# In-hand Rolling Motion Planning using Independent Contact Region (ICR) with Guaranteed Grasp Quality Margin

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Abstract—In this paper, we propose an in-hand motion planning algorithm for 3D objects under rolling contacts. By assuming the in-hand motion is quasi-static, we apply the concept of independent contact region (ICR) to determine stable boundaries of finger contact points during the in-hand motion. A specified level of grasp quality margin representing the ability to bear external load is guaranteed. The overall scheme contains two motion planning phases: one is the motion planning with fingers simultaneously rolling over the object surface, and the other is the planning associated with switching fingers of contact and non-contact. Simulation/experimental results are provided to show the feasibility of the proposed algorithm.

#### I. INTRODUCTION

Robotic grasp planning, which is about how to determine contact locations of finger-tips on object's surface, is one of the most popular issues being studied in the robotics community. In determining the locations of contacts on an object, we often employ its discretized computer model and test the immobility condition by the grasp forces acting on the object surface. This is equivalent to securing the force-closure condition [1], [2], [3].

However, due to the existing discrepancy between the theory and the actual robotic grasp from various sources of uncertainties, the wanted grasp planning strategy may not be easily employed to the real robotic grasping. In order to endow robustness against the finger positioning errors, the concept of ICRs on the object surface was proposed. Nguyen [4] addressed the ICRs by the following sense: if each fingertip is located on its own ICR, such a grasp always satisfies the force-closure condition. The concept was extended to determine ICRs on 2D objects [5] and on 3D objects [6], [7]. Using the ICRs, Roa et. al.[8] developed the grasp map which is a set of robotic grasps satisfying the grasp quality. However, the conventional ICR only means that any choice from a set of contact points in ICRs is a force-closure grasp, but has nothing to do with the grasp quality margin to be met.

In-hand manipulation planning is one of grasp planning methods, but is complicated because the object is manipulated by one hand using the redundancy of finger movements. There have been some number of researches on in-hand motion planning and manipulation using multi-fingered hands and multiple robotic arms. Cole et. al.[9] introduced the rolling kinematics and a control method of a multi-fingered hand during the in-hand motion. They used the net force and velocity of the object for the in-hand(rolling) motion

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control. Hasegawa and Murakami [10] proposed a planning method using an articulated multi-fingered robotic hand. The planning method determined the contact positions and internal grasp forces under the condition of rolling contact. Yoshida et al.[11] proposed a control method for constructing stable grasps of 2D rigid objects with soft fingers, and they also considered the immobility through the rolling constraint. In addition to these works, there are many other works worthy of recognition on in-hand manipulation; please refer to Paljug et. al.[12]), Han et. al.[13], Cherif and Gupta[14], Sudsang et al. [15], Dougleri and Fasoulas [16], just to name a few.

In spite of those previous efforts, most researches on inhand manipulation have been focused on either the kinematic aspects of the contact-related motion or the graspability via force equilibrium, instead of considering the both aspects. Especially, they required a complicated analysis and burdensome computation because the in-hand manipulation and controls had to meet the force-equilibrium condition constantly. So, we propose a new in-hand motion planning algorithm that takes simultaneously into account the kinematic consistency of rolling contacts and the graspability during the in-hand motion. We suggest a way to incorporate a possible grasp quality measure into the planning to robustify the graspability. In the planning, we find acceptable contact positions of fingers using the ICRs considering a priori known object geometry. For the required grasp quality measure, we utilize the distance metric between the absolute grasp wrench space (GWS) and the object wrench space (OWS) borrowed from the authors' former work [17].

The outline of the rest of this paper is as follows. Preliminaries, including the wrench spaces, grasp quality measure, and the concept of the ICRs are provided in section II. Section III presents the main idea of the proposed in-hand motion planning algorithm. Simulation/experimental results of the grasp planning are presented in section IV. Finally, summary and conclusion are addressed in section V.

#### II. PRELIMINARY NOTIONS

#### A. Wrench Spaces and Grasp Quality

1) Grasp Wrench Space: Consider an object which admits *n*-number of contact forces  $\{\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_n\}$  acting in the normal directions of the object surface at positions  $\{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n\}$ , respectively. Each contact force produces a torque with respect to the object's center of mass, such that  $\tau_i = \mathbf{r}_i \times \mathbf{f}_i$ , where  $\mathbf{r}_i$  is the vector from the center of mass to contact point  $\mathbf{p}_i$ . Thus, a complete set of torques

produced by all the contact forces is  $\{\tau_1, \tau_2, \dots, \tau_n\}$ . The primitive wrench is defined as

$$\mathbf{w}_{ij} \triangleq \begin{bmatrix} \mathbf{f}_{ij} \\ \tau_{ij} = \mathbf{r}_i \times \mathbf{f}_{ij} \end{bmatrix}, \quad j = 1, 2, \dots, s, \tag{1}$$

where  $\mathbf{f}_{ij}$  refers to a *j*-th side of pyramid force that approximates the ideal friction cone for the contact force  $\mathbf{f}_i$  [18]. The generic grasp wrench space (GWS) is described as

$$\mathbf{W} = \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{ij} \mathbf{w}_{ij}$$
 (2)

with  $\alpha_{ij} \geq 0$  and  $\sum_{i=1}^{s} \alpha_{ij} \leq 1$ .

2) Object Wrench Space: A grasped object of any shape can allow for disturbance forces, most dominantly normal to its object surface. The object wrench space(OWS) is a wrench space produced by all such disturbance forces of unit length. To define OWS mathematically, let us consider a polygonal(discretized) object with a large number of facets on the surface. For an object with *m*-numbered surface facets, the OWS is created by a set of wrenches such that

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{u}_k \\ \mathbf{l}_k \times \mathbf{u}_k \end{bmatrix}, \quad ||\mathbf{u}_k|| = 1, \quad k = 1, 2, \dots, m,$$
 (3)

where  $\mathbf{l}_k$ ,  $\mathbf{u}_k$ , and  $\mathbf{z}_k$ , respectively, denote the vector from the origin of the center of mass to the k-th facet, the unit normal surface force at  $\mathbf{l}_k$ , and the wrench produced by  $\mathbf{u}_k$ . The generic OWS is represented as

$$\mathbf{Z} = \sum_{k=1}^{m} \beta_k \left( -\mathbf{z}_k \right) \tag{4}$$

with  $\beta_k \ge 0$  and  $\sum_{k=1}^m \beta_k \le 1$ .

3) Grasp Quality: A necessary and sufficient condition of a force-closure grasp is that the convex hull of GWS contains the origin. It means that grasp forces can span the whole external forces. If the convex hull of GWS is denoted as  $\mathcal{H}(\mathbf{W})$ , the force-closure condition is written as,

$$0 \in \mathcal{H}(\mathbf{W}). \tag{5}$$

To evaluate grasp qualities between grasps, a measure is needed. In our previous work [17], a physically meaningful grasp measure was proposed as the scale of the convex hull of OWS with respect to the convex hull of GWS. This is to find the maximum scale factor,  $\alpha$ , such that

$$\alpha \cdot \mathcal{H}(OWS) \subset \mathcal{H}(GWS).$$
 (6)

This grasp quality measure provides the physical meaning of the minimum force that destroys the stability of a grasp. In this paper, we use this measure to define the grasp quality. Refer to [17] for the details of this method.

#### B. Independent Contact Regions (ICRs)

Independent contact regions were proposed to provide robustness to finger positioning error. A method to compute the ICRs on polygonal objects was developed by Roa and Suarez[7]. If each finger-tip is located on its ICR in any contact combination, the grasp always guarantees

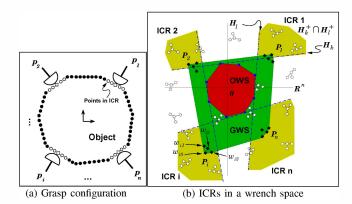


Fig. 1. Independent Contact Regions (ICRs) in a grasp. Open circles in (a) imply contact points within ICRs and solid circles imply points outside ICRs. Solid squares in (b) are primitive wrenches by contacts  $p_i, i = 1 \cdots n$  and open squares are imaginative wrenches through virtual contacts on other surface points. Dashed lines in (b) denote hyperplanes parallel to facets of the convex hull of GWS.

the force-closure. We used this ICRs to find continuous contact positions during the in-hand motion by incorporating a method to meet the bound of grasp quality. Fig.1 shows the schematic of ICRs. Let finger contact points be  $P = \{p_1, p_2, \dots, p_i, \dots, p_n\}$  for *n* number of contacts,  $W_i =$  $\{w_{i0}, w_{i1}, w_{i2}, w_{i3}\}$  be wrenches of the *i*-th friction contact, and  $\alpha$  be the bound of grasp quality measure. When a grasp is force-closure, we find a facet  $F_k$ ,  $k = 1, 2, \dots, \overline{k}$  of the convex hull of GWS, where  $\bar{k}$  denotes the total number of facets made of wrenches from at least two distinct contacts. Then the hyperplane  $H_k$  which is parallel to  $F_k$  is obtained such that  $H_k$  touches the boundary of the convex hull of OWS. Let  $H_k^+$  and  $H_k^-$  be the two half-spaces defined by  $H_k$ . We assign that  $H_k^-$  includes the convex hull of OWS. Finite number of intersections of half-spaces,  $H_k^+$ , for all contacts become the ICRs in wrench space as shown in Fig.1(b). And the open circles in Fig.1(a) correspond to the contact points belonging to the ICRs.

### III. IN-HAND MANIPULATION PLANNING METHOD USING ICRS

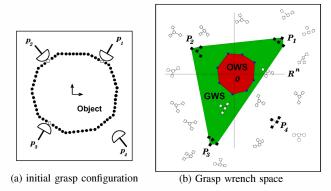


Fig. 2. The pre-phase of in-hand motion planning. Initial grasp configuration is made by three fingers and it satisfies grasp quality.

While the ICRs were originally proposed to provide robustness to positioning errors of finger-tips due to uncertainties of grasp environments, the ICRs may be viewed from a different standpoint: that is, from a context of in-hand motion planning. Physically, the ICRs imply graspable independent regions without destroying the grasp stability (force-closure) condition. If we continue to move the contact locations on an object surface within their ICRs, we can not only assure the grasp stability but also achieve a desired in-hand motion. So, details of how to use the ICRs in the in-hand motion planning is presented below.

#### A. Pre-phase Planning: Initiation of In-Hand Manipulation

Consider a force-closure grasp which involves three contact points as shown in Fig.2. In Fig.2(a), the solid circles denote the object surface, and the open squares denote the current contact points. The contact forces from the three contacts create a grasp wrench space (i.e., the green convex region in Fig. 2(b) constructed by the contact wrenches that are illustrated in solid square(grasp wrenches)). For all surface points on the object except the current contacts, we compute the corresponding wrenches represented by open squares in Fig. 2(b). These points are used as candidate contact points during the in-hand manipulation planning. Now, from the information of GWS and OWS, scale factor  $\alpha$ , defined in the previous section, is selected. This scale factor becomes the guaranteed grasp measure value that implies the maximum resistable external disturbance. The size of ICRs at each time is determined by the magnitude of  $\alpha$ . This initiates the desired in-hand manipulation.

#### B. First Phase of Planning: Rolling Contacts

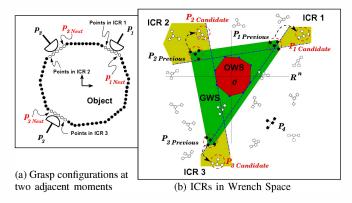


Fig. 3. The first phase of in-hand motion planning. ICRs are computed each time the rolling is continued.

In the first phase of in-hand motion planning, we manipulate the object in the desired direction via rolling using the three fingers in contact. As the first task in this phase, we compute the ICRs of the three contact points. The group of wrenches within each ICR is, normally, a continuously connected set of points (i.e., a region) on the object surface as long as the object surface is assumed smooth. As the rolling is continued, the contact points should be varied smoothly through the desired rolling trajectory within ICRs. We take these subsequent contact points in such a way that all of them lie on the very behind of desired rolling trajectory of the

current contact points, opposite to the tangential direction. By doing this, the candidate wrench for each finger is determined as shown in Fig.3(b), all of which are chosen from the ICRs in the wrench space. In order to find the ICR of each contact, we need to compute the hyperplanes based on the current contact configuration. The hyperplanes are parallel to the facets of the convex hull of GWS where at least two distinct wrenches from different contacts lie on. These hyperplanes can be found, one after another, by applying the following linear-programming (LP) equation:

where d is the offset distance of hyperplane from the origin,  $h_n$  is the normal vector of the facet of GWS (the blue dash lines in Fig.3(b)), and  $-z_k$  is the k-th wrench of OWS. We can obtain the ICRs as the intersections of all the positive half spaces of hyperplanes,  $H_k^+$ . Note that the number of isolated intersections is exactly the number of current contact points as shown in Fig.3(b).

Once the contact points move to the determined candidate points as shown in Fig.4(a), we re-compute the GWS with the new wrenches as in Fig.4(b). During the rolling associated with Fig.4(a), the object may also have an absolute movement such as a rotation about the global frame. Although this does not affect the relative geometry and kinematics of contacts, the external effects from disturbance, gravity, or hand joints would. By the object's absolute movement

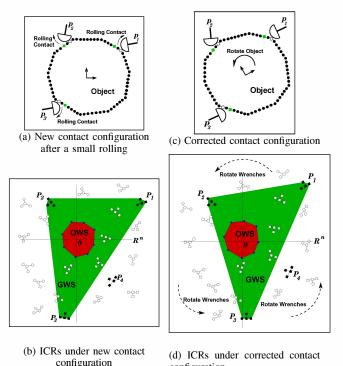


Fig. 4. ICRs and contact configuration after a small rolling.

configuration

such as a rotation, OWS, primitive wrenches in GWS and other wrenches must be changed accordingly, as shown in Fig.4(d). The rolling at the three contacts and the absolute movement of the object with respect to a fixed frame occur at the same time. These sequences of rolling are continued until any finger reaches its kinematic limit.

#### C. Second Phase of Planning: Switching fingers

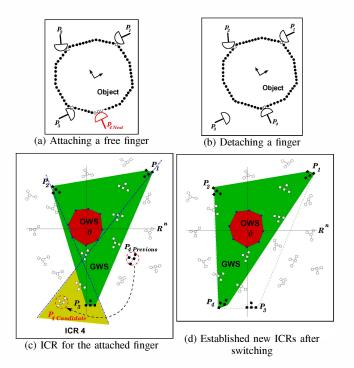


Fig. 5. Finger switching as the second phase of in-hand motion planning

After the first phase of in-hand motion planning, we need to find a rendezvous point of the free finger  $(P_4)$  to be switched with the finger in the limit (here, finger 3  $(P_3)$  in Fig.5(a)). The free finger has to make a contact near the forward region of the next expected finger of detachment, and we also have to consider the finger's workspace to continue the in-hand motion.

In order to select a suitable rendezvous point on the object surface for the free finger, we need to compute a new ICR (i.e., ICR 4, as shown in Fig.5(b)) by incorporating the remaining contacts (i.e.,  $P_1$  and  $P_2$  in Fig.5(b)). To do so, we find hyperplanes, touching the outer boundary of the scaled OWS, where wrenches from  $P_1$  and  $P_2$  lie. And the half space of each hyperplane includes whole object wrenches. See the blue dash lines in Fig.5(b). These hyperplanes can be found, one after another, by the following linear-programming equation:

subject to 
$$\begin{cases} \max d \\ h_n \cdot (\alpha(-z_k)) + d = 0 \\ h_n \cdot w_{jr} + d = 0 \\ h_n \cdot (\alpha(-z_i)) + d \le 0 \\ \text{for } k = 1, \dots, l, \quad k \notin i, \\ d \le 0 \end{cases}$$
(8)

with scale factor  $\alpha$  and wrenches,  $w_{jr}, r = 1, 2, \dots, s$ , of a contact point  $p_j$ . Here,  $h_n$  is the normal vector of the hyperplane containing  $w_{jr}$  and a boundary point of the convex hull of OWS, and halfspaces  $H_k^-$  of the obtained hyperplanes include all object wrenches.

Then, the free finger is moved to the chosen rendezvous location as shown in Fig.5(c), and the finger to be replaced is detached. Because the contacts are changed, we should compute new GWS accordingly as shown in Fig.5(d). This completes a whole procedure of the proposed in-hand motion planning using ICRs. This procedure may run for all fingers recursively. Fig.6 shows a diagram of the in-hand motion planning procedure. When this algorithm is applied to a practical robot hand, it is necessary to take into account the configuration and manipulability of the robot hand. Especially, we consider the mobility of the grasp configuration during the rolling contact motion. If the rolling motion is impossible due to the lack of mobility or the joint limit, the rolling contact motion phase in the planning will be skipped.

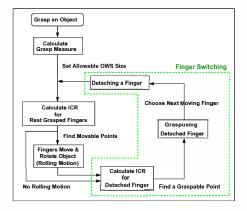
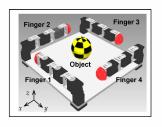
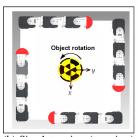


Fig. 6. Diagram of in-hand manipulation planning procedure

## IV. SIMULATION & EXPERIMENTATION STUDY A. Simulation Setup





(a) Simulator view (3D view)

(b) Simulator view (top view)

Fig. 7. Simulation environment for the in-hand manipulation planning

In this section, we present results of the numerical simulation to show the validity of the proposed in-hand manipulation algorithm. We used a 4-fingered hand moving in a horizontal plane as shown in Fig.7. Each finger has 3 degrees of freedom and the finger-tips are rounded by half-sphere shapes of 2.5[cm] radius. The target object is a

sphere of 4.75[cm] radius. The desired task of the simulation was to rotate the object in the counter-clockwise direction seen from the top using the proposed planning algorithm. In this simulation, we assumed that: 1) the contact model is a frictional point contact, 2) the slip between the fingertip and the object surface does not occur, 3) the effect of the gravitational force is not considered, and 4) all external forces act only on the x-y direction (i.e., we just consider the forces in the x-y plane). In order to calculate the convex hulls of OWS and the GWS, we discretize the sphere by 72 vertical & horizontal segments. The scale factor of the OWS and the friction coefficient for in-hand manipulation planning are  $\alpha=0.1$  and  $\mu=0.03$ , respectively.

#### B. Simulation Results

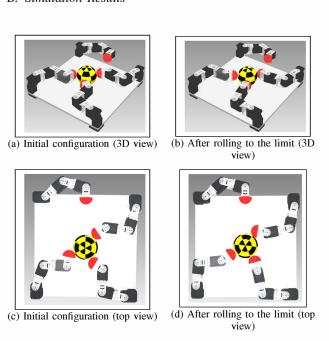


Fig. 8. Simulation result 1: rolling based in-hand motion.

We used finger 1, finger 2 and finger 4 to make an initial grasp pose. Finger 3 remained as a free finger. Initial contact points were  $4.75\angle 30^{\circ}$ ,  $4.75\angle -30^{\circ}$  and  $4.75\angle 140^{\circ}$  for finger 1, finger 2 and finger 3 with respect to object frame (units in [cm]) as depicted in Fig.8(a) and (c). From the initial configuration, we calculated the ICRs of the contacts to start the in-hand rolling motion. Fig.9(a) shows the computed ICRs in the reduced wrench space since the force in the z-axis and the moments in the x-axis and y-axis were not relevant to the situation. In this figure, blue dots('•') denote grasp wrenches at the contacts, opened circles('o') denote imaginative feasible wrenches, and solid squares('a') denote the wrenches in the ICRs. Among the wrenches in each ICR, we chose an adjacent one to the current grasp wrench as the wrenches to be as a result of the small in-hand movement. By continuing the procedures in the first phase of planning, we generated the in-hand rolling motion until a finger reached kinematic limits as shown in Fig.8(b) and (d). When a finger (here finger 4) reached the kinematic limit, the free finger

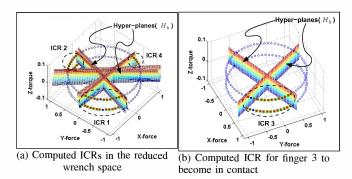


Fig. 9. Computed ICRs during the in-hand motion.

(finger 3) needed to come into contact while the finger 4 was to be detached. We calculated new ICRs by including finger 3 while by excluding finger 4, as shown in Fig.9(b). This result showed that the stable contact range of finger 3 was from  $4.75\angle 130^{\circ}$  to  $4.75\angle 205^{\circ}$ . We chose  $4.75\angle 200^{\circ}$  as the decided contact point of finger 3. Fig.10(a) and (c) shows a configuration when finger 3 was attached, Fig.10(b) and (d) when finger 4 was detached. Fig.11 shows the trajectories of angles (in z-axis) of the finger-tips and object. In this figure, 'RC' regions are at the rolling contact motion and 'FS' region is at the finger-switching. Note that the finger-tips rotated oppositely to the object's rotation. While rolling, the rotation angles of the object and each finger-tip must satisfy the kinematic relation,  $\delta\theta_o = \frac{r_f}{r_o}\theta_f = \frac{0.25}{0.475}\delta\theta_f$ , where  $r_f$  and  $r_o$  are the radii of the finger-tip and the object, respectively, and  $\delta\theta_f$  and  $\delta\theta_o$  are the rotation angles of finger-tip and the object, respectively.

#### C. Experiment

To verify the validity of the proposed method, we also conducted an experiment. In this experiment, we used a planar hand system which has the same conditions and

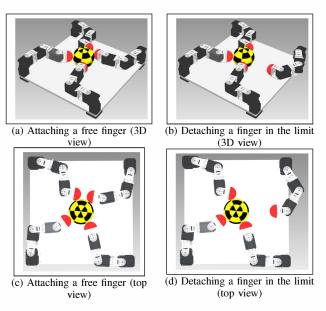


Fig. 10. Simulation result 2: Switching fingers.

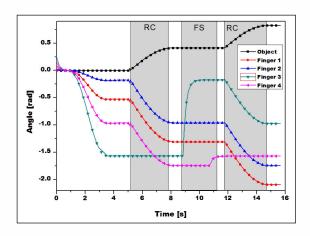


Fig. 11. Angles of finger-tips and object.

parameters as those in the simulation. Fig. 12 shows the snapshots of the experiment in the in-hand manipulation. The experiment showed very close results with the simulation. As a primitive force control, small amount of forces was applied during the in-hand motion to meet a force balance and not to drop the ball.





(a) Initial pose





(c) Attaching a free finger

(d) Detaching a finger in the limit

Fig. 12. Snapshots of experimental result (Top view)

#### V. CONCLUSIONS

In this paper, we proposed a quasi-static in-hand manipulation planning algorithm by using the ICRs. By incorporating the OWS, the proposed algorithm guaranteed a prescribed level of grasp quality margin during the in-hand motion. The ICRs were used in a navel way to easily find contact and switching locations during the in-hand motion. We provided simulation and experimental verification to validate the proposed algorithm. The future work will be taken to reduce the computational cost and to consider systematic finger switching strategies as the humans do.

#### ACKNOWLEDGMENT

This research was supported, in part, by the Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A2042823), by the Intelligent Robotics Development Program, one of the 21st Century Frontier R&D Program, and by the Ministry of Knowledge Economy (MKE), Korea, under the Human Resources Development Program for Convergence Robot Specialists support program supervised by the NIPA(National IT Industry Promotion Agency) (NIPA-2012-H1502-12-1002).

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