unit - III

3/04

Interpolation

If $f(x)$ is given in the form of table within the interval $[x_0, x_n]$ as

x	x_0	x_1	....	x_n
$y=f(x)$	y_0	y_1	....	y_n

Imp

Definition :-

Estimate the function value $f(x) = y$ at $x = x'$ where x' lies b/w the interval $[x_0, x_n]$ is called as "Interpolation."

Extrapolation :-

Estimate the function value $f(x) = y$ at $x = x'$ where x' lies b/w outside the interval $[x_0, x_n]$ is called as Extrapolation.

Newton's forward formula :-

x	y	Δ	Δ^2	Δ^3
x_0	y_0			
		Δy_0		
x_1	y_1		$\Delta^2 y_0$	
		Δy_1		$\Delta^3 y_0$
x_2	y_2		$\Delta^2 y_1$	
		Δy_2		
x_3	y_3			

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \ldots$$

we know that,

$$\boxed{x = x_0 + ph}$$

$$\Rightarrow ph = x - x_0$$

$$\boxed{p = \frac{x - x_0}{h}}$$

where, h = difference b/w 'x' values.

using Newton's forward difference interpolation formula find f(2·5)

x	0	1	2	3	4	5	6
y	0	1	16	81	256	625	1296

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
0	0 (y_0)	1 (Δy_0)	14 ($\Delta^2 y_0$)	36 ($\Delta^3 y_0$)	24 ($\Delta^4 y_0$)	0 ($\Delta^5 y_0$)	0 ($\Delta^6 y_0$)
1	1	15	50	60	24	0	
2	16	65	110	84	24		
3	81	175	194	108			
4	256	369	302				
5	625	671					
6	1296						

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots\cdots$$

we know that,

$$x = x_0 + ph$$

$$ph = x - x_0$$

$$p = \frac{x - x_0}{h}$$

Find (2.5)

$$x = 2.5 \; ; \; x_0 = 0 \; ; \; h = 1$$

$$p = \frac{2.5 - 0}{1} = 2.5$$

$$f(x) = 0 + (2.5)(1) + \frac{2.5(2.5-1)}{2} \times 14 + \frac{2.5(2.5-1)(2.5-2)}{3\times 2} \times 36 + \frac{2.5(2.5-1)(2.5-2)(2.5-3)}{4\times 3\times 2} \times 24$$

$$= 2.5 + 1.875 \times 14 + 11.25 - 0.9375$$

$$= 2.5 + 26.25 + 11.25 - 0.9375$$

$$= 39.0625$$

$$\therefore \boxed{f(x) = 39.0625}$$

using newton forward interpolation formula to Estimate y(0.12) from the following data.

x	0.1	0.15	0.2	0.25	0.3
y	0.656	0.522	0.410	0.316	0.240

x	y	Δ	Δ^2	Δ^3	Δ^4
0.1	0.656 (y_0)				
		-0.134 (Δy_0)			
0.15	0.522		0.022 ($\Delta^2 y_0$)		
		-0.112		-0.004 ($\Delta^3 y_0$)	
0.2	0.410		0.018		0.004 ($\Delta^4 y_0$)
		-0.094		0	
0.25	0.316		0.018		
		-0.076			
0.3	0.240				

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

we know that,

$$x = x_0 + ph$$

$$ph = x - x_0$$

$$p = \frac{x-x_0}{h} = \frac{0.12-0.10}{0.05} = \cancel{0.65}\ 0.4$$

$$p = 0.4$$

Find $y(0.12)$

$$f(x) = 0.656 + 0.4\times(-0.134) + \frac{0.4\,(0.4-1)}{2}\times 0.022$$

$$+ \frac{0.4\,(0.4-1)(0.4-2)}{3\times 2}\times(-0.004) + \cancel{0.4(0.4\cdot 4-1)}$$

$$\frac{0.4\,(0.4-1)(0.4-2)(0.4-3)}{4\times 3\times 2}\times 0.004$$

$$= 0.656 - 0.0536 - 0.00264 - 0.000256$$

$$- 0.0059904$$

$$= 0.5935136$$

$$\boxed{f(x) = 0.593}$$

Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$ and $\sin 60^\circ = 0.8660$, then find $\sin 52^\circ$ using Newton forward interpolation formula.

x	45	50	55	60
Y	0.7071	0.7660	0.8192	0.8660

x	Y	Δ	Δ^2	Δ^3
45	y_0 0.7071			
		Δy_0 0.0589		
50	0.7660		$\Delta^2 y_0$ −0.0057	
		0.0532		$\Delta^3 y_0$ −0.0007
55	0.8192		−0.0064	
		0.0468		
60	0.8660			

$$F(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

we know that,

$$x = x_0 + ph$$

$$ph = x - x_0$$

$$p = \frac{x - x_0}{h}$$

Find $\sin 52^\circ$

$x = 52$; $x_0 = 45$; $h = 5$

$$p = \frac{52-45}{5} = 1.4$$

$$F(x) = 0.7071 + 1.4 \times 0.0589 + \frac{1.4(1.4-1)}{2} \times (-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{3 \times 2} \times (-0.0007)$$

$$= 0.7071 + 0.082246 - 0.00159 - 0.00015$$

$$= 0.78482$$

$$\therefore \boxed{F(x) = 0.78482}$$

using newton's formula find the interpolating polynomial for the data

x	0	1	2	3
y	1	2	1	10

x	y	Δ	Δ^2	Δ^3
0	1 (y_0)			
		+1 (Δy_0)		
1	2		-2 ($\Delta^2 y_0$)	
		-1		12 ($\Delta^3 y_0$)
2	1		10	
		+9		
3	10			

$$p = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$$\boxed{p = x}$$

$$f(x) = 1 + 1\cdot(x) + \frac{(\cancel{-2})(x)(x-1)}{\cancel{2}} + \frac{\cancel{12}^{\,2}\,(x)(x-1)(x-2)}{\cancel{6}}$$

$$= 1 + x - (x^2 - x) + 2(x)(x^2 - 3x + 2)$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$= 2x^3 - 7x^2 + 6x + 1$$

$$\therefore \boxed{f(x) = 2x^3 - 7x^2 + 6x + 1}$$

construct the forward difference table for

$f(x) = x^2$, for $x = 0, 1, 2$

x	y	Δ	Δ^2
	y_0		
0	0	Δy_0	
		1	$\Delta^2 y_0$
1	1		2
		3	
2	4		

we know that,

$$P = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$$

$$\boxed{P = x}$$

From the following data Estimate the no.of pop. earning weekly wages b/w 60 and 70 /-

wages in rupees (x)	below 40	40-60	60-80	80-100	100-120
No.of rupees (y)	250	120	100	70	50

By using Newton's forward formula.

x	y	Δ	Δ^2	Δ^3	Δ^4
below 40	y_0 250	Δy_0 120	$\Delta^2 y_0$ −20	$\Delta^3 y_0$ −10	$\Delta^4 y_0$ 20
below 60	370	100	−30	10	
below 80	470	70	−20		
below 100	540	50			
below 120	590				

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

the no.of persons earnings weekly wages b/w 60 to 70

= below 70 - below 60

below 60 = 370

below 70 = ?

$\therefore \boxed{x = 70}$

we know that,

$ph = x - x_0$

$$p = \frac{x - x_0}{h} = \frac{70-40}{20} = \frac{30}{20}$$

$$\boxed{p = 1.5}$$

$$\frac{1.5(1.5-1)(1.5-2)}{3\times2}\times-10+\frac{1.5(1.5-1)(1.5-2)(1.5-3)}{4\times3\times2}$$

$$=250+180+(-15/2)+\frac{3.75}{6}+\frac{11.25}{24}$$

$$=250+180-7.5+0.625+0.078125$$

$$\boxed{f(x)=423.2031}$$

Below 70 − Below 60 = 423.2031 − 370

= 53.2031

Estimate the missing term in the following table.

x	0	1	2	3	4
f(x)	1	3	9	?	81

x	y	Δ	Δ^2	Δ^3	Δ^4
0	1				
		2			
1	3		4		
		6		a−19	
2	9		a−15		124−4a
		a−9		105−3a	
3	a		90−2a		
		81−a			
4	81				

for the missing term $\Delta^4 y_0 = 0$

$$124 - 4a = 0$$

$$4a = 124$$

$$\boxed{a = 31}$$

find the missing term for the given data

x	1	2	3	4	5
y	1	8	27	?	125

x	y	Δ	Δ^2	Δ^3	Δ^4
1	1				
		7			
2	8		12		
		19		$a-58$	
3	27		$a-46$		$256-4a$
		$a-27$		$198-3a$	
4	a		$152-2a$		
		$125-a$			
5	125				

for the missing term $\Delta^4 y_0 = 0$

$$256 - 4a = 0$$

$$4a = 256$$

$$\boxed{a = 64}$$

find $y(75)$, Given that $y(50) = 205$, $y(60) = 225$
$y(70) = 248$; $y(80) = 274$

x	y	Δ	Δ^2	Δ^3
50	205			
		20		
60	225		3	
		23		0
70	248		3	
		26		
80	274			

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$

we know that,

$$x = x_0 + ph$$

$$ph = x - x_0$$

$$p = \frac{x - x_0}{h} = \frac{75-50}{10} = 2.5$$

$$p = 2.5$$

$$f(x) = 205 + 2.5(20) + \frac{2.5(2.5-1)}{2}(3)$$

$$= 205 + 50 + 5.625$$

$$= 260.625$$

$$f(x) = 260.625$$

Newton backward formula

x	y	Δ	Δ^2	Δ^3
x_0	y_0			
		∇y_1		
x_1	y_1		$\nabla^2 y_2$	
		Δy_2		$\nabla^3 y_3$
x_2	y_2		$\nabla^2 y_3$	
		∇y_3		
x_3	y_3			

$$f(x) = y_n + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n + \cdots$$

we know that,

$$x = x_n + ph$$

$$ph = x - x_n$$

$$\boxed{p = \frac{x - x_n}{h}}$$

problems:-

using Newton's backward difference interpolation formula, find (5.5)

x	0	1	2	3	4	5	6
y	0	1	16	81	256	625	1296

x	y	∇	∇^2	∇^3	∇^4	∇^5	∇^6
0	0						
		1					
1	1		14				
		15		36			
2	16		50		24		
		65		60		0	
3	81		110		24		0
		175		84		0	
4	256		194		24		$\nabla^6 y_n$
		369		108		$\nabla^5 y_n$	
5	625		302		$\nabla^4 y_n$		
		671		$\nabla^3 y_n$			
6	1296		$\nabla^2 y_n$				
		∇y_n					
	y_n						

$$f(x) = y_n + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_n + \ldots$$

we know that,

$$x = x_n + ph$$

$$ph = x - x_n$$

$$p = \frac{x - x_n}{h} = \frac{5.5-6}{1} = -0.5$$

$$\boxed{p = -0.5}$$

$$= 1296 + (-0.5)\times 671 + \frac{(-0.5)((-0.5)-1)}{2}\times 302$$

$$+ \frac{(-0.5)[((-0.5)-1)(-0.5-2)]}{3\times 2}108 + \frac{-0.5[(-0.5-1)(-0.5-2)(-0.5\ldots}{4\times 3\times 2}$$

$= 1296 - 335.5 + 113.25 - 33.75 + 6.5625$

$= 915.0625$

$$\boxed{f(x) = 915.0625}$$

Find newton's backward formula, $f(1.6)$

x	1	1.4	1.8	2.2
y	3.49	4.82	5.96	6.5

x	y	∇	∇^2	∇^3
1	3.49			
		1.33		
1.4	4.82		-0.19	
		1.14		-0.41 ($\nabla^3 y_n$)
1.8	5.96		-0.6 ($\nabla^2 y_n$)	
		0.54 (∇y_n)		
2.2	6.5 (y_n)			

$$f(x) = y_n + P\nabla y_n + \frac{P(P-1)}{2!}\Delta^2 y_n + \frac{P(P-1)(P-2)}{3!}\nabla^3 y_n$$

we know that,

$$x = x_n + ph$$

$$ph = x - x_n$$

$$P = \frac{x - x_n}{h} = \frac{1.6 - 1}{0.4} = -1.5$$

$$\boxed{P = -1.5}$$

$$= 6.5 + (-1.5) \times 0.54 + \frac{(-1.5)(-1.5-1)}{2} \times (-0.6)$$

$$+ \frac{(-1.5)(-1.5-1)(-1.5-2)}{3 \times 2} \times (-0.41)$$

$$= 6.5 + (-0.81) - 1.125 + 0.89687$$

$$= 5.4618$$

$$\boxed{f(x) = 5.4618}$$

find the no. of students who got less than 45 marks and more than 45 marks.

Marks (x)	30-40	40-50	50-60	60-70	70-80
No. of Students (y)	35	48	70	40	22

x	y y_0	∇	∇^2	∇^3	∇^4
below 40	35				
		48 ∇y_0			
below 50	83		22 $\nabla^2 y_0$		
		70		-52 $\nabla^3 y_0$	
below 60	153		-30		64 $\nabla^4 y_0$ / $\Delta^4 y_n$
		40		12 $\Delta^3 y_n$	
below 70	193		-18 $\Delta^2 y_n$		
		22 Δy_n			
below 80	215 y_n				

$$f(x) = y_0 + p\nabla y_0 + \frac{p(p-1)}{2!}\nabla^2 y_0 + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_0$$

$$p = \frac{x - x_0}{h} = \frac{45-40}{10} = 0.5$$

$$f(x) = 35 + 48(0.5) + \frac{22}{2}(0.5)(0.5-1) + \left(\frac{-52}{6}\right)(0.5)(0.5-1)(0.5-2)$$

$$+ \frac{64}{24}(0.5)(0.5-1)(0.5-2)(0.5-3)$$

$$= 35 + 24 - 2.75 - 3.25 - 2.5$$

$$= 50.5$$

The no.of students get more than 45 marks

$$= 215 - 50.5 = 163.5$$

The following table gives population of the town estimate by using interpolation formula. the increase in the population during the period 1946-1948.

x (year)	1911	1921	1931	1941	1951
population (y)	12	13	20	27	39

x	y	Δ	Δ^2	Δ^3	Δ^4
1911	12 (y_0)				
		1 (Δy_0)			
1921	13		6 ($\Delta^2 y_0$)		
		7		-6 ($\Delta^3 y_0$)	
1931	20		0		11 ($\Delta^4 y_0$)
		7		5	
1941	27		5		
		12			
1951	39				

$$P = \frac{x - x_0}{h} = \frac{1946 - 1911}{10} = \frac{35}{10} = 3.5$$

Newton's forward

$$f(1946) = 12 + 3.5 + \frac{6}{2}(3.5)(2.5) + \left(\frac{-6}{6}\right)(3.5)(3.5-1)(3.5-2)$$

$$+ \,^{11}/_{4}(3.5)(3.5-1)(3.5-2)(3.5-3)$$

$$= 12 + 3.5 + 26.25 - 13.125 + 3.0078125$$

$$= 31.6328125$$

we know that,

$$P = \frac{1948-1911}{10} = \frac{37}{10} = 3.7$$

$$f(1948) = 12 + (3.7) + \,^{6}/_{2}(3.7)(2.5) + (\,^{6}/_{6})(3.7)(3.7-1)(3.7-2)$$

$$+ \,^{11}/_{4}(3.7)(3.7-1)(3.7-2)(3.7-3)$$

$$= 12 + 3.7 + 29.97 - 16.983 + 5.44871$$

$$= 45.67 + \cancel{29.97} + 5.4481 - 16.4983$$

$$= 34.135712$$

The increase in the population during the period (1946-1948) = f(1948) − f(1946)

$$= 2.5029.$$

Newton's Backward

x	y	Δ	Δ^2	Δ^3	Δ^4
1911	12				
		1			
1921	13		6		
		7		-6	
1931	20		0		
		7			11 $\Delta^4 y_n$
1941	27			5 $\Delta^3 y_n$	
			5 $\Delta^2 y_n$		
1951	39 y_n	12 Δy_n			

$$f(1946) = y_n + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_n$$

we know that,

$$x = x_n + ph$$

$$ph = \frac{x - x_n}{h} = \frac{-1951+1946}{10} = \frac{-5}{10} = -0.5$$

$$= 39 + 12(-0.5) + 5/2\,(0.5)(0.5-1) + 5/6\,(0.5)$$

$$(0.5-1)(0.5-2) + 11/24\,(0.5)(0.5-1)(0.5-2)(0.5-3)$$

$$= 39 - 6 + 1.875 - 1.5625 + 3.00$$

$$= 36.24$$

we know that

$$p = \frac{x - x_n}{h} = \frac{1948 - 1951}{10} = -3/10 = -0.3$$

$$f(1948) = y_n + \nabla y_n p + \nabla^2 y_n \frac{p(p-1)}{2!} + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n + \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_n$$

$$= 39 + 12(-0.3) + 5/2(-0.3)(-0.3-1) + 5/6(-0.3)(-0.3-1)(-0.3-2) + 11/24(-0.3)(-0.3-1)(-0.3-2)(-0.3-3)$$

$$= 36.984$$

10/04

Gauss-Central difference formula

Gauss forward difference formula :-

x	y	Δ	Δ^2	Δ^3	Δ^4
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}			
x_0	y_0		$\Delta^2 y_{-1}$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-2}$
		Δy_0			
x_1	y_1			$\Delta^3 y_{-1}$	
			$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

The Gauss forward formula is

$$f(x) = y_0 + \Delta y_0 P + \Delta^2 y_{-1} \frac{P(P-1)}{2!} + \Delta^3 y_{-1} \frac{(P+1)P(P-1)}{3!}$$
$$+ \Delta^4 y_{-2} \frac{(P+1)P(P-1)(P-2)}{4!} + \cdots$$

$$x = x_0 + ph$$

$$\boxed{P = \frac{x - x_0}{h}}$$

Gauss Backward difference formula

X	Y	Δ	Δ^2	Δ^3	Δ^4
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
		Δy_0		$\Delta^3 y_{-1}$	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

The Gauss Back ~~forward~~ formula is,

$$f(x) = y_0 + \Delta y_{-1}\, p + \Delta^2 y_{-1} \frac{p(p-1)}{2!} + \Delta^3 y_{-2} \frac{(p-1)p(p+1)}{3!} + \Delta^4 y_{-2} \frac{(p-1)p(p+1)(p+2)}{4!} + \cdots$$

Here, $\boxed{p = \dfrac{x - x_0}{h}}$

Find $y(25)$, given that $y_{20} = 24$; $y_{24} = 32$; $y_{28} = 35$; y_{32} using Gauss forward difference formula.

Difference formula is,

x	y	Δ	Δ^2	Δ^3
20	24			
		8		
$x_0 =$ a 24 forward	y_0 32		$\Delta^2 y_{-1}$ −5	
		3 Δy_0		7 $\Delta^3 y_{-1}$
b 28 backward	35		2	
		5		
32	40			

$$f(x) = y_0 + p\Delta y_0 + \Delta^2 y_{-1} \frac{p(p-1)}{2!} + \Delta^3 y_{-1} \frac{(p+1)p(p}{3!}$$

we know that,

$$x = x_0 + ph$$

$$ph = x - x_0$$

$$\boxed{p = \frac{x - x_0}{h}}$$

$$= \frac{25-24}{4} = 0.25$$

$$\boxed{p = 0.25}$$

$$= 32 + 0.25(3) + (-5)\frac{0.25(0.25-1)}{2} + 7\frac{(0.25+1)(0.25)(0.25-1)}{3 \times 2}$$

$$= 32 + 0.75 + 0.46875 + -0.2734$$

$$= 32.94$$

find by Gauss Backward interpolating formula, the value of y at $x = 1936$ using the following table.

x	1901	1911	1921	1931	1941	1951
y	12	15	20	27	39	52

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1901	12					
		3				
1911	15		2			
		5		0		
1921	20		2		3	
		7		3		-10
1931	27		5		-7	
		12 (Δy_{-1})		-4 ($\Delta^3 y_{-2}$)		
$x_0 =$ 1941	39 (y_0)		1 ($\Delta^2 y_{-1}$)			
		13				
1951	52					

$$f(x) = y_0 + p\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1}$$

$$+ \frac{\cancel{p+1}(p-1)\,p(p+1)}{3!}\Delta^3 y_{-2}$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{1936 - 1941}{10}$$

$$\boxed{p = -0.5}$$

$$= 39 + 12(-0.5) + 1\,\frac{(-0.5)(-0.5-1)}{2} +$$

$$+ \frac{\cancel{(-0.5)}(-0.5-1)(-0.5)(-0.5+1)}{3 \times 2} \times (-4)$$

$$= 39 + (-6) + \cancel{0.1375}\overset{-0.125}{} + -0.25$$

$$= 32.625$$

7. Find $y(30)$ using Gauss forward difference formula, when $y(21) = 18.4708$, $y(25) = 17.8144$, $y(29) = 17.107$, $y(33) = 16.3432$, $y(37) = 15.5154$

The Difference table,

x	y	Δ	Δ^2	Δ^3	Δ^4
21	18.4708	~~1.16864~~ -0.6564			
25	17.8144	-0.7074	-0.051	-0.0054	
a $x_0 =$ 29	y_0 17.107	-0.7638 (Δy_0)	-0.0564 ($\Delta^2 y_{-1}$)	-0.0076 ($\Delta^3 y_{-1}$)	-0.0022 ($\Delta^4 y_{-2}$)
b 33	16.3432	-0.8278	-0.064		
37	15.5154				

$x = 30$

$$F(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \Delta^3 y_{-1}\frac{(p+1)p(p-1)}{3!}$$

$$+ \Delta^4 y_{-2}\frac{(p+1)p(p-1)(p-2)}{4!}$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{30-29}{4} = 0.25$$

$$= 17.107 + 0.25(-0.7638) + \frac{0.25(0.25-1)}{2}(-0.0564)$$

$$+ \frac{(0.25+1)(0.25)(0.25-1)}{6}(-0.0076) + \frac{(0.25+1)(0.25)}{}$$

$$\frac{(0.25-1)(0.25-2)}{24} \times (-0.0022)$$

$$= 17.107 - 0.19095 + 0.00528 + 0.000296$$
$$- 0.0000375$$

$= 16.92$

Find $y(1.91)$, by using Gauss forward difference formula for the given data.

x	1.7	1.8	1.9	2.0	2.1	2.2
y	5.4739	6.0496	6.6859	7.3891	8.1662	9.0250

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1.7	5.4739					
		0.5757				
1.8	6.0496		0.0606			
		0.6363		0.0063		
$x_0 =$ 1.9	6.6859 y_0		0.0669 $\Delta^2 y_{-1}$		0.0007 $\Delta^4 y_{-2}$	
$x =$ 1.91		0.7032 Δy_0		0.007 $\Delta^3 y_{-1}$		0.0001 $\Delta^5 y_{-2}$
2.0	7.3891		0.0739		0.0008	
		0.7771		0.0078		
2.1	8.1662		0.0817			
		0.8588				
2.2	9.0250					

$$f(x) = y_0 + p\Delta y_0 + \Delta^2 y_{-1} \frac{p(p-1)}{2!} + \Delta^3 y_{-1} \frac{(p+1)p(p-1)}{3!}$$

$$\Delta^4 y_{-2} \frac{(p+1)p(p-1)(p-2)}{4!} + \Delta^5 y_{-2} \frac{(p+1)p(p-1)(p-2)(p-3)}{5!}$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{1.91 - 1.9}{0.1} = 0.01$$

$$f(1.91) = 6.6859 + 0.01(0.7032) + 0.0669 \times \frac{0.01(0.01-1)}{2}$$

$$+ 0.007 \times \frac{(0.01+1)(0.01)(0.01-1)}{6} + 0.0007 \times$$

$$\frac{(0.01+1)(0.01)(0.01-2)}{24} + 0.0001 \times \frac{(0.1+1)(0.1)(0.1-1)(0.1-2)(0.1-3)}{120}$$

$$= 6.6859 + 0.07032 + (-0.0063) + (-0.00011) + 0.000005 - 0.0000064$$

$$= 6.7498$$

Find the value of $f(1972)$ using Gauss Forward formula and $f(1976)$ using Gauss Backward formula.

x	1950	1960	1970	1980	1990	2000
Y	17	20	27	32	36	38

Gauss forward formula

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1950	17					
		3				
1960	20		4			
		7		-6		
$x_0 =$ 1970	27 y_0		-2 $\Delta^2 y_{-1}$		7 $\Delta^4 y_{-2}$	
		5 Δy_0		1 $\Delta^3 y_{-1}$		-9 $\Delta^5 y_{-2}$
$x = 1972$ 1980	32		-1		-2	
		4		-1		
1990	36		-2			
		2				
2000	38					

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1}$$

$$+ \Delta^4 y_{-2}\frac{(p+1)p(p-1)(p-2)}{4!} + \Delta^5 y_{-2}\frac{(p+1)p(p-1)(p-2)(p-3)}{5!}$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{1972 - 1970}{10} = \frac{2}{10} = 0.2$$

$$f(1972) = 27 + 0.2(5) + \frac{0.2(0.2-1)}{2}\times(-2) + 1\times\frac{(0.2+1)(0.2)(0.2-1)}{3\times 2} + 7\times\frac{(0.2+1)(0.2)(0.2-1)(0.2-2)}{24}$$

$$+ -9\times\frac{(0.2+1)(0.2)(0.2-1)(0.2-2)(0.2-3)}{120}$$

$$= 27 + 1 + 0.16 - 0.032 + 0.1008 + 0.072576$$

$$= 28.3013$$

Gauss Backward formula

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1950	17					
		3				
1960	20		4			
		7		-6		
1970	27		-2		7	
		5 (Δy_{-1})		1 ($\Delta^3 y_{-2}$)		-9 ($\Delta^5 y_{-3}$)
$x_0 =$ 1980	32 (y_0)		-1 ($\Delta^2 y_{-1}$)		-2 ($\Delta^4 y_{-2}$)	
		4		-1		
1990	36		-2			
		2				
2000	38					

$x = 1976$

$$F(x) = y_0 + p\Delta y_{-1} + \frac{p(p-1)}{2!}\Delta^2_{y-1} + \frac{(p-1)p(p+1)}{3!}\Delta^3_{y-2}$$

$$+ \frac{(p-1)p(p+1)(p+2)}{4!}\Delta^4_{y-2} + \frac{(p-1)p(p+1)(p+2)(p+3)}{5!}\Delta^5 y_{-3}$$

we know that,

$$p = \frac{x-x_0}{h} = \frac{1976-1980}{10} = \frac{-4}{10} = -0.4$$

$$F(1976) = 32 + (-0.4)5 + \frac{(-0.4)(-0.4-1)}{2}\times(-1) + (1)\times\frac{(-0.4-1)(-0.4)(-0.4+1)}{6} + (-2)\frac{(-0.4-1)(-0.4)(-0.4+1)(-0.4+2)}{24}$$

$$+ \frac{(-0.4-1)(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{120}\times(-9)$$

$$= 32+(-2)+0.12+0.056-0.0448-0.1048$$

$$F(1976) = 30.0264$$

12/04

Apply Gauss forward formula find y at $x = 25$ for the following data

x	y	Δ	Δ^2	Δ^3
20	2854			
		308		
x_0 24	y_0 3162		$\Delta^2 y_{-1}$ 74	
$x=25$		382 Δy_0		-8 $\Delta^3 y_{-1}$
28	3544		69	
		448		
32	3992			

$$F(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_{-1} + \frac{(p+1)(p)(p-1)}{3!}\Delta^3 y_{-1}$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{25-24}{4} = 1/4 = 0.25$$

$$= 3162 + 0.25(382) + \frac{0.25(0.25-1)}{2} \times 74 + \frac{(0.25+1)(0.25)(0.25-1)}{6} \times (-8)$$

$$= 3162 + 95.5 + (-6.9375) + 0.3125$$

$$= 3250.875$$

Find the polynomial that is f(x) by using Gauss backward formula from the following table

x	y	Δ	Δ^2	Δ^3	Δ^4
3	6				
		18			
5	24		16		
		34 (Δy_{-1})		0	
$x_0 =$ 7	58 (y_0)		16 ($\Delta^2 y_{-1}$)		0
		50		0	
9	108		16		
		66			
11	174				

$$f(x) = y_0 + p\,\Delta y_{-1} + \frac{p(p-1)}{2}\Delta^2 y_{-1}$$

we know that

$$p = \frac{x-x_0}{h} = \frac{x-7}{2}$$

$$f(x) = 58 + \left(\frac{x-7}{2}\right)34 + \frac{\left(\frac{x-7}{2}\right)\left(\frac{x-7}{2}+1\right)}{\cancel{2}} \times \overset{8}{\cancel{16}}$$

$$= 58 + (x-7)17 + \left(\frac{x-7}{2}\right)\left(\frac{x-7+2}{2}\right) \times \overset{2}{\cancel{8}}$$

$$= 58 + 17x - 119 + 2(x-7)(x-5)$$

$$= 58 + 17x - 119 + 2\left[x^2 - 5x - 7x + 35\right]$$

$$= 58 + 17x - 119 + 2\left[x^2 - 12x + 35\right]$$

$$= 58 + 17x - 119 + 2x^2 - 24x + 70$$

$$= 2x^2 - 7x + 9$$

By using Backward Formula, find u_{11} if

$u_0 = 14 \quad u_4 = 24 \quad u_8 = 32 \quad u_{12} = 35 \quad u_{16} = 40$

x	y	Δ	Δ^2	Δ^3	Δ^4
0	14				
		10			
4	24		-2		
		8		-3	
8	32		-5		10
		3		7	
$x_0 =$ 12	35		2		
		5			
16	40				

we know that,

$$p = \frac{x - x_0}{h} = \frac{11 - 12}{4} = \frac{-1}{4} = -0.25$$

$$y(x) = 35 + 3(-0.25) + (2)\frac{(-0.25)(-0.25+1)}{2!} + 7 \times \frac{(-0.25)(-0.25)+1)(-0.25-1)}{6}$$

$$= 35 - 0.75 - 0.1875 + 0.2734$$

$$y(x) = 34.335$$

Stirling's Interpolation formula

x	y	Δ	Δ^2	Δ^3	Δ^4
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
		Δy_0		$\Delta^3 y_{-1}$	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

$$f(x) = y_0 + \frac{[\Delta y_{-1} + \Delta y_0]}{2} p + \Delta^2 y_{-1} \frac{p^2}{2!} + \frac{(\Delta^3 y_{-2} + \Delta^3 y_{-1})}{2} \frac{p(p^2-1)}{3!}$$

$$+ \frac{\Delta^4 y_{-2}}{4!} p^2(p^2-1)$$

prob

$$x = x_0 + ph$$

$$\boxed{p = \frac{x - x_0}{h}}$$

problems:-

using stirling's formula, find the value of $f(35)$ from the following table.

x	20	30	40	50
y	512	439	346	243

x	y	Δ	Δ^2	Δ^3
20	512			
		-73 (Δy_{-1})		
30	439 (y_0)		-20 ($\Delta^2 y_{-1}$)	
		-93 (Δy_0)		10
40	346		-10	
		-103		
50	243			

$x = 35$

we know that,

$$p = \frac{x - x_0}{h} = \frac{35-30}{10} = \frac{\cancel{5}}{\cancel{10}_2} = \frac{1}{2} = 0.5$$

$$F(x) = y_0 + \frac{(\Delta y_{-1} + \Delta y_0)}{2} p + \Delta^2 y_{-1} \frac{p^2}{2!}$$

$$= 439 + \frac{(-73 - 93)}{2} \times 0.5 + \frac{(0.5)^2}{2} \times -20$$

$$= 439 - 83 \times 0.5 + 0.125 \times -20$$

$$= 439 - 83 \times 0.5 + 0.125 \times -20$$

$$= 439 - 41.5 - 2.5$$

$$= 395.5$$

find the value of $\tan(16^\circ)$ using stirling formula.

θ°	0	5	10	15	20	25	30
$\tan\theta$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
0	0						
		0.0875					
5	0.0875		0.0013				
		0.0888		0.0015			
10	0.1763		0.0028		0.0002		
		Δy_{-1} 0.0916		$\Delta^3 y_{-2}$ 0.0017		$\Delta^5 y_{-3}$ 0.0002	
$x_0 =$ 15	0.2679 y_0		$\Delta^2 y_{-1}$ 0.0045		$\Delta^4 y_{-2}$ 0		$\Delta^6 y_{-3}$ 0.0011
$x = 16$		Δy_0 0.0961		$\Delta^3 y_{-1}$ 0.0017		$\Delta^5 y_{-2}$ 0.0009	
20	0.3640		0.0062		0.0009		
		0.1023		0.0026			
25	0.4663		0.0088				
		0.1111					
30	0.5774						

$$F(x) = y_0 + \frac{[\Delta y_{-1} + \Delta y_0]}{2}\frac{p}{1!} + \Delta^2 y_{-1}\frac{p^2}{2!} + \frac{[\Delta^3 y_{-2} + \Delta^3 y_{-1}]}{2} \times \frac{p(p^2-1)}{3!} + \Delta^4 y_{-2}\frac{p^2(p^2-1^2)}{4!} + \frac{[\Delta^5 y_{-3} + \Delta^5 y_{-2}]}{2} \times \frac{p(p^2-1^2)(p^2-2^2)}{5!} + \Delta^6 y_{-3}\frac{p^2(p^2-1^2)(p^2-2^2)}{6!}$$

we know that,

$$p = \frac{x - x_0}{h} = 0.2$$

$$\tan(16^\circ) = 0.2679 + \frac{[0.0916 + 0.0961]}{2} \times (0.2)$$

$$+ 0.0045 \times \frac{(0.2)^2}{2} + \frac{[0.0017 + 0.0017]}{2} \times \frac{0.2((0.2)^2 - 1)}{6}$$

$$+ \frac{[0.0002 + 0.0009]}{2} \times \frac{0.2[((0.2)^2-1)(0.2)^2-(2)^2]}{120}$$

$$+\ 0.0011 \times \frac{(0.2)^2\left[(0.2)^2-(1)^2\right]\left[(0.2)^2-(2)^2\right]}{720}$$

$$= 0.2679 + 0.01877 + 0.00009 - 0.00028$$

$$+ 0.0000034 + 0.000000232$$

$$= 0.2864$$

Find (1.22) by using Stirlings formula

x	1.0	1.1	1.2	1.3	1.4
y	0.841	0.891	0.932	0.963	0.985

x	y	Δ	Δ^2	Δ^3	Δ^4
1.0	0.841				
		0.05			
1.1	0.891		-0.009		
		Δy_{-1} 0.041		$\Delta^3 y_{-2}$ -0.001	
x_0 1.2	0.932		$\Delta^2 y_{-1}$ -0.01		$\Delta^4 y_{-2}$ 0.002
	Δy_0	0.031		0.001	
1.3	0.963	Δy_0	-0.009	$\Delta^3 y_{-1}$	
		0.022			
1.4	0.985				

$x = 1.22$ (between 1.2 and 1.3)

we know that,

$$p = \frac{1.22 - 1.2}{0.1} = 0.2$$

$$F(x) = y_0 + \frac{[\Delta y_{-1} + \Delta y_0]}{2} \frac{p}{1!} + \Delta^2 y_{-1} \times \frac{p^2}{2!}$$

$$+ \frac{[\Delta^3 y_{-2} + \Delta^3 y_{-1}]}{2} \times \frac{p(p^2-1)}{3!} + \Delta^4 y_{-2} \times \frac{p^2(p^2-1)}{4!}$$

$$= 0.932 + \frac{[0.041 + 0.031]}{2} \times 0.2 + (-0.01) \times \frac{(0.2)^2}{2}$$

$$+ \frac{[-0.001 + 0.001]}{2} \times \frac{0.2((0.2)^2-1)}{6}$$

$$+ 0.002 \times \frac{(0.2)^2[(0.2)^2-1]}{24}$$

$$= 0.932 + 0.0072 - 0.0002 - 0.0000032$$

$$= 0.9389$$

2. Find $y(12.2)$

x	10	11	12	13	14
y	0.2396	0.2806	0.3178	0.3520	0.3836

x	y	Δ	Δ^2	Δ^3	Δ^4
10	0.2396				
		0.041			
11	0.2806		-0.0038		
		Δy_{-1} 0.0372		$\Delta^3 y_{-2}$ -0.0008	
$x_0 =$ 12	0.3178 y_0		$\Delta^2 y_{-1}$ -0.003		$\Delta^4 y_{-2}$ 0.0004
		0.0342 Δy_0		0.0004 $\Delta^3 y_{-1}$	
13	0.3520		-0.0026		
		0.0316			
14	0.3836				

$x = 12.2$

$$\boxed{P = 0.2}$$

$$y(x) = y_0 + \frac{[0.0372 + 0.0342]}{2} \times \frac{P}{1!} + \Delta^2 y_{-1} \times \frac{P^2}{2!} + \frac{[\Delta^3 y_{-2} + \Delta^2 y_{-1}]}{2} \times \frac{P(P^2-1)}{3!} + \Delta^4 y_{-2} \times \frac{P^2(P^2-1)}{4!}$$

$$= 0.3178 + \frac{[0.0372 + 0.0342]}{2} \times \frac{0.2}{1} + -0.003 \times \frac{(0.2)^2}{2} + \frac{[-0.0008 + 0.0004]}{2} \times \frac{0.2[(0.2)^2 - 1]}{6} + 0.0004 \times \frac{(0.2)^2(0.2^2-1)}{24}$$

$$= 0.3178 + 0.0071 - 0.00006 + 0.0000064 + 0.00000064$$

$$= 0.3284$$

Find $f(1.6)$ by using stirling formula

x	1	1.4	1.8	2.2
y	3.49	4.82	5.96	6.5

x	y	Δ	Δ^2	Δ^3
1	3.49			
		Δy_{-1} 1.33		
$x_0 =$ 1.4	4.82 y_0		$\Delta^2 y_{-1}$ −0.19	
		1.14 Δy_0		
1.8	5.96		−0.6	−0.41
		0.54		
2.2	6.5			

$$f(x) = y_0 + \frac{[\Delta y_{-1} + \Delta y_0]}{2} p + \Delta^2 y_{-1} \frac{p^2}{2!}$$

$$x = x_0 + ph$$

$$P = \frac{x - x_0}{h} = \frac{1.6 - 1.4}{0.4} = 0.5$$

$$\boxed{P = 0.5}$$

$$f(1.6) = 4.82 + \frac{[1.33 + 1.14]}{2} 0.5 + (-0.19) \frac{(0.5)^2}{2}$$

$$= 4.82 + (1.235) 0.5 - 0.19(0.125)$$

$$= 5.413$$

By stirling formula

Find u_{11} if $u_0 = 14$; $u_4 = 24$; $u_8 = 32$, $u_{12} = 35$, $u_{16} = 40$

x	y	Δ	Δ^2	Δ^3	Δ^4
0	14				
		10			
4	24		-2		
		8		-3	
8	32		-5		
		Δy_{-1} 3			10
$x_0 = 12$	35 y_0		2 $\Delta^2 y_{-1}$	7 $\Delta^3 y_{-2}$	
		5			
16	40				

$$f(x) = y_0 + \Delta y_{-1}\, p + \Delta^2 y_{-1} \frac{p(p+1)}{2} + \Delta^3 y_{-2} \frac{(p-1)p(p+1)}{6} + \cdots$$

we know that,

$$p = \frac{x - x_0}{h} = \frac{11-12}{4} = -0.25$$

$$\boxed{p = -0.25}$$

$$f(11) = 35 + 3(-0.25) + 2\frac{(-0.25)(-0.25+1)}{2} + 7\frac{(-0.25-1)(-0.25)(-0.25+1)}{6}$$

$$= 35 - 0.75 + 2(-0.09375) + 7(0.0390625)$$

$$= 34.3359.$$

26/04

Bessel's Formula

x	y	Δ	Δ^2	Δ^3	Δ^4
x_{-2}	y_{-2}				
		Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		Δy_{-1}		$\Delta^3 y_{-2}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
		Δy_0		$\Delta^3 y_{-1}$	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

$$x = x_0 + ph$$

$$\therefore \; p = \frac{x - x_0}{h}$$

$$F(x) = \frac{[y_0 + y_1]}{2} + \Delta y_0 (p - 1/2) + \frac{[\Delta^2 y_{-1} + \Delta^2 y_0]}{2} \frac{p(p-1)}{2!}$$

$$+ \Delta^3 y_{-1} \frac{(p - 1/2)\, p(p-1)}{3!} + \frac{[\Delta^4 y_{-2} + \Delta^4 y_{-1}]}{2} \frac{(p+1)\, p\, (p-1)(p-2)}{4!} + \cdots$$

Find the value of the function at points $x = 1.22$ by using Bessel's formula.

x	1	1.1	1.2	1.3	1.4
y	0.841	0.891	0.932	0.963	0.985

x	y	Δ	Δ^2	Δ^3	Δ^4
1	0.841				
		0.050			
1.1	0.891		-0.009		
		0.041		-0.001	
1.2	0.932 y_0		-0.01 $\Delta^2 y_{-1}$		0.002
		0.031 Δy_0		0.001 $\Delta^3 y_{-1}$	
1.3	0.963 y_1		-0.009 $\Delta^2 y_0$		
		0.022			
1.4	0.985				

$$x = x_0 + ph$$

$$p = \frac{x - x_0}{h} = \frac{1.22 - 1.2}{0.1} = 0.2$$

$$\boxed{p = 0.2}$$

$$f(x) = \frac{[y_0 + y_1]}{2} + \Delta y_0 (p - 1/2) + \frac{[\Delta^2 y_{-1} + \Delta^2 y_0]}{2} \frac{p(p-1)}{2!} + \Delta^3 y_{-1} \frac{(p - 1/2) p (p-1)}{3!}$$

$$= \frac{[0.932 + 0.963]}{2} + 0.031 (0.2 - 1/2) + \frac{[-0.01 + 0.009]}{2} \frac{[0.2 (0.2 - 1)]}{2} + 0.001 \frac{(0.2 - 1/2)\, 0.2 (0.2 - 1)}{6}$$

$$= 0.9475 - 0.0093 + 0.00004 + 0.000008$$

$$= 0.938248$$

Using Bessel's Formula find $\cos(0.17)$ from the following table.

x	0	0.05	0.10	0.15	0.20	0.25	0.30
$\cos x$	1	0.9988	0.9950	0.9888	0.9801	0.9689	0.9553

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
0	1						
		-0.0012					
0.05	0.9988		-0.0026				
		-0.0038		0.0002			
0.10	0.9950		-0.0024		-0.0003		
		-0.0062		-0.0001		0.0004	
$x_0 =$ 0.15	0.9888 y_0		-0.0025 $\Delta^2 y_{-1}$		0.0001 $\Delta^4 y_{-2}$		-0.0004
		-0.0087 Δy_0		0 $\Delta^3 y_{-1}$		0 $\Delta^5 y_{-2}$	
0.20	0.9801 y_1		-0.0025 $\Delta^2 y_0$		0.0001 $\Delta^4 y_{-1}$		
		-0.0112		0.0001			
0.25	0.9689		-0.0024				
		-0.0136					
0.30	0.9553						

we know that,

$$p = \frac{x - x_0}{h} = \frac{0.17 - 0.15}{0.05} = 0.4$$

$$f(x) = \frac{[y_0 + y_1]}{2} + \Delta y_0 (p - 1/2) + \frac{[\Delta^2 y_{-1} + \Delta^2 y_0]}{2} \frac{p(p-1)}{2!}$$

$$+ \Delta^3 y_{-1} \frac{(p - 1/2)\, p(p-1)}{3!} + \frac{[\Delta^4 y_{-2} + \Delta^4 y_{-1}]}{2} \frac{(p+1)\, p\, (p-1)(p-2)}{4!}$$

$$+ \Delta^5 y_{-2} (p - 1/2)\, p(p-1)(p-2)$$

$$= \frac{[0.9888 + 0.9801]}{2} + (-0.0087)(0.4-0.5) +$$

$$\frac{[-0.0025 + 0.0025]}{2} \frac{[0.4(0.4-1)]}{2} + \frac{[0.0001 + 0.0001]}{2}$$

$$\frac{(0.4+1)\,0.4\,(0.4-1)(0.4-2)}{24}$$

$$= 0.98445 + 0.00087 + 0.00000224$$

$$= 0.98532$$

Find $f(2.73)$ by using Bessel's formula from data

x	2.5	2.6	2.7	2.8	2.9
y	0.4938	0.4953	0.4965	0.4974	0.4981

x	y	Δ	Δ^2	Δ^3	Δ^4
2.5	0.4938				
		0.0015			
2.6	0.4953		-0.0003		
		0.0012		0	
x_0 2.7	0.4965 y_0		-0.0003 $\Delta^2 y_{-1}$		0.0001
		0.0009 Δy_0		0.0001 $\Delta^3 y_{-2}$	
2.8	0.4974 y_1		-0.0002 $\Delta^2 y_0$		
		0.0007			
2.9	0.4981				

we know that,

$$p = \frac{x - x_0}{h}$$

$$= \frac{2.73 - 2.7}{0.1} = 0.3$$

$$F(x) = \frac{[y_0 + y_1]}{2} + \Delta y_0 (p - 1/2) + \frac{[\Delta^2 y_{-1} + \Delta^2 y_0]}{2} \frac{p(p-1)}{2!}$$

$$+ \frac{\Delta^3 y_{-1} (p - 1/2) p(p-1)}{6}$$

$$= \frac{[0.4965 + 0.4974]}{2} + 0.0009 [0.3 - 0.5]$$

$$+ \frac{[-0.0008 + 0.0002]}{2} \frac{0.3(0.3-1)}{2} + 0.0001$$

$$\frac{(0.3 - 0.5)\, 0.3 (0.3 - 1)}{6}$$

$$= 0.49695 - 0.00018 + 0.00000525 + 0.0000007$$

$$= 0.49677$$

Lagrange's Interpolation Formula

x	x_0	x_1	x_2	$\cdots$	x_n
y	y_0	y_1	y_2	$\cdots$	y_n

Let,

Here the values of 'x' not neccessary equally space.

The Lagrange's formula,

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}$$

$$y_1 + \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} y_n$$

Find $f(3)$ for the following data

	x_0	x_1	x_2
x	1	2	4
	y_0	y_1	y_2
$y=f(x)$	3	2	0

$$f(x) = \frac{(3-2)(3-4)}{(1-2)(1-4)} 3 + \frac{(3-1)(3-4)}{(2-1)(2-4)} 2 + \frac{(3-1)(3-2)}{(3-1)(3-2)} \times 0$$

$$= 1$$

By Lagrange's formula

	x_0	x_1	x_2	x_3	x_4	x_5
x	0	1	2	4	5	6
	y_0	y_1	y_2	y_3	y_4	y_5
f(x)	1	14	15	5	6	19

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} y_4 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_0)(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} y_5$$

$$= 0.05(1) + (-0.3)(14) + 0.75(15) + (0.75)(5) + (-0.3)6 + (0.05)(19)$$

$$= 0.05 - 4.2 + 11.25 + 3.75 - 1.8 + 0.95$$

$$f(3) = 10$$

Find the polynomial p(x) of degree 2 or less for the data using Lagrange's Interpolation.

	x_0	x_1	x_2
x	1	3	4
y	1	27	64
	y_0	y_1	y_2

G. that,

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-3)(x-4)}{(1-3)(1-4)} \cdot 1 + \frac{(x-1)(x-4)}{(3-1)(3-4)} \cdot 27 + \frac{(x-1)(x-3)}{(4-1)(4-3)} 64$$

$$= \frac{x^2-7x+12}{-2\times-3} + \frac{x^2-5x+4}{2\times-1} 27 + \frac{x^2-4x+3}{3\times1} 64$$

$$= \frac{x^2-7x+12}{6} - \frac{x^2-5x+4}{+2} \cdot 27 + \frac{x^2-4x+3}{3} \times 64$$

$$= \frac{x^2-7x+12-81(x^2-5x+4)+128(x^2-4x+3)}{6}$$

$$= \frac{x^2-7x+12-81x^2+405x-324+128x^2-512x+384}{6}$$

$$= \frac{48x^2-114x+72}{6} = 8x^2-19x+12$$

Fit a polynomial to following data using Lagrange's Interpolation formula.

	x_0	x_1	x_2	x_3
t	1	2	3	4
z	2	5	16	41
	y_0	y_1	y_2	y_3

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} 2 + \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)} \times 5$$

$$+ \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)} \times 16 + \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)} \times 41$$

$$= \frac{(x^2-3x-2x+6)(x-4)}{(-1)(-2)(-3)} \times 2 + \frac{(x^2-3x-x+3)(x-4)}{(1)(-1)(-2)} 5$$

$$+ \frac{(x^2-2x-x+2)(x-4)}{(2)(1)(-1)} 16 + \frac{(x^2-2x-x+2)(x-3)}{(3)(2)(1)} \times 41$$

$$= \frac{(x^2-5x+6)(x-4)}{-6} \times 2 + \frac{(x^2-4x+3)(x-4)}{2} \times 5$$

$$+ \frac{(x^2-3x+2)(x-4)}{-2} \times 16 + \frac{(x^2-3x+2)(x-3)}{6} \times 41$$

$$= \frac{6x^3 - 12x^2 + 12x + 6}{6}$$

$$f(x) = x^3 \; 2x^2 + 2x + 1$$

Find the value of $f(10)$ for given

$f(x) = 168, 192, 336$

at $x = 1, 7, 15$

G. that, x : 1 7 15

y : 168 192 336

$$f(x) = \frac{(10-7)(10-15)}{(1-7)(1-15)} 168 + \frac{(10-1)(10-15)}{(7-1)(7-15)} 192$$

$$+ \frac{(10-1)(10-7)}{(15-1)(15-7)} 336$$

$$= \frac{3(-5)}{(-6)(-14)} 168 + \frac{(9)(-5)}{6(-8)} 192 + \frac{(9)(3)}{(14)(8)} 336$$

$$= \frac{-15}{84} \cdot 168 + \frac{-45}{-48} 192 + 81$$

$$= -30 + 180 + 81$$

$$= 231$$

Fit a polynomial to the following data,

	x_0	x_1	x_2	x_3
x	0	1	3	4
y	-12	0	6	12
	y_0	y_1	y_2	y_3

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}(0) +$$

$$\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} \times (6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} \times 12$$

$$= \frac{(x^2-4x+3)(x-4)}{\cancel{-12}}\cancel{(-12)} + \frac{x^2-x(x-4)}{\cancel{-6}}\times\cancel{(6)}$$

$$+ \frac{(x^2-x)(x-3)}{\cancel{12}} \times \cancel{12}$$

$$= (x^2-4x+3)(x-4) - (x^2-x)(x-4) + (x^2-x)(x-3)$$

$$= x^3-4x^2-4x^2+16x+3x-12 - (x^3-4x^2-x^2+4x)$$

$$+ x^3-3x^2-x^2+3x$$

$$= x^3 - 8x^2 + 19x - 12 - x^3 + 4x^2 + x^2 - 4x + x^3 - 4x^2 + 3x$$

$$= x^3 - 7x^2 + 18x - 12$$

01/05

Find the Cubic polynomial with given set of points

x	0	1	2	3
$f(x)$	5	6	3	14

Here, Evaluate $f(0.5)$

$$f(x) = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} \times 5 + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} \times 6$$

$$+ \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} \times 3 + \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)} \times 14$$

$$= \frac{(x^2 - 2x - x + 2)(x-3)}{-6} \times 5 + \frac{(x^2 - 2x)(x-3)}{-\cancel{2}} \times \cancel{6}^{3}$$

$$+ \frac{(x^2 - x)(x-3)}{-2} \times 3 + \frac{(x^2 - x)(x-2)}{\cancel{6}_{3}} \times \cancel{14}^{7}$$

$$= \frac{(x^2 - 3x + 2)(x-3)}{-6} \times 5 + \frac{(x^2 - 2x)(x-3)}{-2} \times 6$$

$$\frac{(x^2 - x)(x-3)}{-2} \times 3 + \frac{(x^2 - x)(x-2)}{6} \times 14$$

$$= \frac{x^3 - 3x^2 - 3x^2 + 9x + 2x - 6}{-6} \times 5 \; -$$

$$\frac{(x^3 - 3x^2 - 2x^2 + 6x)\,3}{\;} + \frac{x^3 - 3x^2 - x^2 + 3x}{-2} \times 3$$

$$+ \frac{x^3 - 2x^2 - x^2 + 2x}{3} \times 7$$

$$= \frac{-5(x^3 - 6x^2 + 19x - 6)}{6} - (x^3 - 5x^2 + 6x)\,3$$

$$- \frac{3(x^3 - 3x^2 - x^2 + 3x)}{2} + \frac{7(x^3 - 3x^2 + 2x)}{3}$$

$$= \frac{-5x^3 + 30x^2 + 55x + 30 + 18x^3 - 90x^2 + 108x - 9x^3 + 36x^2 - 27x + 14x^3 - 42x^2 + 28}{6}$$

$$= \frac{18x^3 - 66x^2 + 54x + 30}{6} = 3x^3 - 11x^2 + 9x + 5$$

$\therefore f(0.5) = 3(0.5)^3 - 11(0.5)^3 + 9(0.5) + 5$

$= 7.125$ //.

problems about operators

forward difference operator (Δ):-

$$\Delta f(x) = f(x+h) - f(x)$$

Backward difference operator (∇):-

$$\nabla f(x) = f(x) - f(x-h)$$

shifting operator :-

$$E f(x) = f(x+h)$$

Formulae:-

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left[\frac{A-B}{1+AB}\right]$$

Compute $\Delta \cos 2x$

$$\Delta \cos 2x = \cos 2(x+h) - \cos 2x$$

$$= \cos(2x+2h) - \cos 2x$$

$$= -2 \overset{\sin}{\cancel{\cos}}\left(\frac{2x+2h+2x}{2}\right) \overset{\sin}{\cancel{\cos}}\left(\frac{2x+2h-2x}{2}\right)$$

$$= -2 \overset{\sin}{\cancel{\cos}}\left(\frac{4x+2h}{2}\right) \overset{\sin}{\cancel{\cos}}\left(\frac{2h}{2}\right)$$

$$= -2 \sin(2x+h) \sin h$$

Compute $\cos x$

$$\Delta \cos x = \cos(x+h) - \cos x$$

$$= -2 \sin\left(\frac{x+h+x}{2}\right) - \sin\left(\frac{x+h-x}{2}\right)$$

$$= -2 \sin\left(\frac{2x+h}{2}\right) - \sin(h/2)$$

Compute $\nabla \sin x$

$$\nabla \sin x = \sin x - \sin(x-h)$$

$$= 2 \cos\left(\frac{x+x-h}{2}\right) \sin\left(\frac{x-x+h}{2}\right)$$

$$= 2 \cos\left(\frac{2x-h}{2}\right) \sin(h/2)$$

Compute $E \log x$

$$E \log x = \log(x+h)$$

Compute $\Delta \tan^{-1} x$

$$\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1} x$$

$$= \tan^{-1}\left[\frac{x+h-x}{1+(x+h)x}\right]$$

$$= \tan^{-1}\left[\frac{h}{1+x^2+hx}\right]$$

prove that $\nabla E = \Delta = E\nabla$

(i) $\nabla E f(x) = \nabla f(x+h)$

$$= f(x+h) - f(x+h-h)$$

$$= f(x+h) - f(x)$$

$$= \Delta f(x)$$

$$\therefore \Delta E = \Delta$$

(ii) $E \nabla f(x) = E\left[f(x) - f(x-h)\right]$

$$= Ef(x) - Ef(x-h)$$

$$= f(x+h) - f(x-h+h)$$

$$= f(x+h) - f(x)$$

$$= \Delta$$

$$\therefore \nabla E = \Delta = E\nabla$$