ (-ve)

$f(3.5) = e^{3.5}[3.2\sin(3.5) - 0.5\cos(3.5)]$

$= -0.019$

$I_2 \Rightarrow x_2 = \frac{(3+3.5)}{2} = 3.25$ (+ve)

$f(3.25) = e^{-3.25}[3.2\sin(3.25) - 0.5\cos(3.25)]$

$= 0.0058$

$I_3 \Rightarrow x_3 \Rightarrow \frac{(3.25+3.5)}{2} = 3.375$ (-ve)

$f(3.375) = e^{-3.375}[3.2\sin(3.375) - 0.5\cos(3.375)]$

$= -0.0086$

$I_4 \Rightarrow x_4 \Rightarrow \frac{(3.25+3.375)}{2} = 3.3125$ (-ve)

$f(3.3125) = e^{-3.3125}[3.2\sin(3.3125) - 0.5\cos(3.3125)]$

$= -0.0087$

$I_5 \Rightarrow x_5 = \dfrac{(3.25 + 3.3125)}{2} = \underline{3.28125}$ (+ve)

$f(3.28125) = e^{-3.28125}[3.2 \sin(3.28125) - 0.5 \cos(3.28125)]$

$= 0.0018$

$I_6 :- x_6 \rightarrow \dfrac{(3.28125 + 3.3125)}{2} = 3.296875$ (−ve)

$f(3.296875) = e^{-3.296875}[3.2 \sin(3.296875) - 0.5 \cos(3.296875)]$

$= -0.0003$

$I_7 :- x_7 \Rightarrow \dfrac{(3.28125 + 3.296875)}{2}$

$= 3.2890$ (−ve)

$f(3.2890) = e^{-3.2890}[3.2 \sin(3.2890) - 0.5 \cos(3.2890)]$

$= -0.0009$

$I_8 :- x_8 \Rightarrow \dfrac{(3.28125 + 3.2890)}{2}$

$= 3.285$

The root of the equa $x_4 = 3.28$

Regula-Falsi method or False position method

1. Let, $f(x)=0$ be the given equation.
2. find an initial interval $[a,b]$
3. find the root of the equa,

$$\boxed{x_1 = \frac{[a\,f(b) - b\,f(a)]}{[f(b) - f(a)]}}$$

problems

~~find~~ find the root of the equa x^3-x-4 by using Regula-falsi method. upto 3 stages.

G. that, $f(x) = x^3-x-4$

To find an Interval

$x=0 \Rightarrow f(0) = (0)^3-0-4$

$= -4$

$x=1 \Rightarrow f(1) = (1)^3-1-4$

$= -4$

$x=2 \Rightarrow f(2) = (2)^3-2-4$

$= 8-6$

$= 2$

Interval $[a,b] = [1,2]$

$x_1 = 1.66666$

$x_2 = 1.78048$

$b=2$

I_1 :- $x_1 \in [1, 2]$

$$= \frac{[a \cdot f(b) - b \cdot f(a)]}{[f(b) - f(a)]}$$

$$= \frac{[1(2) - 2(-4)]}{[2 - (-4)]}$$

$$= \frac{[2 + 8]}{6} = \frac{10}{6} = 1.6666 \ (-ve)$$

$$f(1.66666) = (1.66666)^3 - 1.66666 - 4$$

$$= -1.0370$$

I_2 :- $x_2 \in [1.66666, 2]$

$$= \frac{[1.66666(2) - 2(-1.0370)]}{[2 + 1.0370]}$$

$$= \frac{[3.33332 + 2.074]}{3.037}$$

$$= \frac{5.40732}{3.0370} = 1.78048 \ (-ve)$$

$$f(1.78048) = (1.78048)^3 - 1.78048 - 4$$

$$= -0.13616$$

I_3 :- $x_3 \in [1.78048, 2]$

$$= \frac{1.78048(2) - 2f(1.78048)}{2 - f(1.78048)}$$

$$= \frac{[3.56096 - 2(-0.13616)]}{2 + 0.13616}$$

$$= \frac{[3.56096 + 0.13616]}{2.13616}$$

$$= 1.79447.$$

1. find the root of the equa $x\log_{10} x = 1.2$ by regula-falsi method.

G.that, $f(x) = x\log_{10} x - 1.2$

To find An Interval

$x = 1 \Rightarrow f(1) = 1\log_{10} 1 - 1.2$ (-ve)

$= -1.2$

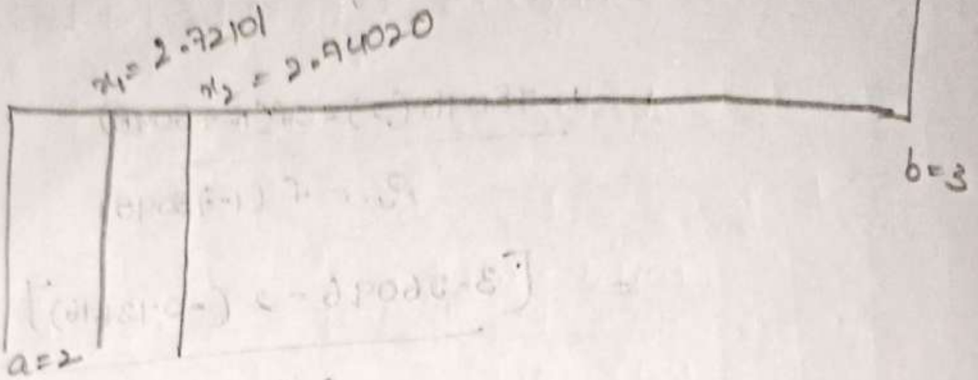
$x = 2 \Rightarrow f(2) = 2\log_{10} 2 - 1.2$

$= -0.59794$ (-ve)

$x = 3 \Rightarrow f(3) = 3\log_{10} 3 - 1.2$

$= 0.23136$

Interval $[a, b] = [2, 3]$



I_1 ie :- $\quad x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$

$$= \frac{[2 f(3) - 3 f(2)]}{[f(3) - f(2)]}$$

$$= \frac{[2 \times 0.23136 - 3(-0.59794)]}{[0.23136 + 0.59794]}$$

$$= 2.72101$$

$$f(2.72101) = 2.72101 \log_{10} 2.72101 - 1.2$$

$$= -0.01709$$

$I_2 \Rightarrow \quad x_2 \in [2.72101, 3]$

$$= \frac{2.72101 f(3) - 3(2.72101)}{f(3) - f(2.72101)}$$

$$\frac{[2.72101\,(0.2336) - 3(2.72 - 0.01709)]}{[0.23136 + 0.01709]}$$

$= 2.74020$ (w.c)

$f(2.74020) = 2.74020 \log_{10} 2.74020 - 1.2$

$= -0.00038$

I_3 :- $x_3 \in [2.74020,\ 3]$

$$= \frac{[2.74020\, f(3) - 3 f(2.74020)]}{[f(3) - f(2.74020)]}$$

$$= \frac{[2.74020(0.23136) - 3(-0.00038)]}{[0.23136 + 0.00038]}$$

$= 2.74062$

The root of equa $(x) = 2.74062$.

find the root of the equa $2x - \log_{10} x = 7$ by regula-falsi method in $[3.5, 4]$.

G. that, $f(x) = 2x - \log_{10} x - 7$

Interval $[a, b] = [3.5, 4]$

$I_1 \Rightarrow x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)} = \cancel{3.56}$

$f(a) = f(3.5) = -0.54406$

$f(b) = f(4) = 0.39794$

$S_1 \Rightarrow x = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$

$= \dfrac{[3.5(0.39794) - 4(-0.54406)]}{[0.39794 + 0.54406]}$

$$\boxed{x_1 = 3.78889}$$

Find the real root of equa $xe^x = 2$ by using regula-falsi method.

G. that, $f(x) = xe^x - 2$

To find An Interval

$x = 0 \Rightarrow f(0) = 0 \cdot e^0 - 2$

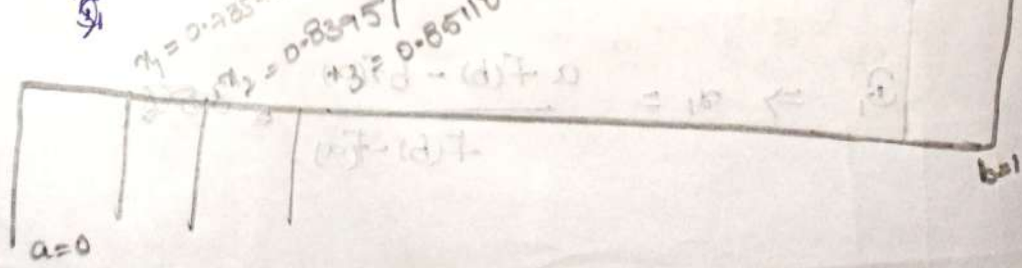
$= -2$

$x = 1 \Rightarrow f(1) = 1 \cdot e^1 - 2$

$= 0.71828$

To find an Interval $[a,b] = [0,1]$

S_1



$I_1 \Rightarrow x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$

$= \dfrac{[0[0.71828] - 1[-2]]}{(0.71828 + 2)}$

$= 0.73575$

$f(0.73575) = 0.73575\, e^{0.73575} - 2$

$= -0.46445$

$I_2 \Rightarrow x_2 \in [0.73575, 1]$

$= \dfrac{0.73575\, f(1) - 1\, f(0.73575)}{f(1) - f(0.73575)}$

$= \dfrac{[0.73575(0.71828) - 1(-0.46445)]}{0.71828 + 0.46445}$

$= \cancel{0.84616} \; 0.83951 \; (-ve)$

$f(0.83951) = 0.83951\, e^{0.83951} - 2$

$= -0.05633$

$I_3 \Rightarrow x = [0.83951, 1]$

$= \dfrac{0.83951\, f(1) - 1 f(0.83951)}{0.71828 - f(0.83951)}$

$= \dfrac{0.83951(0.71828) - 1(-0.05633)}{0.71828 + 0.05633}$

$= 0.85118$ (-ve)

$$f(0.85118) = 0.85118\, e^{0.85118} - 2$$

$$= -0.00618$$

It $4 \Rightarrow x_4 = [0.85118, 1]$

$$= \frac{0.85118\, f(1) - 1\, f(0.85118)}{f(1) - f(0.85118)}$$

$$= \frac{[0.85118\,(0.71828) - 1\,(-0.00618)]}{[0.71828 + 0.00618]}$$

$$= 0.85244$$

$$f(0.85244) = 0.85244\, e^{0.852644} - 2 = -0.00071$$

It $5 \Rightarrow x_5 = [0.85244, 1]$

$$= \frac{[0.85244\, f(1) - 1\, f(0.85244)]}{f(1) - f(0.85244)}$$

$$= \frac{0.85244\,(0.71828) - 1\,(-0.00071)}{0.71828 + 0.00071}$$

$$= 0.85258$$

The root of equa $x_1 = 0.85258$

find the real root of equa $\log x = \cos x$ by using Regula-falsi method in 3 stages.

$$\boxed{\begin{array}{l} \text{Log} = \text{Ln} \\ \text{Log}_{10} = \log \end{array}}$$

G. $f(x) = \text{Log}\, x - \text{Cos}\, x$

$x = 1 \Rightarrow f(1) = \text{Log}\, 1 - \text{Cos}\, 1$

$= -0.54030$

$x = 2 \Rightarrow f(2) = \text{Log}\, 2 - \text{Cos}\, 2$

$= 1.10929$

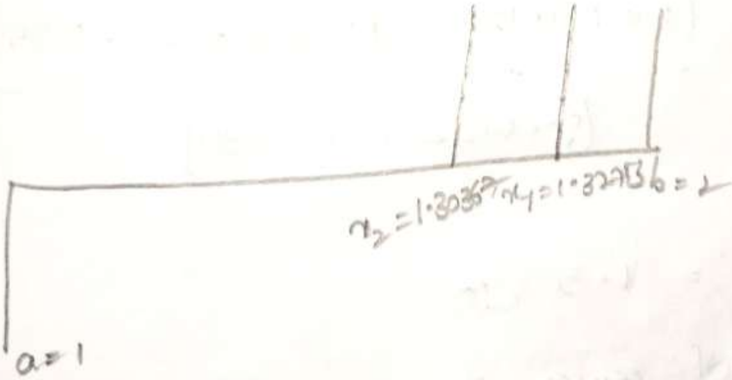
Interval $[a, b] = [1, 2]$

B_1 :- $x_1 \in [1, 2]$

$$x_1 = \frac{a\, f(b) - b\, f(a)}{f(b) - f(a)}$$

$$= \frac{1\, f(2) - 2\, f(1)}{f(2) - f(1)}$$

$$= \frac{[1 \times 1.10929 - 2 \times (-0.54030)]}{1.10929 + 0.54030} = \underline{1.32753} \text{ (+ve)}$$



$F(1.32753) = \text{Log}(1.32753) - \text{Cos}(1.32753)$

$= 0.04244$

B_2 ∈) $x_2 \in [1, 1.32753]$

$$= \frac{[1 \times f(1.32753) - 1.32753\ f(1)]}{[f(1.32753) - f(1)]}$$

$$= \frac{[1 \times 0.042444 - 1.32753(-0.54030)]}{[0.042444 + 0.54030]}$$

$$= \underline{1.30367}\ (+ve)$$

$$f(1.30367) = \log(1.30367) - \cos(1.30367)$$

$$= 0.00122$$

$B_3 \Rightarrow x_3 \in [1, 1.30367]$

$$= \frac{[1 \times f(1.30367) - 1.30367\ f(1)]}{[f(1.30367) - f(1)]}$$

$$= \frac{[1 \times 0.00122 - 1.30367 \times (-0.54030)]}{[0.00122 + 0.54030]}$$

$$= 1.30298$$

The root of equa $x_1 = 1.30298$

find the real root of the equa $e^x \sin x = 1$ upto 4 stages by using regula-falsi method.

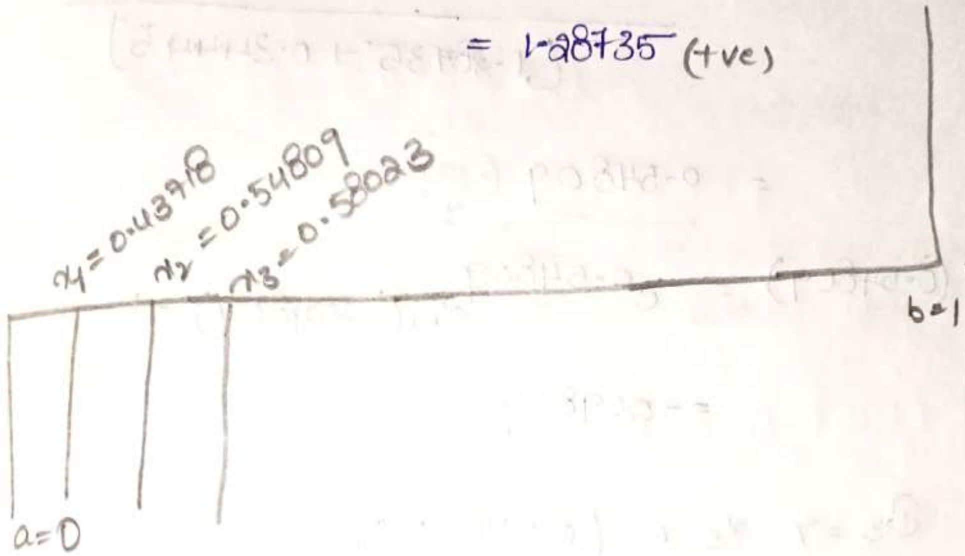
G. $f(x) = e^x \sin x - 1$

$x = 0 \Rightarrow f(0) = e^0 \sin(0) - 1$

$= -1$ (-ve)

$x = 1 \Rightarrow f(1) = e^1 \sin(1) - 1$

$= 1.28735$ (+ve)



I) $\Rightarrow x_1 \in [0, 1]$

$$= \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{[0 \times f(1) - 1 f(0)]}{[f(1) - f(0)]}$$

$$= \frac{0 \times 1.28735 - 1 \times (-1)}{1.28735 + 1} = 0.43718 \text{ (-ve)}$$

$f(0.43718) = e^{0.43718} \sin(0.43718) - 1$

$= -0.34445$

$I_2 \Rightarrow x_2 \in [0.42718, 1]$

$$= \frac{[0.42718\, f(1) - 1 \times f(0.42718)]}{[f(1) - f(0.42718)]}$$

$$= \frac{[0.42718 \times 1.28735 - 1(-0.34445)]}{[1.28735 + 0.34445]}$$

$= \underline{0.54809}$ (-ve)

$f(0.54809) = e^{0.54809} \sin(0.54809) - 1$

$= -0.09859$

$I_3 \Rightarrow x_3 \in [0.54809, 1]$

$$= \frac{[0.54809 \times f(1) - 1 \times f(0.54809)]}{[f(1) - f(0.54809)]}$$

$$= \frac{[0.54809 \times 1.28735 - 1 \times (-0.09859)]}{[1.28735 + 0.09859]}$$

$= \underline{0.58023}$ (-ve)

$f(0.58023) = e^{0.58023} \sin(0.58023) - 1$

$= -0.02063$

$B_4 \Rightarrow x_4 \in [0.58023, 1]$

$$= \frac{[0.58023\, f(1) - 0.58023\, 1 \times f(0.58023)]}{[f(1) - f(0.58023)]}$$

$$= \frac{[0.58023 \times 1.28735 - 1 \times (-0.02063)]}{[1.28735 + 0.02063]}$$

$$= 0.58685$$

The root of equation $x_4 = 0.58685$

find a root of the equa. $f(x) = x^2 + e^x - 5 = 0$ using the regula falsi method correct upto 3 decimal places.

G. that, $f(x) = x^2 + e^x - 5$

$x = 0 \Rightarrow f(0) = (0)^2 + e^0 - 5$

$= -4$

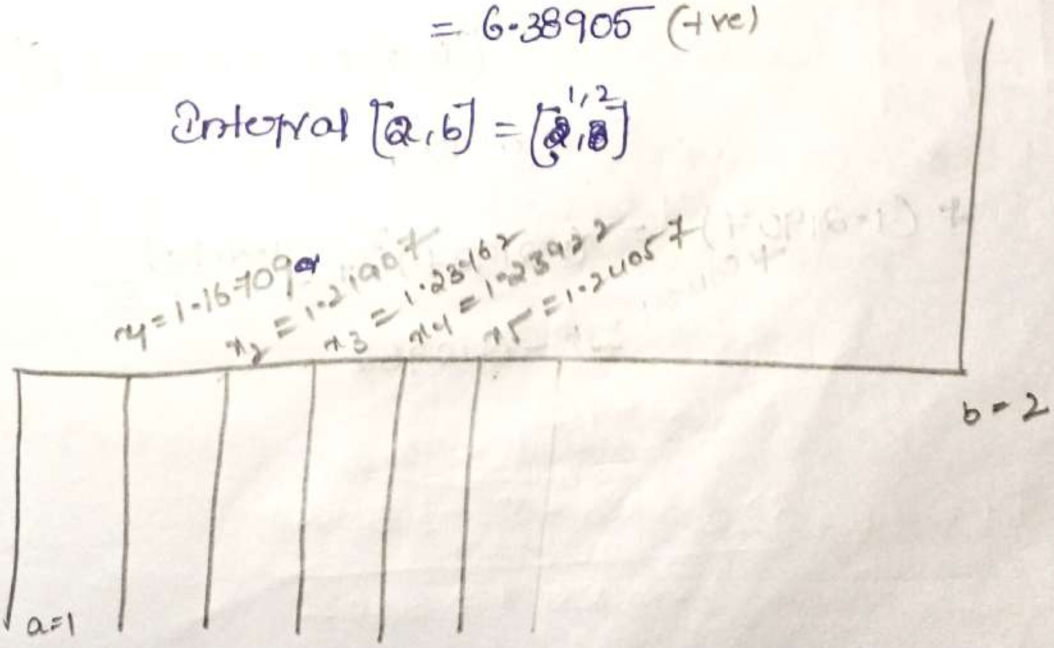
$x = 1 \Rightarrow f(1) = (1)^2 + e^1 - 5$

$= -1.28171$ (-ve)

$x = 2 \Rightarrow f(2) = (2)^2 + e^2 - 5$

$= 6.38905$ (+ve)

Interval $[a, b] = [1, 2]$



$I_1 \Rightarrow x_1 \in [1,2]$

$$= \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{1 \times f(2) - 2 \times f(1)}{f(2) - f(1)}$$

$$= \frac{[1 \times 6.38905 - 2 \times -1.28171]}{[6.38905 + 1.28171]}$$

$$= \underline{1.16709} \ (-ve)$$

$$f(1.16709) = (1.16709)^2 + e^{1.16709} - 5$$

$$= -0.42527$$

$I_2 \Rightarrow x_2 \in [1.16709, 2]$

$$= \frac{1.16709 \times f(2) - 2\,f(1.16709)}{f(2) - f(1.16709)}$$

$$= \frac{[1.16709 \times 6.38905 - 2 \times (-0.42527)]}{[6.38905 + 0.42527]}$$

$$= \underline{1.21907} \ (-ve)$$

$$f(1.21907) = (1.21907)^2 + e^{1.21907} - 5$$

$$= -0.12982$$

$B_3 \Rightarrow x_3 \in (1.21907, 2)$

$$= \frac{1.21907\, f(2) - 2 f(1.21907)}{f(2) - f(1.21907)}$$

$$= \frac{1.21907 \times 6.38905 - 2(-0.12982)}{6.38905 + 0.12982}$$

$= \underline{1.23462}$ (-ve)

$f(1.23462) = (1.23462)^2 + e^{1.23462} - 5 = -0.03864$

$B_4 \Rightarrow x_4 \in (1.23462, 2)$

$$= \frac{1.23462\, f(2) - 2 f(1.23462)}{f(2) - f(1.23462)}$$

$$= \frac{1.23462 \times 6.38905 - 2(-0.03864)}{6.38905 + 0.03864}$$

$= 1.23922$ (-ve)

$f(1.23922) = (1.23922)^2 + e^{1.23922} - 5 = -0.01141$

$B_5 \Rightarrow x_5 \in (1.23922, 2)$

$$\frac{1.23922\, f(2) - 2 f(1.23922)}{f(2) + f(1.23922)}$$

$$= \frac{1.23922 \times 6.38905 - 2(-0.01141)}{6.38905 + 0.01141}$$

$= \underline{1.24057}$ (-ve)

$f(1.24057) = (1.24057)^2 + e^{1.24057} - 5$

$= -0.00340$

$I_6 \Rightarrow x_6 \in (1.24057, 2)$

$$\frac{1.24057\, f(2) - 2\, f(1.24057)}{f(2) - f(1.24057)}$$

$$= \frac{1.24057 \times 6.38905 - 2.(-0.00340)}{6.38905 + (0.00340)}$$

$$= 1.24097$$

∴ The root of the equa is 1.24097

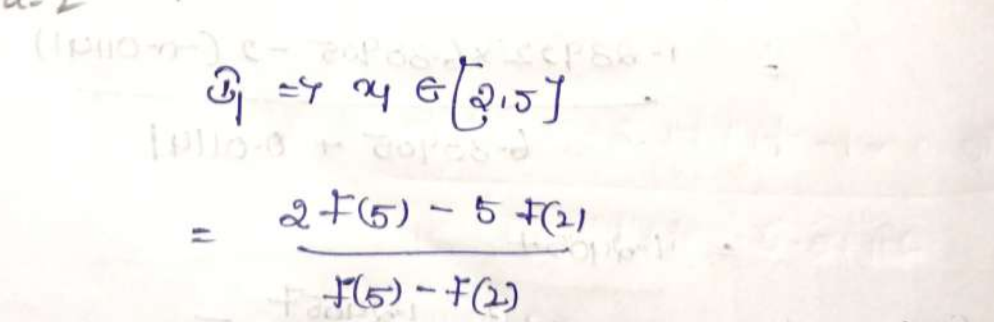
Use regula - falsi method to find a real root of the equation $2x - \log_{10} - 7 = 0$ in $(2,5)$

$f(2) = 2 \times 2 - \log_{10}(2) - 7$

$= -3.30102$

$f(5) = 2 \times 5 - \log_{10}(5) - 7$

$= 2.30102$



$I_1 \Rightarrow x_1 \in [2,5]$

$$= \frac{2 f(5) - 5 f(2)}{f(5) - f(2)}$$

$$= \frac{2 \times 2.30102 - 5(-3.30102)}{2.30102 + 3.30102}$$

$= 3.76775$

$f(3.76775) = 2(3.76775) - \log_{10}(3.76775) - 7$

$= -0.04058$

$I_2 \Rightarrow x_2 \in [3.76775, 5]$

$$= \frac{3.76775 f(5) - 5 f(3.76775)}{f(5) - f(3.76775)}$$

$$= \frac{3.76775 \times 2.30102 - 5 \times (-0.04058)}{2.30102 + 0.04058}$$

$= 3.78910$

$f(3.78910) = -0.00033$

$I_3 \Rightarrow x_3 \in (3.78910, 5)$

$$= \frac{3.78910\, f(5) - 5 \times f(3.78910)}{f(5) - f(3.78910)}$$

$$= \frac{3.78910 \times 2.30102 - 5(-0.00033)}{2.30102 + (0.00033)}$$

$= 3.78932$

The root of the equation is 3.78932

Determine the root of the equa $\cos x - xe^x = 0$ by the method of false position.

G. that, $f(x) = \cos x - xe^x$

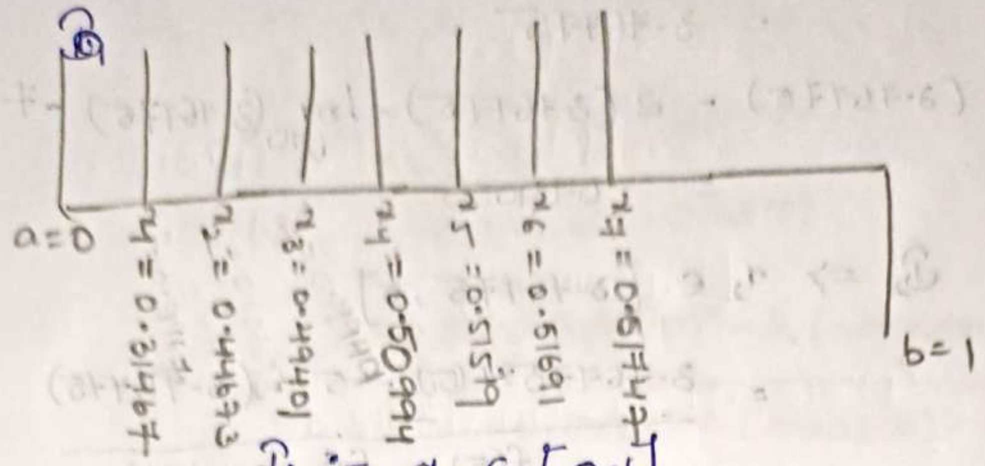
To find an interval

$x = 0 \Rightarrow f(0) = \cos(0) - 0 \cdot e^0$

$= 1$

$x = 1 \Rightarrow f(1) = \cos(1) - 1 \cdot e^1$

$= -2.17797$



$I_1 :- x_1 \in [0,1]$

$$= \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{0 \cdot f(1) - 1 \times f(0)}{f(1) - f(0)}$$

$$= \frac{-1 \times 1}{-2.17797 - 1} = \underline{0.31467} \text{ (+ve)}$$

$f(0.31467) = (0.51986)$

$I_2 :- x_2 \in [0.31467, 1]$

$$= \frac{0.31467 \times f(1) - 1 \times f(0.31467)}{f(1) - f(0.31467)}$$

$$= \frac{0.31467 \times -2.17797 - 1 \times 0.51986}{-2.17797 - 0.51986}$$

$= \underline{0.44673}$ (+ve)

$f(0.44673) = \cos(0.44673) - (0.44673) \times e^{0.44673}$

$= 0.20353$

B_3 :- $x_3 \in [0.44673, 1]$

$$= \frac{0.44673 \times f(1) - 1 \times f(0.44673)}{f(1) - f(0.44673)}$$

$$= \frac{0.44673 \times -2.17797 - 1 \times 0.20353}{-2.17797 - 0.20353}$$

$= 0.49401$ (+ve)

$f(0.49401) = \cos(0.49401) - 0.49401 \times e^{0.49401}$

$= 0.07081$

B_4 :- $x_4 \in [0.49401, 1]$

$$= \frac{0.49401 \times f(1) - 1 \times f(0.49401)}{f(1) - f(0.4901)}$$

$$= \frac{0.49401 \times -2.17797 - 1 \times 0.07081}{-2.17797 - 0.07081}$$

$= 0.50994$ (+ve)

$f(0.50994) = \cos(0.50994) - (0.50994)\, e^{0.50994}$

$= 0.02362$

B_5 :- $x_5 \in [0.50994, 1]$

$$= \frac{0.50994\, f(1) - 1 \times f(0.50994)}{f(1) - f(0.50994)}$$

$$= \frac{0.50994 \times -2.17797 - 1 \times 0.02362}{-2.17797 - 0.02362}$$

$= 0.51519$ +ve

$f(0.51519) = 0.00779$

$\mathbb{D}_6$:- $x_6 \in [0.51519, 1]$

$$= \frac{0.51519 \times f(1) - 1 \times f(0.51519)}{f(1) - f(0.51519)}$$

$$= \frac{0.51519 \times -2.17797 - 1 \times 0.00779}{-2.17797 - 0.00779}$$

$= \underline{0.51691}$ (+ve)

$f(0.51691) = 0.00257$

$\mathbb{D}_7$:- $x_7 \in [0.51691, 1]$.

$$= \frac{0.51691 \times f(1) - 1 \times f(0.51691)}{f(1) - f(0.51691)}$$

$$= \frac{0.51691 \times -2.17797 - 1 \times 0.00257}{-2.17797 - 0.00257}$$

$= \underline{0.51747}$ (+ve)

$f(0.51747) = 0.00087$

I8 :- $x_8 \in (0.51747, 1)$

$$= \frac{0.51747 \times f(1) - 1 \times f(0.51747)}{f(1) - f(0.51747)}$$

$$= \frac{0.51747 \times -2.17797 - 1 \times 0.00087}{-2.17797 - 0.00087}$$

$$= 0.51766$$

The root Equa is 0.51766

(Unit)

27/03

Newton raphson method

This method is very powerful method to find the root of an equa. this method is very fastest method comparatively other methods.

Here, x_0 - Initial root of the equa.

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$$

$$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$$

problems

find a root of the equa $xe^x - \cos x = 0$ by using Newton - raphson method.

G. that, $f(x) = xe^x - \cos x$

$x = 0 \Rightarrow f(0) = 0 \cdot e^0 - \cos 0$

$= -1$

$x = 1 \Rightarrow f(1) = 1 \cdot e^1 - \cos 1$

$= 2.177979$

$[a, b] = [0, 1]$

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$\boxed{\begin{aligned} f(x) &= xe^x - \cos x \\ f'(x) &= e^x + x \cdot e^x + \sin x \end{aligned}}$$

I_1 :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$= 0.5 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = 0.5 \cdot e^{0.5} - \cos 0.5$

$= -0.05322$

$f'(x_0) = e^{0.5} + 0.5 \cdot e^{0.5} + \sin x$

$= 2.95250$

$x_1 = 0.5 - \left[\frac{-0.05322}{2.95250}\right]$

$= 0.51802$

I_2 :- $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$f(x_1) = 0.51802 \times e^{0.51802} - \cos(0.51802)$

$= 0.00079$

$f'(x_1) = e^{0.51802} + 0.51802 \times e^{0.51802} + \sin(0.51802)$

$= 3.04346$

$$x_2 = 0.51802 - \left[\frac{0.00079}{3.04346}\right]$$

$$= 0.51776$$

I_3 :- $$x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$$

$$f(x_2) = 0.51776 \cdot e^{0.51776} - \cos(0.51776)$$

$$= 0.000008$$

$$f'(x_2) = e^{0.51776} + 0.51776 \times e^{0.51776} + \sin(0.51776)$$

$$= 3.04213$$

$$x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$$

$$= 0.51776 - \left[\frac{0.000008}{3.04213}\right]$$

$$= 0.51775$$

$\therefore$ The root of equa $= 0.51775$

Q. find x_2 for the equa $e^x - 3x = 0$, where $a=0$ & $b=1$ using newton raphson method.

G.that, $f(x) = e^x - 3x$

$[a,b] = [0,1]$

$$= \frac{a+b}{2} = 0.5$$

$$\boxed{x_0 = 0.5}$$

$$\boxed{\begin{aligned} f(x) &= e^x - 3x \\ f'(x) &= e^x - 3 \end{aligned}}$$

I_1 :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$= 0.5 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = e^{0.5} - 3 \times 0.5 = 0.14872$

$f'(x_0) = e^{0.5} - 3 = -1.35127$

$x_1 = 0.5 - \left[\frac{0.14872}{-1.35127}\right]$

$= 0.61005$

I_2 :- $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_2)}\right]$

$f(x_1) = e^{0.61005} - 3 \times 0.61005 = 0.01037$

$f'(x_1) = e^{0.61005} - 3 = -1.15947$

$x_2 = 0.61005 - \left(\frac{0.01037}{-1.15947}\right)$

$= 0.61899$

find a root of the equa $x^3 - 2x - 5$ by Newton-Raphson method.

G. that, $f(x) = x^3 - 2x - 5$

$x = 0 \Rightarrow f(0) = (0)^3 - 2(0) - 5 = -5$

$x = 1 \Rightarrow f(1) = (1)^3 - 2(1) - 5 = -6$

$$= 8 - 4 - 5 = -1$$

$$x = 3 \Rightarrow f(3) = (3)^3 - 2(3) - 5$$

$$= 27 - 6 - 5 = 16$$

$$[a, b] = [2, 3]$$

$$x_0 = \frac{2+3}{2} = 5/2$$

$$\boxed{x_0 = 2.5}$$

I_1 :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$$= 2.5 - \frac{f(x_0)}{f'(x_0)}$$

$$\boxed{\begin{aligned} f(x) &= x^3 - 2x - 5 \\ f'(x) &= 3x^2 - 2 \end{aligned}}$$

$$f(x_0) = (2.5)^3 - 2(2.5) - 5$$

$$= 5.625$$

$$f'(x_0) = 3(2.5)^2 - 2$$

$$= 16.75$$

$$x_1 = 2.5 - \left[\frac{5.625}{16.75}\right]$$

$$\boxed{x_1 = 2.16417}$$

I_2 :- $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$$f(x_1) = (2.16417)^3 - 2(2.16417) - 5$$

$$= 0.80783$$

$f'(x_1) = 3(2.16417)^2 - 2$

$= 12.05089$

$x_2 = 2.16417 - \left[\frac{0.80783}{12.05089}\right]$

$$\boxed{x_2 = 2.09713}$$

I_3 :- $x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$

$f(x_2) = (2.09713)^3 - 2(2.09713) - 5$

$= 0.02882$

$f'(x_2) = 3(2.09713)^2 - 2$

$= 11.19386$

$x_3 = 2.09713 - \left[\frac{0.02882}{11.19386}\right]$

$$\boxed{x_3 = 2.09455}$$

I_4 :- $x_4 = x_3 - \left[\frac{f(x_3)}{f'(x_3)}\right]$

$f(x_3) = (2.09455)^3 - 2(2.09455) - 5$

$= -0.000011$

$f'(x_3) = 3(2.09455)^2 - 2$

$= 11.16141$

$$x_4 = 2.09455 - \left[\frac{-0.00001}{11.16141}\right]$$

$$\boxed{x_4 = 2.09455}$$

$\therefore$ The root of the equa $= 2.09455$

obtain Newton Raphson method formula to find the square root of N and hence deduce the value of $\sqrt{8}$.

Let, $x = \sqrt{N}$

Squaring on both sides

$$x^2 = (N^{1/2})^2$$

$$x^2 = N$$

$$f(x) = x^2 - N = 0$$

$$f'(x) = 2x$$

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$\boxed{x_1 = x_0 - \left(\frac{x_0^2 - N}{2x_0}\right)}$$

To find the value of $\sqrt{8}$

Let,

$$x = \sqrt{8}$$

Squaring on both sides

$$x^2 = (8^{1/2})^2$$

$$x^2 = 8$$

$$f(x) = x^2 - 8 = 0$$

$$f'(x) = 2x$$

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$x = 0 \Rightarrow f(0) = 0^2 - 8 = -8$

$x = 1 \Rightarrow f(1) = (1)^2 - 8 = -7$

$x = 2 \Rightarrow f(2) = (2)^2 - 8 = -4$

$x = 3 \Rightarrow f(3) = (3)^2 - 8 = 1$

$$[a, b] = [2, 3]$$

$$x_0 = \frac{2+3}{2} = 5/2 = 2.5$$

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$= 2.5 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$f(x_0) = (2.5)^2 - 8 = \cancel{6.25}\ -1.75$

$f'(x_0) = 2(2.5) = 5$

$$x_1 = 2.5 - \left(\frac{-1.75}{5}\right) = 2.85$$

I_2 :- $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$f(x_1) = (2.85)^2 - 8 = 0.1225$

$f'(x_1) = 2(2.85) = 5.7$

$$= 2.85 \left[\frac{0.1225}{5.7}\right] = 2.82850$$

$$\boxed{x_2 = 2.82850}$$

$$I_3 \Rightarrow x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$$

$$f(x_2) = (2.82850)^2 - 8 = 0.00041$$

$$f'(x_2) = (2.82850)2 = 5.657$$

$$x_3 = 2.82850 - \left[\frac{0.00041}{5.657}\right]$$

$$\boxed{x_3 = 2.82842}$$

$$I_4 \Rightarrow x_4 = x_3 - \left[\frac{f(x_3)}{f'(x_3)}\right]$$

$$f(x_3) = (2.82842)^2 - 8 = -0.00004$$

$$f'(x_3) = 2(2.82842) = 5.65684$$

$$x_4 = 2.82842 \left[\frac{-0.00004}{5.65684}\right]$$

$$\boxed{x_4 = 2.82842}$$

$\therefore$ The root of equa $= 2.82842$

$$\boxed{\sqrt{8} = 2.82842}$$

find the square root of 10 by using newton raphson method.

Let , $x = \sqrt{10}$

S.O.B.S

$x^2 = (10^{1/2})^2$

$$\boxed{\begin{aligned} f(x) &= x^2 - 10 \\ f'x &= 2x \end{aligned}}$$

$a = 3 \Rightarrow f(3) = (3)^2 - 10 = -1$

$x = 4 \Rightarrow f(4) = (4)^2 - 10 = 6$

$[a, b] = [3, 4]$

$$x_0 = \frac{a+b}{2} = \frac{3+4}{2} = 7/2 = 3.5$$

B_1 :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = (3.5)^2 - 10 = 2.25$

$f'(x_0) = 2(3.5) = 7$

$$x_1 = 3.5 - \left[\frac{2.25}{7}\right] = 3.17857$$

$x_1 = 3.17857$

$I_2 \Rightarrow x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$f(x_1) = (3.17857)^2 - 10 = 0.10331$

$f'(x_1) = 2(3.17857) = 6.35714$

$x_2 = 3.17857 - \left[\frac{0.10331}{6.35714}\right]$

$x_2 = 3.16231$

$I_3 \Rightarrow x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$

$f(x_2) = (3.16231)^2 - 10 = 0.00020$

$f'(x_2) = 2(3.16231) = 6.32462$

$x_3 = 3.16231 - \left[\frac{0.00020}{6.32462}\right]$

$= 3.16227$

$I_4 \Rightarrow x_4 = x_3 - \left[\frac{f(x_3)}{f'(x_3)}\right]$

$f(x_3) = (3.16227)^2 - 10 = -0.00004$

$f'(x_3) = 2(3.16227) = 6.32454$

$x_4 = 3.16227 - \left[\frac{-0.00004}{6.32454}\right]$

$= 3.16227$

$\boxed{\therefore \sqrt{10} = 3.16227}$

find the cube root of N by using Newton-raphson method.

Let,

$$x = \sqrt[3]{N}$$

cubic on both side

$$x^3 = ((N)^{1/3})^3$$

$$x^3 = N$$

$$f(x) = x^3 - N = 0$$

$$f'(x) = 3x^2.$$

$$\text{I}_1 :- \quad x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)} \right]$$

$$x_1 = x_0 - \left[\frac{x_0^3 - N}{3x_0^2} \right]$$

find the reciprocal of 18 using newton raphson method.

Let, $x = 1/18$

$$\frac{1}{x} = 18$$

$$\boxed{\begin{array}{l} f(x) = 1/x - 18 \\ f'(x) = -1/x^2 \end{array}}$$

Initial value $x_0 = 0.055$

$$\text{I}_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)} \right]$$

$$f(x_0) = 1/0.055 - 18 = 0.1818$$

$$f'(x_0) = -1/(0.055)^2 = -330.57851$$

$$= 0.055 - \left[\frac{0.18181}{-330.57851}\right]$$

$$= 0.0554$$

I_2 :- $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$f(x_1) = 1/0.05554 - 18 = 0.00504$

$f'(x_1) = -1/(0.05554)^2 = -324.18151$

$$x_2 = 0.05554 - \left[\frac{0.00504}{-324.18151}\right]$$

$$= 0.05555$$

The reciprocal of 18 = 0.05555

find the reciprocal of 22 using newton raphson method.

Let, $x = 1/22$

$1/x = 22$

$$\boxed{\begin{aligned} f(x) &= 1/x - 22 \\ f'(x) &= -1/x^2 \end{aligned}}$$

Initial value $x_0 = 0.04545$

$$I_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$f(x_0) = 1/0.04545 - 22 = 0.00220$

$f'(x_0) = -1/(0.04545)^2 = -484.09681$

$$= 0.04545 - \left[\frac{0.00220}{-484.09681}\right]$$

$$= 0.04545$$

B₂ :- $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$$f(x_1) = \frac{1}{0.04545} - 22$$

$$= 0.00220$$

$$f'(x_1) = \frac{-1}{(0.04545)^2}$$

$$= -484.09681$$

$$x_2 = 0.04545$$

The root of the equa is 0.04545

Find a real root of the equa $\cos x + 1 - 3x = 0$ by using Newton - Raphson method.

G. that, $f(x) = \cos x + 1 - 3x$

To find Interval

$$x = 0 \Rightarrow f(0) = \cos 0 + 1 - 3(0) = 2$$

$$x = 1 \Rightarrow f(1) = \cos(1) + 1 - 3(1) = -1.4596$$

$$\text{Interval } [a, b] = [0, 1]$$

$$x_0 = \frac{0+1}{2} = 0.5$$

$$\boxed{\begin{aligned} f(x) &= \cos x + 1 - 3x \\ f'(x) &= -\sin x - 3 \end{aligned}}$$

$$I_1 \Rightarrow x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$f(x_0) = \cos(0.5) + 1 - 3(0.5) = 0.37758$$

$$f'(x_0) = -\sin(0.5) - 3 = -3.47942$$

$$x_1 = 0.5 - \left(\frac{0.37758}{3.47942}\right)$$

$$\boxed{x_1 = 0.60851}$$

$$I_2 \Rightarrow x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$$

$$f(x_1) = \cos(0.60851) + 1 - 3(0.60851)$$

$$= -0.00502$$

$$f'(x_1) = -\sin(0.60851) - 3 = -3.57164$$

$$\boxed{x_2 = 0.60710}$$

$$I_3 \Rightarrow x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$$

$$f(x_2) = \cos(0.60710) + 1 - 3$$

$$= 0.00000$$

$$f'(x_2) = -\sin(0.60710) - 3$$

$$= \cancel{0.}-3.57049$$

$$\boxed{x_3 = 0.60710}$$

find the real root of equa. $x + \log_{10} x - 2$ using newton raphson method. 1.755757

G. that, $f(x) = x + \log_{10} x - 2$

$$\frac{d}{dx} \log x = 1/x$$

$$\frac{d}{dx} \log_{10} x = \log_{10} e \times 1/x$$

$$\left[\because \log_{10} e = \cancel{2.3026} \; 0.4343\right]$$

To find Interval

$x \neq \cancel{0}$ $x = 1 \Rightarrow f(1) = 1 + \log_{10}(1) - 2$

$$= -1$$

$x = 2 \Rightarrow f(2) = 2 + \log_{10} 2 - 2$

$$= 0.3010$$

Interval $[a, b] = [1, 2]$

$$x_0 = \frac{1+2}{2} = 3/2 = 1.5$$

$$\boxed{x_0 = 1.5}$$

$$\boxed{\begin{aligned} f(x) &= x + \log_{10} x - 2 \\ f'(x) &= 1 + \log_{10} e \; 1/x \\ &= 1 + \frac{\overset{0.4343}{\cancel{2.3026}}}{x} \end{aligned}}$$

I_1 :- $x_1 = x_0 - \left[\dfrac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = 1.5 + \log_{10}(1.5) - 2$

$= -0.32390$

$f'(x_0) = 1 + \dfrac{0.4343}{1.5}$

$= 1.28953$

$x_1 = 1.5 - \left[\dfrac{-0.32390}{1.28953}\right]$

$= 1.75117$

I_2 :- $x_2 = x_1 - \left[\dfrac{f(x_1)}{f'(x_1)}\right]$

$f(x_1) = 1.75117 + \log_{10}(1.75117) - 2$

$= -0.00550$

$f'(x_1) = 1 + \dfrac{0.4343}{1.75117}$

$= 1.24800$

$x_2 = 1.75117 - \left(\dfrac{-0.00550}{1.24800}\right)$

$= 1.75557$

The root of equa is 1.75567 //

find the root of the equa $x + \log_{e} x - 2$ (or) (a_4, x_5)

$x + \log x - 2$

G. that, $\boxed{\begin{array}{l} f(x) = x + \log x - 2 \\ f'(x) = 1 + 1/x \end{array}}$

$x = 1 \Rightarrow f(1) = 1 + \log(1) - 2$

$= -1$

$x = 2 \Rightarrow f(2) = 2 + \log(2) - 2$

$= 0.69314$

Interval $[a, b] = [1, 2]$

$$x_0 = \frac{a+b}{2} = \frac{1+2}{2} = 3/2 = 1.5$$

I_1 :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = 1.5 + \log(1.5) - 2$

$= -0.09453$

$f'(x_0) = 1 + 1/1.5 = 1.66666$

$$x_1 = 1.5 - \left[\frac{-0.09453}{1.66666}\right]$$

$= 1.55671$

I_2 :- $x_2 = x_1 - \left[\dfrac{f(x_1)}{f'(x_1)}\right]$

$$f(x_1) = 1.55671 + \log(1.55671) - 2$$

$$= -0.00071$$

$$f'(x_1) = 1 + 1/1.55671$$

$$= 1.64238$$

$$x_2 = 1.55671 - \left[\frac{-0.00071}{1.64238}\right]$$

$$= 1.55714$$

I_3 :- $x_3 = x_2 - \left[\dfrac{f(x_2)}{f'(x_2)}\right]$

$$f(x_2) = 1.55714 + \log(1.55714) - 2$$

$$= \cancel{0.166689}\ -0.0000091$$

$$f'(x_2) = 1 + 1/1.55714$$

$$= 1.64220$$

$$x_3 = 1.55714 - \left[\frac{0.0000091 \quad \cancel{0.166689}}{1.64220}\right]$$

$$= \cancel{1.557324}\ 1.55714$$

I_4 :- $x_4 = x_3 - \left[\dfrac{f(x_3)}{f'(x_3)}\right]$

$$x_3 = 1.55324 - \left[\frac{-0.00641}{1.64381}\right]$$

$$= 1.55713$$

The root of Equa is 1.55714.

1.4 Comp

find the real root of the equa $x\log_{10}x - 1.2 = 0$ by using newton - raphson method.

$$f(x) = x\log_{10}x - 1.2$$

$f'(x) = \log_{10}x + 2.3025$

$x = 1 \Rightarrow f(1) = 1 = 1\log_{10}(1) - 1.2$

$= -1.2$

$x = 2 \Rightarrow f(2) = 2\log_{10}(2) - 1.2$

$= \cancel{+1.139794} -0.59794$

$x = 3 \Rightarrow f(3) = 3\log_{10}(3) - 1.2$

$= 0.23136$

Interval $[a, b] = [2, 3]$.

$$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

(i)

$$\boxed{\begin{aligned} f(x) &= x\log_{10}x - 1.2 \\ f'(x) &= \log_{10}x + 0.4343 \end{aligned}}$$

(I) :- $x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$f(x_0) = -0.20514$

$f'(x_0) = \log_{10}(2.5) + 0.4343$

$= 0.83224$

$$x_1 = 2.5 - \left[\frac{-0.20514}{0.83224}\right]$$

$$= 2.74649$$

$I_2 \Rightarrow x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$f(x_1) = 2.74649 \log_{10}(2.74649) - 1.2$

$= 0.00509$

$f'(x_1) = \log_{10} x + 0.4343$

$= 0.87307$

$$x_2 = 2.74649 - \left[\frac{0.00509}{0.87307}\right]$$

$$= 2.74065$$

$I_3 :- x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right]$

$f(x_2) = 2.74065 \log_{10}(2.74065) - 1.2$

$= 0.0000034$

$f'(x_2) = \log_{10}(2.74065) + 0.4343$

$= 0.87215$

$$x_3 = 2.74065 - \left[\frac{0.0000034}{0.87215}\right]$$

$$= 2.74065$$

The root of the equa is 2.74065

Iteration method

Consider an equa $f(x)=0$ which can taken in the form of $\boxed{x=\phi(x)}$

where, ϕx satisfies the following conditions.

1. for 2 real numbers a and b $(a \leq x \leq b)$, for which $|\phi'(x)| < 1$.

A formula,

$\boxed{x_n = \phi(x_{n-1})}$ where $\boxed{n>1}$ is called an interative formula.

$$x_1 = \phi(x_0), \quad x_2 = \phi(x_1) \ldots\ldots \quad x_n = \phi(x_{n-1})$$

problems

Using iteration method, solve the equa $x^3+x^2-1=0$ near 0.75 upto 5^{th} stage.

G. that, $f(x) = x^3+x^2-1 = 0 \rightarrow (1)$

G, $x_0 = 0.75$

Eq. (1) Can be written as $x = \phi(x)$

$$x^3 = 1-x^2$$

$$\boxed{x = (1-x^2)^{1/3}}$$

$$\phi x = (1-x^2)^{1/3}$$

$$\phi'(x) = 1/3(1-x^2)^{\frac{-1}{2}}(0-2x)$$

$$= -\frac{2x}{3}(1-x^2)^{-1/2}$$

$$= -0.5879$$

$$|\phi' x| = |0.5879 < 1|$$

$x = \phi(x)$

$x = (1-x^2)^{1/3}$

$\therefore \phi(x) = (1-x^2)^{1/3}$

$x_1 = \phi(x_0) = (1-x_0^2)^{1/3} = (1-(0.75)^2)^{1/3}$

$= 0.75914$

$x_2 = \phi(x_1) = (1-(x_1)^2)^{1/3} = (1-(0.75914)^2)^{1/3}$

$= 0.75108$

$x_3 = \phi(x_2) = (1-(x_2)^2)^{1/3} = (1-0.75108^2)^{1/3}$

$= 0.75820$

$x_4 = \phi(x_3) = (1-(x_3)^2)^{1/3} = (1-0.75820^2)^{1/3}$

$= 0.75192$

$x_5 = \phi(x_4) = (1-(x_4)^2)^{1/3} = (1-0.75192^2)^{1/3}$

$= 0.757417$

The root of equa $x = 0.75744$

by iteration method find root of the equa x^3-2x-5 which Lies near $x=2$.

G. that, $f(x) = x^3-2x-5 \rightarrow (1)$

Gr $x_0 = 2$

Equa (1) Can be written $x = \phi(x)$

$x^3-2x-5 = 0$

$x^3 = 2x+5$

$x = (2x+5)^{1/3}$

$\phi(x) = (2x+5)^{1/3}$

$\phi'(x) = 1/3\,(2x+5)^{-2/3}\,(2)$

$= (2/3)(2x+5)^{-2/3}$

at $x = 2$

$|\phi'(x)| = |0.1540| < 1$

$\therefore \quad x = \phi x$

$x = (2x+5)^{1/3}$

$\therefore \quad \phi(x) = (2x+5)^{1/3}$

$x_1 = \phi(x_0) = (2x+5)^{1/3} = (2(2)+5)^{1/3}$

$= 2.08008$

$x_2 = \phi(x_1) = (2x+5)^{1/3} = (2(2.08008)+5)^{1/3}$

$= 2.09235$

$x_3 = \phi(x_2) = (2x+5)^{1/3} = (2(2.09235)+5)^{1/3}$

$= 2.09421$

$x_4 = \phi(x_3) = (2x+5)^{1/3} = (2(2.09421)+5)^{1/3}$

$= 2.09449$

$x_5 = \phi(x_4) = (2x+5)^{1/3} = (2(2.09449)+5)^{1/3}$

$= 2.09454$

$x_6 = \phi(x_5) = (2x+5)^{1/3} = (2(2.09454)+5)^{1/3}$

$= 2.09454$

The root of the equa $x = 2.09454$

Solve $x = 1 + \tan^{-1} x$ by Iteration method

$f(x) = \cancel{1+\tan^{-1}x}\ x - 1 - \tan^{-1} x$

To find Interval

$x = 0 \Rightarrow f(0) = \cancel{0-1-\tan^{-1}0}\ 0 - 1 - \tan^{-1} 0$

$= -1\ (-ve)$

$x = 1 \Rightarrow f(1) = 1 - 1 - \operatorname{Tan}^{-1}(1)$

$= -0.785$ (-ve)

$x = 2 \Rightarrow f(2) = 2 - 1 - \operatorname{Tan}^{-1}(2)$

$= -0.107$ (-ve)

$x = 3 \Rightarrow f(x) = 3 - 1 - \operatorname{Tan}^{-1}(3)$

$= 0.750$ (+ve)

$x = 1 + \tan^{-1} x$

$\boxed{x_0 = 2.5}$

$\phi(x) = 1 + \operatorname{Tan}^{-1} x$

$\phi'(x) = \frac{1}{1+x^2}$

$|\phi' x| = 0.08163 < 1$

$x_1 = \phi(x_0) = 1 + \tan^{-1} x_0 = 1 + \tan^{-1}(2.5)$

$= 2.19028$

$x_2 = \phi(x_1) = 1 + \tan^{-1} x_1 = 1 + \tan^{-1}(2.19028)$

$= 2.14249$

$x_3 = \phi(x_2) = 1 + \tan^{-1} x_2 = 1 + \tan^{-1}(2.14249)$

$= 2.134103$

$x_4 = \phi(x_3) = 1 + \tan^{-1} x_3 = 1 + \tan^{-1}(2.134103)$

$= 2.13259$

$x_5 = \phi(x_4) = 1 + \tan^{-1} x_4 = 1 + \tan^{-1} 2.13259$

$= 2.13232$

$x_6 = \phi(x_5) = 2.13227$

$x_7 = \phi(x_6) = 2.13226$

The root of equa. $x = 2.13226$

Solve. $\sin x = 5x - 2$

$f(x) = \sin x + 2 - 5x$

To find an interval

$x = 0 \Rightarrow f(0) = \sin(0) - 5(0) + 2$

$= 2$ (+ve)

$x = 1 \Rightarrow f(1) = \sin(1) - 5(1) + 2$

$= -2.15852$ (-ve)

$$x = \frac{\sin x + 2}{5}$$

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$\phi(x) = \frac{\sin x + 2}{5}$$

$$\phi'(x) = \frac{\cos x}{5}$$

$$|\phi' x| = \frac{\cos(0.5)}{5} = 0.17551 < 1$$

$$x_1 = \phi(x_0) = \frac{\sin(0.5) + 2}{5} = 0.49588$$

$$x_2 = \phi(x_1) = \frac{\sin(0.49588) + 2}{5} = 0.49516$$

$$x_3 = \phi(x_2) = \frac{\sin(0.49516) + 2}{5} = 0.49503$$

$$x_4 = \phi(x_3) = \frac{\sin(0.49503) + 2}{5} = 0.49501$$

$x_5 = \phi(x_4) = 0.49500$

$x_6 = \phi(x_5) = 0.49500$

$\therefore$ The root of the equa $x = 0.49500$

Gauss Jordan method

Consider the system of Equations $Ax=B$

The Augmented matrix $[A\ B]$. In this we have to convert A into identity matrix by using Elementary row operations.

Solve the following system of equas by using Gauss-Jordan method $2x+y+z=10$, $3x+2y+3z=18$, $x+4y+9z=16$

G. Equas, $2x+y+z=10$
$3x+2y+3z=18$
$x+4y+9z=16$

This can be written as $Ax=B$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

The Augmented matrix is

$$[A \cdot B] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1$

$R_3 \to 2R_3 - R_1$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$R_3 \rightarrow R_3 - 7R_2$

$$\sim \begin{bmatrix} 2 & 0 & -2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix}$$

$R_1 \rightarrow 4R_1 - 2R_3$

$R_2 \rightarrow 4R_2 + 3R_3$

$$\sim \begin{bmatrix} 8 & 0 & 0 & 56 \\ 0 & 4 & 0 & -36 \\ 0 & 0 & -4 & 20 \end{bmatrix}$$

$R_1 \rightarrow R_1/8$; $R_2 \rightarrow R_2/4$; $R_3 \rightarrow R_3/-4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$A_1 X = B_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \\ 5 \end{bmatrix}$$

$x = 7$; $y = -9$; $z = 5$

Solve the system of Equas $x+y+z=6$, $2x-3y+4z=8$, $x-y+2z=5$ by using Gauss Jordan method.

G. Equas, $x+y+z=6$

$2x-3y+4z=8$

$x-y+2z=5$

This can be written as $AX=B$

$$\begin{bmatrix}1 & 1 & 1\\2 & -3 & 4\\1 & -1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\8\\5\end{bmatrix}$$

The Augmented matrix is

$$[A\cdot B]=\begin{bmatrix}1 & 1 & 1 & 6\\2 & -3 & 4 & 8\\1 & -1 & 2 & 5\end{bmatrix}$$

$R_2 \to 1R_2 - 2R_1$

$R_3 \to 1R_3 - 1R_1$

$$\sim\begin{bmatrix}1 & 1 & 1 & 6\\0 & -5 & 2 & -4\\0 & -2 & 1 & -1\end{bmatrix}$$

$R_1 \to 5R_1 + R_2$

$R_3 \to 5R_3 - 2R_2$

$$\sim\begin{bmatrix}5 & 0 & 7 & 26\\0 & -5 & 2 & -4\\0 & 0 & 1 & 3\end{bmatrix}$$

$R_1 \to R_1 - 7R_3$

$R_2 \to R_2 - 2R_3$

$$\sim \begin{bmatrix} 5 & 0 & 0 & 5 \\ 0 & -5 & 0 & -10 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$R_1 \to R_1/5$

$R_2 \to R_2/-5$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$AX = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x_1 = 1$ (or) $x = 1$

$x_2 = 2$ $\quad y = 2$

$x_3 = 3$ $\quad z = 3$

Solve the system of Equations $x+y+z=8$, $2x+3y+2z=19$, $4x+2y+3z=23$ by using Gauss Jordan method.

G. Equas, $x+y+z=8$

$2x+3y+2z=19$

$4x+2y+3z=23$

This can be written as $AX = B$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 19 \\ 23 \end{bmatrix}$$

The Augmented matrix is,

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 2 & 3 & 2 & 19 \\ 4 & 2 & 3 & 23 \end{bmatrix}$$

$R_2 \rightarrow 1R_2 - 2R_1$

$R_3 \rightarrow 1R_3 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & -1 & -9 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$R_3 \rightarrow R_3 + 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$R_3 \rightarrow R_3/-1$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$A \cdot x = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

$x = 2$

$y = 3$

$z = 3$

Guass seidal Iteration method

consider, the system of equas

$$\left.\begin{aligned} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 &= b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 &= b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 &= b_3 \end{aligned}\right\} \longrightarrow (1)$$

$$x_1 = \frac{1}{a_{11}} \left[b_1 - a_{12} x_2 + a_{13} x_3 \right]$$

$$x_2 = \frac{1}{a_{22}} \left[b_2 - a_{21} x_1 + a_{23} x_3 \right]$$

$$x_3 = \frac{1}{a_{33}} \left[b_3 - a_{31} x_1 + a_{32} x_2 \right]$$

Let, Take initial values as $y=0$ & $z=0$.
find the solutions by using iterations.

Solve by Gauss seidal iteration method,

$83x+11y-4z=95$, $7x+52y+13z=104$, $3x+8y+29z=71$

Given Equations,

$$\left.\begin{aligned} 83x+11y-4z&=95\\ 7x+52y+13z&=104\\ 3x+8y+29z&=71 \end{aligned}\right\} \longrightarrow (1)$$

Equa (1) written as,

$$x = 1/83\,[95-11y+4z]$$

$$y = 1/52\,[104-7x-13z]$$

$$z = 1/29\,[71-3x-8y]$$

I_1 :- $y=0$; $z=0$

$$x = 1/83\,[95-11(0)+4(0)] = 1.14457$$

$$y = 1/52\,[104-7(1.14457)-13(0)] = 1.84592$$

$$z = 1/29\,[71-3(1.14457)-8(1.84592)] = 1.82065$$

	I_1	I_2	I_3	I_4	I_5
x	1.14457	0.98768	1.05175	1.05765	1.05792
y	1.84592	1.41188	1.36926	1.36718	1.36716
z	1.82065	1.95661	1.96174	1.96171	1.96168

I_2 :- $y=1.84592$; $z=1.82065$

$$x = 1/83\,[95-11(1.84592)+4(1.82065)]$$

$$= 0.98768$$

$$y = \frac{1}{52}\left[104 - 7(0.98768) - 13(1.82065)\right]$$

$$= 1.41188$$

$$z = \frac{1}{29}\left[71 - 3(0.98768) - 8(1.\overset{41188}{\cancel{84579}})\right]$$

$$= 1.95661$$

I_3 :- $y = 1.41188$; $z = 1.95661$

$$x = \frac{1}{83}\left[95 - 11(1.41188) + 4(1.95661)\right]$$

$$= 1.05175$$

$$y = \frac{1}{52}\left[104 - 7(1.05175) - 13(1.95661)\right]$$

$$= 1.36926$$

$$z = \frac{1}{29}\left[71 - 3(1.05175) - 8(1.36926)\right]$$

$$= 1.96174$$

I_4 :- $y = 1.36926$; $z = 1.96174$

$$x = \frac{1}{83}\left[95 - 11(1.36926) + 4(1.96174)\right]$$

$$= 1.05765$$

$$y = \frac{1}{52}\left[104 - 7(1.05765) - 13(1.96174)\right]$$

$$= 1.36718$$

$$z = \frac{1}{29}\left[71 - 3(1.05765) - 8(1.36718)\right]$$

$$= 1.96171$$

I_5 :- $y = 1.36718$; $z = 1.96171$

$$x = 1/83 [95 - 11 (1.36718) + 4 (1.96171)]$$

$$= 1.05792$$

$$y = 1/52 [104 - 7 (1.05792) - 13 (1.96171)]$$

$$= 1.36716$$

$$z = 1/29 [71 - 3 (1.05792) - 8 (1.36716)]$$

$$= 1.96168$$

The solution is

$$x = 1.05792 \; ; \; y = 1.36716 \; ; \; z = 1.96168$$

Determine the solution by Gauss Seidal iteration method for $8x_1 - 3x_2 + 2x_3 = 20$; $4x_1 + 11x_2 - x_3 = 33$; $6x_1 + 3x_2 + 12x_3 = 36$.

G. Equas are,

$$\left.\begin{aligned} 8x_1 - 3x_2 + 2x_3 &= 20 \\ 4x_1 + 11x_2 - x_3 &= 33 \\ 6x_1 + 3x_2 + 12x_3 &= 36 \end{aligned}\right\} \rightarrow (1)$$

Equa (1) can be written as,

$$x_1 = 1/8 [20 + 3x_2 - 2x_3]$$

$$x_2 = 1/11 [33 - 4x_1 + x_3]$$

$$x_3 = 1/12 [36 - 6x_1 - 3x_2]$$

I_1 :- $x_2 = 0$; $x_3 = 0$

$x_1 = 1/8 [20 + 3(0) - 2(0)]$

$= 2.5$

$x_2 = 1/11 [33 - 4(2.5) + 0]$

$= 2.09090$

$x_3 = 1/12 [36 - 6(2.5) - 3(2.09090)]$

$= 1.22727$

I_2 :- $x_2 = 2.09090$; $x_3 = 1.22727$

$x_1 = 1/8 [20 + 3(2.09090) - 2(1.22727)]$

$= 2.97727$

$x_2 = 1/11 [33 - 4(2.97727) + 1.22727]$

$= 2.02892$

$x_3 = 1/12 [36 - 6(2.97727) - 3(2.02892)]$

$= 1.00413$

I_3 :- $x_2 = 2.02892$; $x_3 = 1.00413$

$x_1 = 1/8 [20 + 3(2.02892) - 2(1.00413)]$

$= 3.00981$

$x_2 = 1/11 [33 - 4(3.00981) + 1.00413]$

$= 1.99680$

$x_3 = 1/12 [36 - 6(\cancel{1.00413}\ 3.00981) - 3(1.99680)]$

$= 0.99589$

I_4 :- $x_2 = 1.99680$; $x_3 = 0.99589$

$x_1 = 1/8\,[20 + 3(1.99680) - 2(0.99589)]$

$= 2.99982$

$x_2 = 1/11\,[33 - 4(2.99982) + 0.99589]$

$= 1.99969$

$x_3 = 1/12\,[36 - 6(2.99982) - 3(1.99969)]$

$= 1.00016$

I_5 :- $x_2 = 1.99969$; $x_3 = 1.00016$

$x_1 = 1/8\,[20 + 3(1.99969) - 2(1.00016)]$

$= 2.99984$

$x_2 = 1/11\,[33 - 4(2.99984) + 1.00016]$

$= 2.00007$

$x_3 = 1/12\,[36 - 6(2.99984) - 3(2.00007)]$

$= 1.00006$

I_6 :- $x_2 = 2.00007$; $x_3 = 1.00006$

$x_1 = 1/8\,[20 + 3(2.00007) - 2(1.00006)]$

$= 3.00001$

$$x_2 = 1/11 [33 - 4(3.00001) + 1.00006]$$

$$= 2.00000$$

$$x_3 = 1/12 [36 - 6(3.00001) - 3(2.00000)]$$

$$= 0.99999$$

The solutions are

$$x_1 = 3.00001 \; ; \; x_2 = 2.00000 \; ; \; x_3 = 0.99999$$

Solve the system of simultaneous algebraic linear equas $23x_1 + 13x_2 + 3x_3 = 29$; $5x_1 + 23x_2 + 7x_3 = 37$; $11x_1 + x_2 + 23x_3 = 43$ by Gauss-Seidal method.

G. equas, are ;

$$\left.\begin{aligned} 23x_1 + 13x_2 + 3x_3 &= 29 \\ 5x_1 + 23x_2 + 7x_3 &= 37 \\ 11x_1 + x_2 + 23x_3 &= 43 \end{aligned}\right\} \rightarrow (1)$$

Equa (1) can be written as,

$$x_1 = 1/23 [29 - 13x_2 - 3x_3]$$

$$x_2 = 1/23 [37 - 5x_1 - 7x_3]$$

$$x_3 = 1/23 [43 - 11x_1 - x_2]$$

I :- $x_2 = 0$; $x_3 = 0$

$$x_1 = 1/23 [29 - 13(0) - 3(0)]$$

$$= 1.26086$$

$$x_2 = 1/23 [37 - 5(1.26086) - 7(0)]$$

$$= 1.33459$$

$$x_3 = 1/23\left[43 - 11(1.26086) + 1.33459\right]$$

$$= \cancel{1.32454}\ 1.20851$$

I_2 :- $x_2 = 1.33459$; $x_3 = 1.20851$

$$x_1 = 1/23\left[29 - 13(1.33459) - 3(1.20851)\right]$$

$$= 0.34894$$

$$x_2 = 1/23\left[37 - 5(0.34894) - 7(1.20851)\right]$$

$$= 1.165031$$

$$x_3 = 1/23\left[43 - 11(0.34894) - 1.165031\right]$$

$$= 1.65202$$

I_3 :- $x_2 = 1.16503$; $x_3 = 1.65202$

$$x_1 = 1/23\left[29 - 13(1.16503) - 3(1.65202)\right]$$

$$= 0.38689$$

$$x_2 = 1/23\left[37 - 5(0.38689) - 7(1.65202)\right]$$

$$= 1.02180$$

$$x_3 = 1/23\left[43 - 11(0.38689) - 1.02180\right]$$

$$= 1.64010$$

I_4 :- $x_2 = 1.02180$; $x_3 = 1.64010$

$$x_1 = 1/23\left[29 - 13(1.02180) - 3(1.64010)\right]$$

$$= 0.46940$$

$$x_2 = 1/23\,[37 - 5(0.46940) - 7(1.64010)]$$
$$= 1.00749$$
$$x_3 = 1/23\,[43 - 11(0.46940) - 1.00749]$$
$$= 1.60126$$

$It_5 \Rightarrow \quad x_2 = 1.00749 \; ; \; x_3 = 1.60126$

$$x_1 = 1/23\,[29 - 13(1.00749) - 3(1.60126)]$$
$$= 0.48255$$

$$x_2 = 1/23\,[37 - 5(0.482255) - 7(1.60126)]$$
$$= 1.01645$$

$$x_3 = 1/23\,[43 - 11(0.48255) - 1.01645]$$
$$= 1.59468$$

It_6 :- $\quad x_2 = 1.01645 \; ; \; x_3 = 1.59458$

$$x_1 = 1/23\,[29 - 13(1.01645) - 3(1.\overset{59458}{\cancel{01645}})$$
$$= 0.47836$$

$$x_2 = 1/23\,[37 - 5(0.47836) - 7(1.59458)]$$
$$= 1.01939$$

$$x_3 = 1/23\,[43 - 11(0.47836) - 1.01939]$$
$$= 1.59646$$

I_7 :- $x_2 = 1.01939$; $x_3 = 1.59646$

$$x_1 = 1/23 \left[29 - 13(1.01936) - 3(1.59646)\right]$$
$$= 0.47647$$

$$x_2 = 1/23 \left[37 - 5(0.47647) - 7(1.59646)\right]$$
$$= 1.01923$$

$$x_3 = 1/23 \left[43 - 11(0.47647) - 1.01923\right]$$
$$= 1.59737$$

$\therefore$ The solutions are;

$x_1 = 0.47647$; $x_2 = 1.01923$; $x_3 = 1.59737$

Use Gauss-Seidal method to solve the system of equa
$10x + y + z = 12$; $2x + 10y + z = 13$ & $2x + 2y + 10z = 14$.

G. equas are,

$$\left.\begin{array}{l} 10x + y + z = 12 \\ 2x + 10y + z = 13 \\ 2x + 2y + 10z = 14 \end{array}\right\} \longrightarrow (1)$$

Equa (1) can be written as;

$$x = 1/10 \left[12 - y - z\right]$$
$$y = 1/10 \left[13 - 2x - z\right]$$
$$z = 1/10 \left[14 - 2x - 2y\right]$$

I_1 :- $y = 0$; $z = 0$.

$$x_1 = 1/10 \left[12 - y - z\right] \Rightarrow 1/10 \left[12 - 0 - 0\right]$$
$$= 1.2$$

$$y = \frac{1}{10}\left[13 - 2(1.2) - 0\right]$$
$$= 1.06$$

$$z = \frac{1}{10}\left[14 - 2(1.2) - 2(1.06)\right]$$
$$= 0.948$$

I_2 :- $y = 1.06$; $z = 0.948$

$$x = \frac{1}{10}\left[12 - 1.06 - 0.948\right]$$
$$= 0.9992$$

$$y = \frac{1}{10}\left[13 - 2(0.9992) - 0.948\right]$$
$$= 1.00536$$

$$z = \frac{1}{10}\left[14 - 2(0.9992) - 2(1.00536)\right]$$
$$= 0.99908$$

I_3 :- $y = 1.00536$; $z = 0.99908$

$$x = \frac{1}{10}\left[12 - 1.00536 - 0.99908\right]$$
$$= 0.99955$$

$$y = \frac{1}{10}\left[13 - 2(0.99955) - 0.99908\right]$$
$$= 1.000182$$

$$z = \frac{1}{10}\left[14 - 2(0.99956) - 2(1.000182)\right]$$
$$= 1.00005$$

I_4 :- $y = 1.00018$; $z = 1.00005$

$$x = 1/10\,[12 - 1.00018 - 1.00005]$$

$$= \cancel{1.09997}\ 0.99997$$

$$y = 1/10\,[13 - 2(\cancel{1.00}\ \cancel{1.09997}\ 0.99997) - 1.00005]$$

$$= 1.00000$$

$$z = 1/10\,[14 - 2(0.99997) - 2(1.00000)]$$

$$= 1.00000$$

$\therefore$ The solutions are,

$x = 0.99997$; $y = 1.00000$; $z = 1.00000$

Use Gauss-Seidal method to solve the system of Equas $3x + y + z = 1$, $x + 3y - z = 11$, $x - 2y + 4z = 21$.

G. Equas are,

$$\left.\begin{aligned} 3x + y + z &= 1 \\ x + 3y - z &= 11 \\ x - 2y + 4z &= 21 \end{aligned}\right\} \rightarrow (1)$$

Eq (1) Can be written as,

$$x = 1/3\,[1 - y - z]$$

$$y = 1/3\,[11 - x + z]$$

$$z = 1/4\,[21 - x + 2y]$$

I_1 :- $y = 0$; $z = 0$

$$x_1 = 1/3 \,[1 - 0 - 0]$$
$$= 0.33333$$

$$y = 1/3 \,[11 - 0.33333 + 0]$$
$$= 3.55555$$

$$z = 1/4 \,[21 - 0.33333 + 2\,(3.55555)]$$
$$= 6.94444$$

I_2 :- $y = 3.55555$; $z = 6.94444$

$$x = 1/3 \,[1 - 3.55555 - 6.94444]$$
$$= -3.16666$$

$$y = 1/3 \,[11 - (-3.16666) + 6.94444]$$
$$= 7.03703$$

$$z = 1/4 \,[21 + 3.16666 + 2\,(7.03703)]$$
$$= 9.56018$$

I_3 :- $y = 7.03703$; $z = 9.56018$

$$x = 1/3 \,[1 - 7.03703 - 9.56018]$$
$$= -5.19907$$

$$y = 1/3 \,[11 + 5.19907 + 9.56018]$$
$$= 8.58641$$

$z = 1/4\left[21 + 5.19907 + 2(8.58641)\right]$

$= \cancel{2.25856} \; 10.84297$

$\mathfrak{I}_4$:- $y = 8.58641$; $z = 10.84297$

$x = 1/3\left[1 - 8.58641 - 10.84297\right]$

$= -6.14312$

$y = 1/3\left[11 + 6.14312 + 10.84297\right]$

$= 9.32809$

$z = 1/4\left[21 + 6.14312 + 2(9.32809)\right]$

$= 11.44982$

The solutions are,

$x_1 = -6.14312$; $y = 9.32809$; $z = 11.44982$