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THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES[®]

founded in 1964 by N. J. A. Sloane

(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

A031508 $a(n)$ = smallest $k > 0$ such that the elliptic curve $y^2 = x^3 - k$ has rank n , or -1 if no such k exists.

1, 2, 11, 174, 2351, 28279, 975379

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OFFSET 0,2

COMMENTS See [A031507](#) for the smallest $k > 0$ such that the elliptic curve $y^2 = x^3 + k$ has rank n . - [Jonathan Sondow](#), Sep 06 2013

See [A060951](#) for the rank of $y^2 = x^3 - n$. - [Jonathan Sondow](#), Sep 10 2013

Gebel, Pethö, & Zimmer: "One experimental observation derived from the tables is that the rank r of Mordell's curves grows according to $r = O(\log |k| / |\log \log |k||^{2/3})$." Hence this fit suggests $a(n) \gg \exp(n (\log n)^{1/3})$ where \gg is the Vinogradov symbol. - [Charles R Greathouse IV](#), Sep 10 2013

$a(7) \leq 56877643$. $a(8) \leq 2520963512$. $a(9) \leq 463066403167$. $a(10) \leq 56736325657288$. $a(11) \leq 46111487743732324$. $a(12) \leq 6533891544658786928$. See Table 3.3 in [Womack 2003]. - [Jose Aranda](#), Jun 30 2024

The three questions for arbitrary k , positive k , and negative k are not very far from each other because the curves for k and $-27k$ are related by a 3-isogeny and therefore have the same rank. It would be most natural to ask for the minimal $|k|$ for k of either sign [see [A373795](#)]. - [Noam D. Elkies](#), Jul 02 2024

$a(16) \leq 1160221354461565256631205207888$ (Elkies, ANTS-XVI, 2024). The same article also establishes the existence of a value of k which has rank ≥ 17 . - [N. J. A. Sloane](#), Jul 05 2024

REFERENCES Noam D. Elkies, Rank of an elliptic curve and 3-rank of a quadratic field via the Burgess bounds, 2024
Algorithmic Number Theory Symposium, ANTS-XVI, MIT, July 2024.

LINKS [Table of \$n\$, \$a\(n\)\$ for \$n=0..6\$.](#)

J. E. Cremona, [Elliptic Curve Data](#)

Noam D. Elkies and Zev Klagsbrun, [New rank records for elliptic curves having rational torsion](#), ANTS XIV—Proceedings of the Fourteenth Algorithmic Number

	<p>Theory Symposium, 233-250. Mathematical Sciences Publishers, Berkeley, CA, 2020.</p> <p>J. Gebel, Integer points on Mordell curves [Cached copy, after the original web site tnt.math.se.tmu.ac.jp was shut down in 2017]</p> <p>J. Gebel, A. Pethö, and H. G. Zimmer, On Mordell's equation, Compositio Mathematica 110 (1998), 335-367. MR1602064.</p> <p>Tom Womack, Explicit Descent on Elliptic Curves, PhD thesis, University of Nottingham, July 2003</p> <p>Tom Womack, Minimal-known positive and negative k for Mordell curves of given rank (personal web page, latest available snapshot on web.archive.org from Jan. 2017), last modified 10/2002.</p>
FORMULA	$a(n) = \min \{ k \geq 1 \mid A060951(k) = n \}.$ <p>- M. F. Hasler, Jul 01 2024</p>
EXAMPLE	<p>From M. F. Hasler, Jul 01 2024: (Start)</p> <p>Sequence A060951 = (0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 1, 0, 0, 1, ...) gives the analytic rank of the elliptic curve $y^2 = x^3 - k$ for $k = 1, 2, 3, \dots$</p> <p>We can see that:</p> <ul style="list-style-type: none"> - the smallest k that gives rank 0 is $k = 1 = a(0)$; - the smallest k that gives rank 1 is $k = 2 = a(1)$; - the smallest k that gives rank 2 is $k = 11 = a(2)$; <p>etc. (End)</p>
PROG	<pre>(PARI) {a(n) = my(k=1); while(ellanalyticrank(ellinit([0, 0, 0, 0, -k]))[1] <n, k++); k} \\ Seiichi Manyama, Aug 24 2019 (PARI) {A031508(n)=for(k=1, oo, ellrank(ellinit([0, - k]))[1]==n && return(k)))} \\ M. F. Hasler, Jul 01 2024</pre>
CROSSREFS	<p>Cf. A031507, A373795.</p> <p>See also A060950, A002150-A002155, A102833, A179124, A031507, A060951, A081119, A179136, A179137.</p> <p>Sequence in context: A122527 A039747 A049531 * A202140 A011806 A012953</p> <p>Adjacent sequences: A031505 A031506 A031507 * A031509 A031510 A031511</p>
KEYWORD	nonn,nice,hard,more
AUTHOR	Noam D. Elkies
EXTENSIONS	<p>Definition clarified by Jonathan Sondow, Oct 26 2013.</p> <p>Escape clause added to definition by N. J. A. Sloane, Jun 29 2024, because, as John Cremona reminds me, it is not known if k always exists.</p>
STATUS	approved

Last modified August 20 12:51 EDT 2025. Contains 386856 sequences.

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