UNIT-3

Knowledge Based Agents:

The representation of knowledge and the reasoning processes that bring knowledge to life-are central to the entire field of artificial intelligence. Humans, it seems, know things and do reasoning. Knowledge and reasoning are also important for artificial agents because they enable successful behaviors that would be very hard to achieve otherwise. We have seen that knowledge of action outcomes enables problem solving agents to perform well in complex environments. A reflex agent could only find its way from Arad to Bucharest by dumb luck.

The knowledge of problem-solving agents is however, very specific and inflexible. A chess program can calculate the legal moves of its king, but does not know in any useful sense that no piece can be on two different squares at the same time. Knowledge-based agents can benefit from knowledge expressed in very general forms, combiningand recombining information to suit myriad purposes. Often, this process can be quite far removed from the needs of the moment-as when a mathematician proves a theorem or an astronomer calculatesthe earth's life expectancy.

Knowledge and reasoning also play a crucial role in dealing with partially observable environments. A knowledge-based agent can combine general knowledge with current percepts to infer hidden aspects of the current state prior to selecting actions. For example, a physician diagnoses a patient-that is, infers a disease state that is not directly observable prior to choosing a treatment.

Some of the knowledge that the physician uses is in the form of rules learned from textbooks and teachers, and some is in the form of patterns of association that the physician may not be able to consciously describe. If it's inside the physician's head, it counts as knowledge.

Understanding natural language also requires inferring hidden state, namely, the intention of the speaker. When we hear, "John saw the diamond through the window and coveted it," we know "it" refers to the diamond and not the window-we reason, perhaps unconsciously, with our knowledge of relative value. Similarly, when we hear, "John threw the brick through the window and broke it," we know "it" refersto the window.

Reasoning allows us to cope with the virtually infinite variety of utterances using a finite store of commonsense knowledge. Problem- solving agents have difficulty with this kind of ambiguity because their representation of contingency problems is inherently exponential. Our final reason for studying knowledge-based agents is their flexibility. They are able to accept new tasks in the form of explicitly described goals, they can achieve competence quickly by being told or learning new knowledge about the environment, and they can adapt to changes in the environment by updating the relevant knowledge.

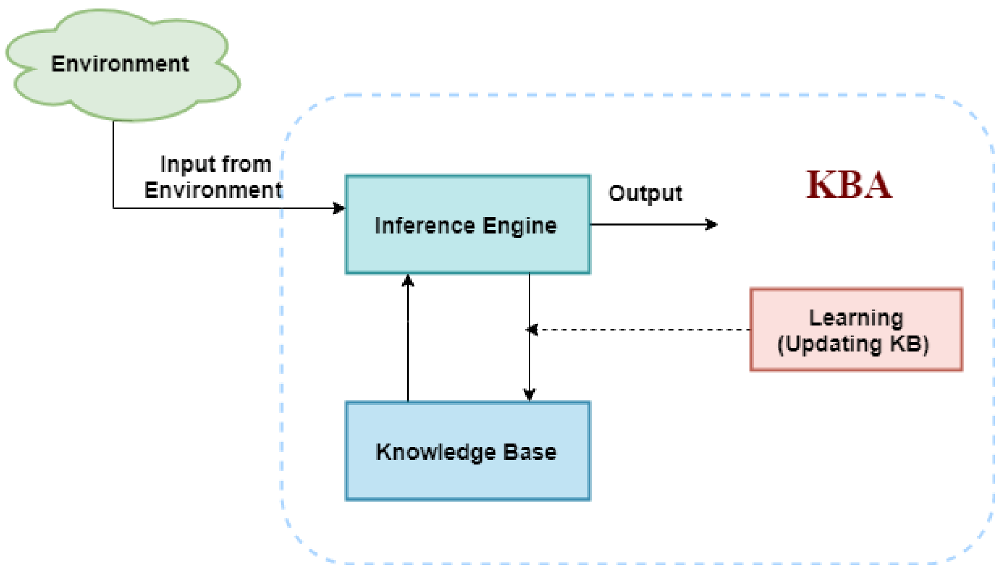
Knowledge-Based Agent in Artificial intelligence

- An intelligent agent needs knowledge about the real world for taking decisions and reasoning to act efficiently.
- Knowledge-based agents are those agents who have the capability of maintaining an internal state of knowledge, reason over that knowledge, update their knowledge after observations and take actions. These agents can represent the world with some formal representation and act intelligently.
- Knowledge-based agents are composed of two main parts:
  - Knowledge-base and
  - Inference system.

A knowledge-based agent must able to do the following:

- An agent should be able to represent states, actions, etc.
- An agent Should be able to incorporate new percepts
- An agent can update the internal representation of the world
- An agent can deduce the internal representation of the world
- An agent can deduce appropriate actions.The

architecture of knowledge-based agent:



The above diagram is representing a generalized architecture for a knowledge-based agent. The knowledge-based agent (KBA) takes input from the environment by perceiving the environment. Theinput is taken by the inference engine of the agent and which also communicate with KB to decide as per the knowledge store in KB.The learning element of KBA regularly updates the KB by learning new knowledge.

Knowledge base: Knowledge-base is a central component of a knowledge-based agent, it is also known as KB. It is a collection of sentences (here 'sentence' is a technical term and it is not identical to sentence in English). These sentences are expressed in a language which is called a knowledge representation language. The Knowledge-base of KBA stores fact about the world.

Why use a knowledge base?

Knowledge-base is required for updating knowledge for an agent to learn with experiences and take action as per the knowledge.

Inference system

Inference means deriving new sentences from old. Inference system allows us to add a new sentence to the knowledge base. A sentence is a proposition about the world. Inference system applies logical rules to the KB to deduce new information.

Inference system generates new facts so that an agent can update the KB. An inference system works mainly in two rules which are given as:

- Forward chaining
- Backward chaining

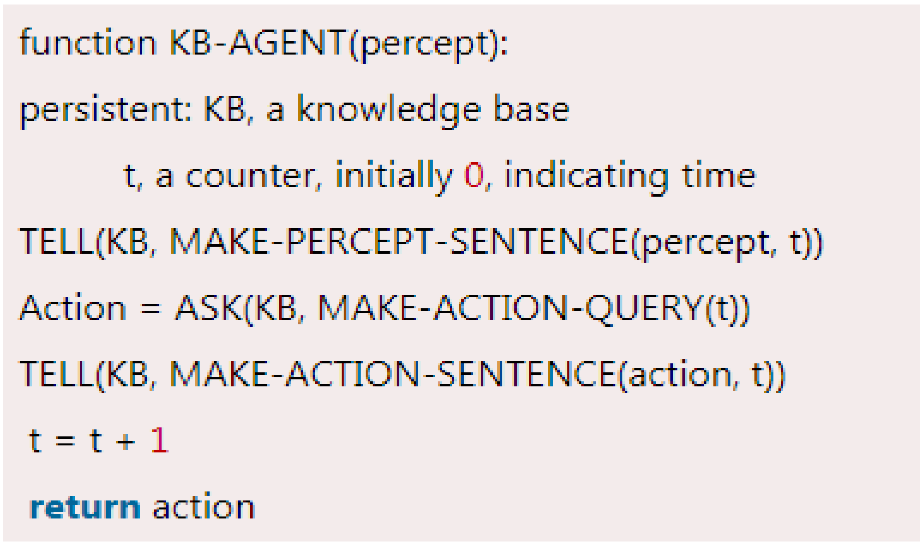
Operations Performed by KBA

Following are three operations which are performed by KBA in orderto show the intelligent behavior:

1. **TELL:** This operation tells the knowledge base what it perceivesfrom the environment.
2. **ASK:** This operation asks the knowledge base what action itshould perform.
3. **Perform:** It performs the selected action.A generic

knowledge-based agent:

Following is the structure outline of a generic knowledge-based agents program:



```
function KB-AGENT(percept):
persistent: KB, a knowledge base
          t, a counter, initially 0, indicating time
TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
Action = ASK(KB, MAKE-ACTION-QUERY(t))
TELL(KB, MAKE-ACTION-SENTENCE(action, t))
 t = t + 1
 return action
```

The knowledge-based agent takes percept as input and returns an action as output. The agent maintains the knowledge base, KB, andit initially has some background knowledge of the real world. It also has a counter to indicate the time for the whole process, and this counter is initialized with zero.

Each time when the function is called, it performs its three operations:

- Firstly it TELLs the KB what it perceives.
- Secondly, it asks KB what action it should take
- Third agent program TELLS the KB that which action waschosen.

The MAKE-PERCEPT-SENTENCE generates a sentence as setting that the agent perceived the given percept at the given time.

The MAKE-ACTION-QUERY generates a sentence to ask which action should be done at the current time.

MAKE-ACTION-SENTENCE generates a sentence which asserts that the chosen action was executed.

Various levels of knowledge-based agent:

A knowledge-based agent can be viewed at different levels which are given below:

1. Knowledge level

Knowledge level is the first level of knowledge-based agent, and in this level, we need to specify what the agent knows, and what the agent goals are. With these specifications, we can fix its behavior. For example, suppose an automated taxi agent needs to go from a stationA to station B, and he knows the way from A to B, so this comes atthe knowledge level.

2. Logical level:

At this level, we understand that how the knowledge representation of knowledge is stored. At this level, sentences are encoded into different logics. At the logical level, an encoding of knowledge into logical sentences occurs. At the logical level we can expect to the automated taxi agent to reach to the destination B.

3. Implementation level:

This is the physical representation of logic and knowledge. At the implementation level agent perform actions as per logical and knowledge level. At this level, an automated taxi agent actually implements his knowledge and logic so that he can reach to the destination.

Approaches to designing a knowledge-based agent:

There are mainly two approaches to build a knowledge-based agent:

1. Declarative approach: We can create a knowledge-based agent by initializing with an empty knowledge base and telling the agent all the sentences with which we want to start with. This approach is called Declarative approach.
2. Procedural approach: In the procedural approach, we directly encode desired behavior as a program code. Which means we just need to write a program that already encodes the desired behavior or agent?

However, in the real world, a successful agent can be built by combining both declarative and procedural approaches, and declarative knowledge can often be compiled into more efficient procedural code.

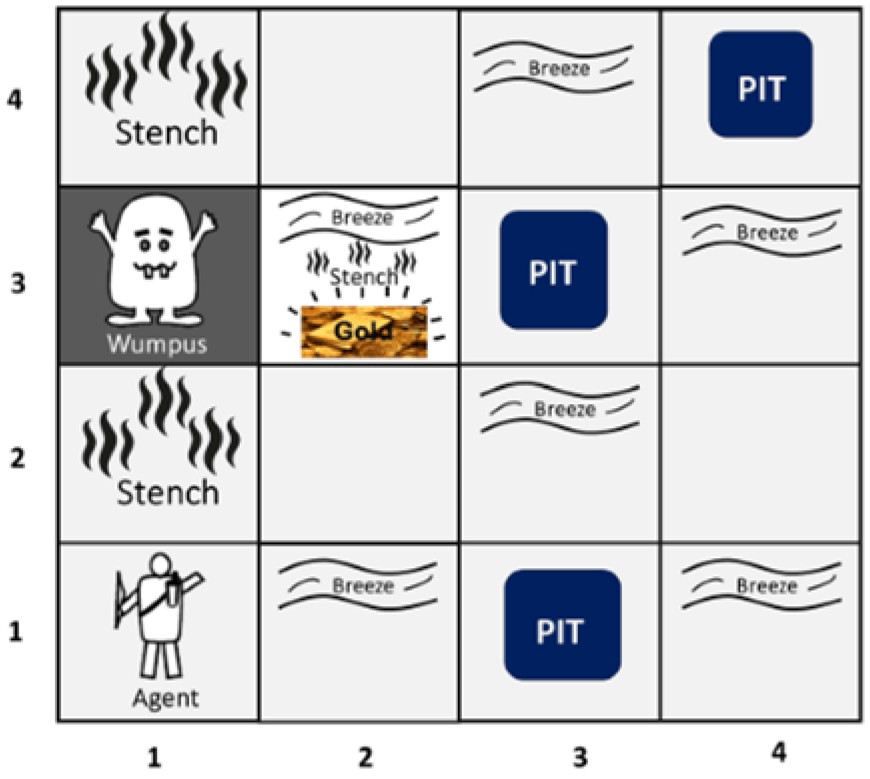
Wumpus world:

The Wumpus world is a simple world example to illustrate the worthof a knowledge-based agent and to represent knowledge representation. It was inspired by a video game Hunt the Wumpus by Gregory Yob in 1973.

The Wumpus world is a cave which has 4/4 rooms connected with passageways. So there are total 16 rooms which are connected with each other. We have a knowledge-based agent who will go forward in this world. The cave has a room with a beast which is called Wumpus, who eats anyone who enters the room. The Wumpus canbe shot by the agent, but the agent has a single arrow.

In the Wumpus world, there are some Pits rooms which are bottomless, and if agent falls in Pits, then he will be stuck there forever. The exciting thing with this cave is that in one room there isa possibility of finding a heap of gold. So the agent goal is to find the gold and climb out the cave without fallen into Pits or eaten by Wumpus. The agent will get a reward if he comes out with gold, and he will get a penalty if eaten by Wumpus or falls in the pit.

Following is a sample diagram for representing the Wumpus world. It is showing some rooms with Pits, one room with Wumpus and oneagent at (1, 1) square location of the world.



There are also some components which can help the agent to navigate the cave. These components are given as follows:

The rooms adjacent to the Wumpus room are smelly, so that it wouldhave some stench.

a. The room adjacent to PITs has a breeze, so if the agent reaches near to PIT, then he will perceive the breeze.
b. There will be glitter in the room if and only if the room has gold.
c. The Wumpus can be killed by the agent if the agent is facing to it, and Wumpus will emit a horrible scream which can be heard anywhere in the cave.

PEAS description of Wumpus world:

To explain the Wumpus world we have given PEAS description asbelow:

Performance measure:

- +1000 reward points if the agent comes out of the cave with thegold.
- -1000 points penalty for being eaten by the Wumpus or fallinginto the pit.

- -1 for each action, and -10 for using an arrow.
- The game ends if either agent dies or came out of the cave.

Environment:

- A 4*4 grid of rooms.
- The agent initially in room square [1, 1], facing toward the right.
- Location of Wumpus and gold are chosen randomly except thefirst square [1,1].
- Each square of the cave can be a pit with probability 0.2 exceptthe first square.

Actuators:

- Left turn,
- Right turn
- Move forward
- Grab
- Release
- Shoot.

Sensors:

- The agent will perceive the stench if he is in the room adjacentto the Wumpus. (Not diagonally).
- The agent will perceive breeze if he is in the room directly adjacent to the Pit.
- The agent will perceive the glitter in the room where the gold ispresent.
- The agent will perceive the bump if he walks into a wall.
- When the Wumpus is shot, it emits a horrible scream which canbe perceived anywhere in the cave.
- These percepts can be represented as five element list, in whichwe will have different indicators for each sensor.

- Example if agent perceives stench, breeze, but no glitter, no bump, and no scream then it can be represented as: [Stench, Breeze, None, None, None].

The Wumpus world Properties:

- Partially observable: The Wumpus world is partially observable because the agent can only perceive the close environment suchas an adjacent room.
- Deterministic: It is deterministic, as the result and outcome of the world are already known.
- Sequential: The order is important, so it is sequential.
- Static: It is static as Wumpus and Pits are not moving.
- Discrete: The environment is discrete.
- One agent: The environment is a single agent as we have oneagent only and Wumpus is not considered as an agent.

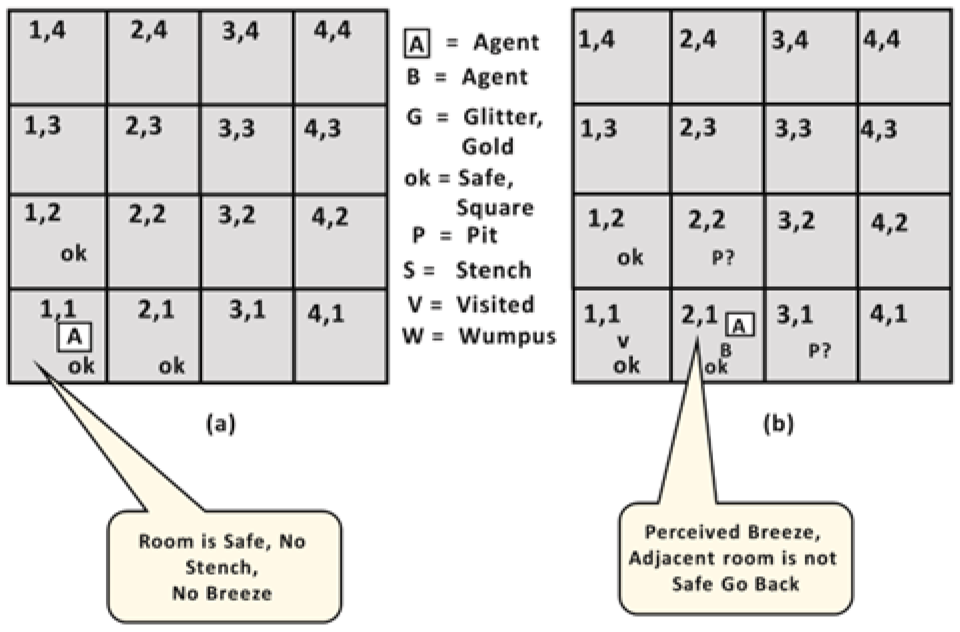
Exploring the Wumpus world:

Now we will explore the Wumpus world and will determine how theagent will find its goal by applying logical reasoning.

Agent's First step:

Initially, the agent is in the first room or on the square [1,1], and we already know that this room is safe for the agent, so to represent on the below diagram (a) that room is safe we will add symbol OK. Symbol A is used to represent agent, symbol B for the breeze, G for Glitter or gold, V for the visited room, P for pits, W for Wumpus.

At Room [1,1] agent does not feel any breeze or any Stench whichmeans the adjacent squares are also OK.



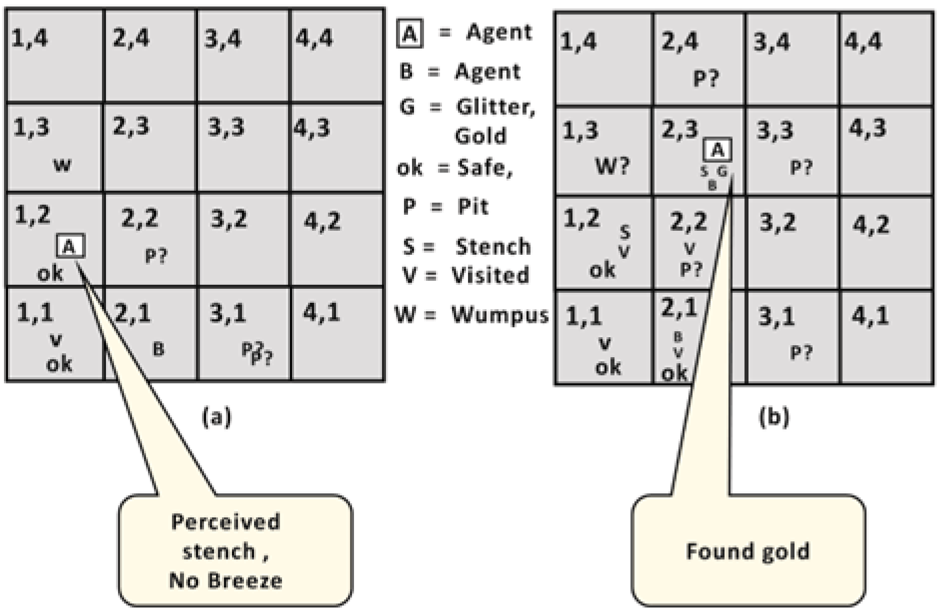
Agent's second Step:

Now agent needs to move forward, so it will either move to [1, 2], or [2,1]. Let's suppose agent moves to the room [2, 1], at this room agent perceives some breeze which means Pit is around this room. The pit can be in [3, 1], or [2,2], so we will add symbol P? to say that,is this Pit room?

Now agent will stop and think and will not make any harmful move. The agent will go back to the [1, 1] room. The room [1,1], and [2,1] are visited by the agent, so we will use symbol V to represent the visited squares.

Agent's third step:

At the third step, now agent will move to the room [1,2] which is OK. In the room [1,2] agent perceives a stench which means there must be a Wumpus nearby. But Wumpus cannot be in the room [1,1] as by rules of the game, and also not in [2,2] (Agent had not detected any stench when he was at [2,1]). Therefore agent infers that Wumpus is in the room [1,3], and in current state, there is no breeze which means in [2,2] there is no Pit and no Wumpus. So it is safe, and we will mark it OK, and the agent moves further in [2,2].



Agent's fourth step:

At room [2,2], here no stench and no breezes present so let's suppose agent decides to move to [2,3]. At room [2,3] agent perceives glitter, so it should grab the gold and climb out of the cave.

Propositional Logic:

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example:

1. a) It is Sunday.
2. b) The Sun rises from West (False proposition)
3. c) 3+3= 7(False proposition)
4. d) 5 is a prime number.

Following are some basic facts about propositional logic:

- Propositional logic is also called Boolean logic as it works on 0and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function,and logical connectives.
- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.
- Connectives can be said as a logical operator which connects two sentences.
- A proposition formula which is always true is called tautology, and it is also called a valid sentence.
- A proposition formula which is always false is called Contradiction.
- A proposition formula which has both true and false values is called
- Statements which are questions, commands, or opinions are not propositions such as "Where is Rohini", "How are you", "What is your name", are not propositions.

Syntax of propositional logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

a. Atomic Propositions
b. Compound propositions

- Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

1.a) 2+2 is 4, it is an atomic proposition as it is a true fact.
2.b) "The Sun is cold" is also a proposition as it is a false fact.

- Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

1. a) "It is raining today, and street is wet."
2. b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1. **Negation:** A sentence such as $\neg$ P is called negation of P. A literal can be either Positive literal or negative literal.

2. **Conjunction:** A sentence which has $\land$ connective such as, P $\land$ Qis called a conjunction. Example: Rohan is intelligent and hardworking. It can be written as, P= Rohan is intelligent, Q= Rohan is hardworking. → P$\land$ Q.

3. **Disjunction**: A sentence which has $\lor$ connective, such as P $\lor$ Q. is called disjunction, where P and Q are the propositions. Example:"Ritika is a doctor or Engineer", Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as P $\lor$ Q.

4. **Implication:** A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as If it is raining, then the street is wet. Let P= It is raining, and Q= Street is wet, so it is represented as P → Q

5. **Biconditional:** A sentence such as P⇔ Q is a Biconditional sentence, example If I am breathing, then I am aliveP= I am breathing, Q= I am alive, it can be represented as P
– Q.

Following is the summarized table for Propositional LogicConnectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Truth Table:

In propositional logic, we need to know the truth values of propositions in all possible scenarios. We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called Truth table. Following are the truth table for all logical connectives:

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

For disjunction:

P	Q	P V Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

For Biconditional:

P	Q	P⇔ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Truth table with three propositions:

We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of 8n Tuples as we have taken threeproposition symbols.

P	Q	R	$\neg R$	$P \lor Q$	$P \lor Q \rightarrow \neg R$
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

Precedence of connectives:

Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators. This order should be followed while evaluating a propositional problem. Following is the list of the precedence order for operators:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Logical equivalence:

Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

Let's take two propositions A and B, so for logical equivalence, we can write it as $A \Longleftrightarrow B$. In below truth table we can see that column for
$\neg A \lor B$ and $A \rightarrow B$, are identical hence A is Equivalent to B

A	B	¬A	¬A∨ B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Properties of Operators:

- Commutativity:
  - PA Q= Q A P, or
  - P ∨ Q = Q ∨ P.
- Associativity:
  - (P A Q) A R= P A (Q A R),
  - (P ∨ Q) ∨ R= P ∨ (Q ∨ R)
- Identity element:
  - P A True = P,
  - P ∨ True= True.
- Distributive:
  - PA (Q ∨ R) = (P A Q) ∨ (P A R).
  - P ∨ (Q A R) = (P ∨ Q) A (P ∨ R).
- DE Morgan's Law:
  - ¬ (P A Q) = (¬P) ∨ (¬Q)
  - ¬ (P ∨ Q) = (¬ P) A (¬Q).
- Double-negation elimination:
  - ¬ (¬P) = P.

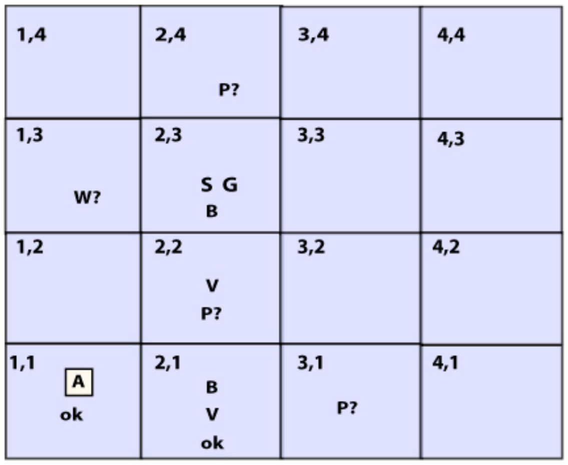
Limitations of Propositional logic:

- We cannot represent relations like ALL, some, or none withpropositional logic. Example:
  a. All the girls are intelligent.
  b. Some apples are sweet.
- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in termsof their properties or logical relationships.

Knowledge-base for Wumpus world

As in the previous topic we have learned about the wumpus world and how a knowledge-based agent evolves the world. Now in this topic, we will create a knowledge base for the wumpus world, and will derive some proves for the Wumpus-world using propositional logic.

The agent starts visiting from first square [1, 1], and we already know that this room is safe for the agent. To build a knowledge base for wumpus world, we will use some rules and atomic propositions. We need symbol [i, j] for each location in the wumpus world, where i is for the location of rows, and j for column location.



atomic proposition variable for Wumpus world:

- Let $\mathbf{P}_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $\mathbf{B}_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $\mathbf{W}_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $\mathbf{S}_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $\mathbf{V}_{i,j}$ be true if that square[i, j] is visited.
- Let $\mathbf{G}_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $\mathbf{OK}_{i,j}$ be true if the room is safe.

Some Propositional Rules for the wumpus world:

(R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

(R2) $\neg S_{21} \rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$

(R3) $\neg S_{12} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{22} \wedge \neg W_{13}$

(R4) $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$

Representation of Knowledgebase for Wumpus world:
Following is the Simple KB for wumpus world when an agent moves from room [1, 1], to room [2,1]:

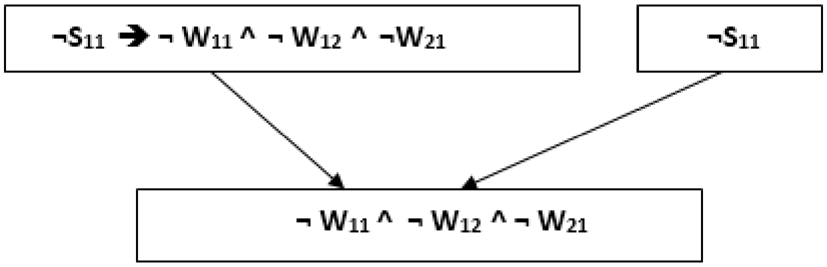
$\neg W_{11}$	$\neg S_{11}$	$\neg P_{11}$	$\neg B_{11}$	$\neg G_{11}$	V_{11}	OK_{11}
$\neg W_{12}$	----	$\neg P_{12}$	-----	----	$\neg V_{12}$	OK_{12}
$\neg W_{21}$	$\neg S_{21}$	$\neg P_{21}$	B_{21}	$\neg G_{21}$	V_{21}	OK_{21}

Here in the first row, we have mentioned propositional variables for room[1,1], which is showing that room does not have wumpus($\neg W_{11}$), no stench ($\neg \mathbf{S}_{11}$), no Pit($\neg \mathbf{P}_{11}$), no breeze($\neg \mathbf{B}_{11}$), no gold ($\neg \mathbf{G}_{11}$), visited ($\mathbf{V}_{11}$), and the room is Safe($\mathbf{OK}_{11}$).
In the second row, we have mentioned propositional variables for room [1,2], which is showing that there is no wumpus, stench and breeze are unknown as an agent has not visited room [1,2], no Pit, not visited yet, and the room is safe.
In the third row we have mentioned propositional variable for room[2,1], which is showing that there is no wumpus($\neg$ W21), no stench ($\neg \mathbf{S}_{21}$), no Pit ($\neg \mathbf{P}_{21}$), Perceives breeze($\mathbf{B}_{21}$), no glitter($\neg \mathbf{G}_{21}$), visited ($\mathbf{V}_{21}$), and room is safe ($\mathbf{OK}_{21}$).
Prove that Wumpus is in the room (1, 3)

We can prove that wumpus is in the room (1, 3) using propositional rules which we have derived for the wumpus world and using inference rule.

- **Apply Modus Ponens with ¬S11 and R1:**

We will firstly apply MP rule with R1 which is $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, and $\mathbf{\neg S_{11}}$ which will give the output $\neg W_{11} \wedge W_{12} \wedge W_{12}$.

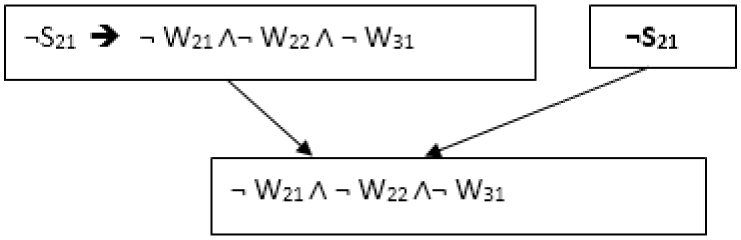


- **Apply And-Elimination Rule:**

After applying And-elimination rule to $\neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, we will get three statements:
$\mathbf{\neg W_{11}, \neg W_{12},}$ **and** $\mathbf{\neg W_{21}}$.

- **Apply Modus Ponens to $\mathbf{\neg S_{21}}$, and R2:**

Now we will apply Modus Ponens to $\neg S_{21}$ and R2 which is $\neg S_{21} \rightarrow \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$, which will give the Output as $\neg \mathbf{W_{21}}$ □ $\neg \mathbf{W_{22}}$ □$\neg \mathbf{W_{31}}$

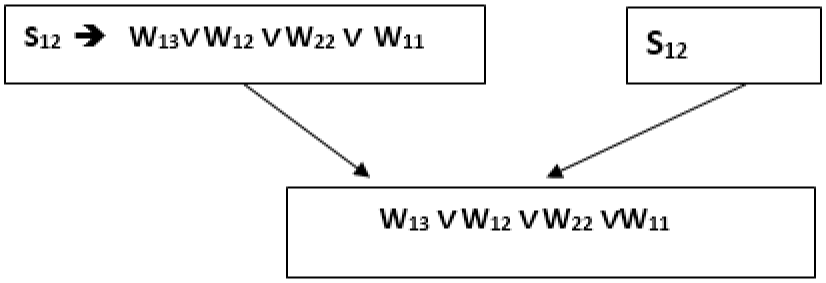


- **Apply And -Elimination rule:**

Now again apply And-elimination rule to $\neg \mathbf{W_{21}}$ □ $\neg \mathbf{W_{22}}$ □$\neg \mathbf{W_{31}}$, We will get three statements:
$\mathbf{\neg W_{21}, \neg W_{22},}$ **and** $\mathbf{\neg W_{31}}$.

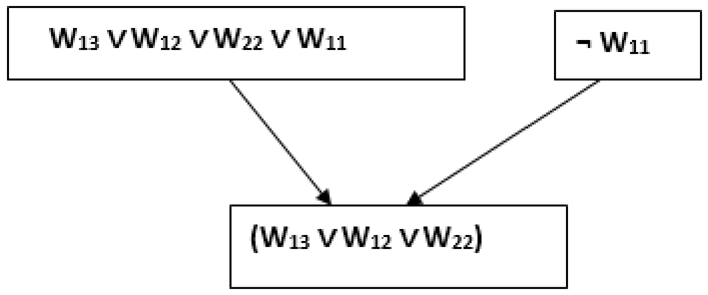
- **Apply MP to $\mathbf{S_{12}}$ and R4:**

Apply Modus Ponens to $\mathbf{S_{12}}$ and $\mathbf{R_4}$ which is $\mathbf{S_{12}} \rightarrow \mathbf{W_{13}}$ □. $\mathbf{W_{12}}$ □. $\mathbf{W_{22}}$ □.$\mathbf{W_{11}}$, we will get the output as $\mathbf{W_{13}}$□ $\mathbf{W_{12}}$ □ $\mathbf{W_{22}}$ □.$\mathbf{W_{11}}$.



- **Apply Unit resolution on $\mathbf{W_{13}}$ □ $\mathbf{W_{12}}$ □ $\mathbf{W_{22}}$ □$\mathbf{W_{11}}$ and $\mathbf{\neg W_{11}}$:**

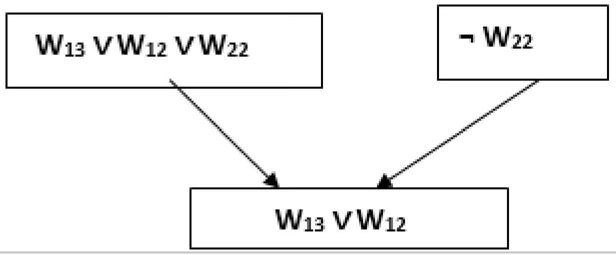
After applying Unit resolution formula on $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ and $\neg W_{11}$ we will get $W_{13} \vee W_{12} \vee W_{22}$.



$W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$
$\neg W_{11}$
$(W_{13} \vee W_{12} \vee W_{22})$

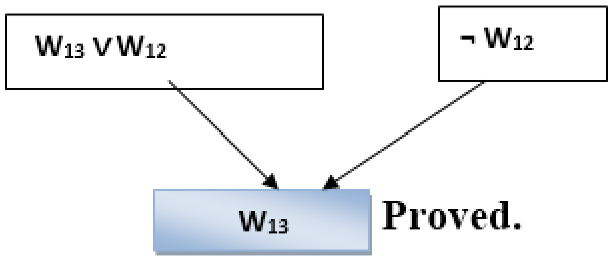
- **Apply Unit resolution on W_{13} □ W_{12} □ W_{22} and ¬ W_{22} :**

After applying Unit resolution on **W_{13} □ W_{12} □ W_{22}**, and **¬W_{22}**, we will get **W_{13} □ W_{12}** as output.



- **Apply Unit Resolution on W_{13} □ W_{12} and ¬ W_{12} :**

After Applying Unit resolution on **W_{13} □ W_{12} and ¬ W_{12}**, we will get **W_{13}** as an output, hence it is proved that the Wumpus is in the room [1, 3].



First-Order Logic in Artificial intelligence

In the topic of Propositional logic, we have seen that how to represent statements using propositional logic. But unfortunately, in propositional logic, we can only represent the facts, which are either true or false. PL is not sufficient to represent the complex sentences or natural language statements. The propositional logic has very limited expressive power. Consider the following sentence, which we cannot represent using PL logic.

- "Some humans are intelligent", or
- "Sachin likes cricket."

To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

First-Order logic:

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as Predicate logic or First-order predicate logic. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.
- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
    - Objects: A, B, people, numbers, colors, wars, theories, squares, pits, wumpus ...
    - Relations: It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
    - Function: Father of, best friend, third inning of, end of, ...

As a natural language, first-order logic also has two main parts:

d. Syntax

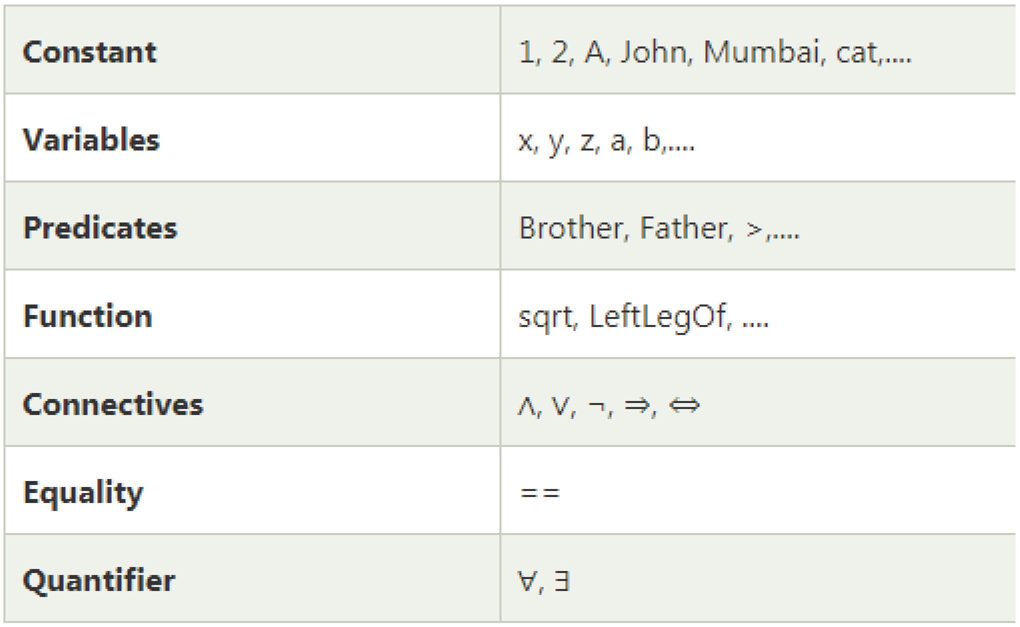
e. Semantics Syntax of

First-Order logic:

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first- order logic are symbols. We write statements in short-hand notationin FOL.

Basic Elements of First-order logic:

Following are the basic elements of FOL syntax:



Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

Atomic sentences:

- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as Predicate (term1, term2, ..., term n).

Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).
Chinky is a cat: => cat (Chinky).

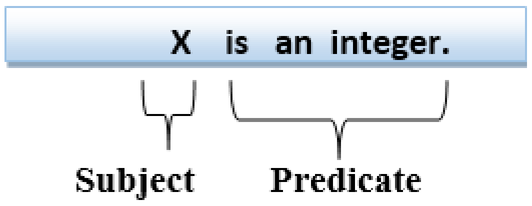
Complex Sentences:

- Complex sentences are made by combining atomic sentencesusing connectives.

First-order logic statements can be divided into two parts:

- Subject: Subject is the main part of the statement.
- Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.

Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



Quantifiers in First-order logic:

- A quantifier is a language element which generates quantification, and quantification specifies the quantity ofspecimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:

a. Universal Quantifier, (for all, everyone, everything)

b. Existential quantifier, (for some, at least one).Universal

Quantifier:

Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.

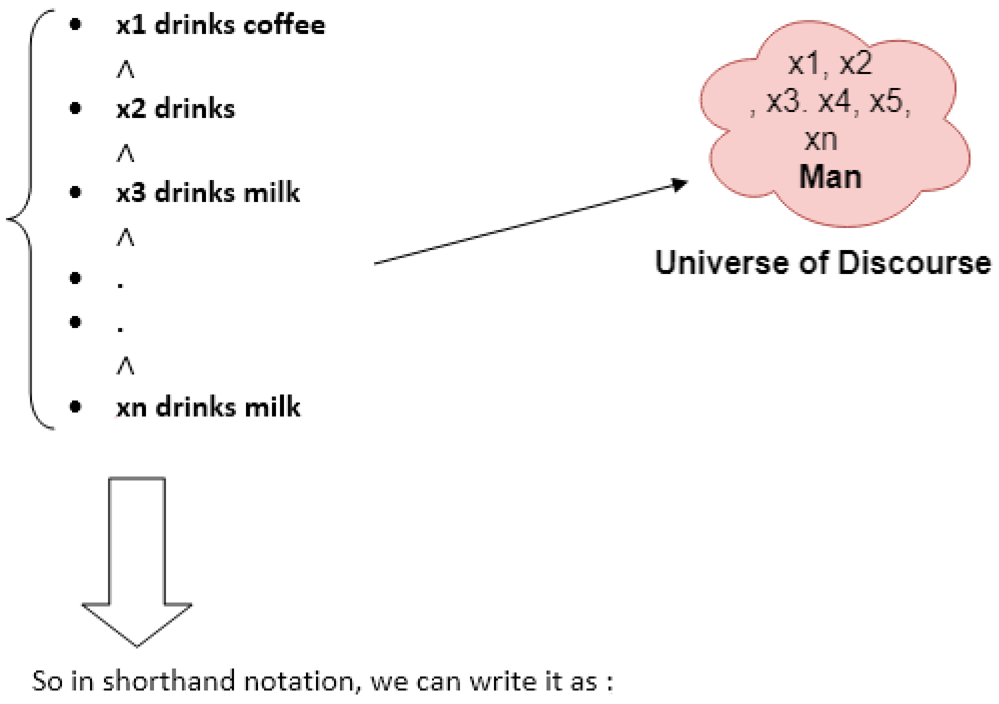
If x is a variable, then ∀x is read as:

- For all x
- For each x
- For every x.

Example:

All man drink coffee.

Let a variable x which refers to a cat so all x can be represented inUOD as below:



∀x man(x) → drink (x, coffee).

It will be read as: There are all x where x is a man who drink coffee. Existential

Quantifier:

Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.

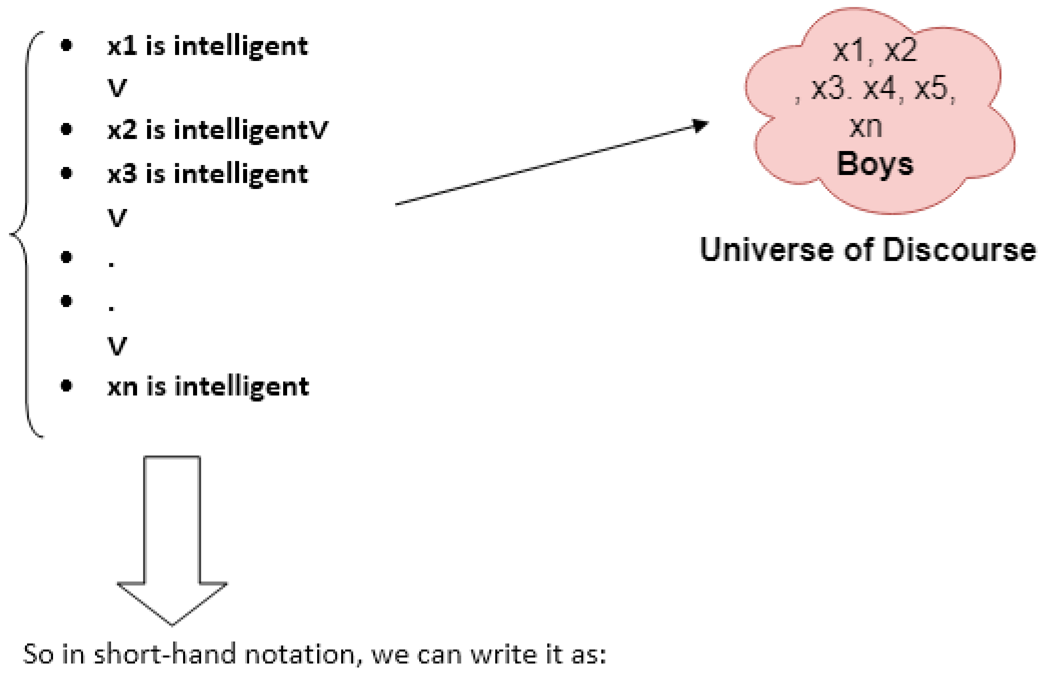
It is denoted by the logical operator ∃, which resembles as inverted E.When it is used with a predicate variable then it is called as an existential quantifier.

If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:

- There exists a 'x.'
- For some 'x.'
- For at least one 'x.'

Example:

Some boys are intelligent.



Ex: boys(x) ∧ intelligent(x)

It will be read as: There are some x where x is a boy who is intelligent.

- The main connective for universal quantifier ∀ is implication →.
- The main connective for existential quantifier ∃ is and ∧.

Properties of Quantifiers:

- In universal quantifier, ∀x∀y is similar to ∀y∀x.
- In Existential quantifier, ∃x∃y is similar to ∃y∃x.
- ∃x∀y is not similar to ∀y∃x.

Some Examples of FOL using quantifier:

1. All birds fly.
   In this question the predicate is "fly(bird)."
   And since there are all birds who fly so it will be representedas follows.
   ∀x bird(x) →fly(x).

2. Every man respects his parent.
   In this question, the predicate is "respect(x, y)," where x=man,and y= parent.

   Since there is every man so will use ∀, and it will be represented
   ∀x man(x) → respects (x, parent).

3. Some boys play cricket.
   In this question, the predicate is "play(x, y)," where x= boys, and y= game.
   Since there are some boys so we will use ∃, andit will be represented as:
   ∃x boys(x) → play(x, cricket).

4. Not all students like both Mathematics and Science.
   In this question, the predicate is "like(x, y)," where x= student,and y= subject.
   Since there are not all students, so we will use ∀ withnegation, so
   following representation for this:
   ¬∀ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].

5. Only one student failed in Mathematics.
   In this question, the predicate is "failed(x, y)," where x=student, and y=
   subject.
   Since there is only one student who failed in Mathematics, sowe will use
   following representation for this:
   ∃(x) [student(x) → failed (x, Mathematics) ∧∀ (y) [¬(x==y) ∧
   student(y) → ¬failed (x, Mathematics)].

Free and Bound Variables:

The quantifiers interact with variables which appear in a suitable way. There are two types of variables in First-order logic which are given below:

Free Variable: A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

Example: $\forall x\ \exists(y)[P(x, y, z)]$, where z is a free variable.

Bound Variable: A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

Example: $\forall x\ [A(x)\ B(y)]$, here x and y are the bound variables.

Inference in First-Order Logic

Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used inFOL.

Substitution:

Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write F[a/x], so it refers to substitute a constant "a" in place of variable "x".

Equality:

First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality inFOL. For this, we can use equality symbols which specify that thetwo terms refer to the same object.

Example: Brother (John) = Smith.

As in the above example, the object referred by the Brother (John) is similar to the object referred by Smith. The equality symbol can also be used with negation to represent that two terms are not the same objects.

Example: $\neg(x=y)$ which is equivalent to $x \neq y$.FOL

inference rules for quantifier:

As propositional logic we also have inference rules in first-order logic,so following are some basic inference rules in FOL:

- Universal Generalization
- Universal Instantiation
- Existential Instantiation
- Existential introduction

1. Universal Generalization:

- Universal generalization is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x).
- It can be represented as: $\frac{P(c)}{\forall x\, P(x)}$.
- This rule can be used if we want to show that every element has a similar property.
- In this rule, x must not appear as a free variable.

Example: Let's represent, P(c): "A byte contains 8 bits", so for ∀ xP(x) "All bytes contain 8 bits.", it will also be true.

2. Universal Instantiation:

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.
- The new KB is logically equivalent to the previous KB.
- As per UI, we can infer any sentence obtained by substituting a ground term for the variable.
- The UI rule state that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from ∀ x P(x) for any object in the universe of discourse.
- It can be represented as: $\frac{\forall x\, P(x)}{P(c)}$.

Example:1.

IF "Every person like ice-cream"=> ∀x P(x) so we can infer that"John likes ice-cream" => P(c)

Example: 2.

Let's take a famous example,

"All kings who are greedy are Evil." So let our knowledge basecontains this detail as in the form of FOL:

∀x king(x) ∧ greedy (x) → Evil (x),

So from this information, we can infer any of the following statements using Universal Instantiation:

- King(John) ∧ Greedy (John) → Evil (John),
- King(Richard) ∧ Greedy (Richard) → Evil (Richard),

- King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John)),

3. Existential Instantiation:

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.
- The new KB is not logically equivalent to old KB, but it will besatisfiable if old KB was satisfiable.
- This rule states that one can infer P(c) from the formula given inthe form of ∃x P(x) for a new constant symbol c.
- The restriction with this rule is that c used in the rule must bea new term for which P(c) is true.
- It can be represented as: $\frac{\exists x\, P(x)}{P(c)}$

Example:

From the given sentence: ∃x Crown(x) ∧ OnHead(x, John),

So we can infer: Crown(K) ∧ OnHead(K, John), as long as K does notappear in the knowledge base.

- The above used K is a constant symbol, which is called Skolemconstant.
- The Existential instantiation is a special case of Skolemizationprocess.

4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P, then we can infer that there exists something in the universe which has the propertyP.
- It can be represented as: $\frac{P(c)}{\exists x P(x)}$
- Example: Let's say that,
  "Priyanka got good marks in English." "Therefore, someone
  got good marks in English."

Generalized Modus Ponens Rule:

For the inference process in FOL, we have a single inference rule which is called Generalized Modus Ponens. It is lifted version of Modus ponens.

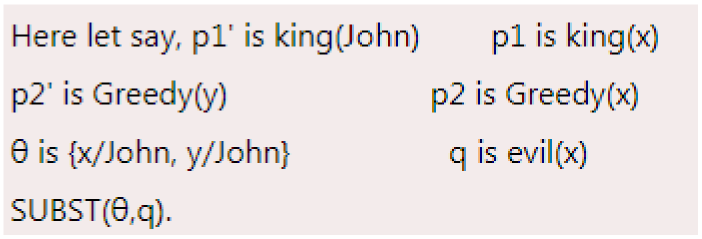
Generalized Modus Ponens can be summarized as, " P implies Q and P is asserted to be true, therefore Q must be True."

According to Modus Ponens, for atomic sentences pi, pi', q. Where there is a substitution θ such that SUBST (θ, pi',) = SUBST(θ, pi), it can be represented as:

$$\frac{p1', p2', \ldots, pn', (p1 \land p2 \land \ldots \land pn \Rightarrow q)}{\mathrm{SUBST}(\theta, q)}$$

Example:

We will use this rule for Kings are evil, so we will find some x suchthat x is king, and x is greedy so we can infer that x is evil.



Here let say, p1' is king(John)	p1 is king(x)
p2' is Greedy(y)	p2 is Greedy(x)
θ is {x/John, y/John}	q is evil(x)
SUBST(θ,q).	

Forward Chaining and Backward chaining

In artificial intelligence, forward and backward chaining is one of the important topics, but before understanding forward and backward chaining lets first understand that from where these two terms came.

Inference engine:

The inference engine is the component of the intelligent system in artificial intelligence, which applies logical rules to the knowledge base to infer new information from known facts. The first inference engine was part of the expert system. Inference engine commonly proceeds in two modes, which are:

- ❖ Forward chaining
- ❖ Backward chaining Horn

Clause and Definite clause:

Horn clause and definite clause are the forms of sentences, which enables knowledge base to use a more restricted and efficient inference algorithm. Logical inference algorithms use forward and backward chaining approaches, which require KB in the form of the first-order definite clause.

Definite clause: A clause which is a disjunction of literals with exactly one positive literal is known as a definite clause or strict horn clause.

Horn clause: A clause which is a disjunction of literals with at most one positive literal is known as horn clause. Hence all the definite clauses are horn clauses.

Example: $(\neg p \vee \neg q \vee k)$. It has only one positive literal k. It is

equivalent to $p \wedge q \rightarrow k$.

A. Forward Chaining

Forward chaining is also known as a forward deduction or forward reasoning method when using an inference engine. Forward chaining is a form of reasoning which start with atomic sentences in the knowledge base and applies inference rules (Modus Ponens) in the forward direction to extract more data until a goal is reached.

The Forward-chaining algorithm starts from known facts, triggers all rules whose premises are satisfied, and add their conclusion to the known facts. This process repeats until the problem is solved.

Properties of Forward-Chaining:

- o It is a down-up approach, as it moves from bottom to top.
- o It is a process of making a conclusion based on known facts or data, by starting from the initial state and reaches the goal state.
- o Forward-chaining approach is also called as data-driven as we reach to the goal

using available data.

- Forward -chaining approach is commonly used in the expert system, such as CLIPS, business, and production rule systems.

Consider the following famous example which we will use in both approaches:

Example:

"As per the law, it is a crime for an American to sell weapons tohostile nations. Country A, an enemy of America, has some missiles, and all the missiles were sold to it by Robert, who is an American citizen."

Prove that "Robert is criminal."

To solve the above problem, first, we will convert all the above facts into first-order definite clauses, and then we will use a forward- chaining algorithm to reach the goal.

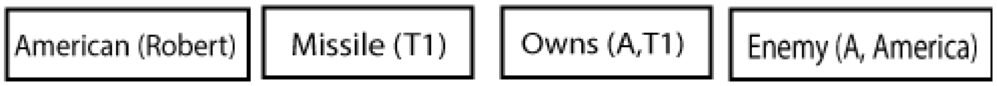
Facts Conversion into FOL:

- It is a crime for an American to sell weapons to hostile nations.(Let's say p, q, and r are variables)
  American (p) ∧ weapon(q) ∧ sells (p, q, r) ∧ hostile(r) →Criminal(p) ...(1)
- Country A has some missiles. ?p Owns(A, p) ∧ Missile(p). It canbe written in two definite clauses by using Existential Instantiation, introducing new Constant T1.
  Owns(A, T1)..............................(2)
  Missile(T1)...............................(3)
- All of the missiles were sold to country A by Robert.
  ?p Missiles(p) ∧ Owns (A, p) → Sells (Robert, p, A)...............................(4)
- Missiles are weapons.
  Missile(p) → Weapons (p)....................................(5)
- Enemy of America is known as hostile. Enemy(p, America) →Hostile(p)..(6)
- Country A is an enemy of America. Enemy (A, America)..(7)
- Robert is American American(Robert) ......(8)

Forward chaining proof:

Step-1:

In the first step we will start with the known facts and will choose the sentences which do not have implications, such as: American(Robert), Enemy(A, America), Owns(A, T1), and Missile(T1). All these facts will be represented as below.



Step-2:

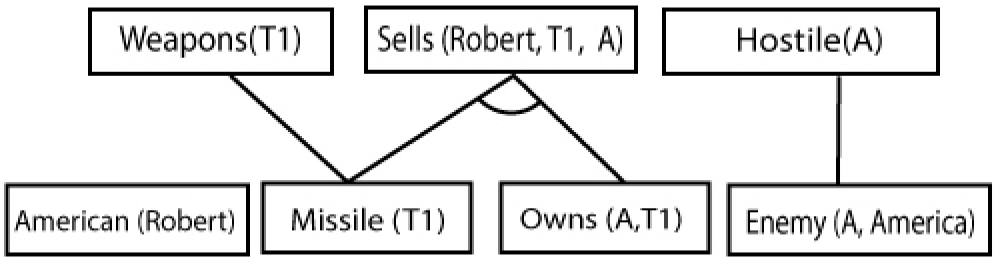
At the second step, we will see those facts which infer from availablefacts and with satisfied premises.

Rule-(1) does not satisfy premises, so it will not be added in the firstiteration.

Rule-(2) and (3) are already added.

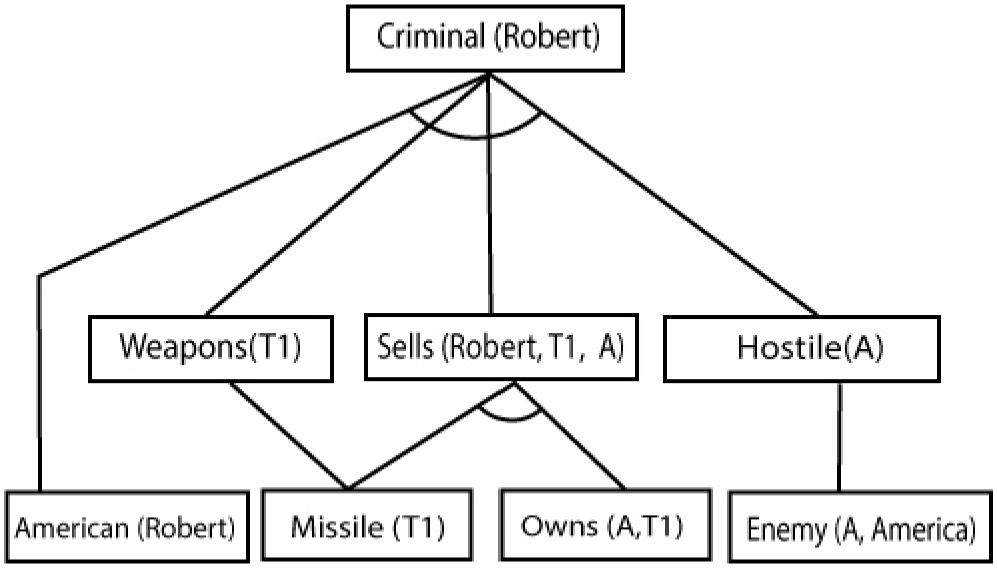
Rule-(4) satisfy with the substitution {p/T1}, so Sells (Robert, T1,A) is added, which infers from the conjunction of Rule (2) and (3).

Rule-(6) is satisfied with the substitution(p/A), so Hostile(A) is addedand which infers from Rule-(7).



Step-3:

At step-3, as we can check Rule-(1) is satisfied with the substitution {p/Robert, q/T1, r/A}, so we can add Criminal(Robert) which infers all the available facts. And hence we reached our goal statement.



Hence it is proved that Robert is Criminal using forward chaining approach.

B. Backward Chaining:

Backward-chaining is also known as a backward deduction or backward reasoning method when using an inference engine. A backward chaining algorithm is a form of reasoning, which starts with the goal and works backward, chaining through rules to find known facts that support the goal.

Properties of backward chaining:

- It is known as a top-down approach.
- Backward-chaining is based on modus ponens inference rule.
- In backward chaining, the goal is broken into sub-goal or sub- goals to prove the facts true.
- It is called a goal-driven approach, as a list of goals decides which rules are selected and used.
- Backward -chaining algorithm is used in game theory, automated theorem proving tools, inference engines, proof assistants, and various AI applications.
- The backward-chaining method mostly used a depth-first search strategy for proof.

Example:

In backward-chaining, we will use the same above example, and willrewrite all the rules.

- American (p) Λ weapon(q) Λ sells (p, q, r) Λ hostile(r) →Criminal(p) ...(1)
  Owns(A, T1)..(2)
- Missile(T1)
- ?p Missiles(p) Λ Owns (A, p) → Sells (Robert, p, A).......................................(4)
- Missile(p) → Weapons (p) .... (5)

- Enemy(p, America) →Hostile(p) ....(6)
- Enemy (A, America) .... (7)
- American(Robert) .... (8)

Backward-Chaining proof:

In Backward chaining, we will start with our goal predicate, whichis Criminal (Robert), and then infer further rules.

Step-1:

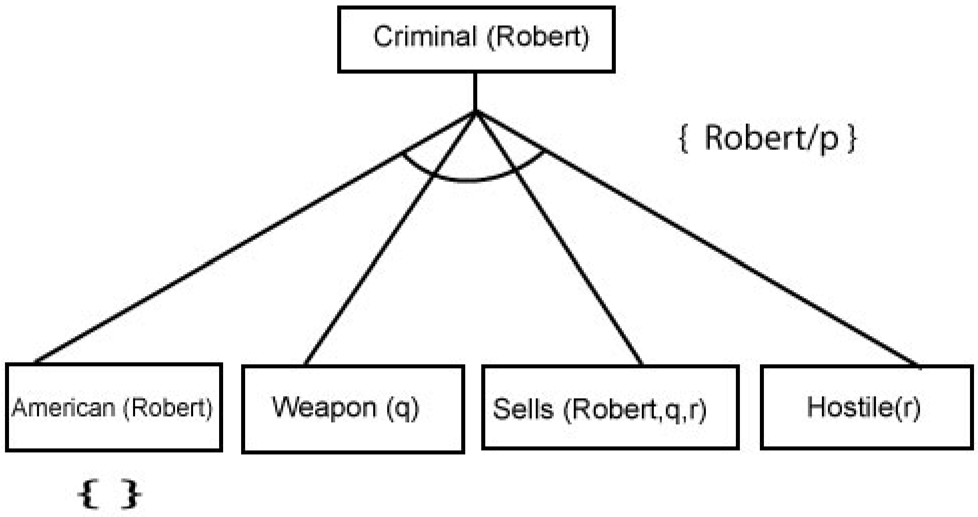
At the first step, we will take the goal fact. And from the goal fact, we will infer other facts, and at last, we will prove those facts true. So our goal fact is "Robert is Criminal," so following is the predicate of it.

Criminal (Robert)

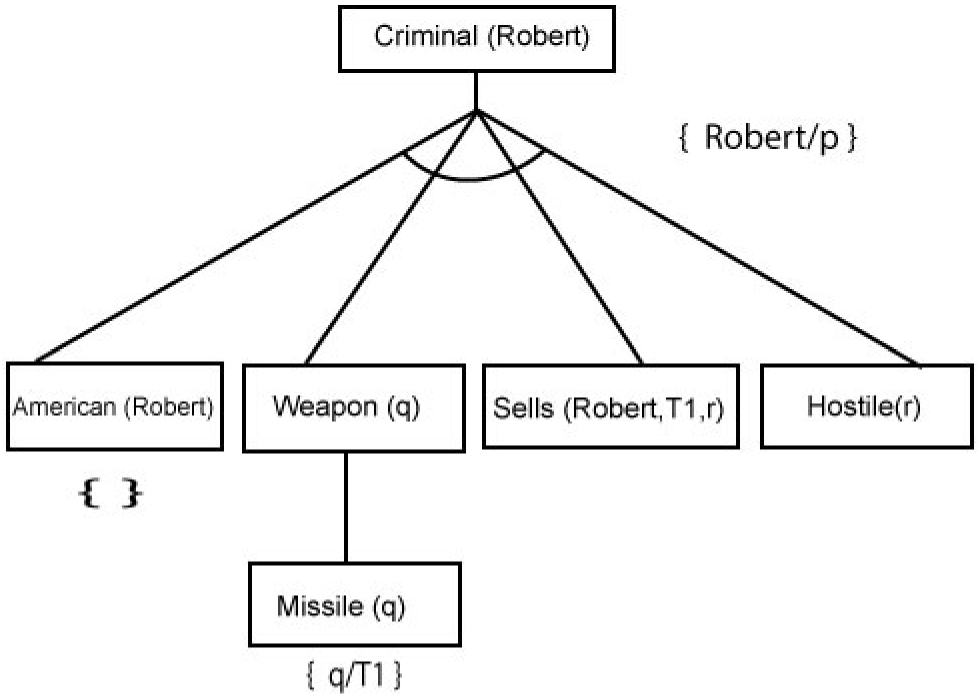
Step-2:

At the second step, we will infer other facts form goal fact which satisfies the rules. So as we can see in Rule-1, the goal predicate Criminal (Robert) is present with substitution {Robert/P}. So we will add all the conjunctive facts below the first level and will replace p with Robert.

Here we can see American (Robert) is a fact, so it is proved here.

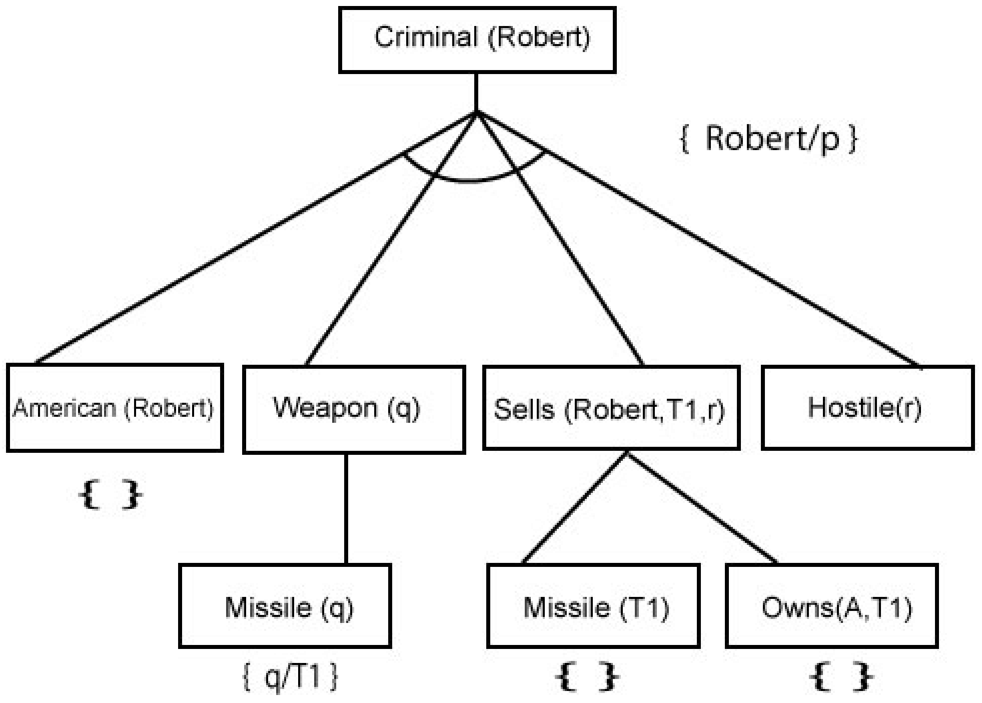


Step-3:t At step-3, we will extract further fact Missile(q) which infer from Weapon(q), as it satisfies Rule-(5). Weapon (q) is also true with the substitution of a constant T1 at q.



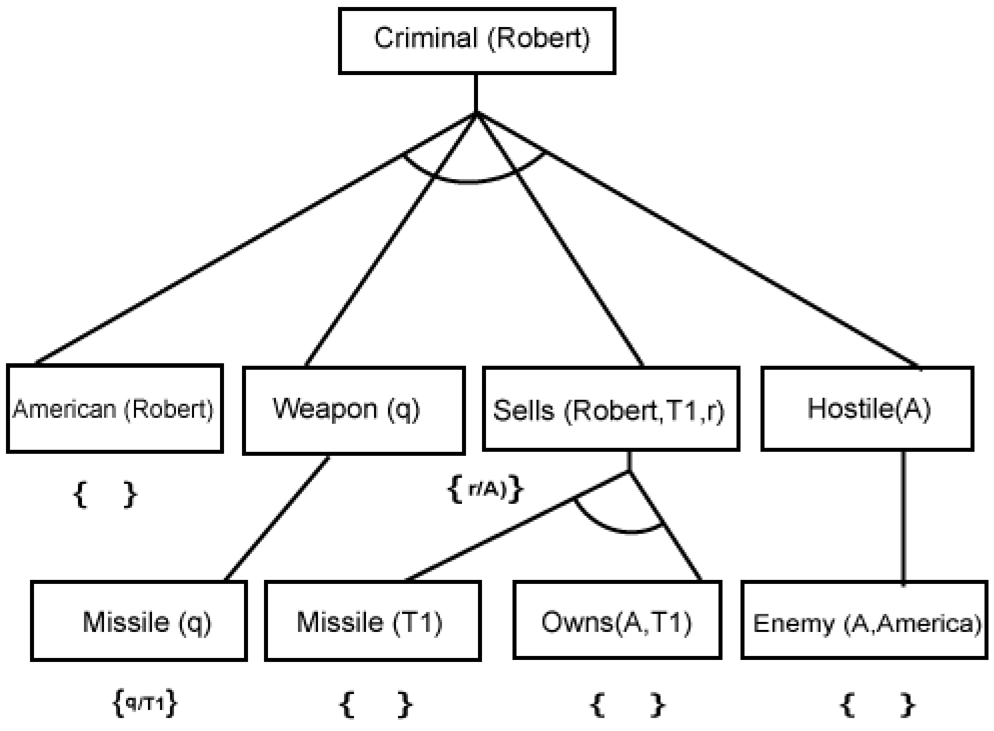
Step-4:

At step-4, we can infer facts Missile(T1) and Owns(A, T1) form Sells(Robert, T1, r) which satisfies the Rule- 4, with the substitution of A in place of r. So these two statements are proved here.



Step-5:

At step-5, we can infer the fact Enemy(A,America) from Hostile(A) whichsatisfies Rule-6. And hence all the statements are proved true using backward chaining.



Difference between backward chaining and forward chaining

S. No.	Forward Chaining	Backward Chaining
1.	Forward chaining starts from known facts and applies inference rule to extract more data unit it reaches to the goal.	Backward chaining starts from the goal and works backward through inference rules to find the required facts that support the goal.
2.	It is a bottom-up approach	It is a top-down approach
3.	Forward chaining is known as data-driven inference technique as we reach to the goal using the available data.	Backward chaining is known as goal-driven technique as we start from the goal and divide into sub-goal to extract the facts.
4.	Forward chaining reasoning applies a breadth-first search strategy.	Backward chaining reasoning applies a depth-first search strategy.
5.	Forward chaining tests for all the available rules	Backward chaining only tests for few required rules.
6.	Forward chaining is suitable forthe planning, monitoring, control, and interpretation application.	Backward chaining is suitable for diagnostic, prescription, and debugging application.
7.	Forward chaining can generate an infinite number of possible conclusions.	Backward chaining generates a finite number of possible conclusions.
8.	It operates in the forward direction.	It operates in the backward direction.
9.	Forward chaining is aimed for any conclusion.	Backward chaining is only aimed for the required data.

Unification

- Unification is a process of making two different logical atomic expressions identical by finding a substitution. Unification depends on the substitution process.
- It takes two literals as input and makes them identical using substitution.
- Let $\Psi 1$ and $\Psi 2$ be two atomic sentences and σ be a unifier such that, $\Psi 1\sigma = \Psi 2\sigma$, then it can be expressed as UNIFY($\Psi 1$, $\Psi 2$).
- Example: Find the MGU for Unify{King(x), King(John)}

Let Ψ_1 = King(x), Ψ_2 = King (John),

Substitution θ = {John/x} is a unifier for these atoms and applying this substitution, and both expressions will be identical.

- The UNIFY algorithm is used for unification, which takes two atomic sentences and returns a unifier for those sentences (If any exist).
- Unification is a key component of all first-order inference algorithms.
- It returns fail if the expressions do not match with each other.
- The substitution variables are called Most General Unifier or MGU.

E.g. Let's say there are two different expressions, P(x, y), and P(a,f(z)).

In this example, we need to make both above statements identical to each other. For this, we will perform the substitution.

P(x, y).............. (i)
P(a, f(z)).................(ii)

- Substitute x with a, and y with f(z) in the first expression, and it will be represented as a/x and f(z)/y.
- With both the substitutions, the first expression will be identical to the second expression and the substitution set will be: [a/x, f(z)/y].

Conditions for Unification:

Following are some basic conditions for unification:

- Predicate symbol must be same, atoms or expression withdifferent predicate symbol can never be unified.
- Number of Arguments in both expressions must be identical.
- Unification will fail if there are two similar variables present inthe same expression.

Unification Algorithm:

Algorithm: Unify(Ψ1, Ψ2)

```
Step. 1: If Ψ1 or Ψ2 is a variable or constant, then:
        a) If Ψ1 or Ψ2 are identical, then return NIL.
        b) Else if Ψ1is a variable,
                a. then if Ψ1 occurs in Ψ2, then return FAILURE
                b. Else return { (Ψ2/ Ψ1)}.
        c) Else if Ψ2 is a variable,
                a. If Ψ2 occurs in Ψ1 then return FAILURE,
                b. Else return {( Ψ1/ Ψ2)}.
        d) Else return FAILURE.

Step.2: If the initial Predicate symbol in Ψ1 and Ψ2 are not same,then return
            FAILURE.
Step. 3: IF Ψ1 and Ψ2 have a different number of arguments, thenreturn
            FAILURE.
Step. 4: Set Substitution set(SUBST) to NIL. Step. 5: For
i=1 to the number of elements in Ψ1.
        a) Call Unify function with the ith element of Ψ1 and ithelement of
           Ψ2, and put the result into S.
        b) If S = failure then returns Failure
        c) If S ≠ NIL then do,
                a. Apply S to the remainder of both L1 and L2.
                b. SUBST= APPEND(S, SUBST).
Step.6: Return SUBST.
```

Implementation of the Algorithm

Step.1: Initialize the substitution set to be empty.Step.2:

Recursively unify atomic sentences:

a. Check for Identical expression match.

b. If one expression is a variable v_i, and the other is a termt_i which does not contain variable v_i, then:

a. Substitute t_i / v_i in the existing substitutions

b. Add t_i /v_i to the substitution setlist.

c. If both the expressions are functions, then function name mustbe similar, and the number of arguments must be the same in both the expression.

For each pair of the following atomic sentences find the most generalunifier (If exist).

1. Find the MGU of {p(f(a), g(Y)) and p(X, X)}

Sol: S_0 => Here, Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(X, X)SUBST
θ= {f(a) / X}
S_1 => Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(f(a), f(a))SUBST
θ= {f(a) / g(y)}, Unification failed.

Unification is not possible for these expressions.

2. Find the MGU of {p(b, X, f(g(Z))) and p(Z, f(Y), f(Y))}Here,

Ψ_1 = p(b, X, f(g(Z))) , and Ψ_2 = p(Z, f(Y), f(Y))
S_0 => { p(b, X, f(g(Z))); p(Z, f(Y), f(Y))}
SUBST θ={b/Z}

S_1 => { p(b, X, f(g(b))); p(b, f(Y), f(Y))}
SUBST θ={f(Y) /X}

S_2 => { p(b, f(Y), f(g(b))); p(b, f(Y), f(Y))}SUBST θ=
{g(b) /Y}

S_2 => { p(b, f(g(b)), f(g(b)); p(b, f(g(b)), f(g(b))} Unified Successfully.And Unifier = { b/Z, f(Y) /X , g(b) /Y}.

3. Find the MGU of {p (X, X), and p (Z, f(Z))}Here,

Ψ_1 = {p (X, X), and Ψ_2 = p (Z, f(Z))
S_0 => {p (X, X), p (Z, f(Z))}
SUBST θ= {X/Z}
S1 => {p (Z, Z), p (Z, f(Z))}
SUBST θ= {f(Z) / Z}, Unification Failed.

Hence, unification is not possible for these expressions.

4. Find the MGU of UNIFY(prime (11), prime(y))

Here, Ψ1 = {prime(11) , and Ψ2 = prime(y)}S0 =>
{prime(11) , prime(y)}
SUBST θ= {11/y}

S1 => {prime (11) , prime(11)} , Successfully unified.
Unifier: {11/y}.

5. Find the MGU of Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)}

Here, Ψ1 = Q(a, g(x, a), f(y)), and Ψ2 = Q(a, g(f(b), a), x)
S0 => {Q(a, g(x, a), f(y)); Q(a, g(f(b), a), x)}
SUBST θ= {f(b)/x}
S1 => {Q(a, g(f(b), a), f(y)); Q(a, g(f(b), a), f(b))}

SUBST θ= {b/y}
S1 => {Q(a, g(f(b), a), f(b)); Q(a, g(f(b), a), f(b))}, Successfully Unified.Unifier: [a/a,

f(b)/x, b/y].

6. UNIFY(knows(Richard, x), knows(Richard, John))

Here, Ψ1 = knows(Richard, x), and Ψ2 = knows(Richard, John)S0 => {
knows(Richard, x); knows(Richard, John)}
SUBST θ= {John/x}
S1 => { knows(Richard, John); knows(Richard, John)}, SuccessfullyUnified.
Unifier: {John/x}.

Resolution

Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions. It was invented by a Mathematician John Alan Robinson in the year 1965.

Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements. Unification is a key concept in proofs by resolutions. Resolution is a single inference rule which can efficiently operate on the conjunctive normal form or clausal form.

Clause: Disjunction of literals (an atomic sentence) is called a clause. It is also known as a unit clause.

Conjunctive Normal Form: A sentence represented as a conjunctionof clauses is said to be conjunctive normal form or CNF.

The resolution inference rule:

The resolution rule for first-order logic is simply a lifted version of the propositional rule. Resolution can resolve two clauses if they contain complementary literals, which are assumed to be standardized apartso that they share no variables.

$$\frac{l_1 \vee \ldots\ldots \vee l_k, \qquad m_1 \vee \ldots\ldots \vee m_n}{\mathrm{SUBST}(\theta, l_1 \vee \ldots\ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots\ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

Where l_i and m_j are complementary literals.

This rule is also called the binary resolution rule because it onlyresolves exactly two literals.

Example:

We can resolve two clauses which are given below:

[Animal (g(x) V Loves (f(x), x)]   and   [¬Loves(a, b) V ¬Kills(a, b)]

Where two complimentary literals are: Loves (f(x), x) and ¬ Loves (a,b)

These literals can be unified with unifier θ= [a/f(x), and b/x] , and itwill generate a resolvent clause:

[Animal (g(x) V ¬Kills(f(x), x)].Steps

for Resolution:

1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

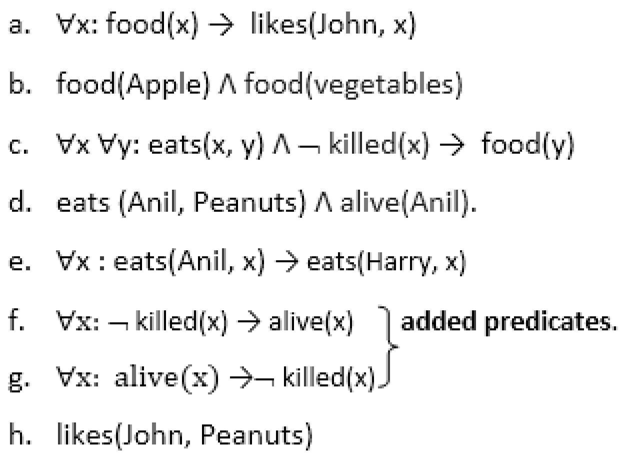
To better understand all the above steps, we will take an example inwhich we will apply resolution.

Example:

a. John likes all kind of food.
b. Apple and vegetable are food
c. Anything anyone eats and not killed is food.
d. Anil eats peanuts and still alive
e. Harry eats everything that Anil eats.Prove by resolution that:
f. John likes peanuts.

Step-1: Conversion of Facts into FOL

In the first step we will convert all the given statements into its firstorder logic.



a. $\forall x: \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$
b. $\text{food}(\text{Apple}) \land \text{food}(\text{vegetables})$
c. $\forall x \forall y: \text{eats}(x, y) \land \neg \text{killed}(x) \rightarrow \text{food}(y)$
d. $\text{eats}(\text{Anil}, \text{Peanuts}) \land \text{alive}(\text{Anil})$.
e. $\forall x : \text{eats}(\text{Anil}, x) \rightarrow \text{eats}(\text{Harry}, x)$
f. $\forall x: \neg \text{killed}(x) \rightarrow \text{alive}(x)$ **added predicates.**
g. $\forall x: \text{alive}(x) \rightarrow \neg \text{killed}(x)$
h. $\text{likes}(\text{John}, \text{Peanuts})$

Step-2: Conversion of FOL into CNF

In First order logic resolution, it is required to convert the FOL intoCNF as CNF form makes easier for resolution proofs.

- Eliminate all implication (→) and rewrite

    a. $\forall x \neg \text{food}(x) \lor \text{likes}(\text{John}, x)$

b. food(Apple) Λ food(vegetables)

c. ∀ x ∀ y ¬ [eats(x, y) Λ ¬ killed(x)] V food(y)

d. eats (Anil, Peanuts) Λ alive(Anil)

e. x ¬ eats(Anil, x) V eats(Harry, x)

f. ∀ x¬ [¬ killed(x)] V alive(x)

g. ∀ x ¬ alive(x) V ¬ killed(x)

h. likes(John, Peanuts).

- Move negation (¬)inwards and rewrite

  . ∀ x ¬ food(x) V likes(John, x)

  a. food(Apple) Λ food(vegetables)

  b. ∀ x ∀ y ¬ eats(x, y) V killed(x) V food(y)

  c. eats (Anil, Peanuts) Λ alive(Anil)

  d. ∀ x ¬ eats(Anil, x) V eats(Harry, x)

  e. ∀ x ¬killed(x)] V alive(x)

  f. ∀ x ¬ alive(x) V ¬ killed(x)

  g. likes(John, Peanuts).

- Rename variables or standardize variables

. ∀ x ¬ food(x) V likes(John, x)

  a. food(Apple) Λ food(vegetables)

  b. ∀ y ∀ z ¬ eats(y, z) V killed(y) V food(z)

  c. eats (Anil, Peanuts) Λ alive(Anil)

  d. ∀ w¬ eats(Anil, w) V eats(Harry, w)

  e. ∀ g ¬killed(g)] V alive(g)

  f. ∀ k ¬ alive(k) V ¬ killed(k)

  g. likes(John, Peanuts).

- Eliminate existential instantiation quantifier by elimination. In this step, we will eliminate existential quantifier ∃ , and this process is known as Skolemization. But in this example problem since there is no existential quantifier so all the statements will remain same in this step.

Drop Universal quantifiers.

In this step we will drop all universal quantifier since all thestatements are not implicitly quantified so we don't need it.

. ¬ food(x) V likes(John, x)

a. food(Apple)

b. food(vegetables)

c. ¬ eats(y, z) V killed(y) V food(z)

d. eats (Anil, Peanuts)

e. alive(Anil)

f. ¬ eats(Anil, w) V eats(Harry, w)

g. killed(g) V alive(g)

h. ¬ alive(k) V ¬ killed(k)

i. likes(John, Peanuts).

- Distribute conjunction □ over disjunction ¬.

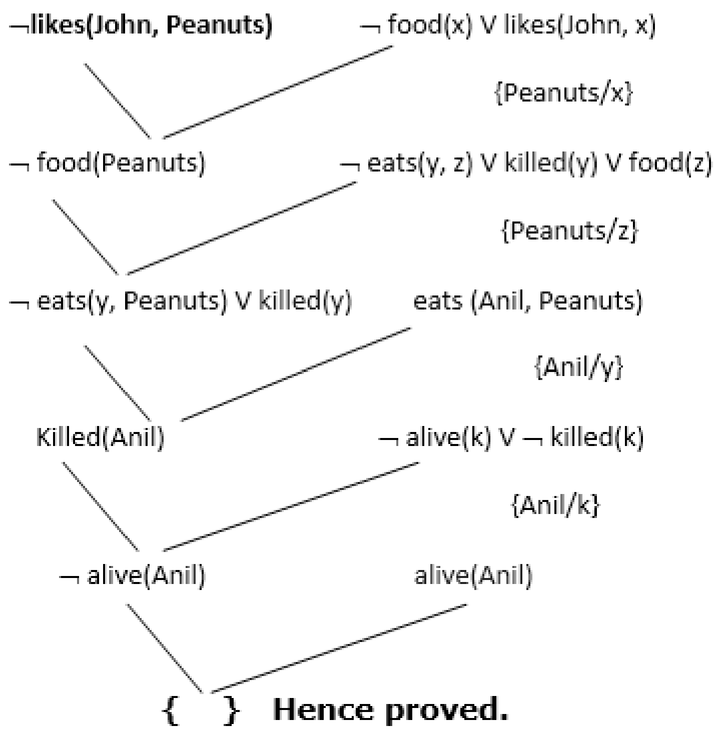
  This step will not make any change in this problem.

Step-3: Negate the statement to be proved

In this statement, we will apply negation to the conclusion statements, which will be written as ¬likes(John, Peanuts)

Step-4: Draw Resolution graph:

Now in this step, we will solve the problem by resolution tree usingsubstitution. For the above problem, it will be given as follows:



Hence the negation of the conclusion has been proved as a completecontradiction with the given set of statements.

Explanation of Resolution graph:

- In the first step of resolution graph, ¬likes(John, Peanuts) ,and likes(John, x) get resolved(canceled) by substitution of {Peanuts/x}, and we are left with ¬ food(Peanuts)
- In the second step of the resolution graph, ¬ food(Peanuts) , and food(z) get resolved (canceled) by substitution of { Peanuts/z}, and we are left with ¬ eats(y, Peanuts) V killed(y) .
- In the third step of the resolution graph, ¬ eats(y, Peanuts) and eats (Anil, Peanuts) get resolved by substitution {Anil/y}, and we are left with Killed(Anil) .
- In the fourth step of the resolution graph, Killed(Anil) and ¬ killed(k) get resolve by substitution {Anil/k}, and we are left with ¬ alive(Anil) .
- In the last step of the resolution graph ¬ alive(Anil) and alive(Anil) get resolved.