Complex Nonlinearities for Audio Signal Processing

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Thanks

- Julius Smith
- Dave Berners
- Viraga Perera
- GASP

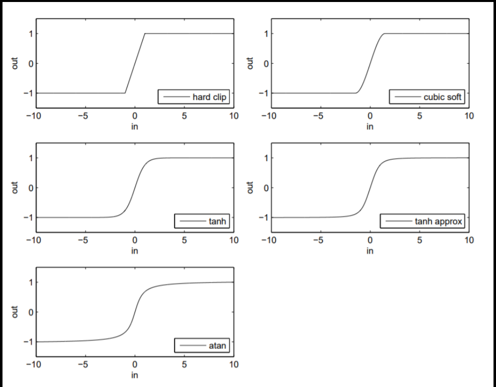
Motivation

Flashback to 2016…

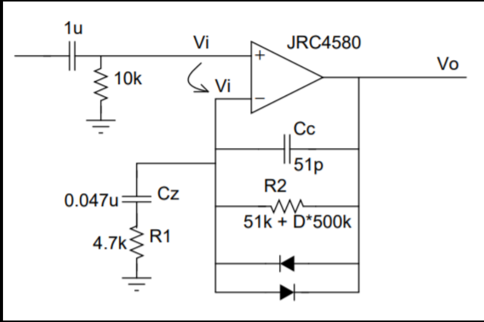
Trying to make a distortion effect…

Motivation

Static Nonlinearities



Circuit Modelling



Motivation

Solution: $\tanh$ approximation[1]

$$f(x) = \frac{x}{(1+|x|^n)^{1/n}}, \quad n = 2.5 \tag{1}$$

[1] Yeh, "Digital Implementation of Musical Distortion Circuits by Analysis and Simulation".

Motivation

Trying to fill the gap between:

- Simple static nonlinear systems
- Physically modelled nonlinear systems

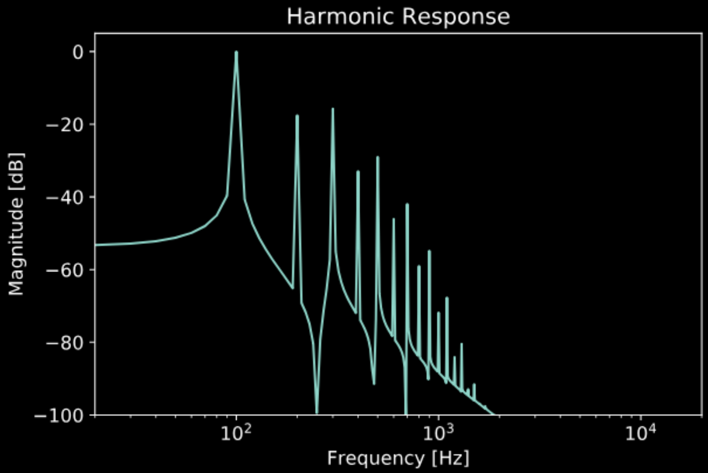
Outline

- Basics of nonlinear signal processing
- Complex nonlinearities
  - Double Soft Clipper
  - Exciter
  - Hysteresis
  - Nonlinear biquad filters
  - Wavefolding
  - Gated Recurrent Distortion

Building Blocks

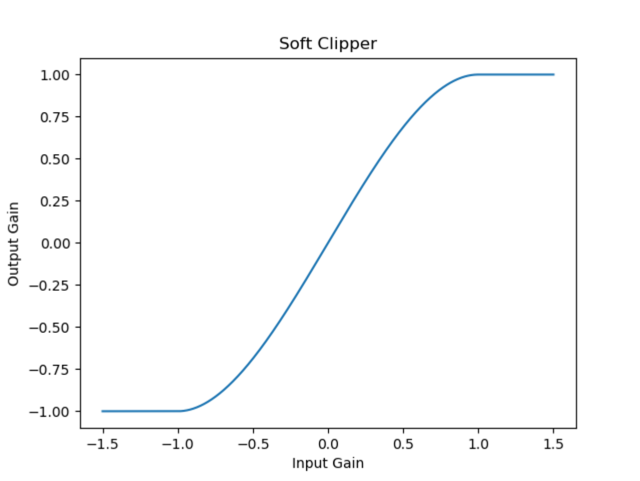
What is a nonlinear system?

Frequency domain: you get out more than what you put in



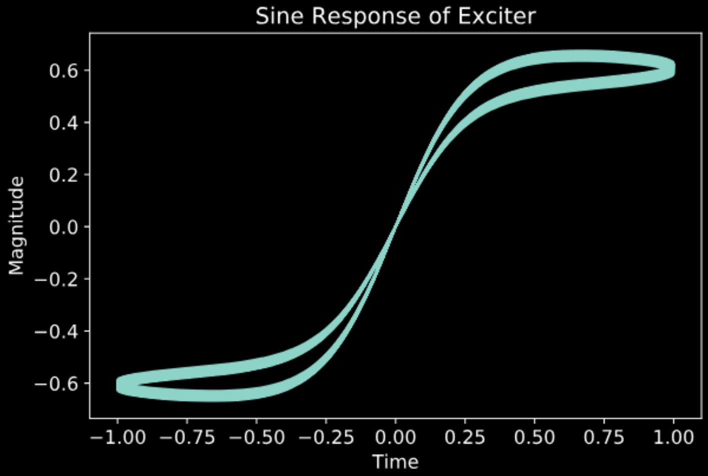
What is a nonlinear system?

Time domain: static response

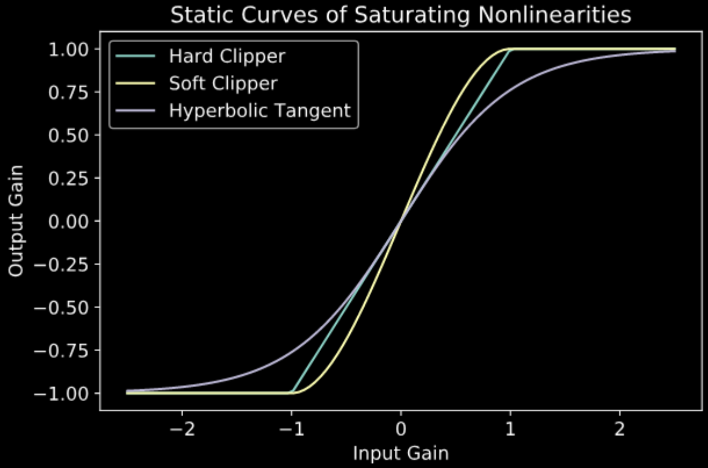


What is a nonlinear system?

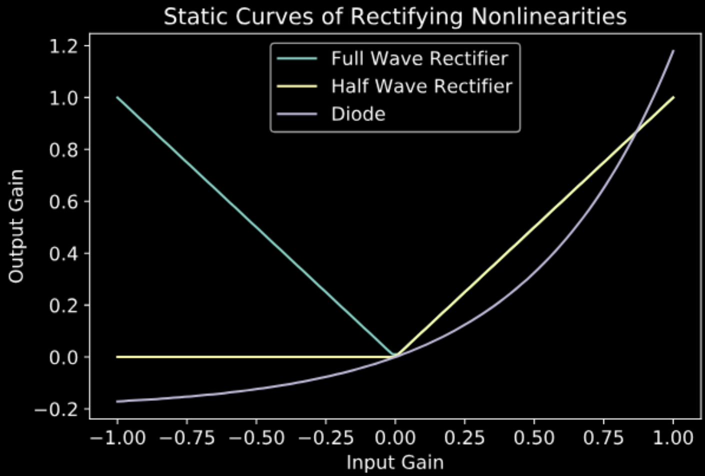
Time domain: dynamic response



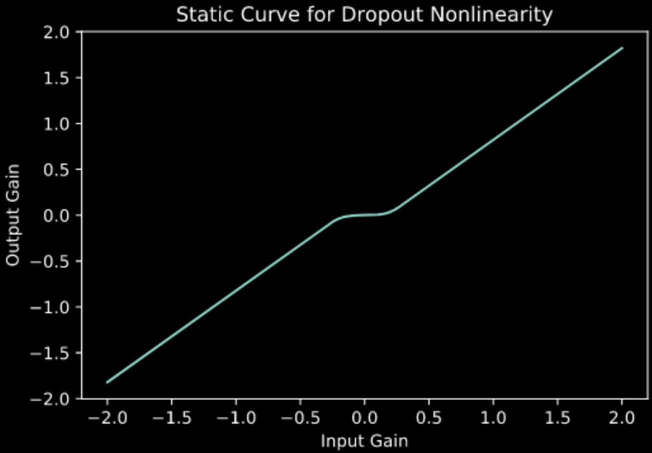
Saturating Nonlinearities



Rectifying Nonlinearities



Dropout Nonlinearities



(also called “dead-zone”)

What is a “Complex Nonlinearity”

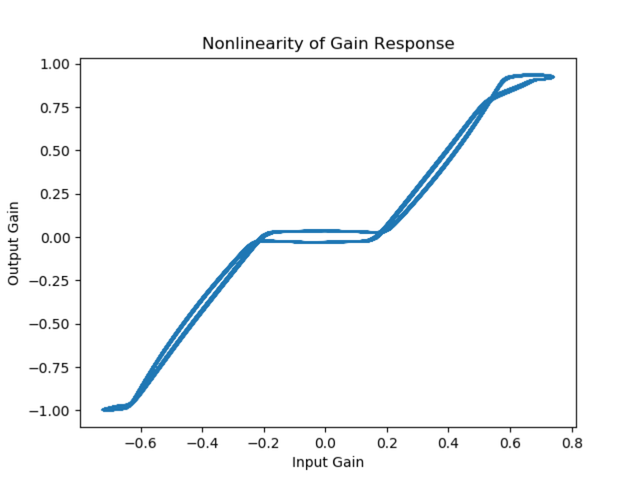
Has one of the following properties:

- Is not memoryless (has some memory of past states)
- Has an interesting harmonic response
- Has interesting parameters

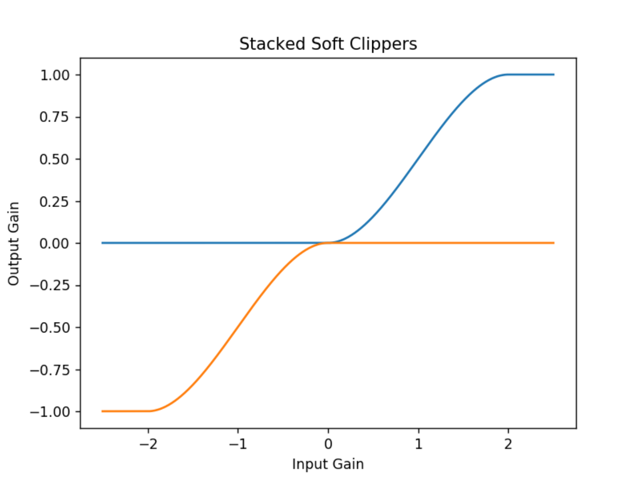
Double Soft Clipper

Double Soft Clipper: Inspiration

Measured speaker response



Double Soft Clipper



Double Soft Clipper: Original Soft Clipper

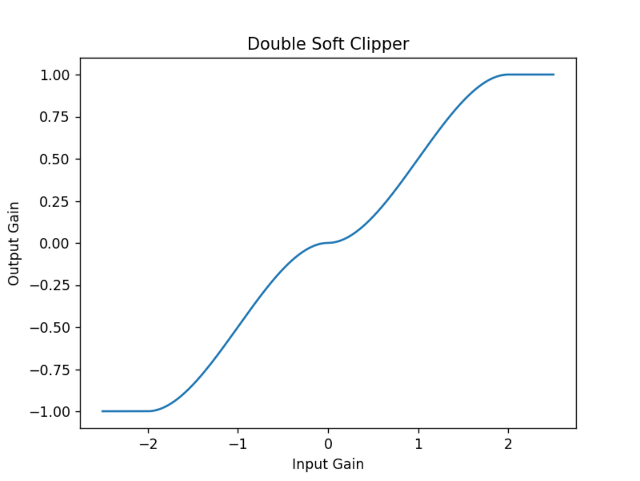
$$f_{SC}(x) = \begin{cases} 1 & x \geq 1 \\ \frac{3}{2}(x - x^3/3) & -1 < x < 1 \\ -1 & x \leq -1 \end{cases} \tag{2}$$

Double Soft Clipper

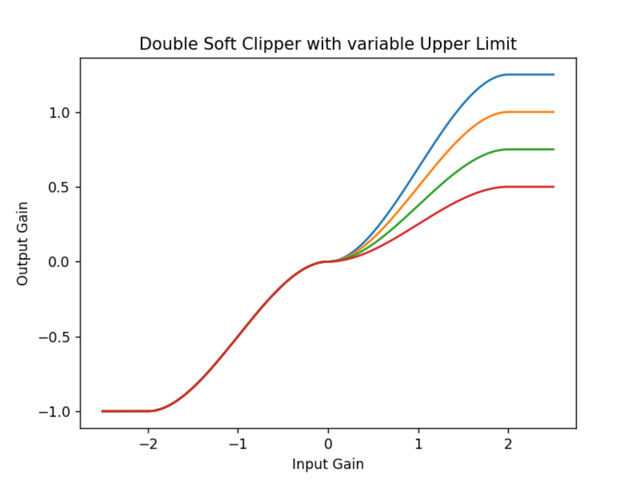
$$f_{DSC}(x) = \begin{cases} 1 & u \geq 1 \\ \frac{3}{4}(u - u^3/3) + 0.5 & 0 < u < 1 \\ \frac{3}{4}(u - u^3/3) - 0.5 & -1 < u < 0 \\ -1 & u \leq -1 \end{cases} \tag{3}$$

$$u(x) = \begin{cases} x - 0.5 & x > 0 \\ x + 0.5 & x < 0 \end{cases} \tag{4}$$

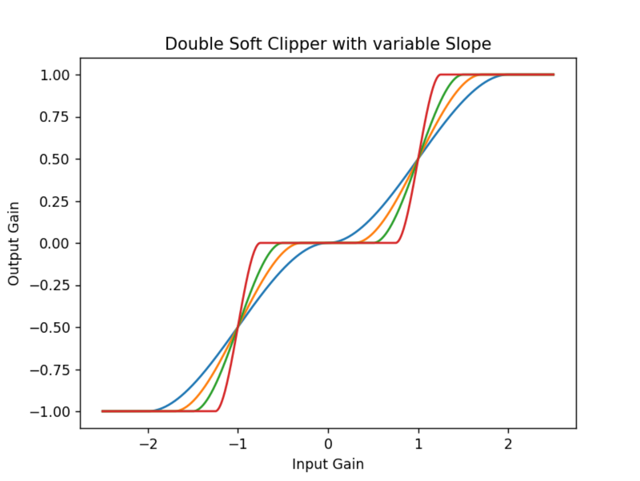
Double Soft Clipper



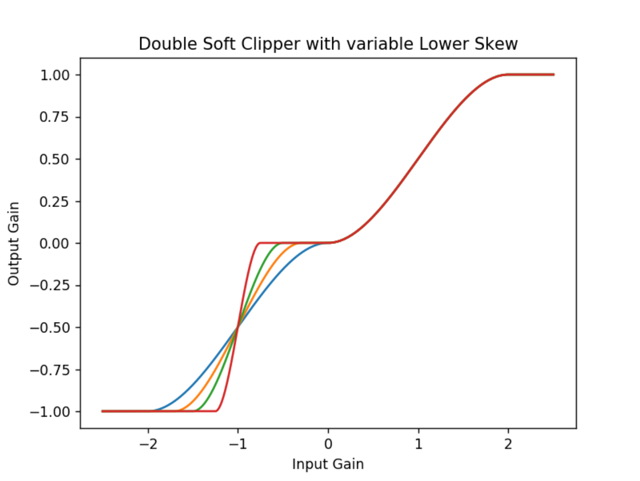
Double Soft Clipper: Parameters



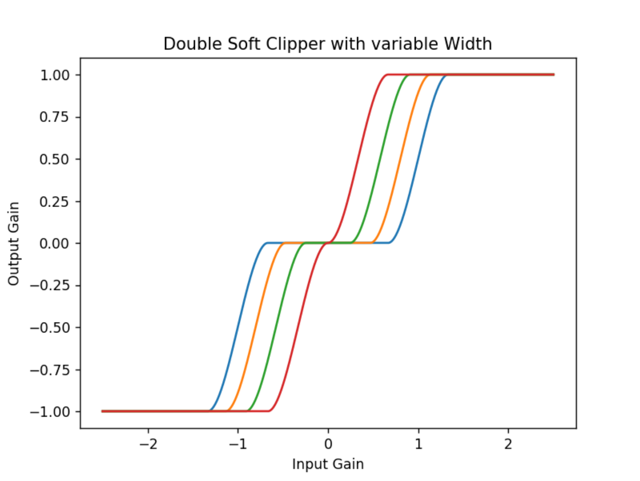
Double Soft Clipper: Parameters



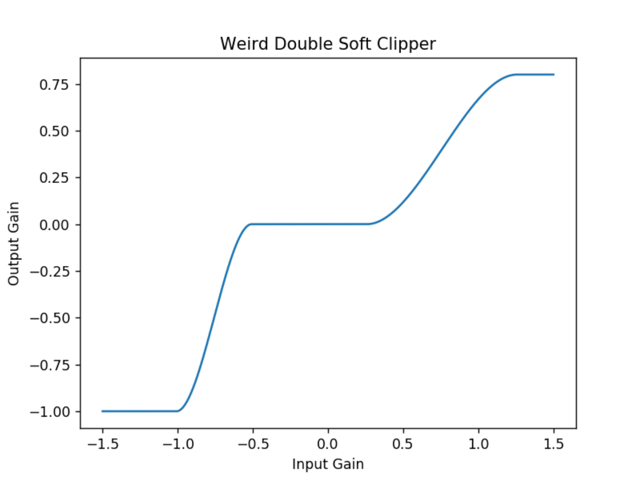
Double Soft Clipper: Parameters



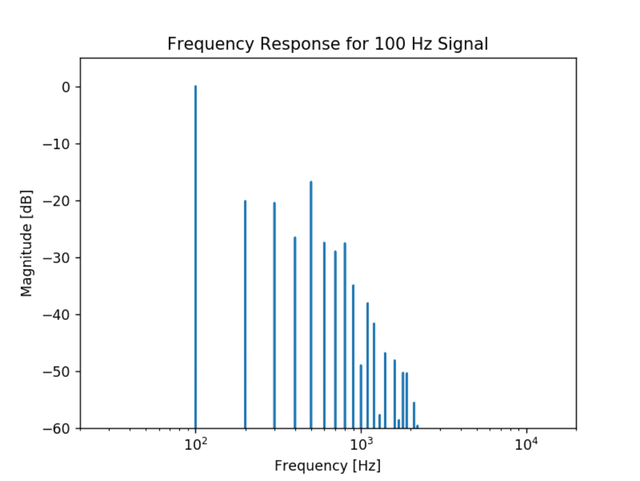
Double Soft Clipper: Parameters



Double Soft Clipper: Weird



Double Soft Clipper: Harmonic Response



Harmonic Exciter

Harmonic Exciter

What is a harmonic exciter?

- Add subtle harmonic distortion
- Make audio sound “shiny”, “brighter”, “enhanced”
- Used for mixing, live broadcasts, restoring old recordings with missing spectral content

Aphex Aural Exciter[2]

- Introduced in the mid-1970’s
- Used by Jackson Browne, The Four Seasons, Linda Ronstadt, and more
- Originally rented to studios for **$30 per minute**

[2] Ltd., *Aphex Aural Exciter Type B: Operating Guide*.

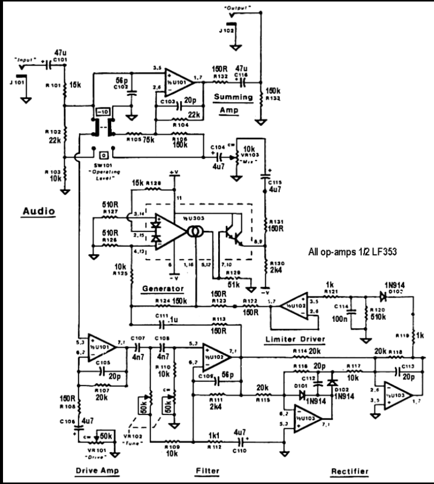
Harmonic Exciter

Goals:

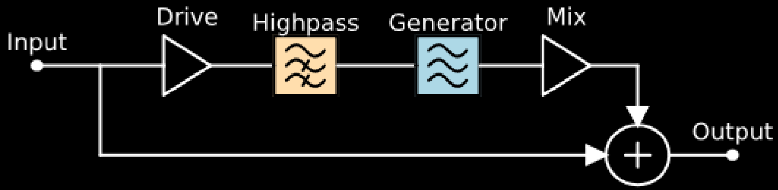
- Exciter model that sounds “smooth”
- Generalize the model to transcend circuit modelling[3]

[3] Giannoulis, Massberg, and Reiss, “Digital Dynamic Range Compressor Design - A Tutorial and Analysis”.

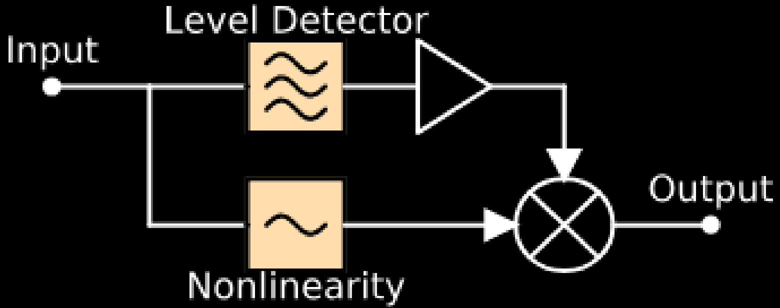
Harmonic Exciter



Harmonic Exciter

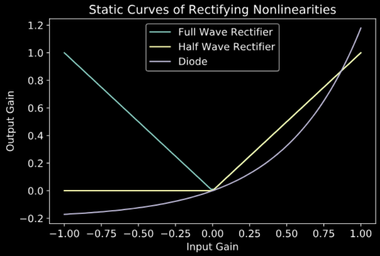


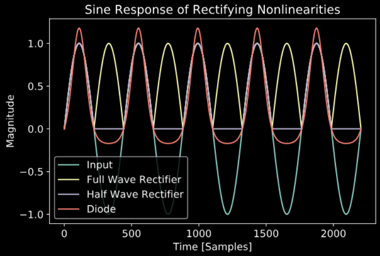
Harmonic Exciter: Generator



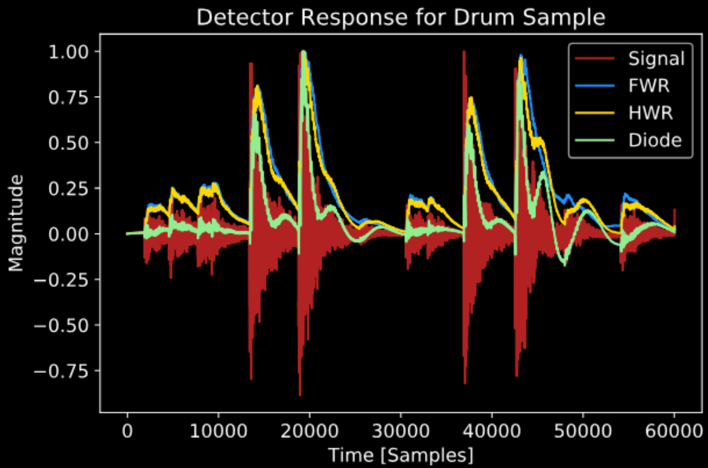
Harmonic Exciter: Level Detector

Rectifying nonlinearity $\rightarrow$LPF

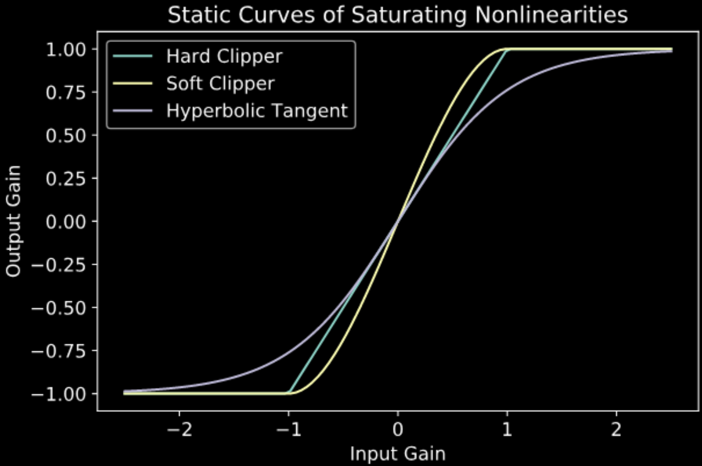




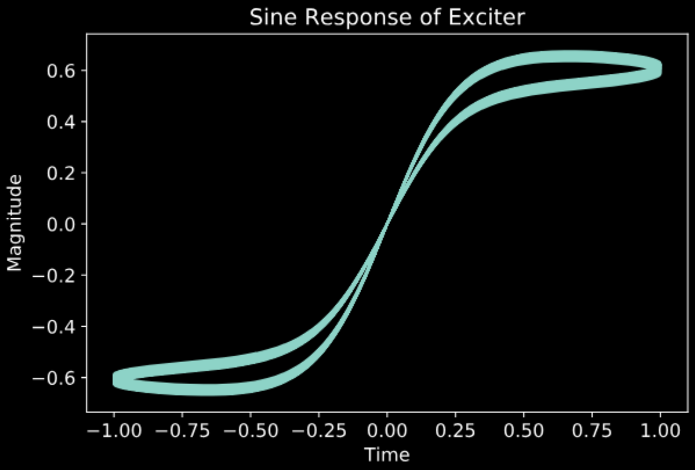
Harmonic Exciter: Level Detector



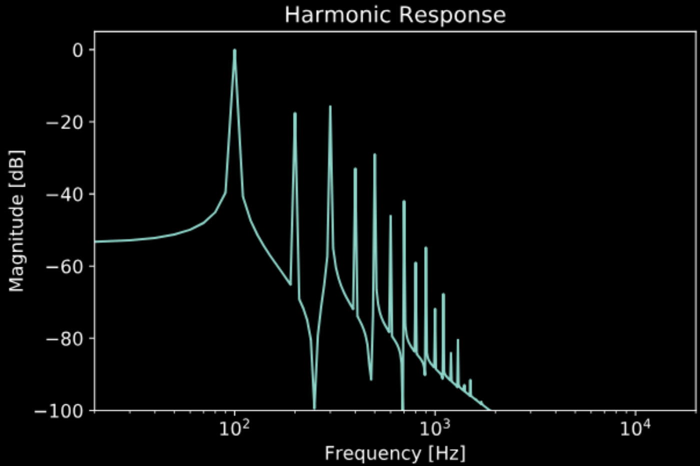
Harmonic Exciter: Nonlinearity



Harmonic Exciter: Generator



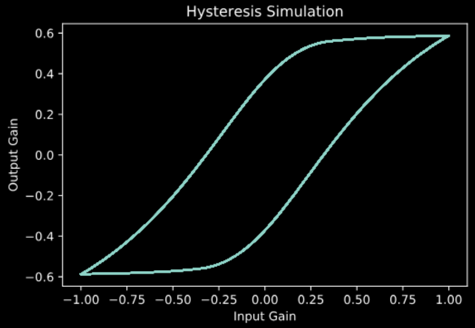
Harmonic Exciter: Generator



Hysteresis

Hysteresis

- Stateful nonlinearity that describes magnetisation (magnetic tape, transformers, ...)
- Also models processes in civil engineering, economics, and more



Hysteresis

Jiles-Atherton Hysteresis model[4]

$$\dot{y} = \frac{\frac{(1-c)\delta_y(SL(Q)-y)}{(1-c)\delta_x k-\alpha(SL(Q)-y)}\dot{x} + c\frac{S}{a}\dot{x}L'(Q)}{1 - c\alpha\frac{S}{a}L'(Q)} \tag{5}$$

$$Q(x,y) = \frac{x+\alpha y}{a} \tag{6}$$

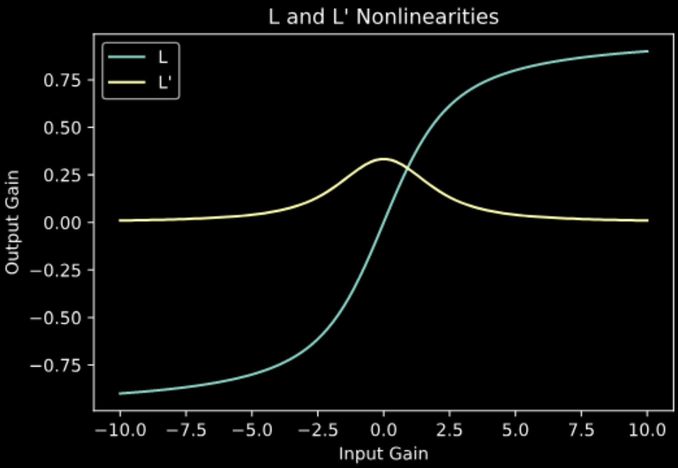
Langevin function:

$$L(x) = \coth(x) - \frac{1}{x} \tag{7}$$

[4] Jiles and Atherton, "Theory of ferromagnetic hysteresis".

Hysteresis

Langevin function (and derivative)



Hysteresis

Digitize hysteresis model[5]

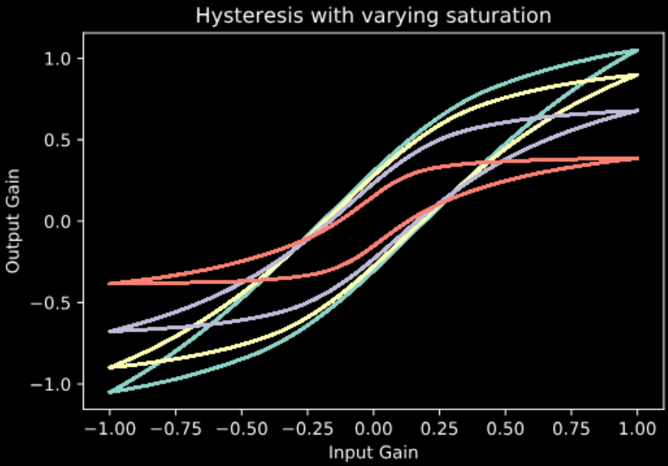
Use S, a, c as parameters

$$\dot{y} = \frac{\frac{(1-c)\delta_y (SL(Q)-y)}{(1-c)\delta_x k - \alpha(SL(Q)-y)}\dot{x} + c\frac{S}{a}\dot{x}L'(Q)}{1 - c\alpha\frac{S}{a}L'(Q)} \tag{8}$$

[5]Holters and Zolzer, “Circuit Simulation with Inductors and Transformers Based on the Jiles-Atherton Model of Magnetization”; Chowdhury, “Real-Time Physical Modelling For Analog Tape Machines”.

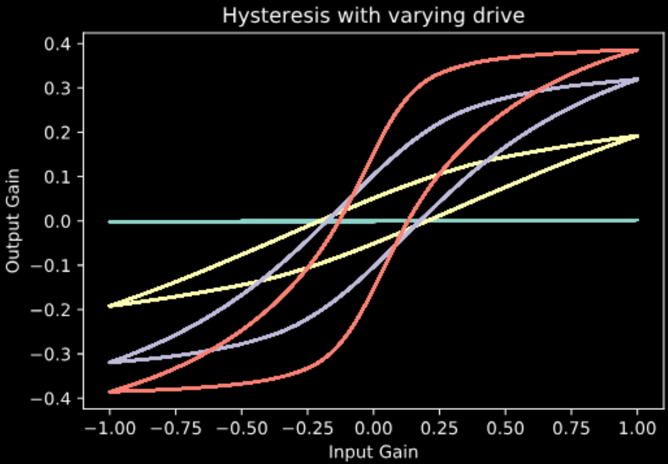
Hysteresis: Parameters

Saturation



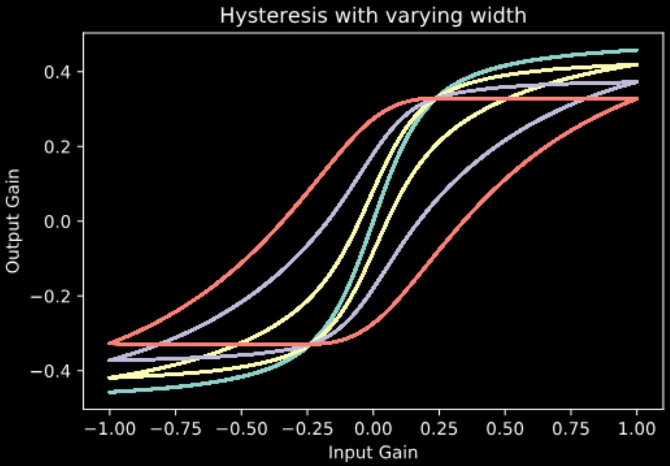
Hysteresis: Parameters

Drive



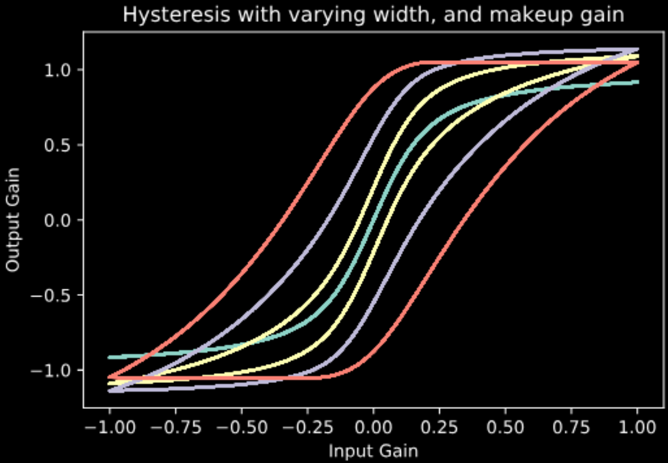
Hysteresis: Parameters

Width

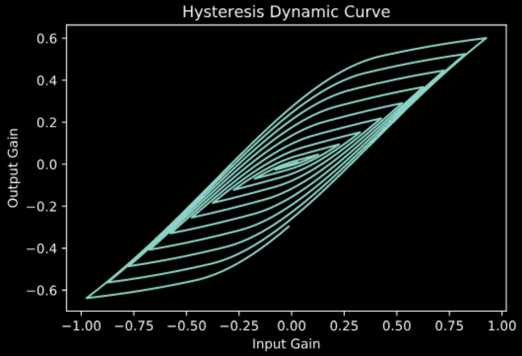


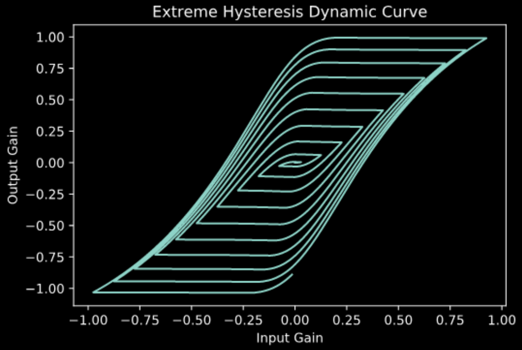
Hysteresis: Parameters

Width (with makeup gain)



Hysteresis

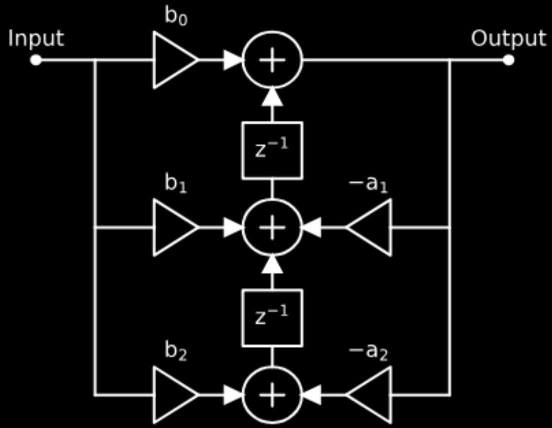

Hysteresis Dynamic Curve
Output Gain
Input Gain
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Extreme Hysteresis Dynamic Curve
Output Gain
Input Gain
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

Nonlinear Filters

Biquad Filter

Transposed Direct Form II



Biquad Filter

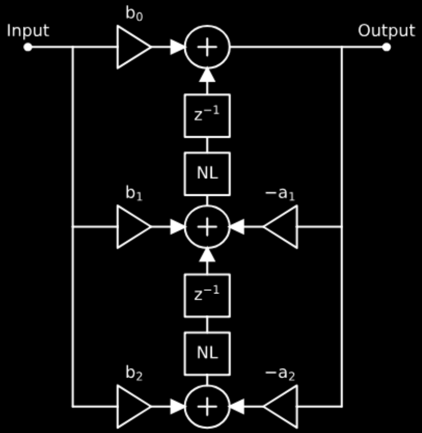
Difference equation:

$$y[n] = b_0 u[n] + b_1 u[n-1] + b_2 u[n-2] - a_1 y[n-1] - a_2 y[n-2] \tag{9}$$

State space formulation:

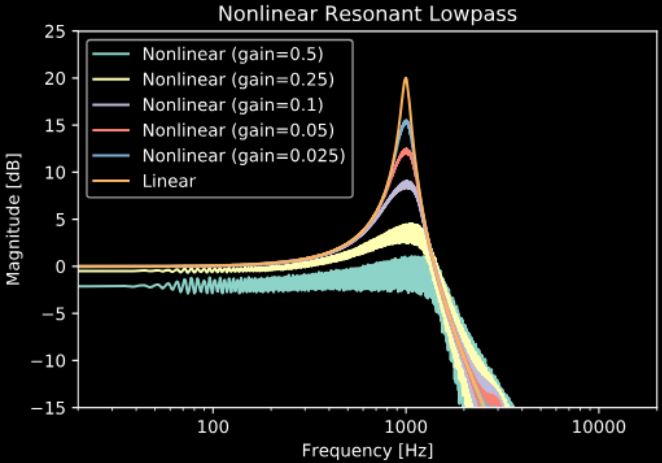
$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & -a_1 \\ 0 & 0 & -a_2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \tag{10}$$

Nonlinear Biquad Filter



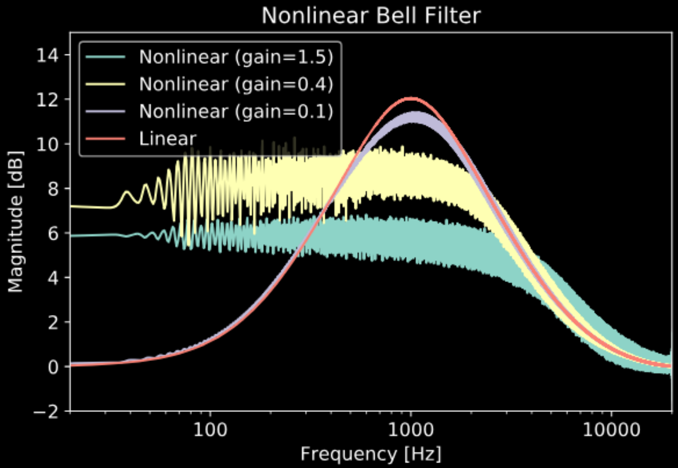
Nonlinear Biquad Filter

Saturating nonlinearities $\rightarrow$ nonlinear resonance



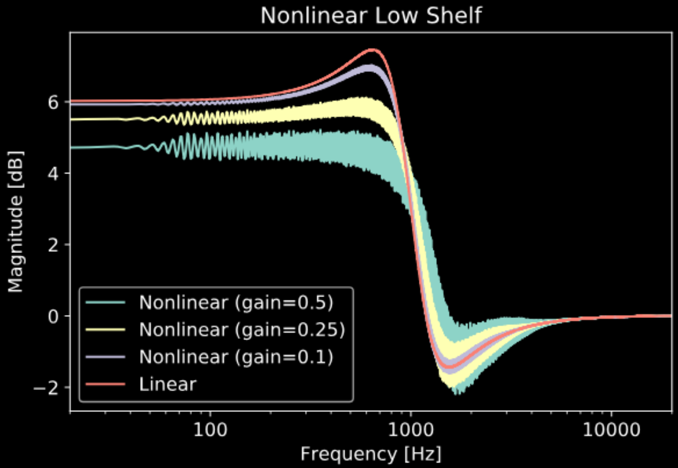
Nonlinear Biquad Filter

Saturating nonlinearities →nonlinear resonance



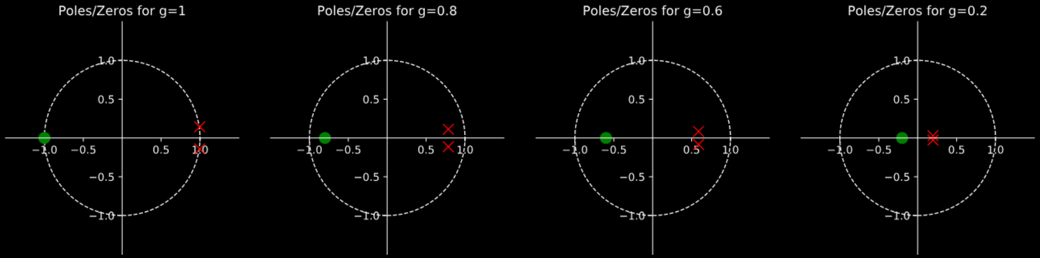
Nonlinear Biquad Filter

Saturating nonlinearities $\rightarrow$nonlinear resonance

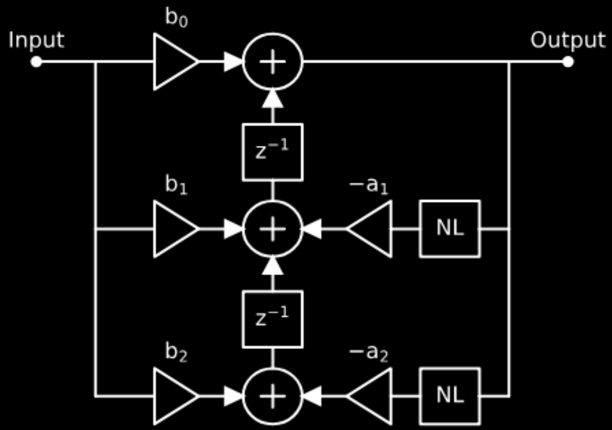


Nonlinear Biquad Filter

Pole/zero movement

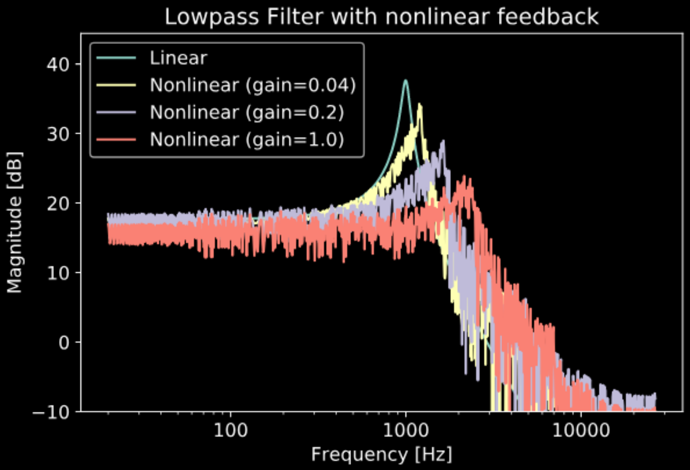


Nonlinear Feedback Filter



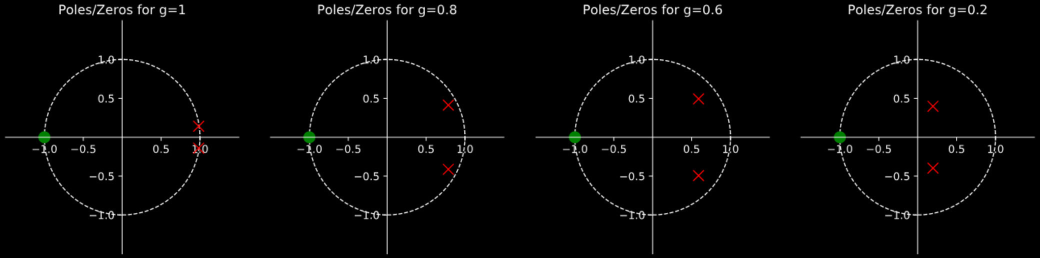
Nonlinear Feedback Filter

Saturating nonlinearity →cutoff frequency modulation



Nonlinear Feedback Filter

Pole/zero movement



Nonlinear Biquad Stability

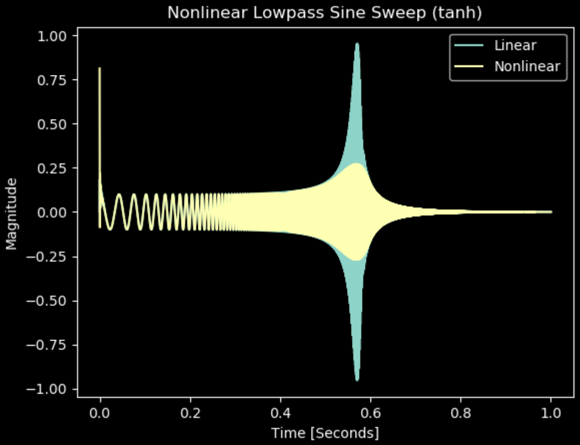
Questions:

Can we guarantee that a nonlinear filter will be stable given that its linear corrolary is stable?

For what subset of nonlinear functions is this guaranteed?

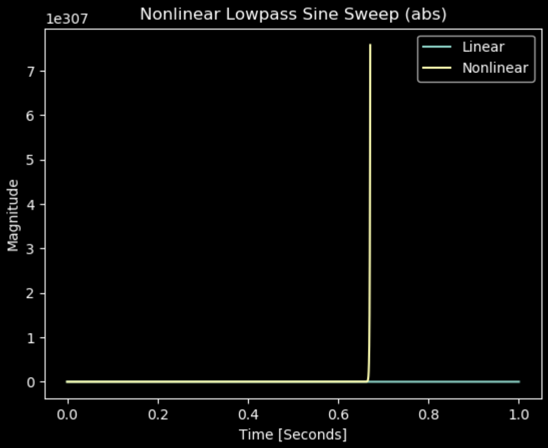
Nonlinear Biquad Stability

Test case: saturating nonlinearity, $f_{NL} = \tanh(x)$ →STABLE!



Nonlinear Biquad Stability

Test case: full wave rectifier, $f_{NL} = 0.45|x|$ →UNSTABLE!



Lyapunov Stability[6]

1. Form state space equation:

$$\mathbf{x}[n+1] = \mathbf{f}(\mathbf{x}[n]) \tag{11}$$

2. Find Jacobian $\mathbf{J}$ of $\mathbf{f}$

3. If every element of $\mathbf{J}$ is less than 1 at some operating point, the system is Lyapunov stable about that point.

[6] Chen, "Stability of Nonlinear Systems".

Nonlinear Biquad Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h}\left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} \right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \tag{12}$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) =& f_{NL}(x_2[n]) - a_1 y[n] \\ h_2(x_1[n], x_2[n], y[n]) =& - a_2 y[n] \\ h_3(x_1[n], x_2[n], y[n]) =& f_{NL}(x_1[n]) \end{aligned} \tag{13}$$

Nonlinear Biquad Stability

$$\mathbf{J} = \begin{bmatrix} 0 & f'_{NL}(x_2[n]) & -a_1 \\ 0 & 0 & -a_2 \\ f'_{NL}(x_1[n]) & 0 & 0 \end{bmatrix} \tag{14}$$

Note that if f'_{NL} does not exist at some point, the system is NOT stable at that point.

Nonlinear Feedback Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h}\left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix}\right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \tag{15}$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) =& x_2[n] - a_1 f_{NL}(y[n]) \\ h_2(x_1[n], x_2[n], y[n]) =& -a_2 f_{NL}(y[n]) \\ h_3(x_1[n], x_2[n], y[n]) =& x_1[n] \end{aligned} \tag{16}$$

Nonlinear Feedback Stability

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & -a_1 f'_{NL}(y[n]) \\ 0 & 0 & -a_2 f'_{NL}(y[n]) \\ 1 & 0 & 0 \end{bmatrix} \tag{17}$$

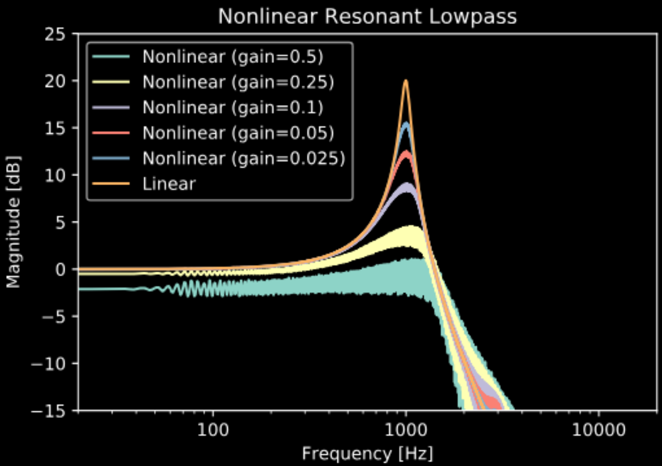
Nonlinear Biquad Stability

General stability contstraint:

$$|f'_{NL}(x)| \leq 1 \tag{18}$$

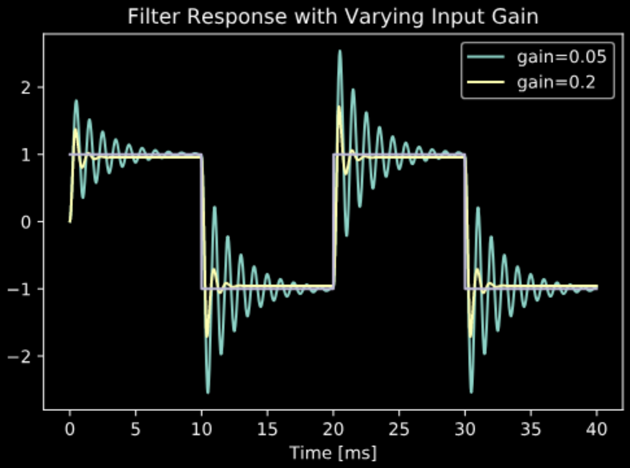
Nonlinear Biquad Filter

Can we use this for analog modelling?



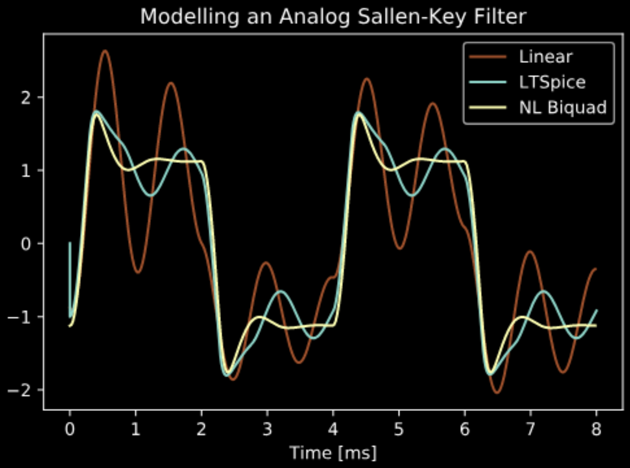
Nonlinear Biquad Filter

Parameters: nonlinearities, input gain



Nonlinear Biquad Filter

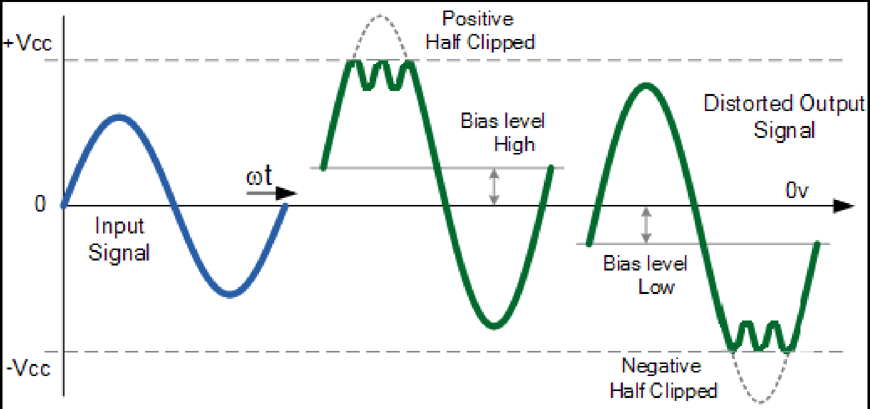
Modelling an overdriven Sallen-Key lowpass filter



Wavefolding

Wavefolder

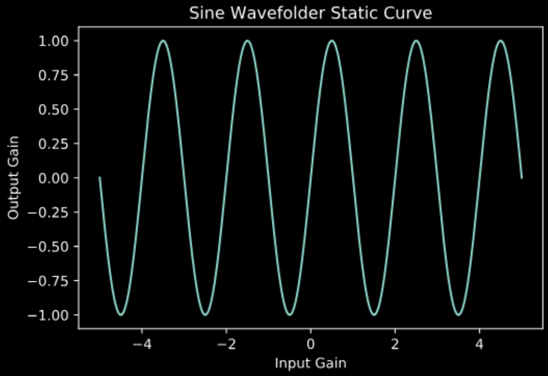
What is wavefolding?



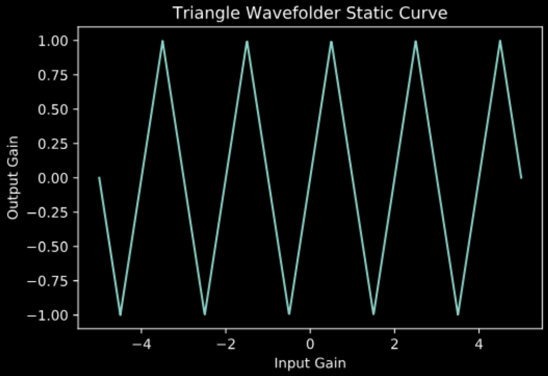
Wavefolder

Standard digital wavefolding:

$$f_{NL}(x) = sin(x)$$

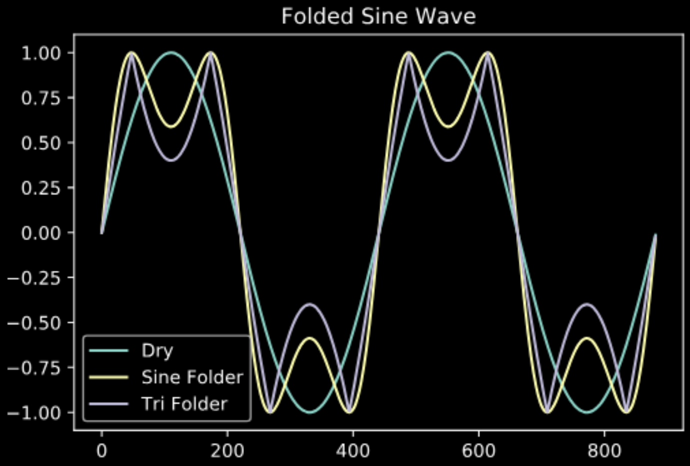


$$f_{NL}(x) = tri(x)$$



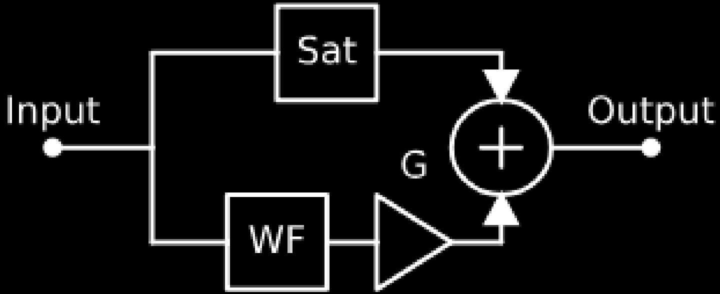
Wavefolder

Standard digital wavefolding:



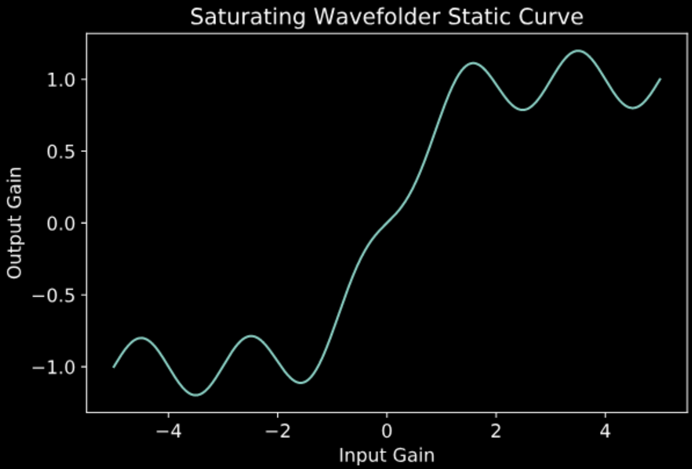
Wavefolder

Saturating wavefolder:



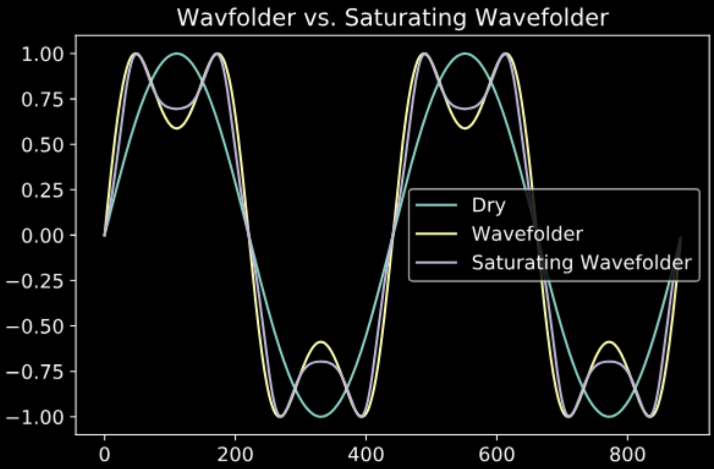
Wavefolder

Saturating wavefolder:



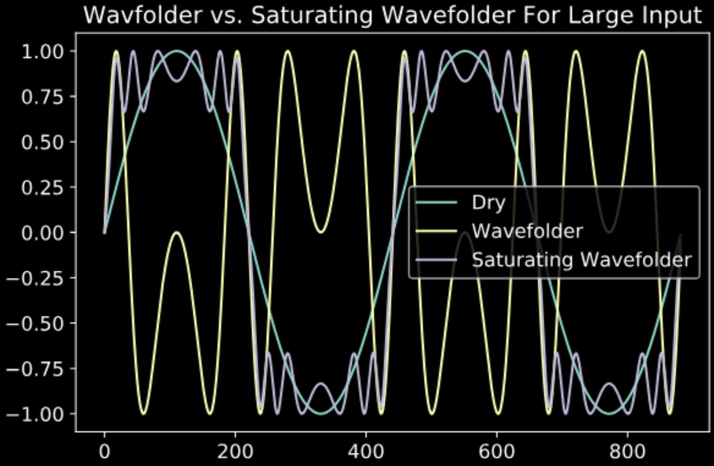
Wavefolder

Saturating wavefolder:



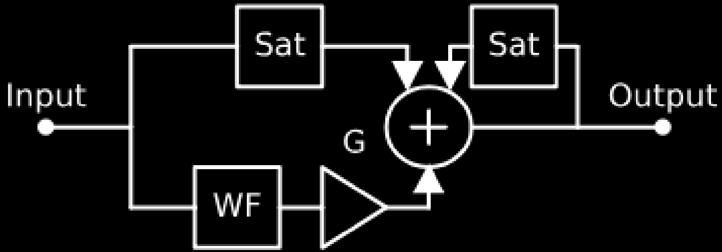
Wavefolder

Saturating wavefolder:



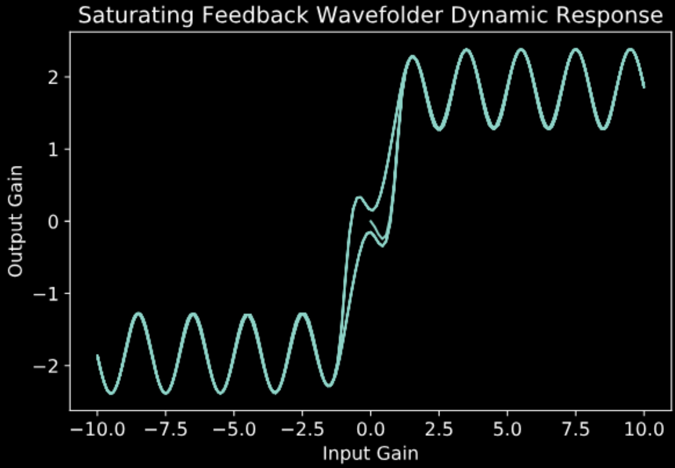
Wavefolder

Feedback wavefolder:



Wavefolder

Feedback wavefolder:



Subharmonics

Subharmonics: Motivation

Most nonlinear audio effects add higher harmonics to the signal.

What if we want lower harmonics …

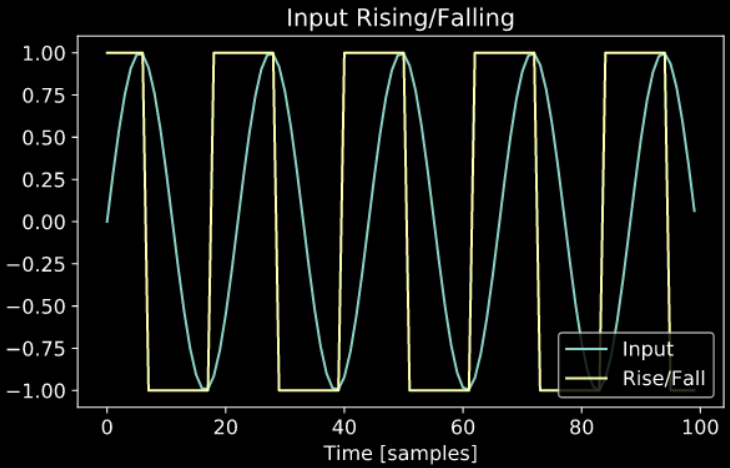
Subharmonics

Goals:

- Generate subharmonic content
- Avoid circuit modelling
- Avoid relying on high-quality pitch detection

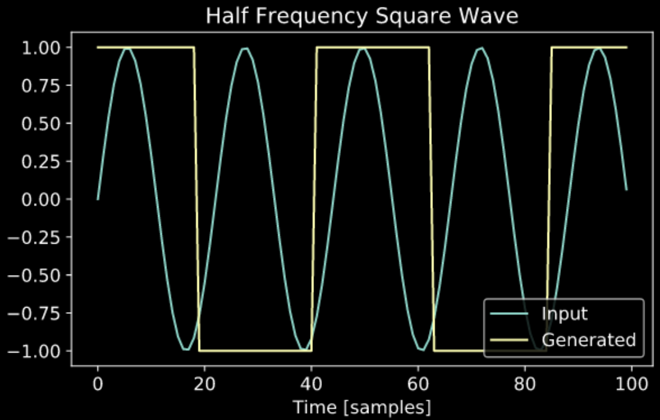
Subharmonics

Step 1: Detect when the input signal switches directions



Subharmonics

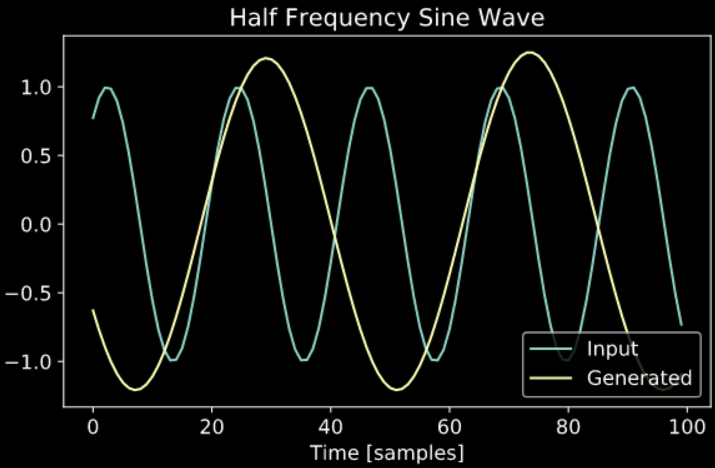
Step 2: Flip detector signal every *other* time



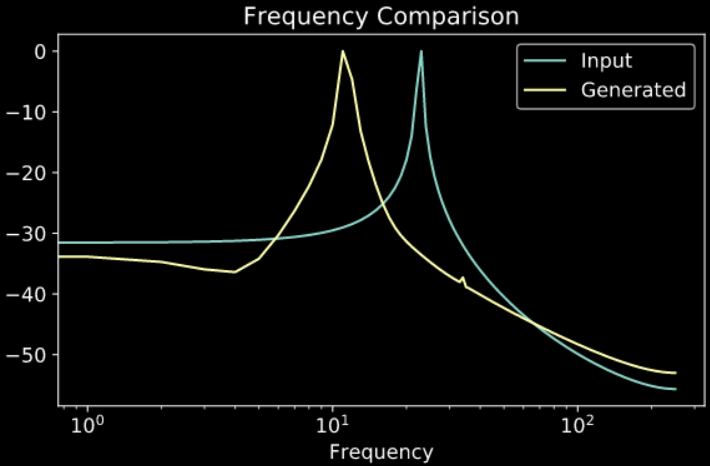
Result: half-frequency square wave!

Subharmonics

Step 3: Lowpass Filter

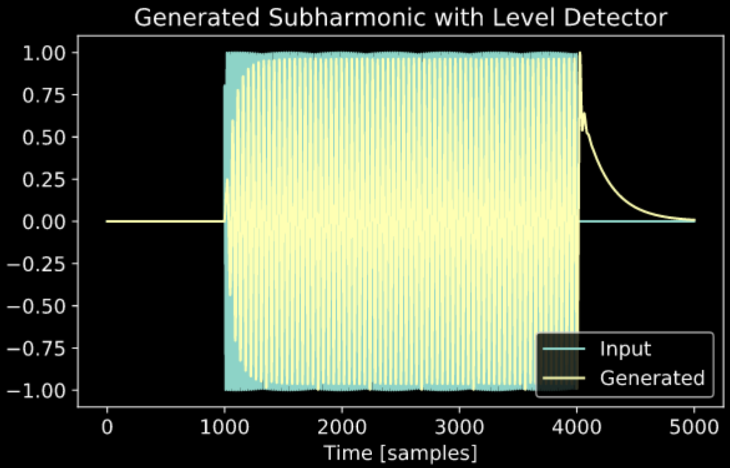


Subharmonics

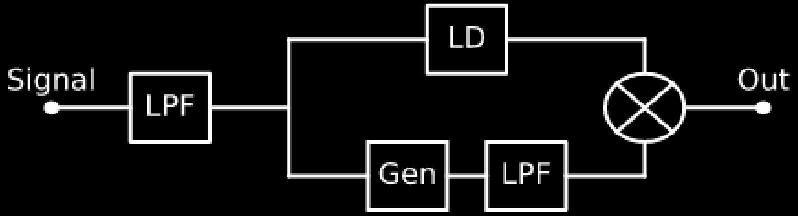


Subharmonics

Step 4: Apply level detector



Subharmonics



Gated Recurrent Distortion

Gated Recurrent Distortion

Gated Recurrent Unit:

- Building block for recurrent neural networks[7]
- Here we examine a variation: "minimal gated unit"[8]

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[7] Cho et al., "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation".

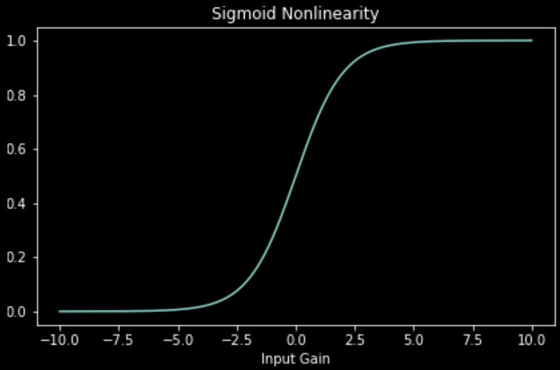
[8] Zhou et al., "Minimal Gated Unit for Recurrent Neural Networks".

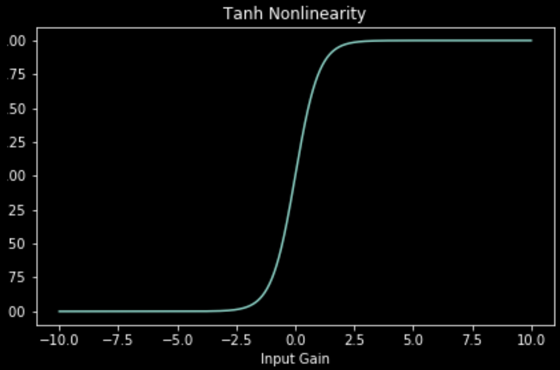
Gated Recurrent Distortion

$$\begin{aligned}\Gamma_f &= \sigma(W_f x[n] + U_f y[n-1] + b_f)\\ y[n] &= \Gamma_f y[n-1] + (1-\Gamma_f)\tanh(W_h x[n] + U_h \Gamma_f y[n-1] + b_h)\end{aligned} \tag{19}$$

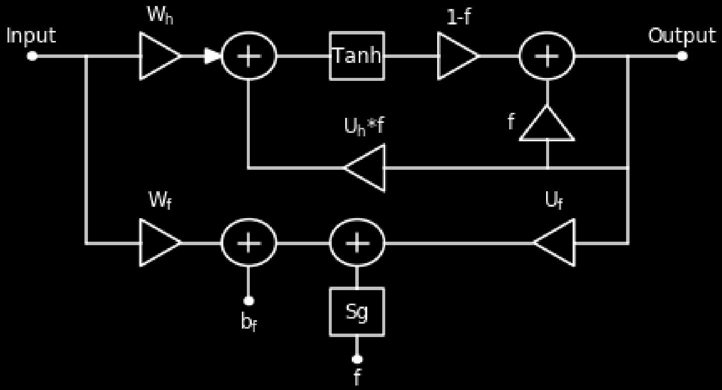
$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{20}$$

Gated Recurrent Distortion

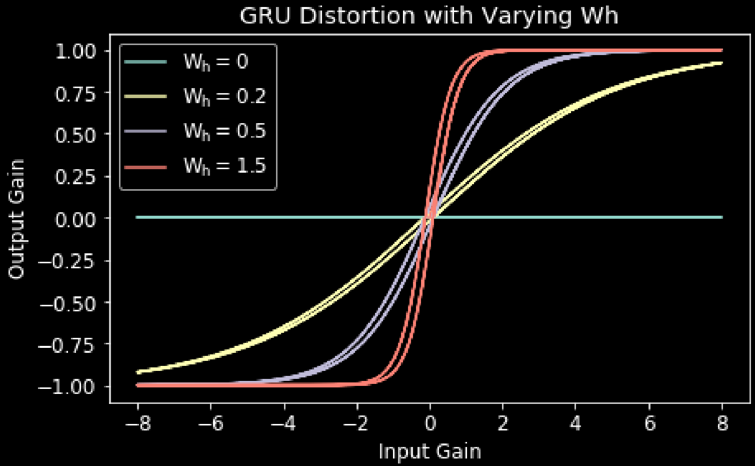




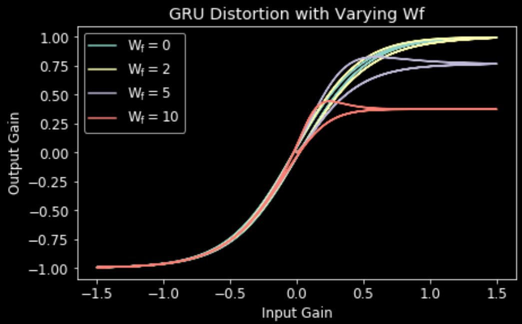
Gated Recurrent Distortion

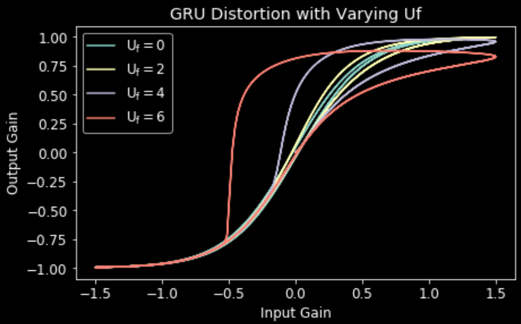


Gated Recurrent Distortion: Parameters

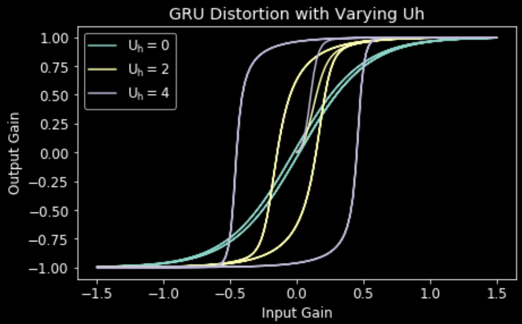


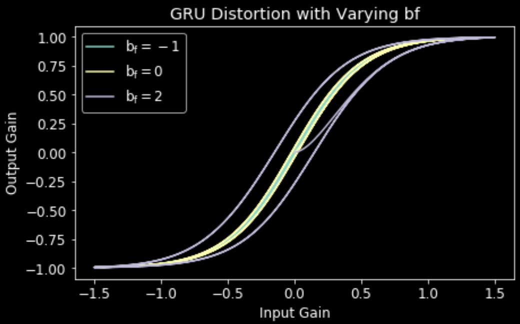
Gated Recurrent Distortion: Parameters



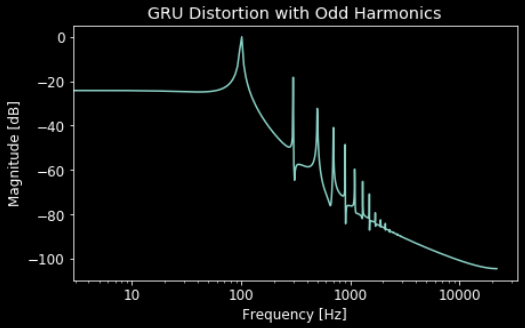


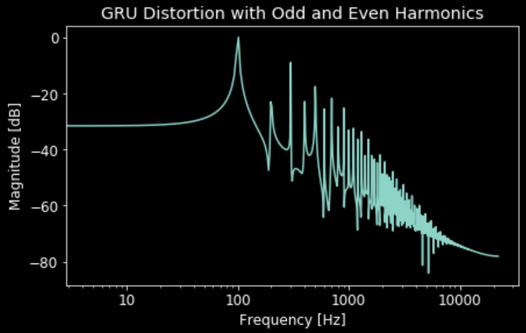
Gated Recurrent Distortion: Parameters





Gated Recurrent Distortion: Harmonic Response





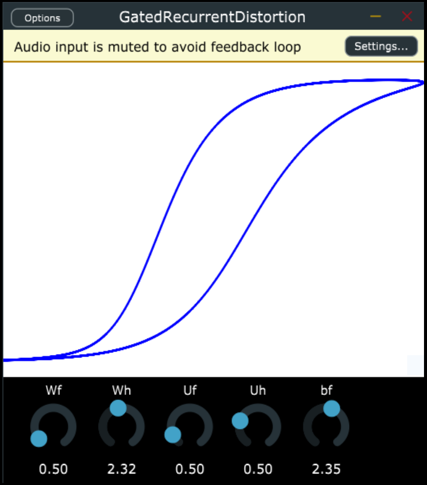
Conclusion

Goals

- Tools for musicians/mixing engineeers
- Inspiration/explanations for audio effect makers
- A academic paper (or two)

Presentation

Audio plugins (VST/AU)



Presentation

Medium articles

M

Upgrade

Complex Nonlinearities Episode 4: Nonlinear Biquad Filters

Jatin Chowdhury
Oct 10, 2019 · 6 min read

For today's article, we'll be talking about filters. So far in this series I haven't spoken too much about filters, which might seem odd considering how much of signal processing in general is all about filters. The reason I've avoided filters is that most filters in audio signal processing are implemented as linear processors, and I've been focusing on nonlinear processing concepts.

Thank you!