Complex Nonlinearities for Audio Signal Processing

Jatin Chowdhury

Center for Computer Research in Music and Acoustics (CCRMA)

Thanks

- Julius Smith
- Dave Berners
- Viraga Perera
- GASP

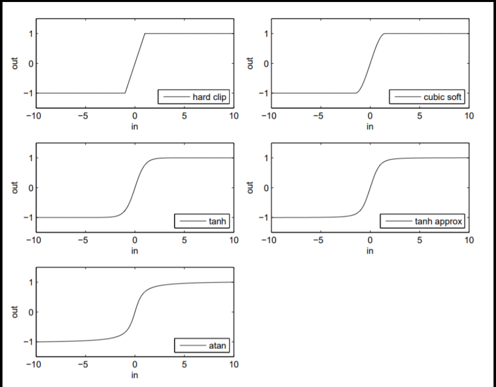
Motivation

Flashback to 2016…

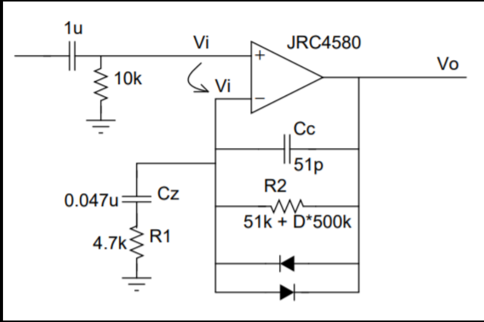
Trying to make a distortion effect…

Motivation

Static Nonlinearities



Circuit Modelling



Motivation

Solution: $\tanh$ approximation[1]

$$f(x) = \frac{x}{(1 + |x|^n)^{1/n}}, \quad n = 2.5 \tag{1}$$

[1] Yeh, "Digital Implementation of Musical Distortion Circuits by Analysis and Simulation".

Motivation

Trying to fill the gap between:

- Simple static nonlinear systems
- Physically modelled nonlinear systems

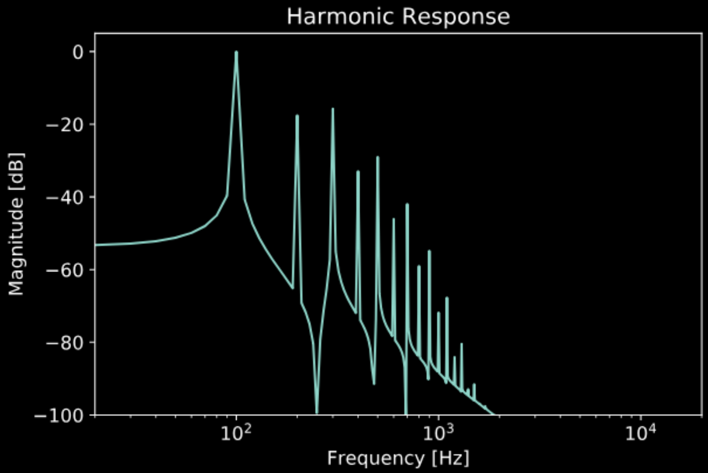
Outline

- Basics of nonlinear signal processing
- Complex nonlinearities
  - Double Soft Clipper
  - Exciter
  - Nonlinear biquad filters
  - Wavefolding
  - Subharmonics
  - Gated Recurrent Distortion
- Audio Examples
- Conclusion

Building Blocks

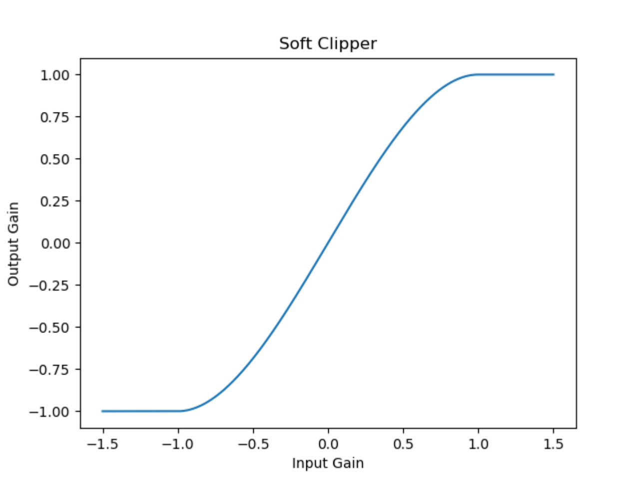
What is a nonlinear system?

Frequency domain: you get out more than what you put in



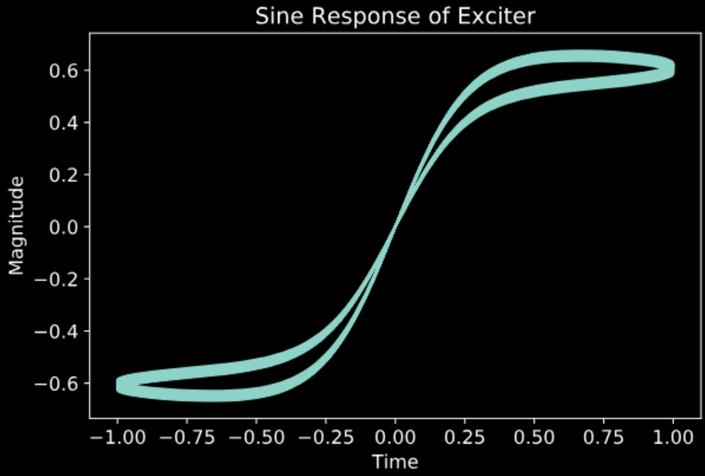
What is a nonlinear system?

Time domain: static response

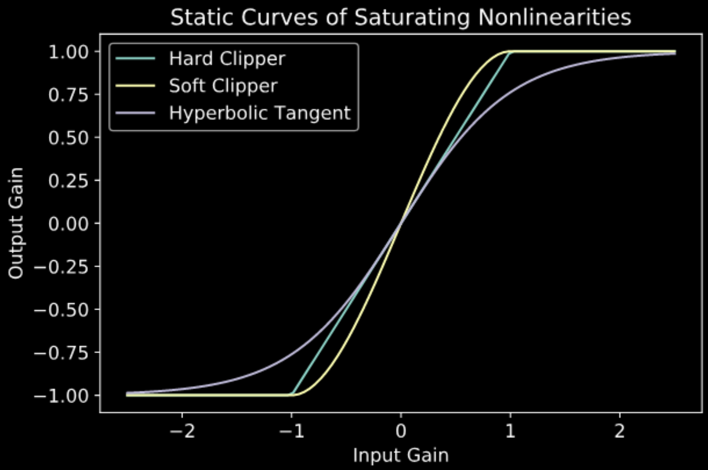


What is a nonlinear system?

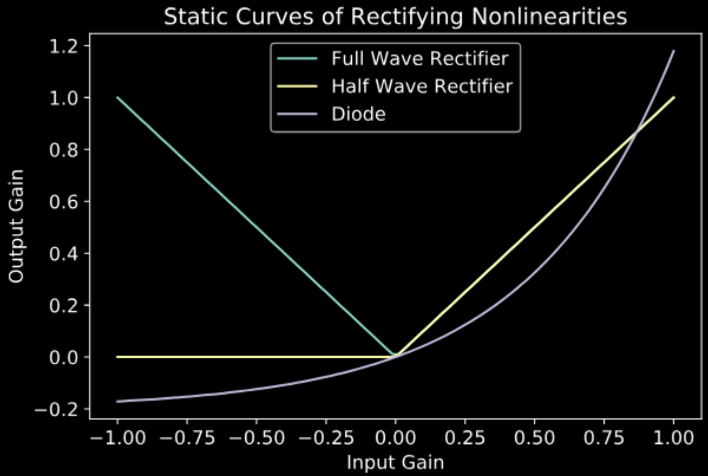
Time domain: dynamic response



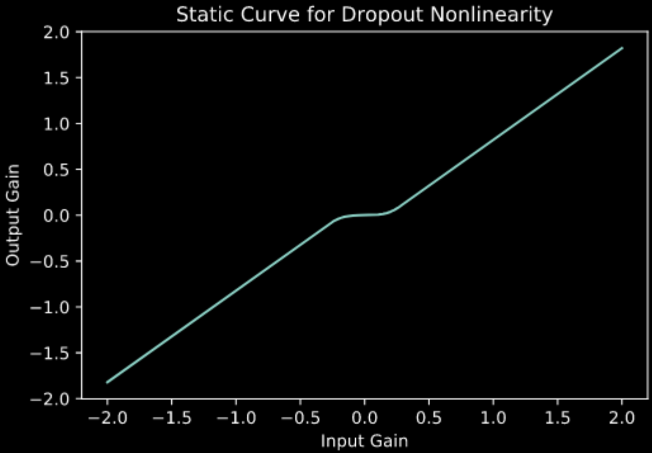
Saturating Nonlinearities



Rectifying Nonlinearities



Dropout Nonlinearities



(also called “dead-zone”)

What is a “Complex Nonlinearity”

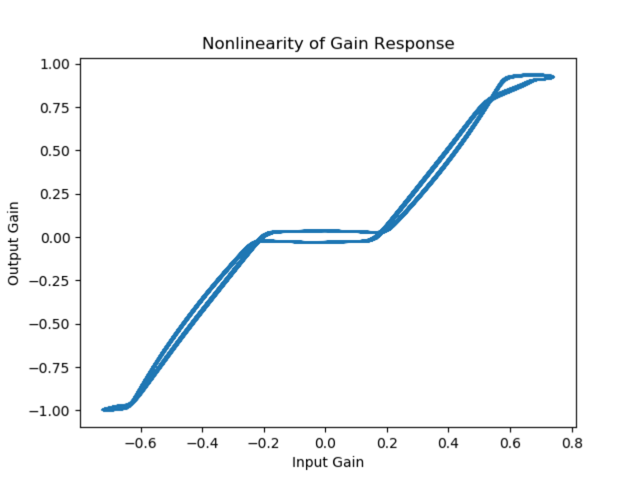
Has one of the following properties:

- Is not memoryless (has some memory of past states)
- Has an interesting harmonic response
- Has interesting parameters

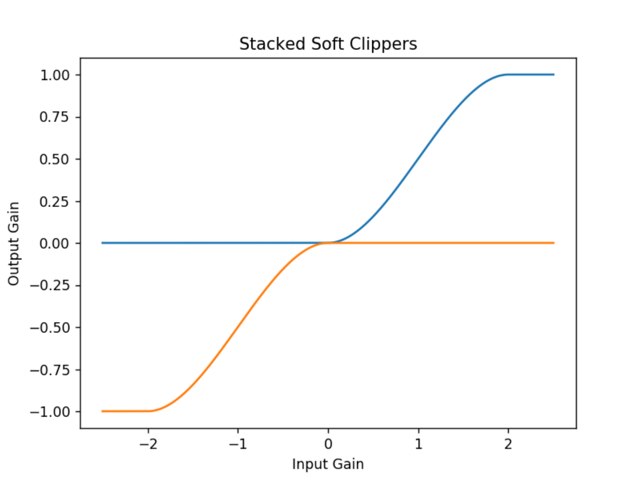
Double Soft Clipper

Double Soft Clipper: Inspiration

Measured speaker response



Double Soft Clipper



Double Soft Clipper: Original Soft Clipper

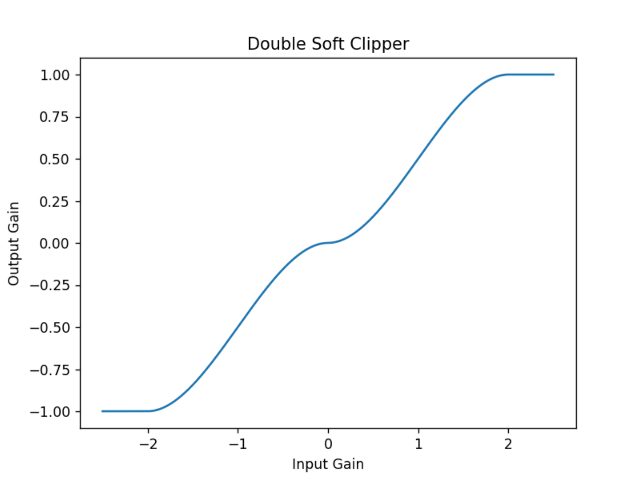
$$f_{SC}(x) = \begin{cases} 1 & x \geq 1 \\ \frac{3}{2}(x - x^3/3) & -1 < x < 1 \\ -1 & x \leq -1 \end{cases} \tag{2}$$

Double Soft Clipper

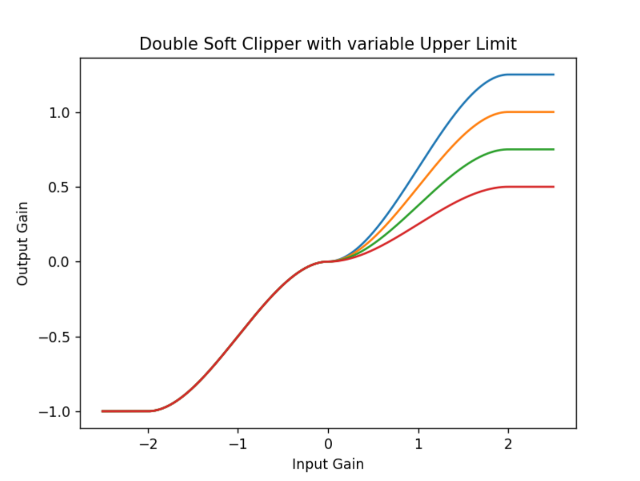
$$f_{DSC}(x) = \begin{cases} 1 & u \geq 1 \\ \frac{3}{4}(u - u^3/3) + 0.5 & 0 < u < 1 \\ \frac{3}{4}(u - u^3/3) - 0.5 & -1 < u < 0 \\ -1 & u \leq -1 \end{cases} \tag{3}$$

$$u(x) = \begin{cases} x - 0.5 & x > 0 \\ x + 0.5 & x < 0 \end{cases} \tag{4}$$

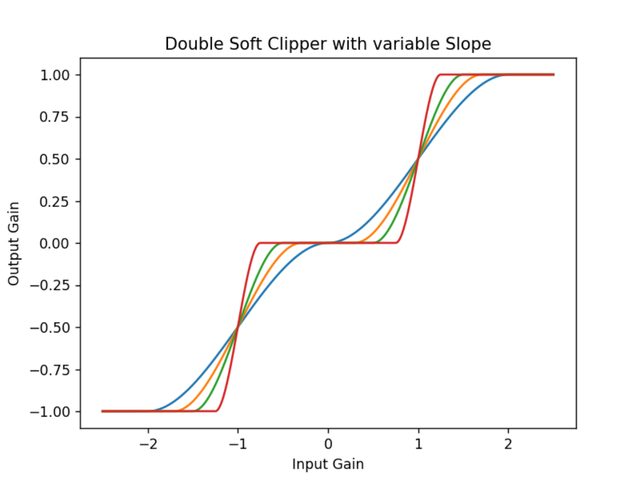
Double Soft Clipper



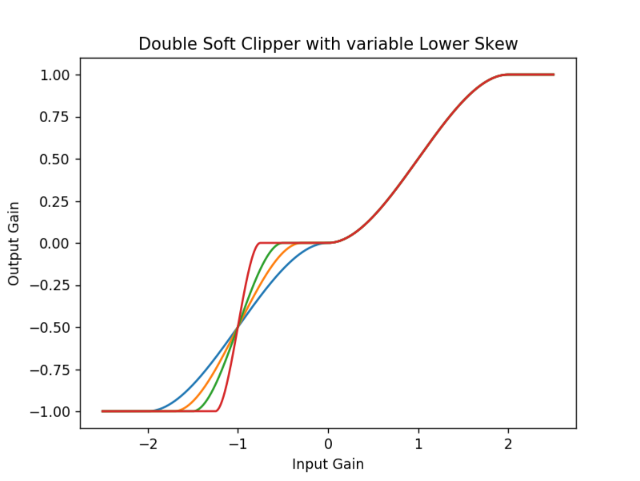
Double Soft Clipper: Parameters



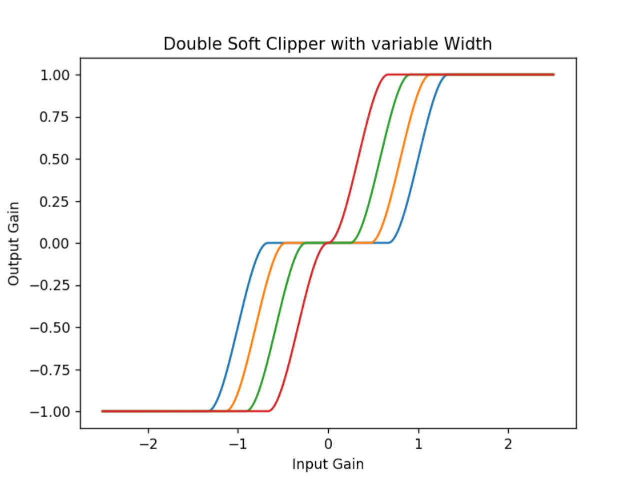
Double Soft Clipper: Parameters



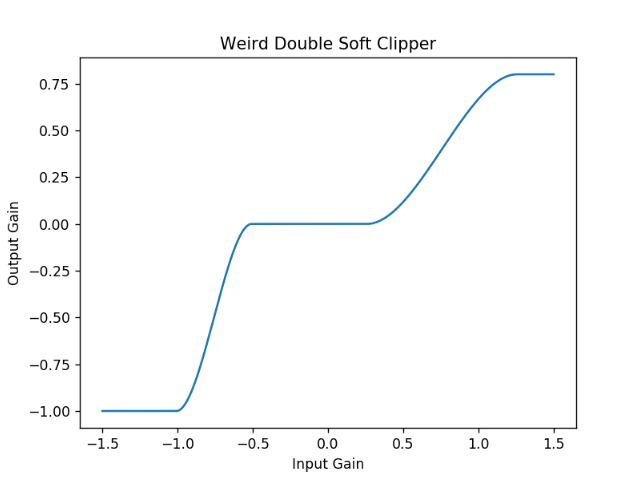
Double Soft Clipper: Parameters



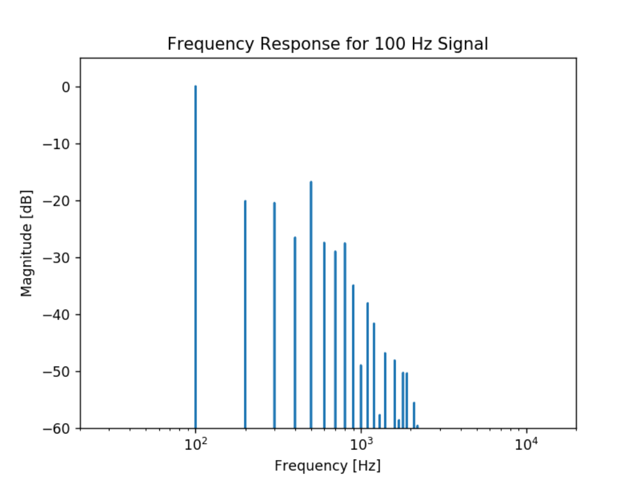
Double Soft Clipper: Parameters



Double Soft Clipper: Weird



Double Soft Clipper: Harmonic Response



Harmonic Exciter

Harmonic Exciter

What is a harmonic exciter?

- Add subtle harmonic distortion
- Make audio sound “shiny”, “brighter”, “enhanced”
- Used for mixing, live broadcasts, restoring old recordings with missing spectral content

Aphex Aural Exciter[2]

- Introduced in the mid-1970's
- Used by Jackson Browne, The Four Seasons, Linda Ronstadt, and more
- Originally rented to studios for **$30 per minute**

[2] Ltd., *Aphex Aural Exciter Type B: Operating Guide*.

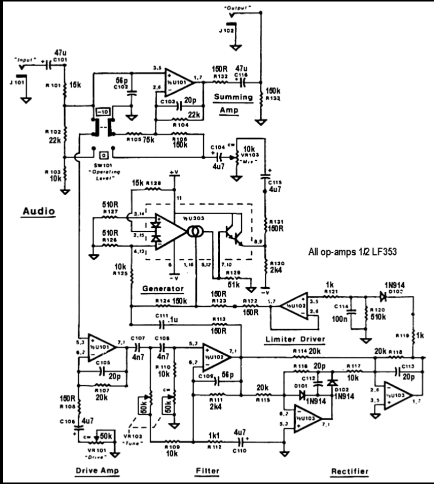
Harmonic Exciter

Goals:

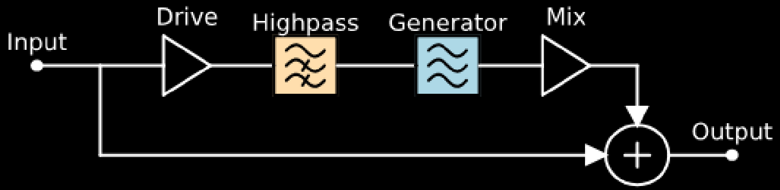
- Exciter model that sounds “smooth”
- Generalize the model to transcend circuit modelling[3]

[3] Giannoulis, Massberg, and Reiss, “Digital Dynamic Range Compressor Design - A Tutorial and Analysis”.

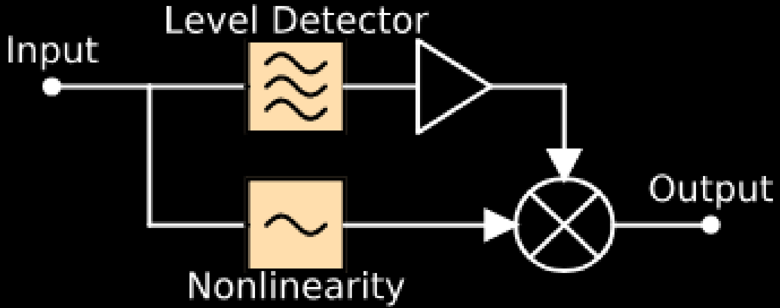
Harmonic Exciter



Harmonic Exciter

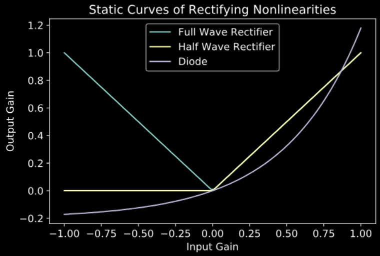


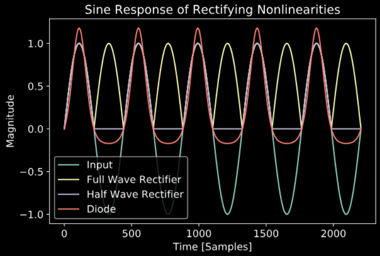
Harmonic Exciter: Generator



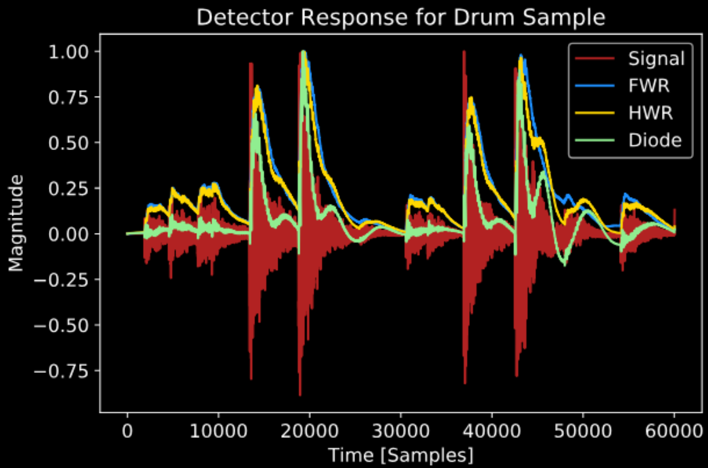
Harmonic Exciter: Level Detector

Rectifying nonlinearity $\rightarrow$LPF

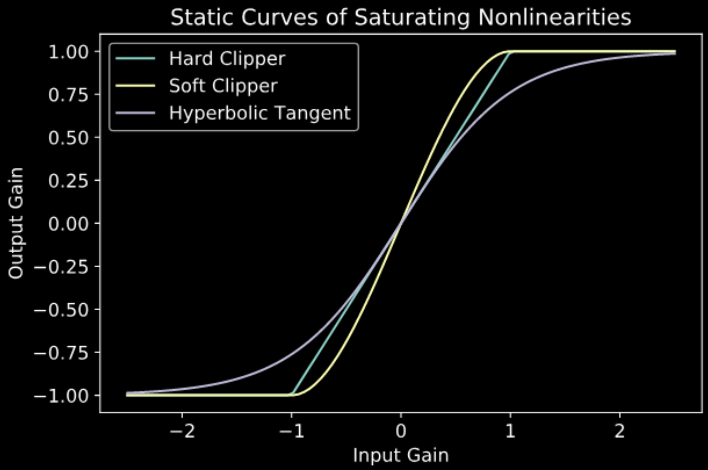




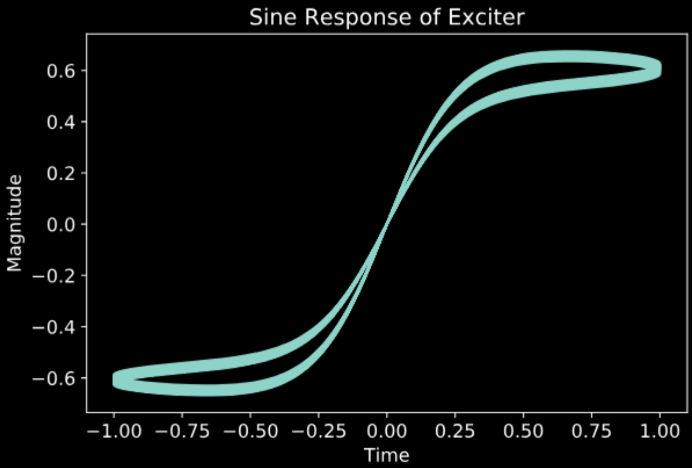
Harmonic Exciter: Level Detector



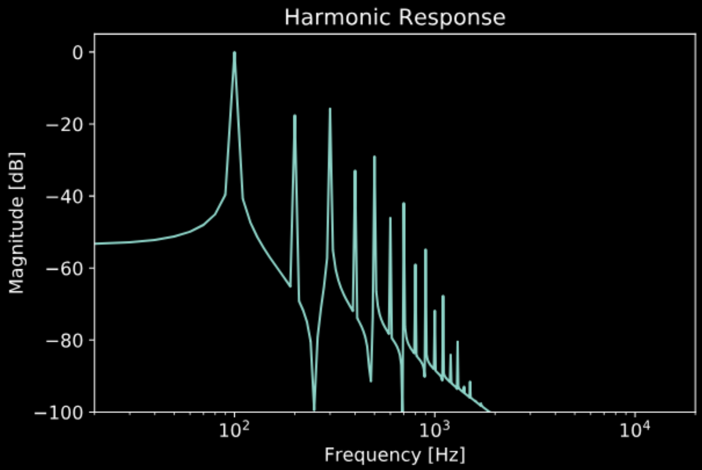
Harmonic Exciter: Nonlinearity



Harmonic Exciter: Generator



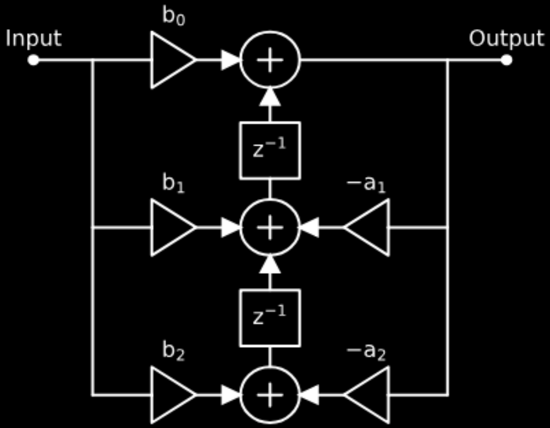
Harmonic Exciter: Generator



Nonlinear Filters

Biquad Filter

Transposed Direct Form II



Biquad Filter

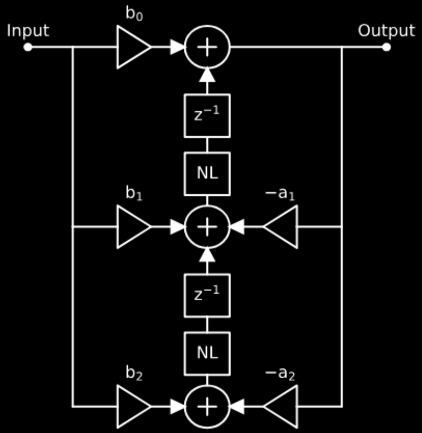
Difference equation:

$$y[n] = b_0 u[n] + b_1 u[n-1] + b_2 u[n-2] - a_1 y[n-1] - a_2 y[n-2] \quad (5)$$

State space formulation:

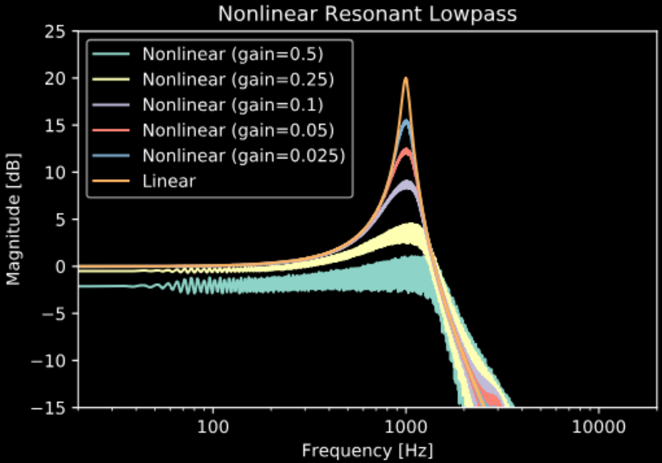
$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & -a_1 \\ 0 & 0 & -a_2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \quad (6)$$

Nonlinear Biquad Filter



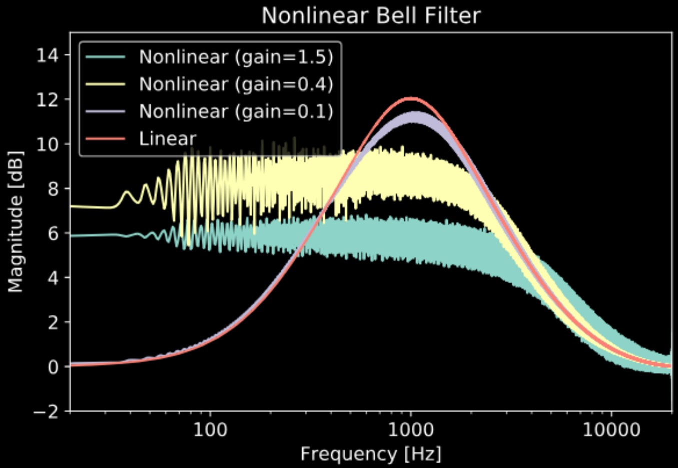
Nonlinear Biquad Filter

Saturating nonlinearities →nonlinear resonance



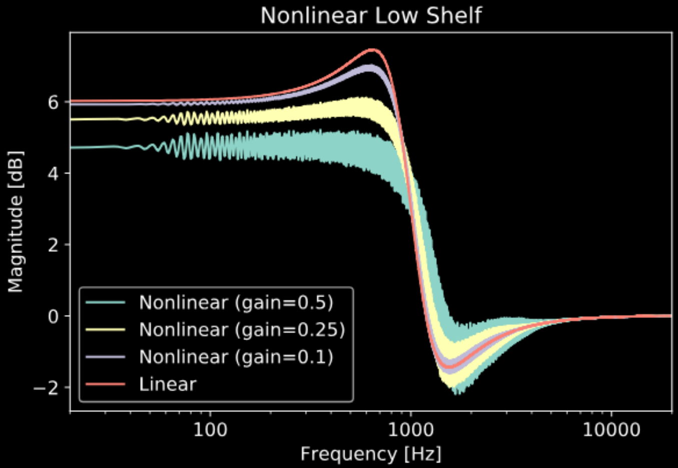
Nonlinear Biquad Filter

Saturating nonlinearities →nonlinear resonance



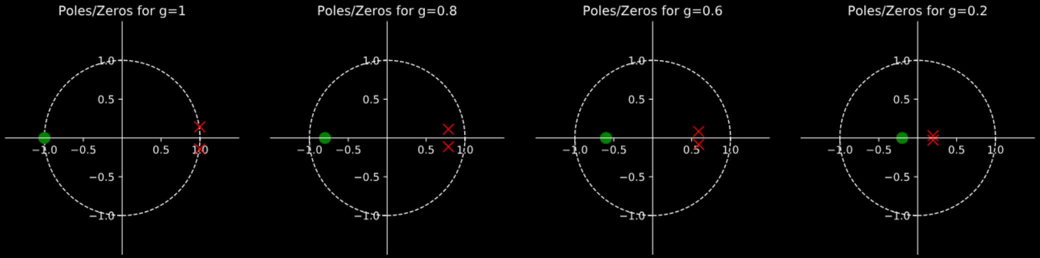
Nonlinear Biquad Filter

Saturating nonlinearities →nonlinear resonance

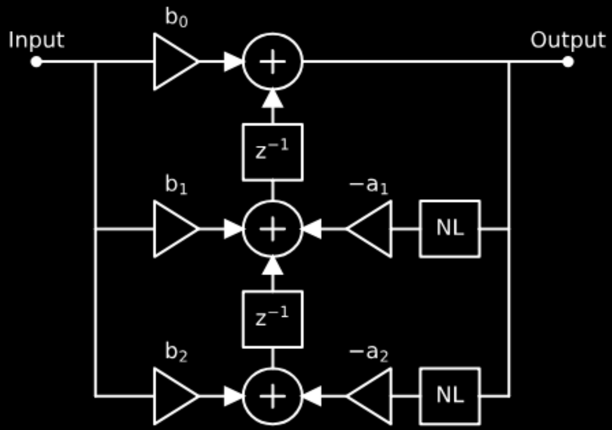


Nonlinear Biquad Filter

Pole/zero movement

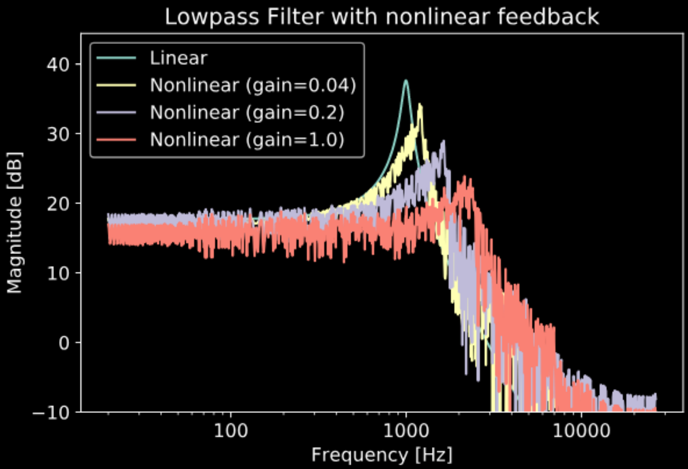


Nonlinear Feedback Filter



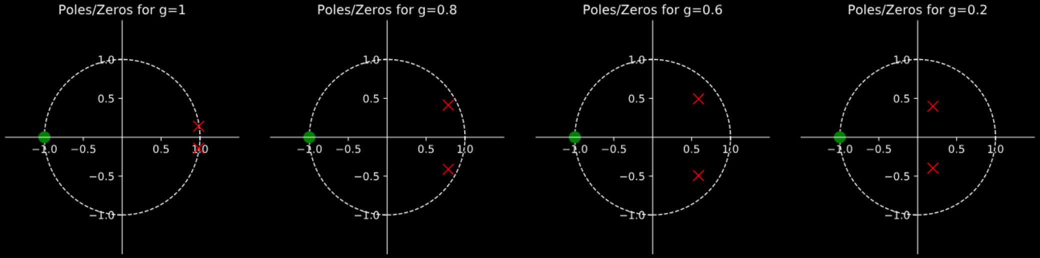
Nonlinear Feedback Filter

Saturating nonlinearity →cutoff frequency modulation



Nonlinear Feedback Filter

Pole/zero movement



Nonlinear Biquad Stability

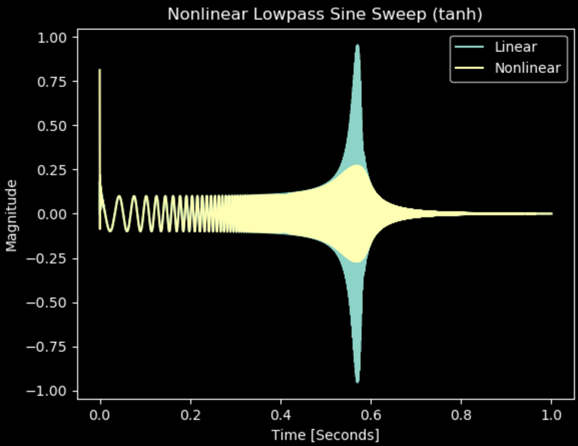
Questions:

Can we guarantee that a nonlinear filter will be stable given that its linear corrolary is stable?

For what subset of nonlinear functions is this guaranteed?

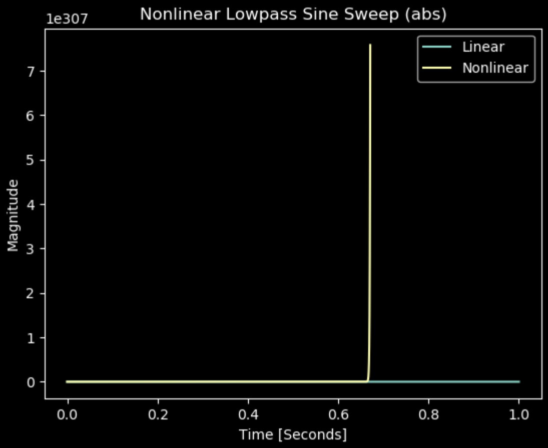
Nonlinear Biquad Stability

Test case: saturating nonlinearity, $f_{NL} = \tanh(x)$ →STABLE!



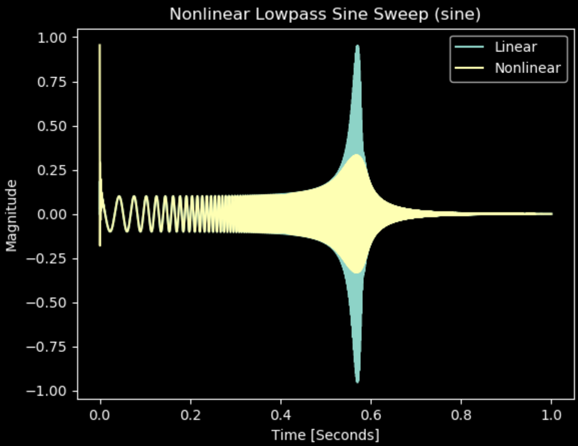
Nonlinear Biquad Stability

Test case: full wave rectifier, $f_{NL} = 0.45|x| \rightarrow$UNSTABLE!



Nonlinear Biquad Stability

Test case: sine, $f_{NL} = \sin(x)$ $\rightarrow$STABLE!



Lyapunov Stability[4]

1. Form state space equation:

$$\mathbf{x}[n+1] = \mathbf{f}(\mathbf{x}[n]) \tag{7}$$

2. Find Jacobian $\mathbf{J}$ of $\mathbf{f}$

3. If every element of $\mathbf{J}$ is less than 1 at some operating point, the system is Lyapunov stable about that point.

[4] Chen, “Stability of Nonlinear Systems”.

Nonlinear Biquad Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h}\left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix}\right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \tag{8}$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) =& f_{NL}(x_2[n]) - a_1 y[n] \\ h_2(x_1[n], x_2[n], y[n]) =& - a_2 y[n] \\ h_3(x_1[n], x_2[n], y[n]) =& f_{NL}(x_1[n]) \end{aligned} \tag{9}$$

Nonlinear Biquad Stability

$$\mathbf{J} = \begin{bmatrix} 0 & f'_{NL}(x_2[n]) & -a_1 \\ 0 & 0 & -a_2 \\ f'_{NL}(x_1[n]) & 0 & 0 \end{bmatrix} \tag{10}$$

Note that if f'_{NL} does not exist at some point, the system is NOT stable at that point.

Nonlinear Feedback Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h}\left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} \right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \tag{11}$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) =& x_2[n] - a_1 f_{NL}(y[n]) \\ h_2(x_1[n], x_2[n], y[n]) =& - a_2 f_{NL}(y[n]) \\ h_3(x_1[n], x_2[n], y[n]) =& x_1[n] \end{aligned} \tag{12}$$

Nonlinear Feedback Stability

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & -a_1 f'_{NL}(y[n]) \\ 0 & 0 & -a_2 f'_{NL}(y[n]) \\ 1 & 0 & 0 \end{bmatrix} \tag{13}$$

Nonlinear Biquad Stability

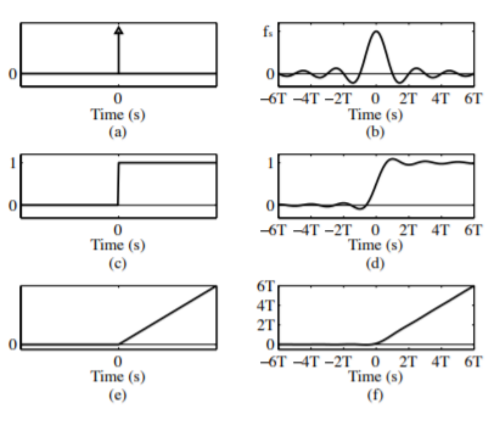
General stability contstraint:

$$|f'_{NL}(x)| \leq 1 \tag{14}$$

Note: if the $f'_{NL}(x)$ does not exist, the filter is not guaranteed stable.

Nonlinear Biquad Stability

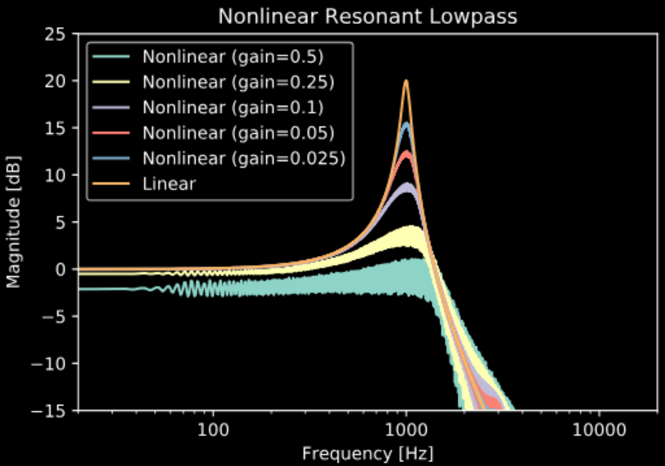
If derivative doesn't exist at every point: use BLAMP[5]!



[5] Esqueda, Valimaki, and Bilbao, "Rounding Corners with BLAMP".

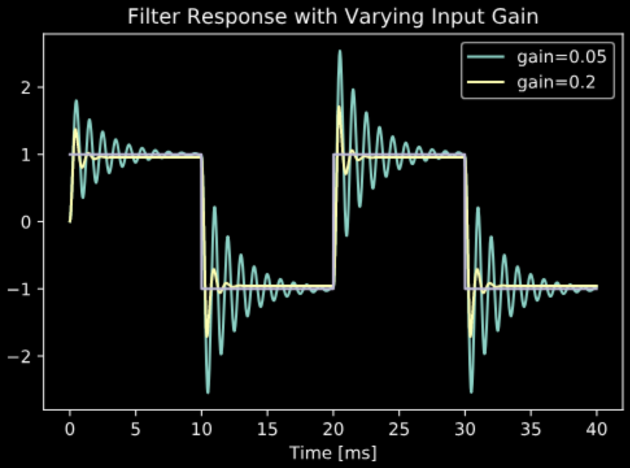
Nonlinear Biquad Filter

Can we use this for analog modelling?



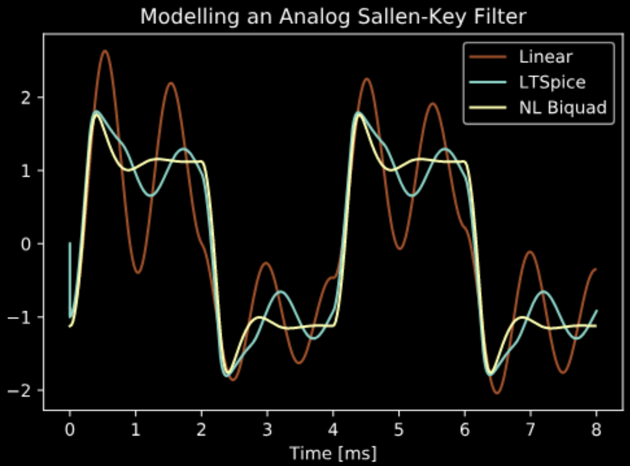
Nonlinear Biquad Filter

Parameters: nonlinearities, input gain



Nonlinear Biquad Filter

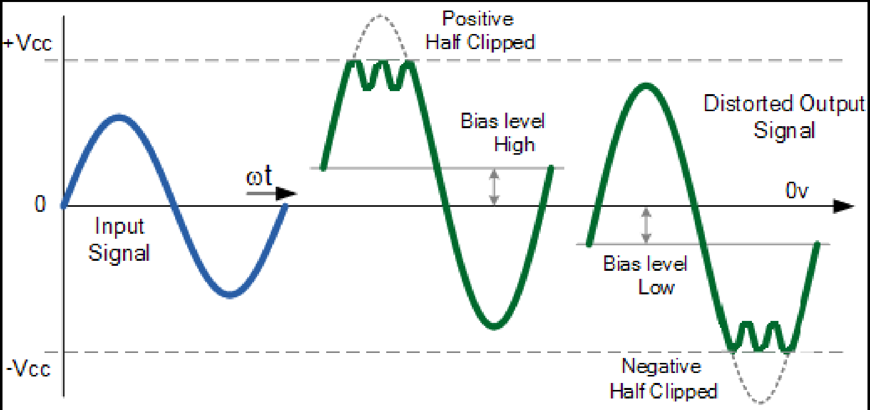
Modelling an overdriven Sallen-Key lowpass filter



Wavefolding

Wavefolder

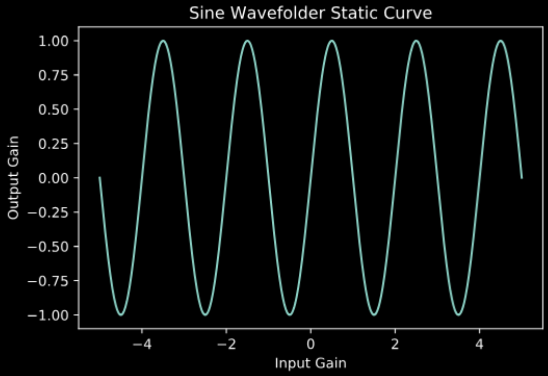
What is wavefolding?



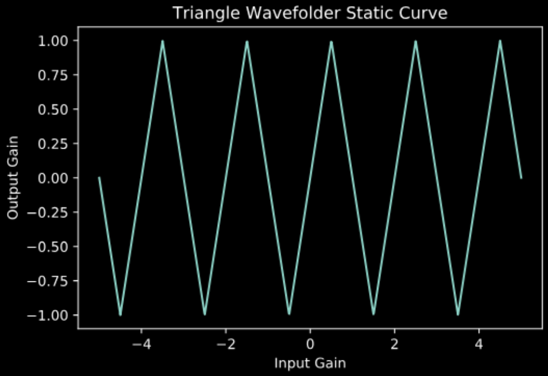
Wavefolder

Standard digital wavefolding:

$$f_{NL}(x) = sin(x)$$

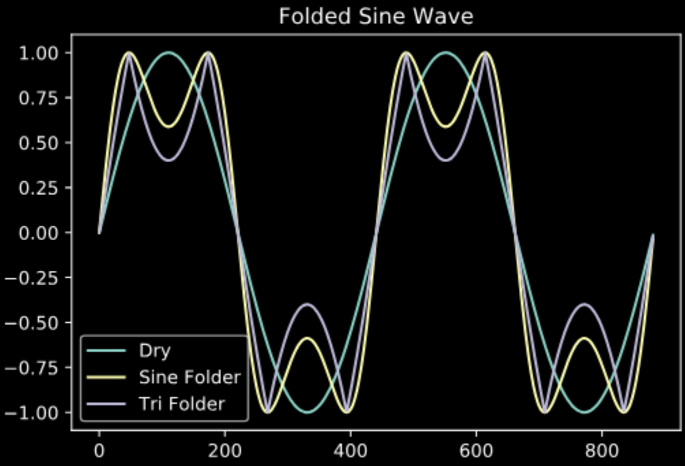


$$f_{NL}(x) = tri(x)$$



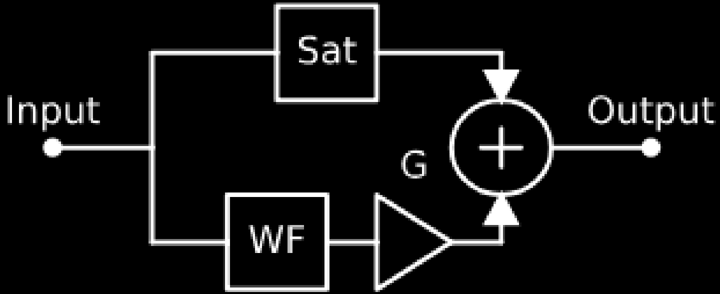
Wavefolder

Standard digital wavefolding:



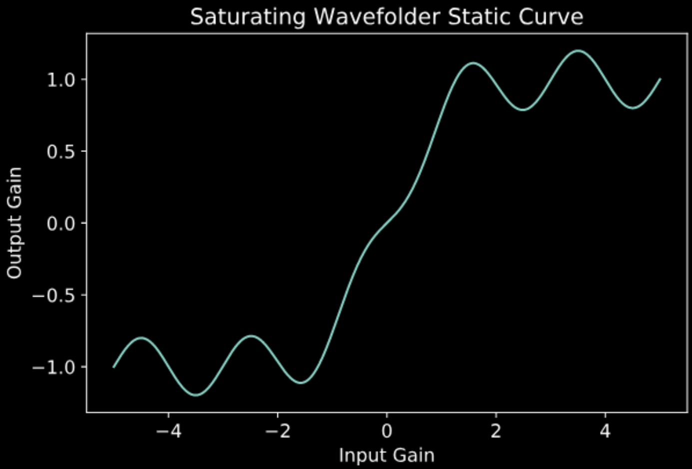
Wavefolder

Saturating wavefolder:



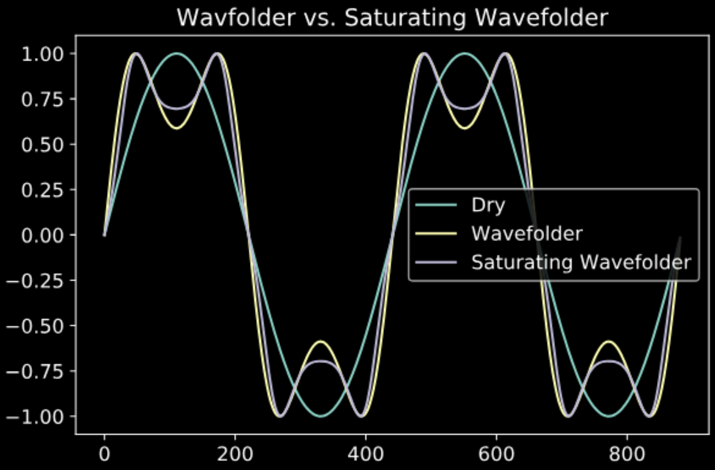
Wavefolder

Saturating wavefolder:



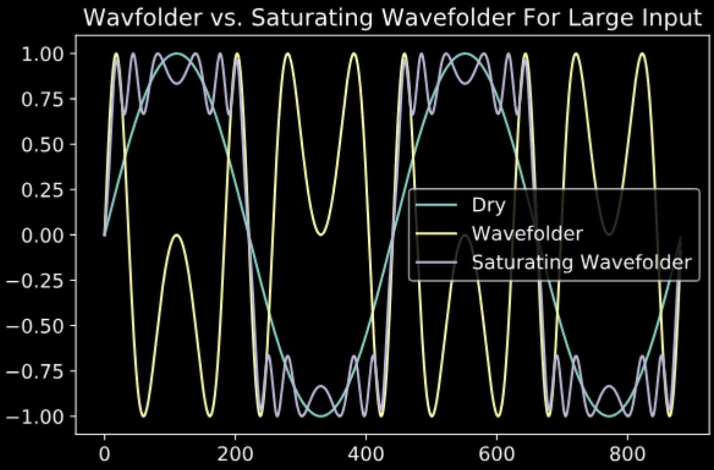
Wavefolder

Saturating wavefolder:



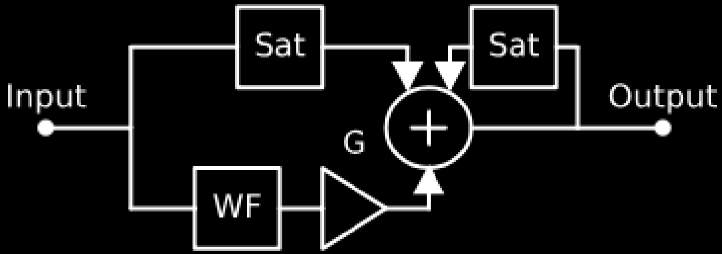
Wavefolder

Saturating wavefolder:



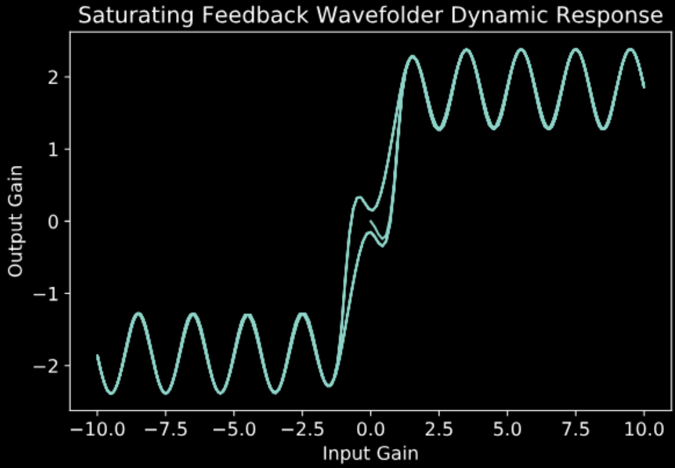
Wavefolder

Feedback wavefolder:



Wavefolder

Feedback wavefolder:



Subharmonics

Subharmonics: Motivation

Most nonlinear audio effects add higher harmonics to the signal.

What if we want lower harmonics …

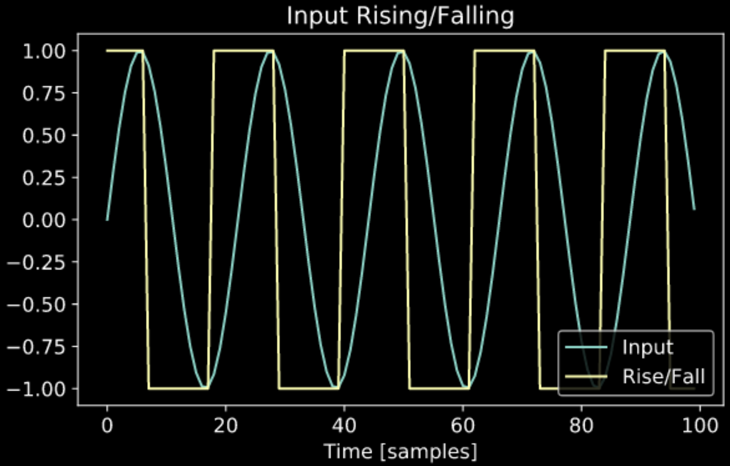
Subharmonics

Goals:

- Generate subharmonic content
- Avoid circuit modelling
- Avoid relying on high-quality pitch detection

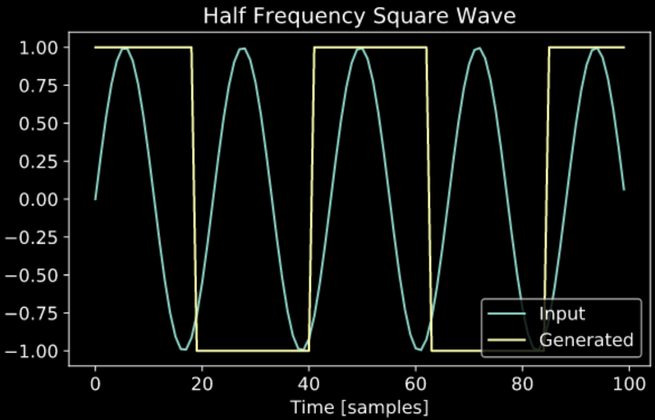
Subharmonics

Step 1: Detect when the input signal switches directions



Subharmonics

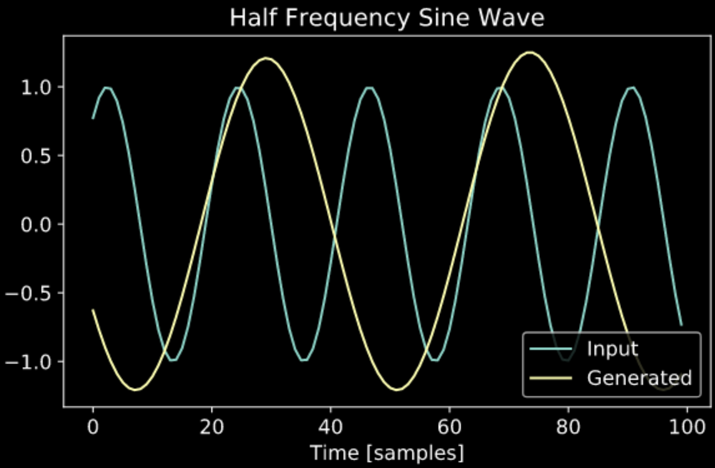
Step 2: Flip detector signal every *other* time



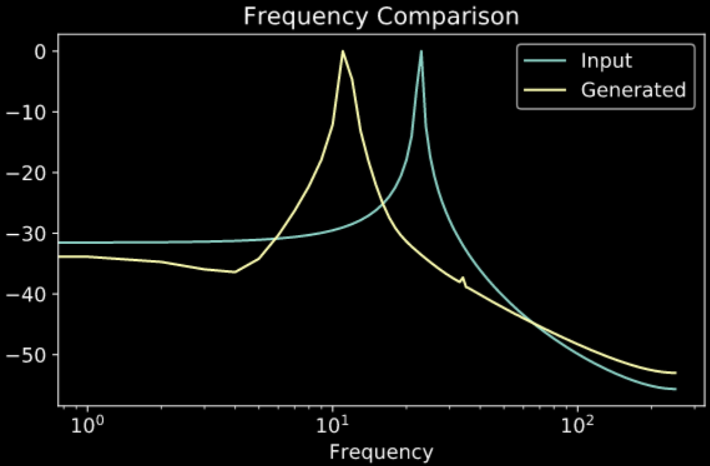
Result: half-frequency square wave!

Subharmonics

Step 3: Lowpass Filter

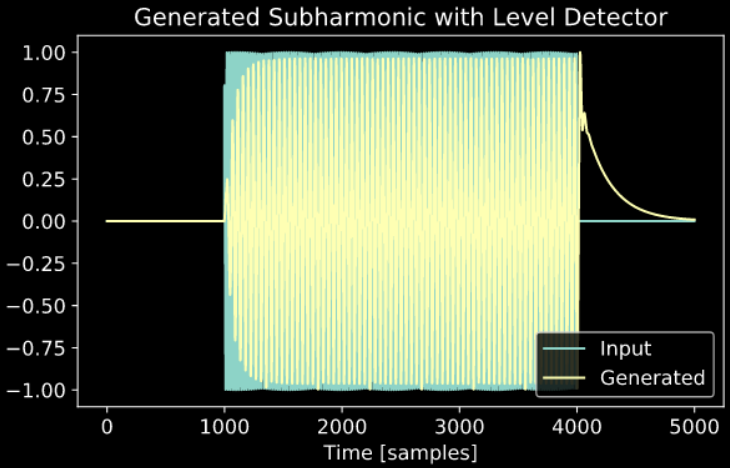


Subharmonics

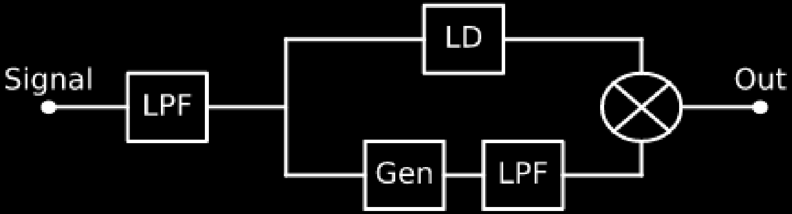


Subharmonics

Step 4: Apply level detector



Subharmonics



Gated Recurrent Distortion

Gated Recurrent Distortion

Gated Recurrent Unit:

- Building block for recurrent neural networks[6]
- Here we examine a variation: “minimal gated unit”[7]

[6] Cho et al., “Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation”.

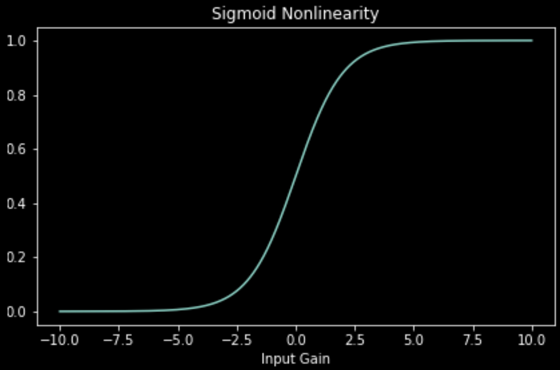
[7] Zhou et al., “Minimal Gated Unit for Recurrent Neural Networks”.

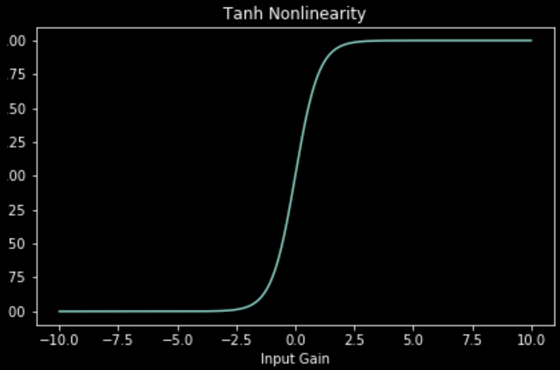
Gated Recurrent Distortion

$$
\begin{aligned}
\Gamma_f &= \sigma(W_f x[n] + U_f y[n-1] + b_f) \\
y[n] &= \Gamma_f y[n-1] + (1 - \Gamma_f) \tanh(W_h x[n] + U_h \Gamma_f y[n-1] + b_h)
\end{aligned}
\tag{15}
$$

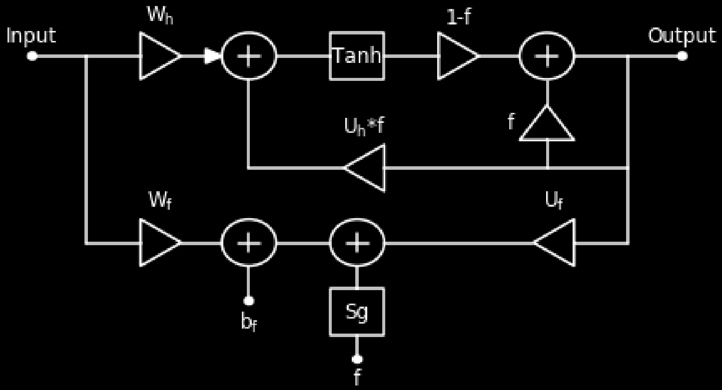
$$
\sigma(x) = \frac{1}{1 + e^{-x}} \tag{16}
$$

Gated Recurrent Distortion

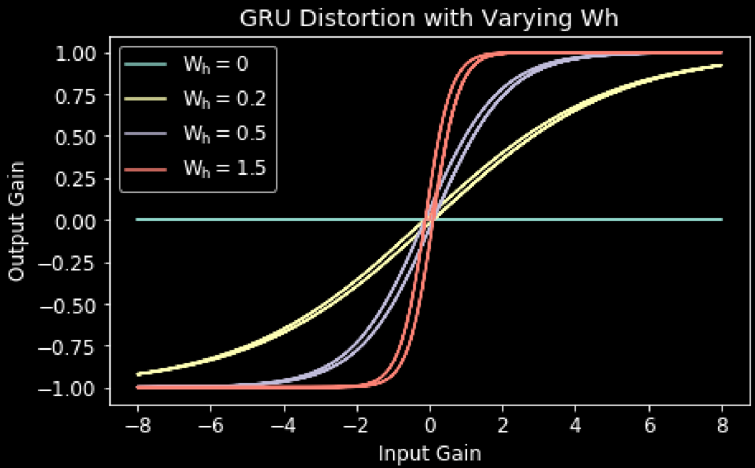




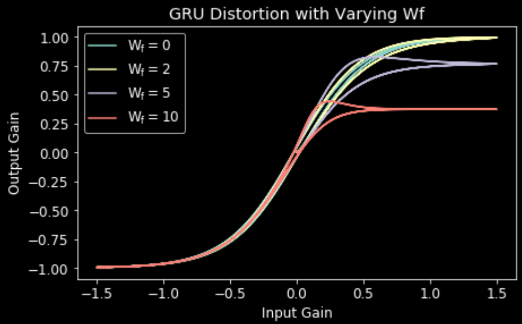
Gated Recurrent Distortion

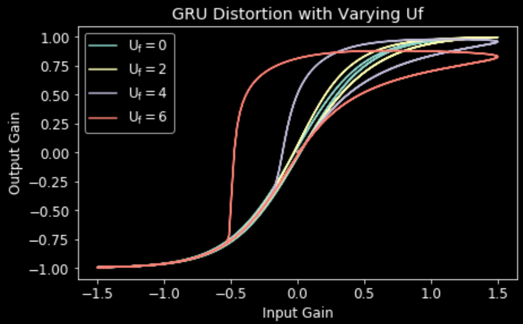


Gated Recurrent Distortion: Parameters

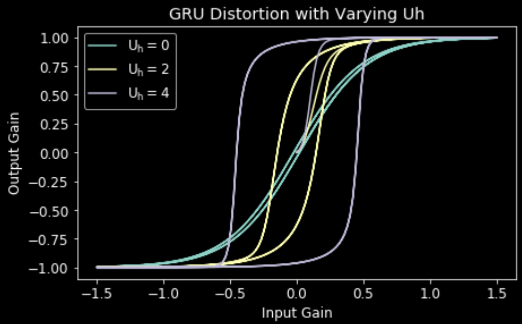


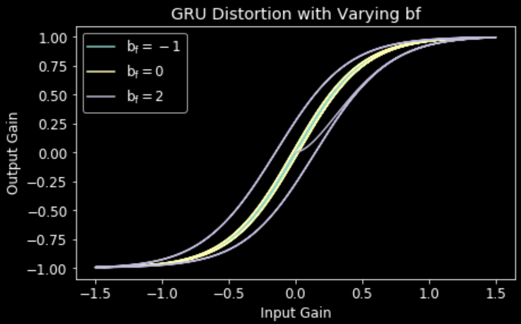
Gated Recurrent Distortion: Parameters



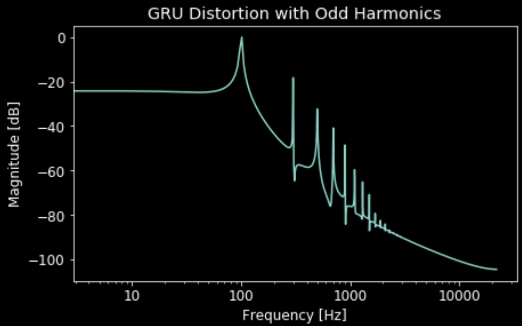


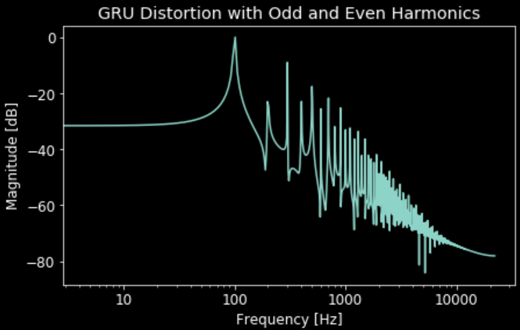
Gated Recurrent Distortion: Parameters





Gated Recurrent Distortion: Harmonic Response





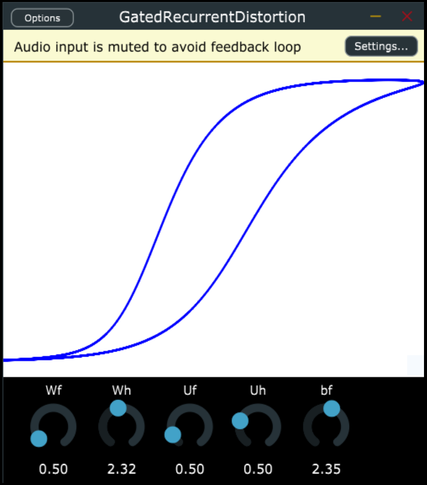
Conclusion

Goals

- Tools for musicians/mixing engineeers
- Inspiration/explanations for audio effect makers
- A academic paper (or two)

Presentation

Audio plugins (VST/AU)



Presentation

Medium articles

Upgrade

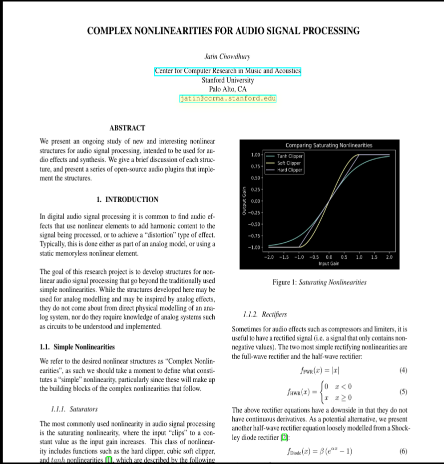
Complex Nonlinearities Episode 4: Nonlinear Biquad Filters

Jatin Chowdhury
Oct 10, 2019 · 6 min read

For today's article, we'll be talking about filters. So far in this series I haven't spoken too much about filters, which might seem odd considering how much of signal processing in general is all about filters. The reason I've avoided filters is that most filters in audio signal processing are implemented as linear processors, and I've been focusing on nonlinear processing concepts.

Presentation

Papers



COMPLEX NONLINEARITIES FOR AUDIO SIGNAL PROCESSING

Jatin Chowdhury

Center for Computer Research in Music and Acoustics
Stanford University
Palo Alto, CA
jatin@ccrma.stanford.edu

ABSTRACT

We present an ongoing study of new and interesting nonlinear structures for audio signal processing, intended to be used for audio effects and synthesis. We give a brief discussion of each structure, and present a series of open-source audio plugins that implement the structures.

1. INTRODUCTION

In digital audio signal processing it is common to find audio effects that use nonlinear elements to add harmonic content to the signal being processed, or to achieve a "distortion" type of effect. Typically, this is done either as part of an analog model, or using a static memoryless nonlinear element.

The goal of this research project is to develop structures for nonlinear audio signal processing that go beyond the traditionally used simple nonlinearities. While the structures developed here may be used for analog modelling and may be inspired by analog effects, they do not come about from direct physical modelling of an analog system, nor do they require knowledge of analog systems such as circuits to be understood and implemented.

1.1. Simple Nonlinearities

We refer to the desired nonlinear structures as "Complex Nonlinearities", as such we should take a moment to define what constitutes a "simple" nonlinearity, particularly since these will make up the building blocks of the complex nonlinearities that follow.

1.1.1. Saturators

The most commonly used nonlinearity in audio signal processing is the saturating nonlinearity, where the input "clips" to a constant value as the input gain increases. This class of nonlinearity includes functions such as the hard clipper, cubic soft clipper, and *tanh* nonlinearities [1], which are described by the following

Figure 1: *Saturating Nonlinearities*

1.1.2. Rectifiers

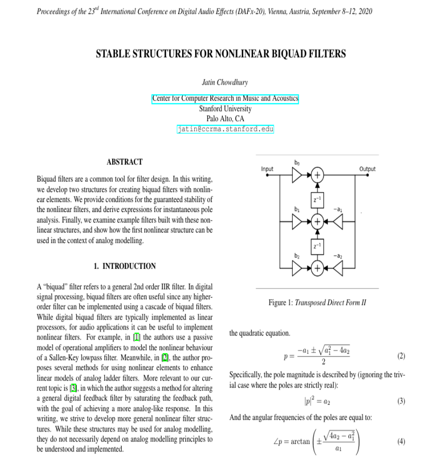
Sometimes for audio effects such as compressors and limiters, it is useful to have a rectified signal (i.e. a signal that only contains non-negative values). The two most simple rectifying nonlinearities are the full-wave rectifier and the half-wave rectifier:

$$f_{\text{FWR}}(x) = |x| \tag{4}$$

$$f_{\text{HWR}}(x) = \begin{cases} 0 & x < 0 \\ x & x \geq 0 \end{cases} \tag{5}$$

The above rectifier equations have a downside in that they do not have continuous derivatives. As a potential alternative, we present another half-wave rectifier equation loosely modelled from a Shockley diode rectifier [2]:

$$f_{\text{Diode}}(x) = \beta\left(e^{\alpha x} - 1\right) \tag{6}$$



Proceedings of the 23rd International Conference on Digital Audio Effects (DAFx-20), Vienna, Austria, September 8–12, 2020

STABLE STRUCTURES FOR NONLINEAR BIQUAD FILTERS

Jatin Chowdhury

Center for Computer Research in Music and Acoustics
Stanford University
Palo Alto, CA
jatin@ccrma.stanford.edu

ABSTRACT

Biquad filters are a common tool for filter design. In this writing, we develop two structures for creating biquad filters with nonlinear elements. We provide conditions for the guaranteed stability of the nonlinear filters, and derive expressions for instantaneous pole analysis. Finally, we examine example filters built with these nonlinear structures, and show how the first nonlinear structure can be used in the context of analog modelling.

1. INTRODUCTION

A "biquad" filter refers to a general 2nd order IIR filter. In digital signal processing, biquad filters are often useful since any higher-order filter can be implemented using a cascade of biquad filters. While digital biquad filters are typically implemented as linear processors, for audio applications it can be useful to implement nonlinear filters. For example, in [1] the authors use a passive model of operational amplifiers to model the nonlinear behaviour of a Sallen-Key lowpass filter. Meanwhile, in [2], the author proposes several methods for using nonlinear elements to enhance linear models of analog ladder filters. More relevant to our current topic is [3], in which the author suggests a method for altering a general digital feedback filter by saturating the feedback path, with the goal of achieving a more analog-like response. In this writing, we strive to develop more general nonlinear filter structures. While these structures may be used for analog modelling, they do not necessarily depend on analog modelling principles to be understood and implemented.

Figure 1: *Transposed Direct Form II*

the quadratic equation.

$$p = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} \tag{2}$$

Specifically, the pole magnitude is described by (ignoring the trivial case where the poles are strictly real):

$$|p|^2 = a_2 \tag{3}$$

And the angular frequencies of the poles are equal to:

$$\angle p = \arctan\left(\pm\frac{\sqrt{4a_2 - a_1^2}}{a_1}\right) \tag{4}$$

Presentation

Links:

- `https://github.com/jatinchowdhury18/ComplexNonlinearities`

- `https://medium.com/@jatinchowdhury18`

Thank you!