

Complex Nonlinearities for Audio Signal Processing

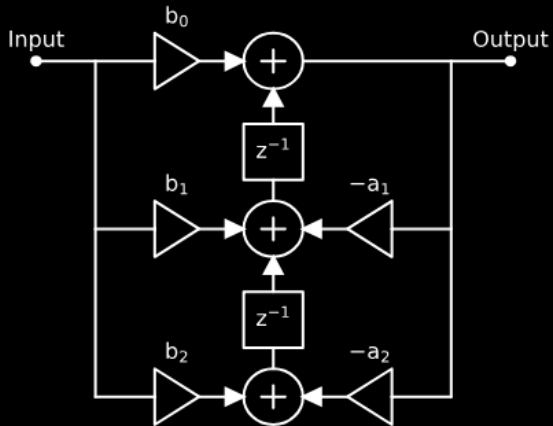
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Nonlinear Filters

Biquad Filter

Transposed Direct Form II



Biquad Filter

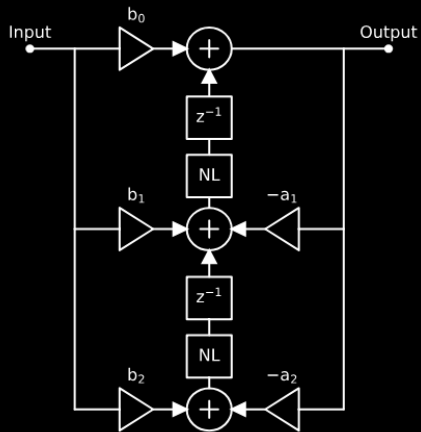
Difference equation:

$$y[n] = b_0u[n] + b_1u[n-1] + b_2u[n-2] - a_1y[n-1] - a_2y[n-2] \quad (1)$$

State space formulation:

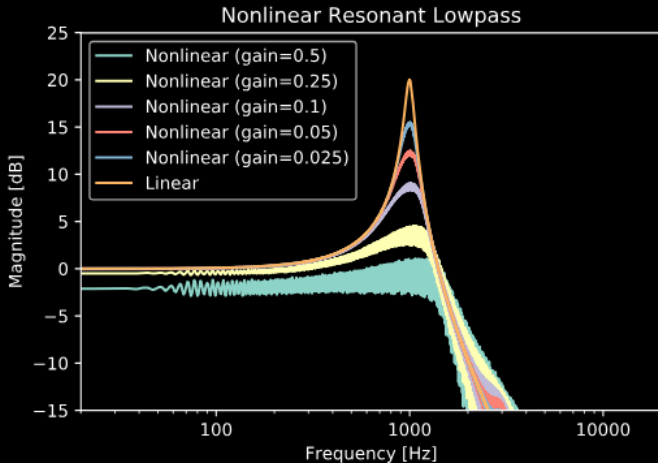
$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & -a_1 \\ 0 & 0 & -a_2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \quad (2)$$

Nonlinear Biquad Filter



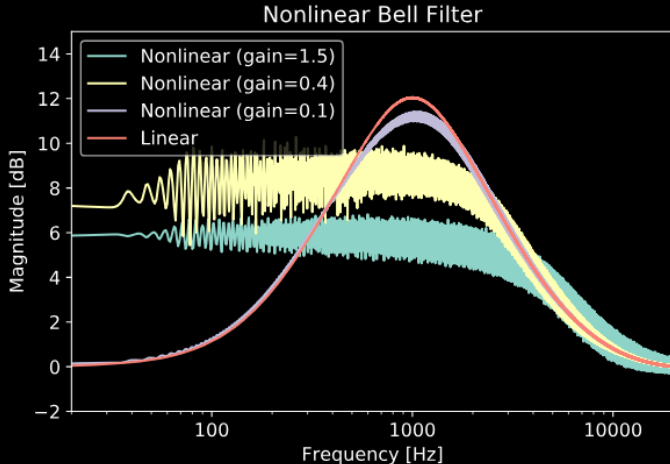
Nonlinear Biquad Filter

Saturating nonlinearities \rightarrow nonlinear resonance



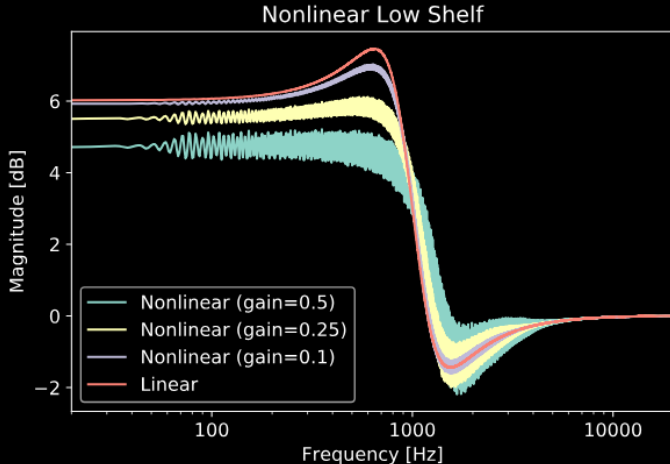
Nonlinear Biquad Filter

Saturating nonlinearities \rightarrow nonlinear resonance



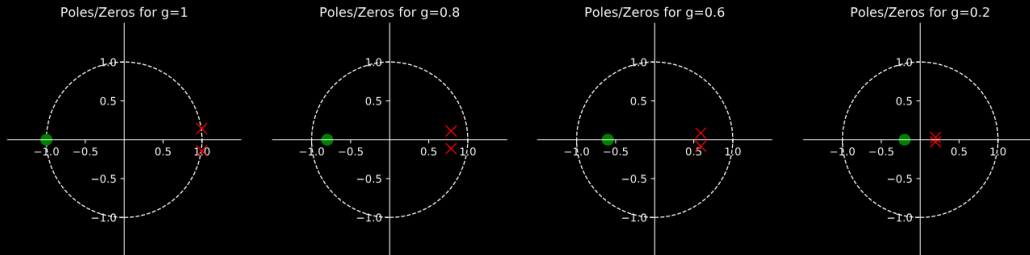
Nonlinear Biquad Filter

Saturating nonlinearities \rightarrow nonlinear resonance

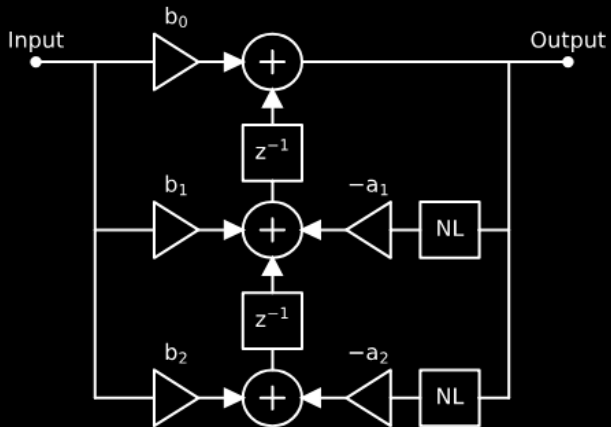


Nonlinear Biquad Filter

Pole/zero movement

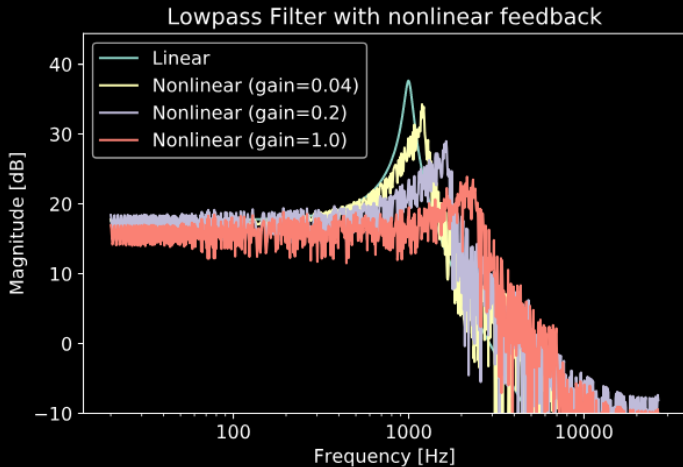


Nonlinear Feedback Filter



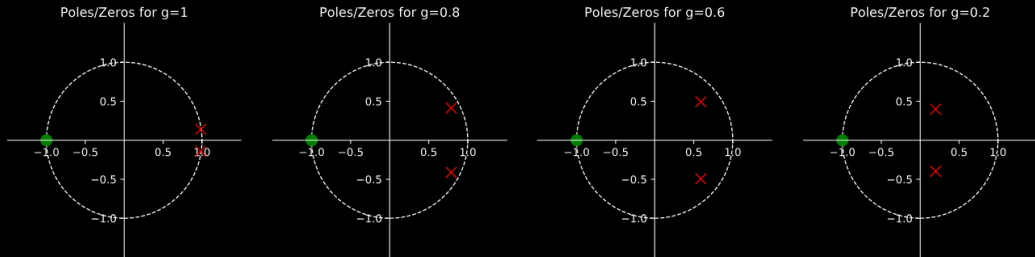
Nonlinear Feedback Filter

Saturating nonlinearity \rightarrow cutoff frequency modulation



Nonlinear Feedback Filter

Pole/zero movement



Nonlinear Biquad Stability

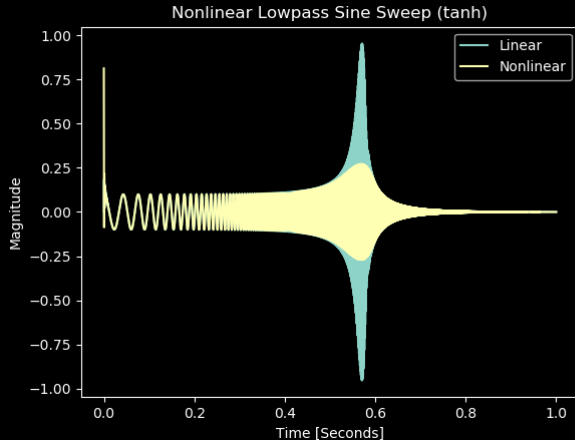
Questions:

Can we guarantee that a nonlinear filter will be stable given that its linear corrolary is stable?

For what subset of nonlinear functions is this guaranteed?

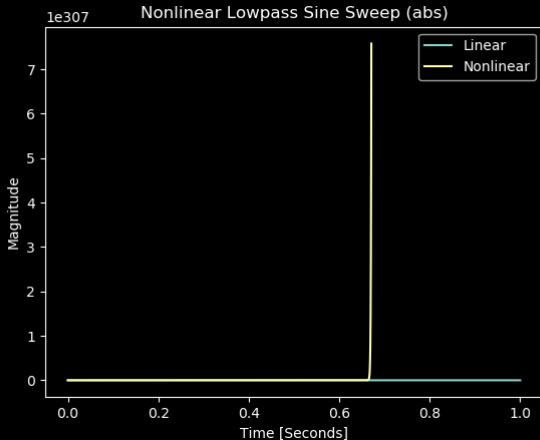
Nonlinear Biquad Stability

Test case: saturating nonlinearity, $f_{NL} = \tanh(x) \rightarrow$ **STABLE!**



Nonlinear Biquad Stability

Test case: full wave rectifier, $f_{NL} = 0.45|x| \rightarrow$ **UNSTABLE!**



Lyapunov Stability¹

1. Form state space equation:

$$\mathbf{x}[n + 1] = \mathbf{f}(\mathbf{x}[n]) \quad (3)$$

2. Find Jacobian \mathbf{J} of \mathbf{f}

3. If every element of \mathbf{J} is less than 1 at some operating point, the system is Lyapunov stable about that point.

¹Chen, "Stability of Nonlinear Systems".

Nonlinear Biquad Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h} \left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} \right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \quad (4)$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) &= f_{NL}(x_2[n]) - a_1 y[n] \\ h_2(x_1[n], x_2[n], y[n]) &= -a_2 y[n] \\ h_3(x_1[n], x_2[n], y[n]) &= f_{NL}(x_1[n]) \end{aligned} \quad (5)$$

Nonlinear Biquad Stability

$$\mathbf{J} = \begin{bmatrix} 0 & f'_{NL}(x_2[n]) & -a_1 \\ 0 & 0 & -a_2 \\ f'_{NL}(x_1[n]) & 0 & 0 \end{bmatrix} \quad (6)$$

Note that if f'_{NL} does not exist at some point, the system is NOT stable at that point.

Nonlinear Feedback Stability

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ y[n+1] \end{bmatrix} = \mathbf{h} \left(\begin{bmatrix} x_1[n] \\ x_2[n] \\ y[n] \end{bmatrix} \right) + \begin{bmatrix} b_1 \\ b_2 \\ b_0 \end{bmatrix} u[n] \quad (7)$$

$$\begin{aligned} h_1(x_1[n], x_2[n], y[n]) &= x_2[n] - a_1 f_{NL}(y[n]) \\ h_2(x_1[n], x_2[n], y[n]) &= -a_2 f_{NL}(y[n]) \\ h_3(x_1[n], x_2[n], y[n]) &= x_1[n] \end{aligned} \quad (8)$$

Nonlinear Feedback Stability

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & -a_1 f'_{NL}(y[n]) \\ 0 & 0 & -a_2 f'_{NL}(y[n]) \\ 1 & 0 & 0 \end{bmatrix} \quad (9)$$

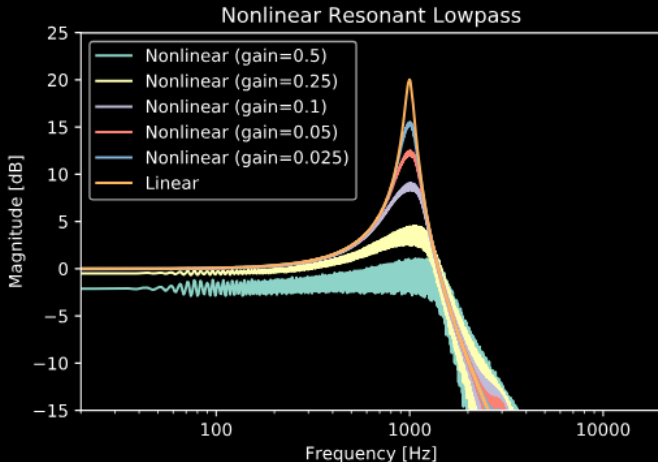
Nonlinear Biquad Stability

General stability constraint:

$$|f'_{NL}(x)| \leq 1 \quad (10)$$

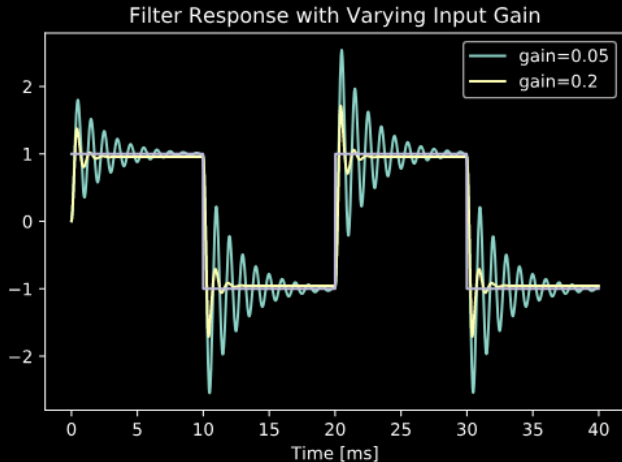
Nonlinear Biquad Filter

Can we use this for analog modelling?



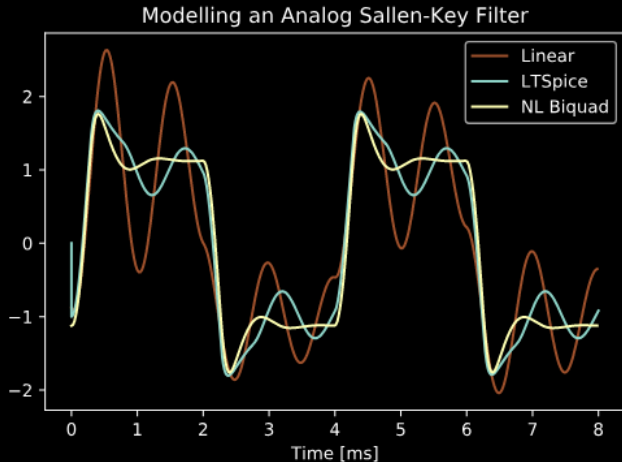
Nonlinear Biquad Filter

Parameters: nonlinearities, input gain



Nonlinear Biquad Filter

Modelling an overdriven Sallen-Key lowpass filter



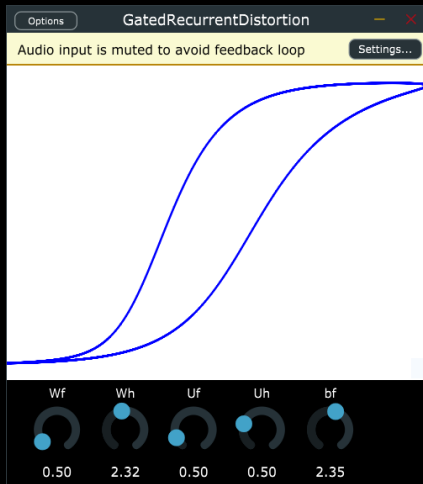
Conclusion

Goals

- Tools for musicians/mixing engineers
- Inspiration/explanations for audio effect makers
- A academic paper (or two)


Presentation



Audio plugins (VST/AU)




Presentation

Medium articles










Upgrade



Complex Nonlinearities Episode 4: Nonlinear Biquad Filters



Jatin Chowdhury
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For today's article, we'll be talking about filters. So far in this series I haven't spoken too much about filters, which might seem odd considering how much of signal processing in general is all about filters. The reason I've avoided filters is that most filters in audio signal processing are implemented as linear processors, and I've been focusing on nonlinear processing concepts.

Thank you!