Robot Dynamics and Control – Assignment 2

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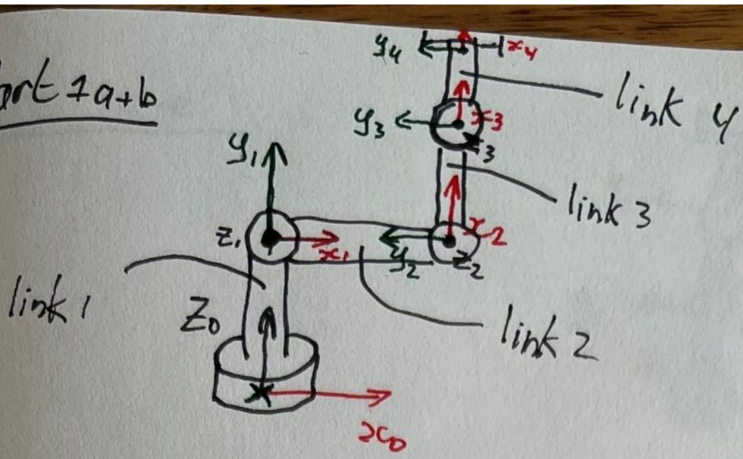
Part 1a-b DH Table first Robot

Part 1a+b

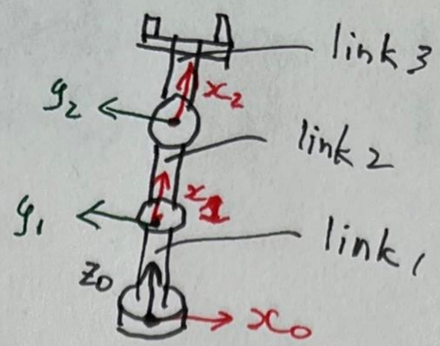


	θ	d	a	α
link 1	q_1	l_1	0	$\frac{\pi}{2}$
link 2	q_2	0	l_2	0
link 3	$q_3 + \frac{\pi}{2}$	0	l_3	0
link 4	q_4	0	l_4	0

$$Z\text{-}X = {}^{i-1}_{\;\;i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & \cos\theta_i\cdot a_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & \sin\theta_i\cdot a_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part 1c-e DH Table Second Robot

Part 1c-e



	θ	d	a	α
link 1	q_1	l_1	0	$\frac{\pi}{2}$
link 2	$q_2 + \frac{\pi}{2}$	0	l_2	0
link 3	q_3	0	l_3	0

Part 1c-e Transformation Matrices second robot

$$
{}^0T_1 = \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ \sin q_1 & 0 & -\cos q_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1T_2 = \begin{bmatrix} -\sin q_2 & -\cos q_2 & 0 & l_2 \cdot (-\sin q_2) \\ \cos q_2 & -\sin q_2 & 0 & l_2 \cdot \cos q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_2 = {}^0T_1 \cdot {}^1T_2
$$

$$
{}^0T_2 = \begin{bmatrix} \cos q_1(-\sin q_2) & \cos q_1(-\cos q_2) & \sin q_1 & l_2 \cos q_1(-\sin q_2) \\ \sin q_1(-\sin q_2) & \sin q_1(-\cos q_2) & -\cos q_1 & l_2 \sin q_1(-\sin q_2) \\ \cos q_2 & -\sin q_2 & 0 & l_1 + l_2 \cos q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2T_3 = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & l_3 \cos q_3 \\ \sin q_3 & \cos q_3 & 0 & l_3 \sin q_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_3 = {}^0T_2 \cdot {}^2T_3
$$

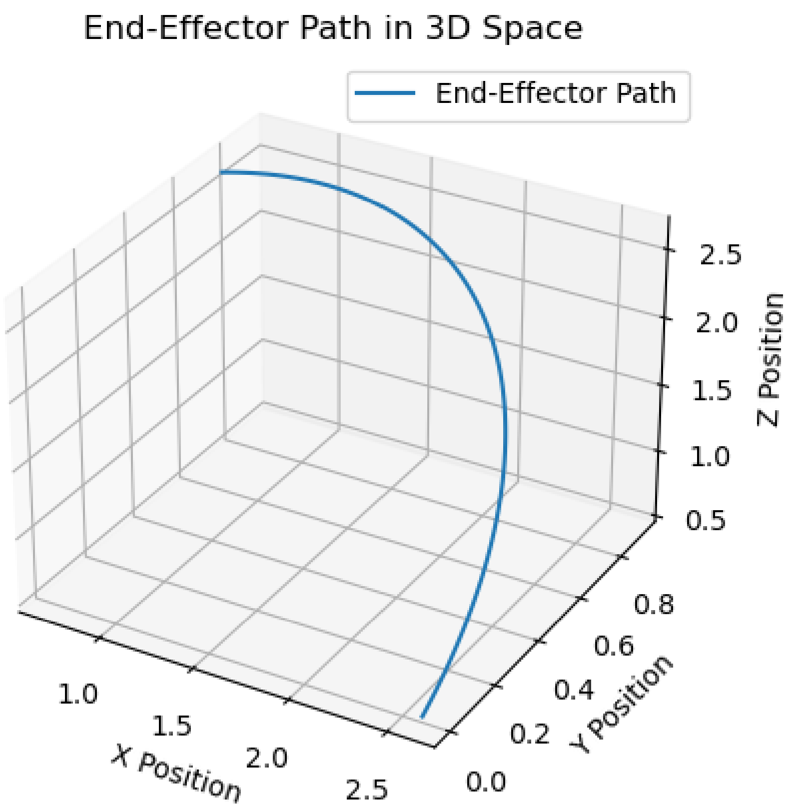
$$
{}^0T_3 = \begin{bmatrix} \cos q_1 \cos q_2 \cos q_3 - \cos q_1 \sin q_2 \sin q_3 & -\cos q_1 \cos q_2 \sin q_3 - \cos q_1 \sin q_2 \cos q_3 & \sin q_1 & l_2 \cos q_1 \cos q_2 + l_3(\cos q_1 \cos q_2 \cos q_3 - \cos q_1 \sin q_2 \sin q_3) \\ \sin q_1 \cos q_2 \cos q_3 - \sin q_1 \sin q_2 \sin q_3 & -\sin q_1 \cos q_2 \sin q_3 - \sin q_1 \sin q_2 \cos q_3 & -\cos q_1 & l_2 \sin q_1 \cos q_2 + l_3(\sin q_1 \cos q_2 \cos q_3 - \sin q_1 \sin q_2 \sin q_3) \\ \sin q_2 \cos q_3 + \cos q_2 \sin q_3 & -\sin q_2 \sin q_3 + \cos q_2 \cos q_3 & 0 & l_1 + l_2 \sin q_2 + l_3(\sin q_2 \cos q_3 + \cos q_2 \sin q_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Part 2 Robot end-effector motion

2b) Final transform matrix when all joint angles are set to 0:

1.000000e+00	0.000000e+00	0.000000e+00	2.600000e+00
0.000000e+00	6.123234e-17	-1.000000e+00	0.000000e+00
0.000000e+00	1.000000e+00	6.123234e-17	5.000000e-01
0.000000e+00	0.000000e+00	0.000000e+00	1.000000e+00

2c) End-effector motion



Part 3 The Jacobian including z and t vectors

Part 3

	θ	d	a	α
J_1	q_1	h_1	0	$\frac{\pi}{2}$
J_2	q_2	0	l_2	0
J_3	q_3	0	l_3	0

$$
{}^{0}_{0}T = \begin{array}{c} \begin{array}{cccc} & & z_0 & t_0 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
\qquad
{}^{0}_{1}T = \begin{array}{c} \begin{array}{cccc} & & z_1 & t_1 \end{array} \\ \begin{bmatrix} cq_1 & 0 & sq_1 & 0 \\ sq_1 & 0 & -cq_1 & 0 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

$$
{}^{0}_{2}T = \begin{array}{c} \begin{array}{cccc} & & z_2 & t_2 \end{array} \\ \begin{bmatrix} cq_1 cq_2 & -cq_1 sq_2 & sq_1 & l_2 cq_1 cq_2 \\ sq_1 cq_2 & -sq_1 sq_2 & -cq_1 & l_2 sq_1 cq_2 \\ sq_2 & cq_2 & 0 & h_1 + l_2 sq_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

$$
{}^{0}_{3}T = \begin{array}{c} \begin{array}{cccc} \phantom{cq_1 cq_{23}} & \phantom{-cq_1 sq_{23}} & z_3 & t_3 \end{array} \\ \begin{bmatrix} cq_1 cq_{23} & -cq_1 sq_{23} & sq_1 & cq_1(l_2 cq_2 + l_3 cq_{23}) \\ sq_1 cq_{23} & -sq_1 cq_{23} & -cq_1 & sq_1(l_2 cq_2 + l_3 cq_{23}) \\ sq_{23} & cq_{23} & 0 & h_1 + l_2 sq_2 + l_3 sq_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} sq_1 \\ -cq_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} sq_1 \\ -cq_1 \\ 0 \end{bmatrix}$$

$$t_1 = \begin{bmatrix} 0 \\ 0 \\ h_1 \end{bmatrix} \quad t_2 = \begin{bmatrix} l_2 cq_1 cq_2 \\ l_2 sq_1 cq_2 \\ h_1 + l_2 sq_2 \end{bmatrix}$$

$$t_3 = \begin{bmatrix} cq_1 \cdot (l_2 cq_2 + l_3 cq_{23} \\ sq_1 \cdot (l_2 cq_2 + l_3 cq_{23} \\ h_1 + l_2 sq_2 + l_3 sq_{23} \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times t_3 \cdot \dot{q}_1 & z_1 \times (t_3 - t_1) \cdot \dot{q}_2 & z_2 \times (t_3 - t_2) \cdot \dot{q}_3 \\ z_0 \cdot \dot{q}_1 & z_1 \cdot \dot{q}_2 & z_2 \cdot \dot{q}_3 \end{bmatrix}$$

$$J = \begin{bmatrix} -sq_1 (l_2 cq_2 + l_3 cq_{23}) & -cq_1 (l_2 sq_2 + l_3 sq_{23}) & -l_3 cq_1 cq_{23} \\ cq_1 (l_2 cq_2 + l_3 cq_{23}) & -sq_1 (l_2 sq_2 + l_3 sq_{23}) & -l_3 sq_1 cq_{23} \\ 0 & l_2 cq_2 + l_3 cq_{23} & l_3 cq_{23} \\ 0 & sq_1 & sq_1 \\ 0 & -cq_1 & -cq_1 \\ 1 & 0 & 0 \end{bmatrix}$$

Part 3c jacobian_fromVP() == jacobianMoreJoints()

```python
    print("Jacobian matrix by using jacobian_fromVP() at joint angles q = [0, 0, 0]:")
    print(J1)

# Part (3c): Fill in what you need to run part (3c) and compare the two Jacobian functions you have created.
# I recommend testing them at the 0 position first and then trying others positions. Testing 3-5 positions is sufficient.
# Remember to use your dh() and fk_calc() functions as needed.

    J2 = jacobianMoreJoints(q, dh_paras, numjoints=3)

    print("Jacobian matrix by using jacobianMoreJoints() at joint angles q = [0, 0, 0]:")
    print(J2)
```

Python Console × P4_and_P5 × P4_and_P5 (1) × P4_and_P5 (2) × frame × P2b × P2c ×

```
Python 3.8.8 (default, Apr 13 2021, 15:08:03) [MSC v.1916 64 bit (AMD64)]
Jacobian matrix by using jacobian_fromVP() at joint angles q = [0, 0, 0]:
[[ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00]
 [ 2.60000000e+00  1.59204084e-16  7.34788079e-17]
 [ 0.00000000e+00  2.60000000e+00  1.20000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.00000000e+00 -1.00000000e+00]
 [ 1.00000000e+00  6.12323400e-17  6.12323400e-17]]
Jacobian matrix by using jacobianMoreJoints() at joint angles q = [0, 0, 0]:
[[ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00]
 [ 2.60000000e+00  1.59204084e-16  7.34788079e-17]
 [ 0.00000000e+00  2.60000000e+00  1.20000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.00000000e+00 -1.00000000e+00]
 [ 1.00000000e+00  6.12323400e-17  6.12323400e-17]]
```

Part 3d

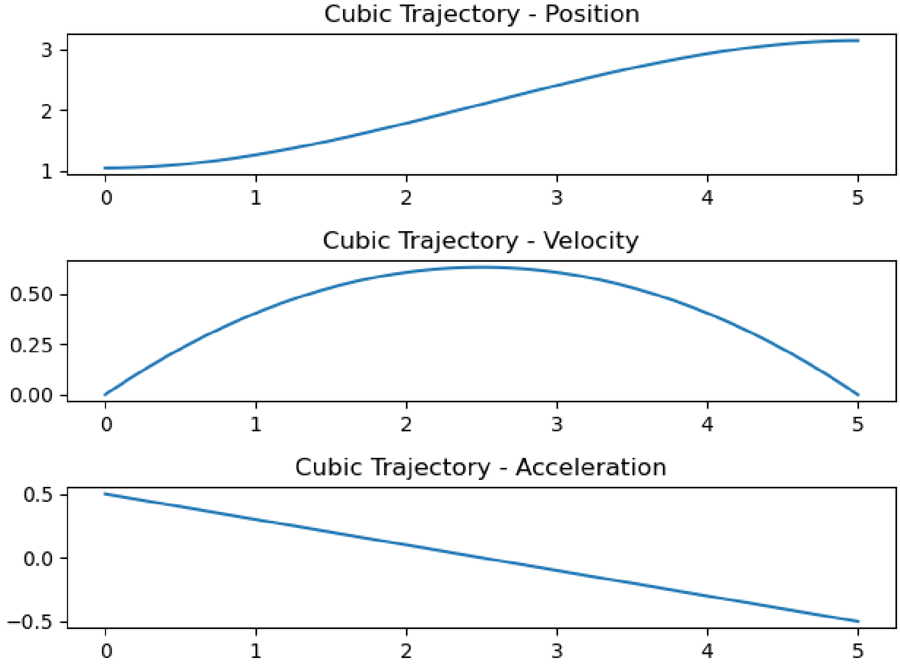
```
Position 1: Joint angles set to [0, 0, 0, 0, 0, 0]
End-effector position (T1):
[-0.4569  -0.19425  0.06655]
Jacobian at Position 1:
```

0.19425	0.08535	0.08535	0.08535	-0.0819	5.01492864e-18
-0.4569	-2.79770561e-17	-1.30577965e-17	0.00000000e+00	0.00000000e+00	0.00000000e+00
0.00000000e+00	-0.4569	-0.21325	0.00000000e+00	0.00000000e+00	0.00000000e+00
0.00000000e+00	0.00000000e+00	0.00000000e+00	0.00000000e+00	0.00000000e+00	0.00000000e+00
0.00000000e+00	-1.00000000e+00	-1.00000000e+00	-1.00000000e+00	-1.22464680e-16	-1.00000000e+00
1.00000000e+00	6.12323400e-17	6.12323400e-17	6.12323400e-17	-1.00000000e+00	6.12323400e-17

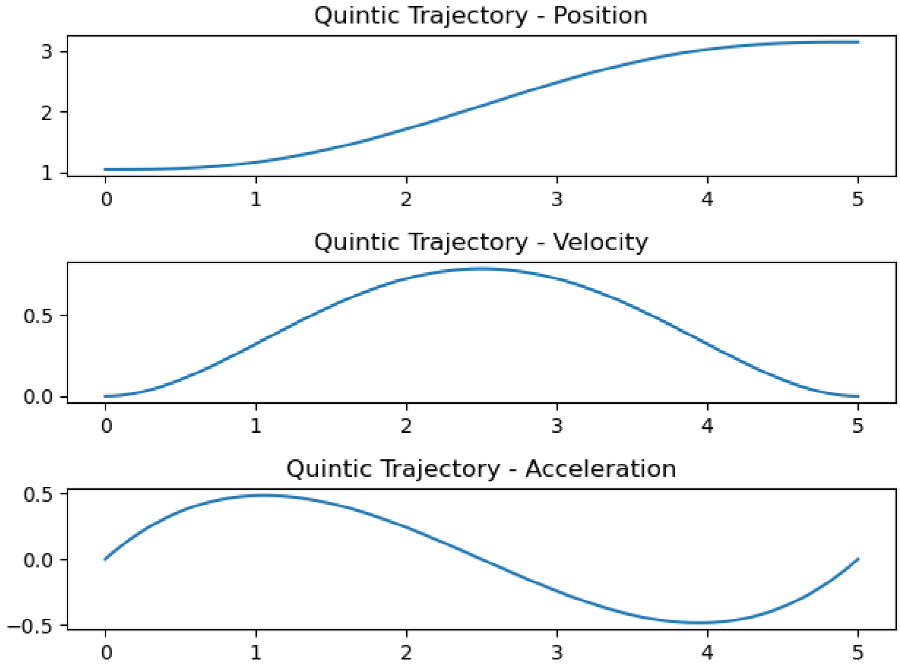
```
Position 2: Joint angles set to [0, pi/3, pi/3, 0, pi/4, 0]
End-effector position (T2):
[ 0.08767129 -0.17026205 -0.25126531]
Jacobian at Position 2:
```

0.17026205	0.403165309	0.192158220	0.00747830248	0.0289560227	2.08166817e-17
0.0876712909	5.36831829e-18	1.28279481e-17	6.29904986e-18	0.0579120454	6.93889390e-18
-0.00000000e+00	0.0876712909	0.209496291	0.102871291	-0.0501533025	3.46944695e-18
0.00000000e+00	0.00000000e+00	0.00000000e+00	0.00000000e+00	0.866025404	0.353553391
0.00000000e+00	-1.00000000e+00	-1.00000000e+00	-1.00000000e+00	-3.06161700e-17	-0.707106781
1.00000000e+00	6.12323400e-17	6.12323400e-17	6.12323400e-17	0.500000000	-0.612372436

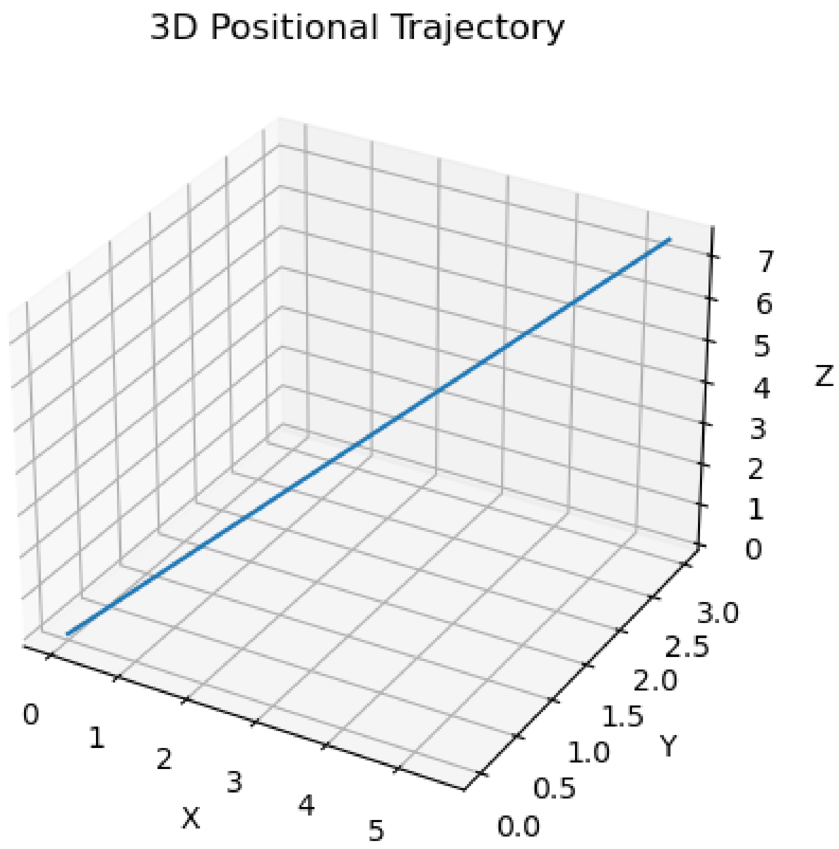
Part 4 Cubic Trajectory



Part 4 Quintic Trajectory



Part 5 3D Trajectory



Part 6 Frame

```
Homogeneous Transformation Matrix of Frame B:
Transformation matrix of frame 'Frame B':
[[ 0.90630779 -0.39713126  0.14454396  1.        ]
 [ 0.42261826  0.85165074 -0.30997552  3.        ]
 [ 0.          0.34202014  0.93969262  5.        ]
 [ 0.          0.          0.          1.        ]]

Quaternion representation of Frame B rotation:
[0.96146388 0.16953202 0.03758434 0.21315141]

Axis-Angle representation of Frame B rotation:
Axis: [0.61663416 0.1367045  0.77528975]
Angle (radians): 0.5570364713258218
Angle (degrees): 31.91583884182969

Frame from Quaternion Transformation Matrix:
Transformation matrix of frame 'Frame from Quaternion':
[[ 0.90630779 -0.39713126  0.14454396  1.        ]
 [ 0.42261826  0.85165074 -0.30997552  3.        ]
 [ 0.          0.34202014  0.93969262  5.        ]
 [ 0.          0.          0.          1.        ]]
```

```
Frame from Axis-Angle Transformation Matrix:
Transformation matrix of frame 'Frame from Axis-Angle':
[[ 9.06307787e-01 -3.97131262e-01  1.44543958e-01  1.00000000e+00]
 [ 4.22618262e-01  8.51650740e-01 -3.09975519e-01  3.00000000e+00]
 [ 4.16333634e-17  3.42020143e-01  9.39692621e-01  5.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]]

Difference between original Frame B and Frame from Quaternion:
[[ 0.00000000e+00  5.55111512e-17 -5.55111512e-17  0.00000000e+00]
 [-5.55111512e-17  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -5.55111512e-17  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]

Difference between original Frame B and Frame from Axis-Angle:
[[ 0.00000000e+00  1.11022302e-16 -1.11022302e-16  0.00000000e+00]
 [-1.11022302e-16  1.11022302e-16  5.55111512e-17  0.00000000e+00]
 [-4.16333634e-17 -1.11022302e-16  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
Maximum absolute error in degrees: 2.349231920106831e-13
Mean absolute error in degrees: 1.1430856261540611e-14
```

Part 6 Plot Rotational Error