A031507 a(n) = smallest k>0 such that the elliptic curve y^2 = x^3 + k has rank n, or -1 if no such k exists. [13]

1, 2, 15, 113, 2089, 66265, 1358556

(list; graph; refs; listen; history; text; internal format)

OFFSET 0,2

COMMENTS See A031508 for the smallest negative k. - Artur Jasinski, Nov 21 2011

See A060950 for the rank of y^2 = x^3 + n. - Jonathan Sondow, Sep 10 2013

Gebel, Pethö, & Zimmer: "One experimental observation derived from the tables is that the rank r of Mordell's curves grows according to r = O(log |k|/|log log |k||^(2/3))." Hence this fit suggests a(n) >> exp(n (log n)^(1/3)) where >> is the Vinogradov symbol. - Charles R Greathouse IV, Sep 10 2013

The curves for k and -27*k are isogenous (as Noam Elkies points out---see Womack), so they have the same rank. - Jonathan Sondow, Sep 10 2013

Womack (2003) gives further upper bounds: a(7) <= 47550317, a(8) <= 1632201497, a(9) <= 185418133372, a(10) <= 68513487607153. - M. F. Hasler, Jul 01 2024

The three questions for arbitrary k, positive k, and negative k are not very far from each other because the curves for k and -27k are related by a 3-isogeny and therefore have the same rank. It would be most natural to ask for the minimal |k| for k of either sign [see A373795]. - Noam D. Elkies, Jul 02 2024

a(16) <= 116022135446156525663120520788 (Elkies, ANTS-XVI, 2024). The same article also establishes the existence of a value of k which has rank >= 17. - N. J. A. Sloane, Jul 05 2024

REFERENCES Noam D. Elkies, Rank of an elliptic curve and 3-rank of a quadratic field via the Burgess bounds, 2024 Algorithmic Number Theory Symposium, ANTS-XVI, MIT, July 2024.

LINKS Table of n, a(n) for n=0..6.

J. E. Cremona, Elliptic Curve Data

Noam D. Elkies and Zev Klagsbrun, New rank records for elliptic curves having rational torsion, ANTS XIV–Proceedings of the Fourteenth Algorithmic Number

Theory Symposium, 233-250. Mathematical Sciences Publishers, Berkeley, CA, 2020.
J. Gebel, Integer points on Mordell curves, web.archive.org copy of the "MORDELL+" file on the SIMATH web site shut down in 2017. [Locally cached copy].
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J. Quer, Corps quadratiques de 3-rang 6 et courbes elliptiques de rang 12, C. R. Acad. Sc. Paris I, 305 (1987), 215-218.
Tom Womack, Explicit Descent on Elliptic Curves, PhD thesis, University of Nottingham, July 2003.
Tom Womack, Minimal-known positive and negative k for Mordell curves of given rank (personal web page, latest available snapshot on web.archive.org from Jan. 2017), last modified Oct. 2002.

FORMULA a(n) <= 27*A031508(n) and A031508(n) <= 27*a(n). - Jonathan Sondow, Sep 10 2013

EXAMPLE a(12) <= 27*A031508(12) <= 27*6533891544658786928 = 176415071705787247056 (from Quer 1987 and Womack). - Jonathan Sondow, Sep 10 2013

PROG (PARI) {A031507(n)=for(k=1, oo, ellrank(ellinit([0, k]))[1]==n && return(k))} \\ Use ellanalyticrank() for PARI version < 2.14. - M. F. Hasler, Jul 01 2024

CROSSREFS Cf. A031508, A373795.
See also A060950, A002150-A002155, A102833, A179124, A031507, A060951, A081119, A179136, A179137.
Sequence in context: A360432 A376327 A074622 * A207998 A246570 A052861
Adjacent sequences: A031504 A031505 A031506 * A031508 A031509 A031510

KEYWORD nonn,nice,hard,more

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EXTENSIONS Definition clarified by Jonathan Sondow, Oct 26 2013
Escape clause added to definition by N. J. A. Sloane, Jun 29 2024, because, as John Cremona reminds me, it is not known if k always exists.

STATUS approved